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Infinite-dimensional Lie algebra cohomology and the cohomology of invariant Euler-Lagrange complexes: a preliminary report. (English summary)
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Let $\pi\colon E\to M^n$ be a fiber bundle and G a Lie pseudogroup of π-projectable transformations on E. On the infinite jet bundle $\pi^\infty\colon J^\infty(E)\to M$ of E there is a natural variational bicomplex $\Omega^{*,*}=\Omega^{*,*}(J^\infty(E))$. The edge complex of $\Omega^{*,*}$ is the Euler-Lagrange complex

$$0\to\mathbf{R}\to\Omega^{0,0}\xrightarrow{d_H}\Omega^{1,0}\xrightarrow{d_H}\cdots\xrightarrow{d_H}\Omega^{n-1,0}\xrightarrow{d_H}\Omega^{n,0}\xrightarrow{\delta_V}\mathcal{F}^1\xrightarrow{\delta_V}\mathcal{F}^2\to\cdots,$$

where $d=d_V+d_H$ is the exterior differentiation on $J^\infty(E)$, and $\mathcal{F}^s:=\{\omega\in\Omega^{n,s}\colon\ I\omega=\omega\}$ with $I\colon\Omega^{n,s}\to\Omega^{n,s}$ being the integration by parts operator and $\delta_V:=I\circ d_V$. Note that $\delta_V\colon\Omega^{n,0}\to\mathcal{F}^1$ is precisely the classical Euler-Lagrange operator.

If $\varphi\colon E\to E$ is a π-projectable map, its infinite prolongation $\varphi^\infty\colon J^\infty(E)\to J^\infty(E)$ commutes with π^∞ and preserves the contact ideal on $J^\infty(E)$. Thus in particular, φ^∞ is a transformation on each $\Omega^{r,s}$ and commutes with d_H, d_V and I. The G-invariant variational bicomplex on E is $\Omega^{r,s}_G(J^\infty(E)):=\{\omega\in\Omega^{r,s}(J^\infty(E))\colon\ (\varphi^\infty)^*\omega=\omega$ for all $\varphi\in G\}$ with the same d_V, d_H and I as before, and the G-invariant Euler-Lagrange complex is

$$0\to\mathbf{R}\to\Omega^{0,0}_G\xrightarrow{d_H}\Omega^{1,0}_G\xrightarrow{d_H}\cdots\xrightarrow{d_H}\Omega^{n-1,0}_G\xrightarrow{d_H}\Omega^{n,0}_G\xrightarrow{\delta_V}\mathcal{F}^1_G\xrightarrow{\delta_V}\mathcal{F}^2_G\to\cdots.$$

In this expository article, a general method of computing the cohomology of the G-invariant Euler-Lagrange complex in terms of the continuous Lie algebra cohomology of the formal infinitesimal operators of G is described, and the method is illustrated by a couple of interesting examples.

{For the entire collection see MR1406316 (97c:53004)}

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