A Practical Stopping Criterion For Iterative Non-Cartesian SENSE Reconstruction

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Introduction

In non-Cartesian SENSE reconstruction based on conjugate gradient (CG) iteration method [1], the iteration very often exhibits a "semi-convergence" behavior which can be characterized as initial convergence towards the exact solution and later divergence. This is because the encoding matrix is ill-conditioned and the CG method has a regularization effect with the degree of regularization deceasing along the iteration [2, 3]. This phenomenon makes a suitable stopping criterion very critical for automatic implementation of this reconstruction strategy. Due to the convergence-divergence behavior, classic δ-criterion for CG iteration is not applicable for SENSE reconstruction in many cases. In this study a practical stopping rule for iterative SENSE reconstruction is proposed. By monitoring the residual norm along the iteration, this stopping criterion can automatically stop the process where artifact and SNR are well compromised and good overall image quality are achieved.

Method

During the CG iterative reconstruction, the residual norm decrease monotonically. At a closer look, it is observed that the residual norm declines very fast at early stages of iteration and then level off. For a typical reconstruction, the plot of the residual norm $\Delta(k)$ in *log* scale versus the iteration count k usually exhibits an L-curve characteristic, as shown in Fig. 1. In this fashion the iteration procedure can be divided into three phases. At the left of the L-corner, the residual norm declines sharply and it is referred to as a "dropping phase"; at the right side, the residual norm level off and it is a "level phase". The vicinity of the distinct L-corner then represents the "transition phase" where the artifact and SNR are generally well compromised. We suggest that the iteration should be stopped at the L-corner position.

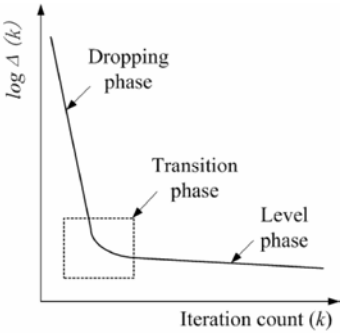

Fig. 1. Plot of residual norm vs. iteration count for CG iterative SENSE reconstruction.

An operational definition of the "corner" is the point on the curve that has maximum curvature [3]. Let $\delta(k) = \log \Delta(k)$. The curvature is then defined by $\kappa(k) = \frac{\delta''}{((\delta')^2 + 1)^{3/2}}$, where differentiation is with respect to the regularization parameter k. Since the residual norm $\Delta(k)$ can be computed as a by-product in each CG iteration loop, the L-corner can be identified at very modest computational cost.

Results

Axial brain images were acquired with a 2D spiral sequence using an 8-element head coil array. A full dataset with 4 interleaves × 3906 samples was acquired and later decimated offline by extracting 1 or 2 interleaves to simulate 4X or 2X acceleration. Iterative SENSE reconstruction was performed using these undersampled spiral datasets. Images after 1, 5, 15, 50 and 99 iterations are presented in Fig. 2. Apparent semi-convergence behavior is observed. Artifact dominates in early stages, while noise booming occurs in later stages of iteration. Fig. 3 show the residual norm (in *log* scale) varying with iteration count for the two reconstructions corresponding to Fig. 2. Both curves exhibit L-shapes. The L-corners of these two curves correspond to 21 iterations and 55 iterations, respectively, which are marked by triangles on the curves. By corresponding Fig. 2 to Fig. 3, it is easy to see that the intermediate images within the transition phase, for instance the image after 15 iterations in Fig. 4a and the one after 50 iteration in Fig. 4b, exhibit good compromises between SNR and artifacts and they show the best overall image quality in their respective image sequences.

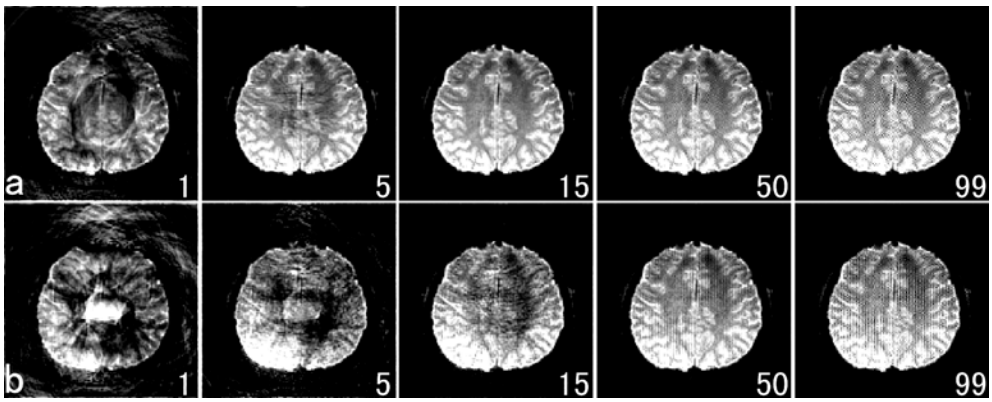

Fig. 2. Sensitivity-encoded spiral brain imaging. (**a**) 2X acceleration; (**b**) 4X acceleration

Conclusion

A practical stopping rule for iterative SENSE reconstruction has been proposed. In vivo experiments have shown that this stopping rule can automatically achieve good compromise between artifact and SNR.

Acknowledgement

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References

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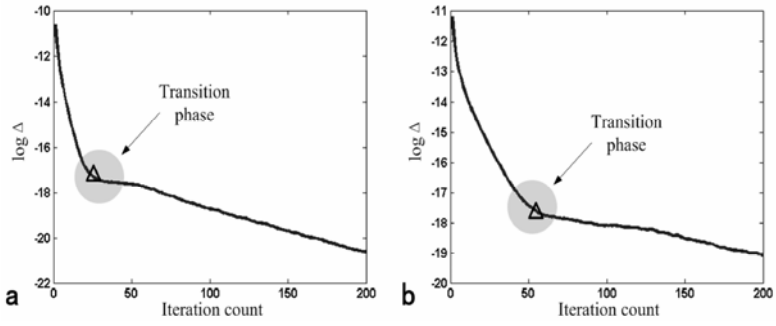

Fig. 3 Plot of residual norm (in *log* scale) vs. iteration count for the reconstructions corresponding to Fig. 2