This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the WCNC 2009 proceedings.

Bayesian CFO Estimation in OFDM Systems

Kun Cai, Xiao Li and Yik-Chung Wu
Department of Electrical and Electronic Engineering
The University of Hong Kong, Hong Kong, China
Email: {caikun, lixiao, ycwu}@eee.hku.hk

***Abstract*—This paper addresses the problem of carrier frequency offset (CFO) estimation in orthogonal frequency division multiplexing (OFDM) systems using Bayesian method. Depending on the availability of the noise variance, two general CFO estimators are derived. Furthermore, the two general maximum a posteriori (MAP) estimators are developed into several special cases based on different degrees of prior information on parameters. The relationships between the proposed estimators and existing estimators are comprehensively investigated. Finally, numerical results demonstrate the effects of employing different prior information on the estimation performances.**

I. Introduction

Because of the robustness against frequency selective fading channels, orthogonal frequency division multiplexing (OFDM) has been widely used in many communication systems such as wireless metropolitan area networks (WMAN), wireless local area networks (WLAN) and digital broadcasting (DAB and DVB) systems [1] .

However, OFDM systems are highly sensitive to the carrier frequency offset (CFO) caused by mismatch of the local oscillators in transceivers. To tackle this problem, most of the existing CFO estimation algorithms employ the information provided by a priori known training sequences [2], the redundancy in cyclic prefix [3], or the statistics of the transmitted signals [4].

As can be seen from the CFO estimation methods mentioned above, the attainable performances are dependent on the efficient use of information obtained through the inherent structure of the signals. However, in practice, some prior information of the underlying parameters might be known in advance, such as the variance of CFO and channel statistical information. As an initial work, [5] applied Bayesian analysis to synchronization problem in OFDM systems based on transmitted signal statistics. Unfortunately, [5] provides little insights into the effects of prior information on CFO and channel statistics. Recently, in [6], the CFO estimation problem of ignorant and well-informed receivers have been discussed, showing that the CFO estimation performance could be fairly enhanced with the incorporation of previous channel estimate as prior. Although [6] sheds some light on the effect of including prior information in CFO estimation, a comprehensive framework of Bayesian CFO estimation under various prior information scenarios is still missing.

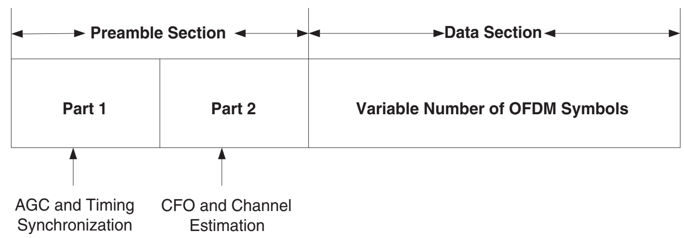

Fig. 1. The packet structure for OFDM systems

In order to present a comprehensive analysis towards CFO estimation using Bayesian method, this paper proposes two general MAP estimators for CFO in OFDM systems, one with known noise variance and the other with unknown noise variance. These two estimators include existing estimators, such as Conditional Maximum Likelihood (CML) [9] and Unconditional Maximum Likelihood (UML) estimators [10], as special cases. The relationship between various special cases are further discussed in details.

Notation : The operator $\mathrm{diag}(\mathbf{x})$ denotes a diagonal matrix with the elements of $\mathbf{x}$ located on the main diagonal. Superscripts $(\cdot)^H$ and $(\cdot)^T$ denote the conjugate transpose and the transpose operators respectively. Notation $\mathbf{I}$ is the identity matrix and $\det(\mathbf{A})$ takes the determinant of matrix $\mathbf{A}$.

II. System Description

A packet-based OFDM system with N subcarriers is considered. For each data packet, it is preceded by some training blocks as shown in Figure 1. It is assumed that timing synchronization will be completed by exploiting the training blocks at the first part of the preamble

978-1-4244-2948-6/09/$25.00 ©2009 IEEE

(Part 1). The CFO and channel estimation tasks will be carried out on the second portion of the preamble (Part 2). Without loss of generality, we assume there is only one OFDM training block in Part 2 of the preamble. Similar training structures have been proposed in many standards, such as IEEE802.11a [7] and HiperLAN2 [8].

At the transmitter, an OFDM symbol is generated by passing the data symbols $\mathbf{d} = [d(0), d(1), \ldots, d(N-1)]^T$ through an inverse fast Fourier transform (IFFT). A cyclic prefix (CP) of length L_{cp} is inserted ahead of the OFDM symbol to cope with the inter-symbol interference (ISI) caused by multipath channel. The discrete-time composite channel impulse response (encompassing the transmit/receive filters and the transmission medium) is denoted as $\mathbf{h} = [h_0, \ldots, h_{L-1}]^T$, and is quasi-static over one data packet. The CP length L_{cp} is assumed to be larger than the channel order L. The normalized CFO between the transmitter and receiver is assumed to be ε_o.

At the receiver after timing synchronization and CP removal, the received vector after FFT is given by

$$\mathbf{x} = \mathbf{\Gamma}(\omega_o)\mathbf{F}^H\mathbf{D}\mathbf{W}\mathbf{h} + \mathbf{v} \tag{1}$$

where

$$\mathbf{\Gamma}(\omega_o) \triangleq \text{diag}[1, \ldots, e^{j(N-1)\omega_o}] \tag{2}$$

$$\mathbf{D} \triangleq \text{diag}(\mathbf{d}) \tag{3}$$

$$\omega_o \triangleq \frac{2\pi\varepsilon_o}{N} \tag{4}$$

$$\mathbf{v} \triangleq [v(0), v(1), \cdots, v(N-1)]^T. \tag{5}$$

In the above equations, $\mathbf{F}$ is the FFT matrix with $\mathbf{F}(k,l) = e^{-\frac{j2\pi kl}{N}}/\sqrt{N}$ and $\mathbf{W} = \sqrt{N}\mathbf{F}(:, 0\colon(L-1))$ is a $N \times L$ matrix containing the first L columns of $\mathbf{F}$ scaled by $\sqrt{N}$; $\mathbf{D}$ is a diagonal matrix formed from $\mathbf{d}$. Vector $\mathbf{v}$ denotes the complex white Gaussian noise with zero mean and variance σ^2.

In the previous works on CFO estimation in OFDM systems, the CFO ω_o and channel $\mathbf{h}$ in (1) are normally assumed as deterministic unknowns [9]. However in practice, statistical information about the CFO as well as channel may be available. For example, the channel vector $\mathbf{h}$ is usually modeled as a complex Gaussian random vector and the channel power delay profiles for typical environments have been measured and documented [11] [12]. Thus the distribution of $\mathbf{h}$ can be represented as

$$P(\mathbf{h}) = \frac{1}{\pi^L \det(\mathbf{Q})} \exp(-\mathbf{h}^H\mathbf{Q}^{-1}\mathbf{h}), \tag{6}$$

where $\mathbf{Q}$ is the channel covariance matrix, which contains the power delay profile information. Furthermore, in cooperative communications systems, at the receiver, an initial maximum likelihood (ML) estimate of the CFO could be obtained from the cooperation among users, as illustrated in [13]. According to the properties of ML estimator, the estimator is shown to asymptotically follow a Gaussian distribution

$$P(\omega_o) = \frac{1}{\sqrt{2\pi}\sigma_{\omega_o}} \exp(-\frac{\omega_o^2}{2\sigma_{\omega_o}^2}), \tag{7}$$

where $\sigma_{\omega_o}^2$ is the variance of the CFO distribution, which is related to the estimation accuracy of the initial estimate (i.e., CRB). Note that this is a general prior for CFO because when there is no prior CFO information, we can set $\sigma_{\omega_o}^2 \to \infty$ and the distribution becomes uninformative and flat.

III. CFO ESTIMATION USING BAYESIAN METHOD

As discussed in the previous section, we may have prior information on the CFO and channel statistics, therefore, Bayesian framework can be used for CFO estimation problems. Hereby we propose two general maximum a posteriori (MAP) estimators depending on the availability of noise variance. In order to derive the MAP estimator, the posterior distribution of ω_o is needed and calculated by the Bayes rule as

$$P(\omega_o|\mathbf{x}) = \frac{P(\mathbf{x}|\omega_o)P(\omega_o)}{P(\mathbf{x})}, \tag{8}$$

where $P(\mathbf{x}|\omega_o)$ is the likelihood function of $\mathbf{x}$ given ω_o. Since $P(\mathbf{x})$ is a constant, the MAP estimator only requires the maximization of $P(\mathbf{x}|\omega_o)P(\omega_o)$.

In the following, we will first present the derivation of the MAP estimator with known σ^2, and then consider the case when σ^2 is unknown.

A. MAP CFO Estimator with known σ^2

When the noise variance σ^2 is known, the CFO posterior distribution $P(\omega_o|\mathbf{x})$ is given by

$$\begin{aligned} P(\omega_o|\mathbf{x},\sigma) &\propto P(\mathbf{x}|\omega_o,\sigma)P(\omega_o) \\ &= \left(\int P(\mathbf{x}|\omega_o,\mathbf{h},\sigma)P(\mathbf{h})d\mathbf{h}\right)P(\omega_o). \end{aligned} \tag{9}$$

The likelihood function of $\mathbf{x}$ given ω_o, $\mathbf{h}$ and σ^2 is

$$\begin{aligned} &P(\mathbf{x}|\omega_o,\mathbf{h},\sigma) \\ &= \frac{1}{(\pi\sigma^2)^N} \exp\left\{ -\frac{[\mathbf{x}-\mathbf{G}(\omega_o)\mathbf{h}]^H[\mathbf{x}-\mathbf{G}(\omega_o)\mathbf{h}]}{\sigma^2}\right\} \end{aligned} \tag{10}$$

where $\mathbf{G}(\omega_o) = \mathbf{\Gamma}(\omega_o)\mathbf{F}^H\mathbf{D}\mathbf{W}$. As shown in Appendix, (9) can be shown to be

$$P(\omega_o|\mathbf{x},\sigma) \propto \exp\left\{ -\frac{\omega_o^2}{2\sigma_{\omega_o}^2} - \frac{\mathbf{x}^H\mathbf{C}(\omega_o)\mathbf{x}}{\sigma^2} \right\} \quad (11)$$

where

$$\mathbf{C}(\omega_o) = \mathbf{I} - \mathbf{G}(\omega_o)[\mathbf{G}(\omega_o)^H\mathbf{G}(\omega_o) + \sigma^2\mathbf{Q}^{-1}]^{-1}\mathbf{G}(\omega_o)^H.$$

Then the MAP estimate of ω_o is obtained by maximizing (11), which is equivalent to

$$\hat{\omega}_o = \arg\min_{\omega_o} \left\{ \frac{\omega_o^2}{2\sigma_{\omega_o}^2} + \frac{\mathbf{x}^H\mathbf{C}(\omega_o)\mathbf{x}}{\sigma^2} \right\}, \quad (12)$$

which represents a trade-off between prior information on CFO and channel statistics.

1) Special Case 1 : Estimator without prior knowledge of CFO : When there is no prior information on CFO, this corresponds to the case that σ_{ω_o} goes into infinity. Therefore, the MAP estimator with no CFO prior information is

$$\hat{\omega}_o = \arg\min_{\omega_o} \{\mathbf{x}^H\mathbf{C}(\omega_o)\mathbf{x}\}, \quad (13)$$

which is the Unconditional Maximum Likelihood (UML) estimator [10].

2) Special Case 2: Estimator without prior knowledge of channel statistics: Similar to special case 1, the lack of knowledge on channel covariance could be addressed by assigning uninformative prior to $\mathbf{h}$. Here we assign $\mathbf{Q} = \mathrm{diag}[\delta_1^2, \delta_2^2, \cdots, \delta_{L-1}^2]$ with $\delta_1^2, \delta_2^2, \cdots, \delta_{L-1}^2$ approaching infinity, where δ_i^2 is the variance of the ith tap of the channel. Therefore, when $\delta_1^2, \delta_2^2, \cdots, \delta_{L-1}^2 \rightarrow \infty$, we have $\mathbf{Q}^{-1} \rightarrow \mathbf{0}$ and

$$\begin{aligned}\mathbf{C}(\omega_o) &\approx \mathbf{I} - \mathbf{G}(\omega_o)[\mathbf{G}(\omega_o)^H\mathbf{G}(\omega_o)]^{-1}\mathbf{G}(\omega_o)^H \\ &\triangleq \mathbf{P}_{\mathbf{G}}^{\perp}(\omega_o).\end{aligned} \quad (14)$$

Then the MAP estimator without knowledge of channel covariance is

$$\hat{\omega}_o = \arg\min_{\omega_o} \left\{ \frac{\omega_o^2}{2\sigma_{\omega_o}^2} + \frac{\mathbf{x}^H\mathbf{P}_{\mathbf{G}}^{\perp}(\omega_\mathbf{o})\mathbf{x}}{\sigma^2} \right\}, \quad (15)$$

which also represents a trade-off between CFO prior information and the information inferred from the data.

3) Special Case 3: Estimator without prior knowledge of both CFO and channel statistics: Under this scenario, we can simply assign both $\sigma_{\omega_o}^2 \rightarrow \infty$ and $\delta_1^2, \delta_2^2, \cdots, \delta_{L-1}^2 \rightarrow \infty$. Thus, the estimator without any prior information is obtained as

$$\hat{\omega}_o = \arg\min_{\omega_o} \left\{ \mathbf{x}^H\mathbf{P}_{\mathbf{G}}^{\perp}(\omega_\mathbf{o})\mathbf{x} \right\}, \quad (16)$$

which turns out to be the Conditional Maximum Likelihood (CML) estimator [9]. Note that although the derivation starts with assuming that σ^2 is known, this estimator can be implemented without knowledge of σ^2.

B. MAP CFO Estimator with unknown σ^2

In the previous section, all the cases require the knowledge of σ^2. When the noise variance is unknown, we need to further average out σ^2, which is given by

$$P(\omega_o|\mathbf{x}) \propto \left[\int \underbrace{\left[\int P(\mathbf{x}|\omega_o, \mathbf{h}, \sigma)P(\mathbf{h})d\mathbf{h} \right]}_{P(\mathbf{x}|\omega_o,\sigma)} P(\sigma)d\sigma \right] P(\omega_o). \quad (17)$$

In this paper, it is assumed that we have little knowledge on σ^2, thus σ^2 obeys the Jeffrey's prior $P(\sigma) = 1/\sigma$ [14]. Furthermore, together with $P(\mathbf{x}|\omega_o, \sigma)$ in (11), the integral (17) is expressed as

$$\begin{aligned}&\int P(\mathbf{x}|\omega_o, \sigma)P(\sigma)d\sigma \\ &\propto \int \frac{1}{(\pi\sigma^2)^{(N-L)}} \exp\left\{ \frac{\mathbf{x}^H\mathbf{C}(\omega_o)\mathbf{x}}{\sigma^2} \right\} \frac{1}{\sigma} d\sigma\end{aligned} \quad (18)$$

Unfortunately, for $\mathbf{Q}$ with finite values, the integration in (18) is complicated and needs to be calculated using Markov Chain Monte Carlo (MCMC) method. Due to the space limitation, we are not exploring this direction, but we focus on two cases where we have no knowledge on channel statistics.

1) Special Case 4: Estimator without knowledge of the channel statistics: In this situation, let $\delta_1^2, \delta_2^2, \cdots, \delta_{L-1}^2 \rightarrow \infty$ or $\mathbf{Q}^{-1} \rightarrow \mathbf{0}$. Then from (14)

$$\mathbf{C}(\omega_o) \approx \mathbf{P}_{\mathbf{G}}^{\perp}(\omega_\mathbf{o}). \quad (19)$$

Furthermore, with the well-known result

$$\int_0^\infty x^{-(p+1)} \exp\{-\frac{u}{x^2}\}dx = \frac{1}{2}u^{-(p/2)}\Gamma(\frac{p}{2}), \quad (20)$$

it can be readily obtained that

$$P(\omega_o|\mathbf{x}) \propto \exp\left(\frac{\omega_o^2}{2\sigma_{\omega_o}^2} \right) \left(\mathbf{x}^H\mathbf{P}_{\mathbf{G}}^{\perp}(\omega_\mathbf{o})\mathbf{x} \right)^{-(N-L)} \quad (21)$$

After taking logarithm of (21) and dropping some irrelevant terms, maximizing the above expression becomes

$$\hat{\omega}_o = \arg\min_{\omega_o} \left\{ \frac{\omega_o^2}{2\sigma_{\omega_o}^2} + (N-L)\ln\left(\mathbf{x}^H\mathbf{P}_{\mathbf{G}}^{\perp}(\omega_o)\mathbf{x} \right) \right\}, \quad (22)$$

which represents a trade-off between the observation data and CFO prior information. Notice that the weighing of prior and observations in (15) and (22) are different.

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the WCNC 2009 proceedings.

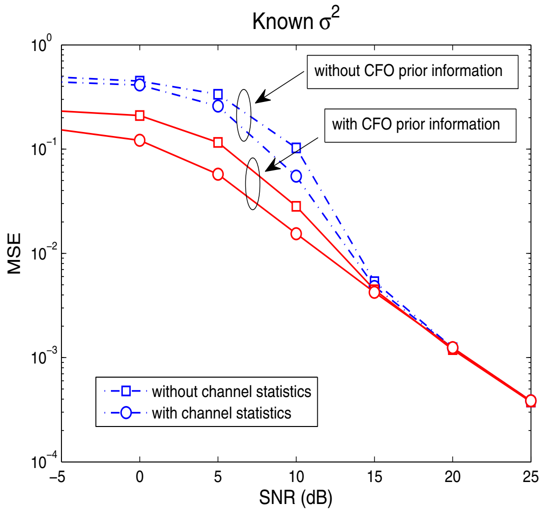

Fig. 2. MSE of MAP estimators with known σ^2

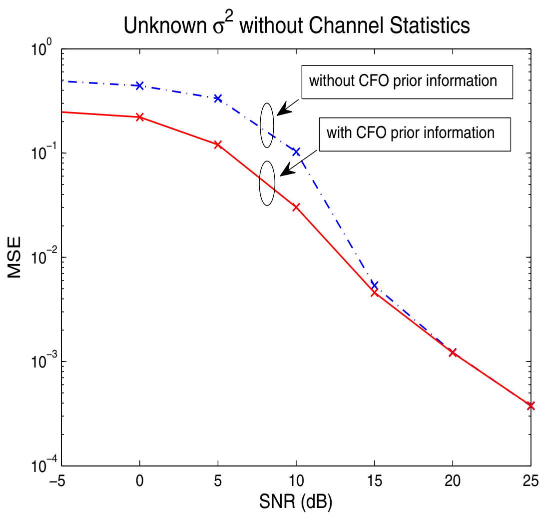

Fig. 3. MSE of MAP estimators with unknown σ^2

2) Special Case 5: Estimator without knowledge of both CFO and channel statistics: As to this case, the corresponding estimator can be obtained by letting $\sigma^2_{\omega_o} \to \infty$ in (22), and we have

$$\hat{\omega}_o = \arg\min_{\omega_o} \left\{ \mathbf{x}^H \mathbf{P}^{\perp}_{\mathbf{G}}(\omega_o)\mathbf{x} \right\} \tag{23}$$

which is also the Conditional Maximum Likelihood (CML) [9] estimator and corresponds to the ignorant receiver in [6].

IV. Simulation Results

In this section, numerical results are provided to demonstrate the effectiveness of the proposed CFO estimators under different prior knowledge, with each point

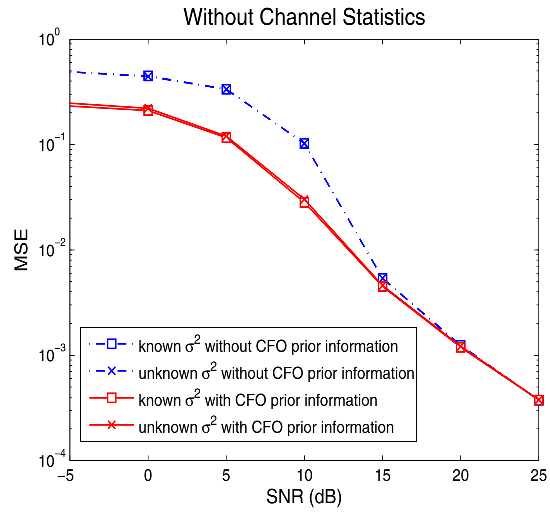

Fig. 4. MSE comparison of MAP estimators with known and unknown σ^2

obtained from 10^4 simulation runs. In all simulations, the considered OFDM system has the following parameters: $N = 64, L_{cp} = 16$, which is consistent with the WLAN standard [7]. The second part of the preamble section is constructed by a Chu-sequence [15]. A multipath Rayleigh fading channel with $L = 8$ and exponential power delay profile (normalized to unit power) is assumed. The normalized CFO for each packet ω_o is generated as a Gaussian random variable with zero mean and variance $\sigma^2_{\omega_o} = 0.1$.

In Figure 2, the performance of the MAP estimators with known σ^2 in Section III-A are plotted as a function of signal-to-noise ratio (SNR). Comparing the estimators with and without CFO prior, it can be observed that the CFO prior provides significant performance improvements, especially at low SNR. With regard to the knowledge of channel statistics, it provides some performance improvements, but not as significant as the CFO priors.

In Figure 3, the performance of the MAP estimators with unknown σ^2 in Section III-B are plotted as a function of SNR. As can be seen from the figure, a performance gap similar to that in Figure 2 can also be observed, which again demonstrates the significance of CFO prior knowledge on estimation performance.

Finally, comparisons between estimators with and without the knowledge of σ^2 are shown in Figure 4. It is clear that the knowledge of noise variance has basically no effect on the performances of the estimators, provided that the same prior information on CFO and channel statistics has been used.

V. CONCLUSIONS

In this paper, MAP CFO estimators in OFDM systems have been derived assuming different prior information. It was shown that CML and UML estimators can be obtained from the Bayesian estimation framework. The effects of different prior information on the CFO estimation performances have further been demonstrated by simulation results.

APPENDIX
PROOF OF (11)

The integral to be computed is

$$\int P(\mathbf{x}|\omega_o, \mathbf{h}, \sigma)P(\mathbf{h})d\mathbf{h}. \tag{24}$$

Using (6) and (10), we have

$$\begin{aligned}&\int P(\mathbf{x}|\omega_o, \mathbf{h}, \sigma)P(\mathbf{h})d\mathbf{h}\\ &= \int \frac{1}{(\pi\sigma^2)^N} \exp\Big\{ - \frac{[\mathbf{x}-\mathbf{G}(\omega_o)\mathbf{h}]^H[\mathbf{x}-\mathbf{G}(\omega_o)\mathbf{h}]}{\sigma^2}\Big\}\\ &\quad \times \frac{1}{\pi^L \det(\mathbf{Q})} \exp(-\mathbf{h}^H\mathbf{Q}^{-1}\mathbf{h})d\mathbf{h}. \end{aligned}\tag{25}$$

By combining the terms related to $\mathbf{h}$ into a quadratic form, (25) can be simplified as

$$\begin{aligned}\int P(\mathbf{x}|\omega_o, \mathbf{h}, \sigma)P(\mathbf{h})d\mathbf{h} &\propto \frac{1}{(\pi\sigma^2)^N} \exp\Big\{ - \frac{\mathbf{x}^H\mathbf{C}(\omega_o)\mathbf{x}}{\sigma^2}\Big\}\\ &\times \int \exp\Big\{ - \frac{[\mathbf{h}-\mathbf{B}(\omega_o)\mathbf{x}]^H\mathbf{A}^{-1}[\mathbf{h}-\mathbf{B}(\omega_o)\mathbf{x}]}{\sigma^2}\Big\}d\mathbf{h}\end{aligned}$$

where

$$\begin{aligned}\mathbf{C}(\omega_o) &= \mathbf{I} - \mathbf{G}(\omega_o)[\mathbf{G}(\omega_o)^H\mathbf{G}(\omega_o) + \sigma^2\mathbf{Q}^{-1}]^{-1}\mathbf{G}(\omega_o)^H\\ \mathbf{B}(\omega_o) &= [\mathbf{G}(\omega_o)^H\mathbf{G}(\omega_o) + \sigma^2\mathbf{Q}]^{-1}\mathbf{G}(\omega_o)^H\\ \mathbf{A} &= \underbrace{\mathbf{G}(\omega_o)^H\mathbf{G}(\omega_o)}_{=\mathbf{F}_L^H\mathbf{D}^H\mathbf{D}\mathbf{F}_L.\text{independent of }\omega_o} +\sigma^2\mathbf{Q}.\end{aligned}$$

Note that

$$\begin{aligned}&\int \frac{1}{(\pi\sigma^2)^L \det(\mathbf{A})}\\ &\times \exp\Big\{ - \frac{[\mathbf{h}-\mathbf{B}(\omega_o)\mathbf{x}]^H\mathbf{A}^{-1}[\mathbf{h}-\mathbf{B}(\omega_o)\mathbf{x}]}{\sigma^2}\Big\}d\mathbf{h}\\ &= 1,\end{aligned}\tag{26}$$

and

$$\begin{aligned}&\int P(\mathbf{x}|\omega_o, \mathbf{h}, \sigma)P(\mathbf{h})d\mathbf{h}\\ &\propto \frac{1}{(\pi\sigma^2)^{(N-L)}} \exp\Big\{ - \frac{\mathbf{x}^H\mathbf{C}(\omega_o)\mathbf{x}}{\sigma^2}\Big\}.\end{aligned}\tag{27}$$

Hence, the posterior distribution of ω_o becomes

$$\begin{aligned}&P(\omega_o|\mathbf{x}, \sigma) \propto\\ &\frac{1}{(\pi\sigma^2)^{(N-L)}} \exp\Big(- \frac{\omega_o^2}{2\sigma_{\omega_o}^2}\Big) \exp\Big\{ - \frac{\mathbf{x}^H\mathbf{C}(\omega_o)\mathbf{x}}{\sigma^2}\Big\}.\end{aligned}$$

REFERENCES

[1] A. R. S. Bahai, B. R. Saltzberg and M. Ergen "Multi-Carrier Digital Communications: Theory and Applications of OFDM", *IEEE Springer*, 2005.

[2] P. H. Moose, "A technique for orthogonal frequency division multiplexing frequency offset correction", *IEEE Trans. on Commun.*, vol. 41, pp. 2908–2914, Nov. 1994.

[3] J. J. Van de Beek, M. Sandell, and P. O. Borjesson, "ML estimation of time and frequency offset in OFDM systems", *IEEE Trans. on Commun.*, vol. 45, pp. 1800–1805, Nov. 1997.

[4] U. Tureli, H. Liu and M. D. Zoltowski, "OFDM blind carrier offset estimation: ESPRIT", *IEEE Trans. on Commun.*, vol. 48, pp. 1459–1461, no. 9, Nov. 2000.

[5] M. C. Bromberg and I. F. Progri, "Bayesian Parameter Estimation for Time and Frequency synchronization", *IEEE WTS*, pp. 126-130 , Apr. 2005.

[6] E. Bjornemo, "Frequency synchronisation in OFDM- a Bayesian analysis", *IEEE VTC*, vol. 2, pp. 1077-1081, Jun. 2005.

[7] *IEEE Std 802.11a, Supplement to Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) specification: High-speed Physical Layer in the 5GHz Band* IEEE Std. 802.11a-1999, 1999.

[8] ETSI. (2001).BRAN; HIPERLAN Type2; Physical (PHY) Layer Specification (2nd ed.) Technical Specification 101 475.[Online]. Available: http://www.etsi.org

[9] J.-W. Chen, Y.-C. Wu, S.-D. Ma and T.-S. Ng, "ML joint CFO and channel estimation in OFDM systems with timing ambiguity", *IEEE Trans. on Wireless Commun.*, vol. 7, pp. 2436-2440, Jul. 2008.

[10] C. Tao and C. Tellambura, "Robust joint frequency offset and channel estimation for OFDM systems", *VTC*, Fall 2004.

[11] Commission of the European Communities, COST 207: Digital Land Mobile Radion communications. Luxembourg: Final Report, Office for Official Publications of European Communities, 1989.

[12] T. S. Rappaport, *Wireless Communications: Principles and Practice*Englewood Cliffs, NJ: Prentice-Hall, 1996.

[13] P. A. Parker, P. Mitran, D. W. Bliss and V. Tarokh, "On Bounds and Algorithms for Frequency Synchronization for Collaborative Communication Systems", *IEEE Trans. Signal Processing*, vol. 56, pp. 3742-3752, Aug. 2008.

[14] G. L. Bretthorst, *Bayesian Spectrum Analysis and Parameter Estimation, Lecture Notes in Statistics* Springer-Verlag press, 1988.

[15] D. C. Chu, "Polyphase codes with good periodic correlation properties", *IEEE Trans. Inform. Theory*, vol. 18, pp. 531–532, July. 1972.