H_∞ Fuzzy Filtering of Nonlinear Systems With Intermittent Measurements

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***Abstract*—This paper is concerned with the problem of H_∞ fuzzy filtering of nonlinear systems with intermittent measurements. The nonlinear plant is represented by a Takagi–Sugeno (T–S) fuzzy model. The measurements transmission from the plant to the filter is assumed to be imperfect, and a stochastic variable satisfying the Bernoulli random binary distribution is utilized to model the phenomenon of the missing measurements. Attention is focused on the design of an H_∞ filter such that the filter error system is stochastically stable and preserves a guaranteed H_∞ performance. A basis-dependent Lyapunov function approach is developed to design the H_∞ filter. By introducing some slack matrix variables, the coupling between the Lyapunov matrix and the system matrices is eliminated, which greatly facilitates the filter-design procedure. The developed theoretical results are in the form of linear matrix inequalities (LMIs). Finally, an illustrative example is provided to show the effectiveness of the proposed approach.**

***Index Terms*—Basis-dependent Lyapunov function, H_∞ filter design, intermittent measurements, nonlinear systems, Takagi–Sugeno (T–S) fuzzy systems.**

I. Introduction

In recent years, there has been a growing interest in the Takagi–Sugeno (T–S) fuzzy model since it is a powerful solution that bridges the gap between linear control and complex nonlinear systems [4], [36], [37]. The important advantage of the T–S fuzzy model is its universal approximation of any smooth nonlinear function by a "blending" of some local linear system models. Based on that local linearity, many complex nonlinear problems can be simplified by employing the Lyapunov function approach [9]. The earlier approach employs quadratic Lyapunov functions, which has shown great effectiveness and has been widely used up until now [5]–[7], [27], [30]. This approach attempts to find a common positive definite matrix to satisfy a set of linear matrix inequalities (LMIs), which is recognized to be conservative, and for some highly nonlinear complex systems, the common Lyapunov matrix even does not exist [45]. This has motivated the development of the more recent approach, which employs basis-dependent Lyapunov functions. Results in many papers have shown that this approach is less conservative because the basis-dependent Lyapunov function is also a "blending" of some piecewise Lyapunov functions [15], [44].

Since the state variables in control systems are not always available, state estimation is another important problem that has been attracting attention from researchers around the world, and a great number of important results have been reported. To mention a few, the filtering problem has been solved for linear systems for uncertain systems [35], Markovian jumping systems [3], [17], [25], [38], sample-data systems [21], [23], [33], systems with singular perturbation [18], [20], and systems with time delays [11], [38]. Different norms have been used to measure the filtering performance (see, for instance, the H_∞ norm [12], [16], the L_1 norm [1], and the L_2–L_∞ norm [19]). There are also some results investigating the filtering problems for nonlinear systems [10], [20], [24].

Among the aforementioned references, H_∞ filtering is one of the most important strategies [14], [38], [41]. The advantage of H_∞ filtering lies in that no statistical assumption on the noise signals is needed, and thus, it is more general than classical Kalman filtering [12]. Due to the powerful approximation property of T–S fuzzy model, recently, there have been a number of results on H_∞ filtering for T–S fuzzy systems [8], [39], [45]. A robust H_∞ filter design for continuous T–S fuzzy models based on the notion of quadratic stability proposed in [8], [13], and [45] are concerned with the H_∞ filtering problem for a class of discrete-time fuzzy systems using basis-dependent Lyapunov functions with reduced conservatism. It is worth noting that all these results are based on the implicit assumption that the communication between the physical plant and filter is perfect, that is, the signals transmitted from the plant will arrive at the filter simultaneously and perfectly.

On another research front, networked control systems have drawn much attention due to their great advantages over traditional systems such as low cost, reduced weight and power requirements, simple installation and maintenance, and high reliability. However, the utilization of networks as communication channels brings us new challenges, and the analysis and synthesis problems become more difficult and complicated due to their limited transmission capacity. Among a few other important problems, data packet dropout is an important issue to be addressed. So far, there have been a number of results focusing on stability analysis of networked systems [29], [43]. Recently,

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increasing attention has been paid to the synthesis problems. For example, state-feedback control is investigated in [40], and H_∞ control is developed in [22] and [42]. It is noted that most of these results focus on the control-related problems. More recently, there have been a few results on the filtering problem for networked systems: References [31] and [32] consider the filtering problem for stochastic systems with missing measurements, and [26] investigates the problem of performing Kalman filtering with intermittent observations, while [11] and [34] discuss that for stochastic systems with time delays. To the best of the author's knowledge, up until now, there has been no research on the filter design for T–S fuzzy systems in the presence of intermittent measurements, which still remains important and challenging. This motivates the present study.

In this paper, we investigate the problem of H_∞ filter design for nonlinear systems with intermittent measurements. The nonlinear plant is represented by a T–S fuzzy model. The measurements transmitted between the plant and the filter are assumed to be imperfect, and the phenomenon of the missing measurements is assumed to satisfy the Bernoulli random binary distribution. Given a T–S fuzzy system, our objective is to design an H_∞ filter such that the filter error system is stochastically stable and preserves a guaranteed H_∞ performance. A basis-dependent Lyapunov function approach is developed to design a desired H_∞ filter. The introduction of some slack matrix variables eliminates the coupling between the system matrices and Lyapunov matrix, which simplifies the filter design. The theoretical results are in the form of LMIs, which can be solved by standard numerical software. An example shows the effectiveness of the proposed approach.

The rest of this paper is organized as follows. Section II formulates the problem under consideration. The stability condition and H_∞ performance of the filter error system are given in Section III. The filter design problem is solved in Section IV. An illustrative example is given in Section V, and we conclude the paper in Section VI.

The notation used in the paper is standard. The superscript "T" stands for matrix transposition; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, and the notation $P > 0$ (≥ 0) means that P is real symmetric and positive definite (semidefinite). $l_2[0,\infty)$ is the space of square-integrable vector functions over $[0,\infty)$; the notation $|\cdot|$ refers to the Euclidean vector norm, and $\|\cdot\|_2$ stands for the usual $l_2[0,\infty)$ norm. In symmetric block matrices or complex matrix expressions, we use an asterisk $(*)$ to represent a term that is induced by symmetry, and diag$\{\ldots\}$ stands for a block-diagonal matrix. In addition, $E\{x\}$ and $E\{x|\,y\}$ will, respectively, mean expectation of x and expectation of x conditional on y. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

II. Problem Formulation

The filtering problem with intermittent measurements is shown in Fig. 1, where the physical plant is represented by a T–S fuzzy model, and the data missing phenomenon occurs intermittently from the plant to the filter. In the following, we model the whole problem mathematically.

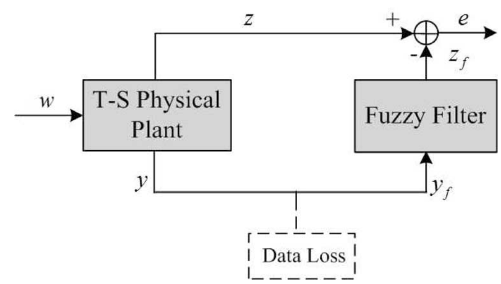

Fig. 1. Filtering problem with intermittent measurements.

A. Physical Plant

The plant under consideration is a nonlinear discrete-time system that is represented by the T–S fuzzy model as follows:

1) Plant Rule i: IF $\theta_1(k)$ is M_{i1} and $\theta_2(k)$ is M_{i2} and $\cdots$ and $\theta_p(k)$ is M_{ip}, THEN

$$\begin{aligned} x_{k+1} &= A_i x_k + B_i w_k \\ y_k &= C_i x_k + D_i w_k \\ z_k &= L_i x_k \\ i &= 1,\ldots,r. \end{aligned} \tag{1}$$

Here, M_{ij} are the fuzzy sets, $x_k \in \mathbb{R}^n$ is the state vector, $w_k \in \mathbb{R}^p$ is the noise signal that is assumed to be arbitrary signal in $l_2[0,\infty)$, $z_k \in \mathbb{R}^q$ is the signal to be estimated, $y_k \in \mathbb{R}^m$ is the measurement output, A_i, B_i, C_i, D_i, and L_i are known constant matrices with appropriate dimensions, r is the number of IF–THEN rules, and $\theta(k) = [\,\theta_1(k),\ \theta_2(k),\ldots,\theta_p(k)\,]$ is the premise variable vector and measurable. The fuzzy basis functions are given by

$$h_i(\theta(k)) = \frac{\prod_{j=1}^{p} M_{ij}(\theta_j(k))}{\sum_{i=1}^{r}\prod_{j=1}^{p} M_{ij}(\theta_j(k))}$$

with $M_{ij}(\theta_j(k))$ representing the grade of membership of $\theta_j(k)$ in M_{ij}. In what follows, we will drop the argument of $h_i(\theta_k)$ for brevity. Therefore, for all k, we have

$$\begin{aligned} &h_i \geq 0, \quad i = 1,2,\ldots,r \\ &\sum_{i=1}^{r} h_i = 1. \end{aligned} \tag{2}$$

Let ρ be a set of basis functions satisfying (2). A more compact presentation of the T–S discrete-time fuzzy model is given by

$$\begin{aligned} x_{k+1} &= A(h)x_k + B(h)w_k \\ y_k &= C(h)x_k + D(h)w_k \\ z_k &= L(h)x_k \end{aligned} \tag{3}$$

where

$$\begin{aligned} &A(h) = \sum_{i=1}^{r} h_i A_i, \quad B(h) = \sum_{i=1}^{r} h_i B_i, \quad C(h) = \sum_{i=1}^{r} h_i C_i \\ &D(h) = \sum_{i=1}^{r} h_i D_i, \quad L(h) = \sum_{i=1}^{r} h_i L_i \end{aligned} \tag{4}$$

and $h \triangleq (h_1, h_2, \ldots, h_r) \in \rho$.

B. Filter

In this paper, we consider the following fuzzy filter to estimate z_k.

1) Filter Rule i: IF $\theta_1(k)$ is M_{i1} and $\theta_2(k)$ is M_{i2} and $\cdots$ and $\theta_p(k)$ is M_{ip}, THEN

$$\begin{aligned}\hat{x}_{k+1} &= A_{fi}\hat{x}_k + B_{fi}y_{fk}\\ \hat{z}_k &= L_{fi}\hat{x}_k\\ i &= 1,\ldots,r.\end{aligned}\tag{5}$$

Here, $\hat{x}_k \in \mathbb{R}^n$, and $\hat{z}_k \in R^q$, and A_{fi}, B_{fi}, and L_{fi} are to be determined. Thus, the filter can be represented by the following input–output form:

$$\begin{aligned}\hat{x}_{k+1} &= A_f(h)\hat{x}_k + B_f(h)y_{fk}\\ \hat{z}_k &= L_f(h)\hat{x}_k.\end{aligned}\tag{6}$$

C. Communication Link

It is assumed that measurements are intermittent, that is, the data may be lost during their transmission. In this case, the input y_{fk} of the filter is no longer equivalent to the output y_k of the plant (that is, $y_k \neq y_{fk}$). In this paper, the data loss phenomenon is modeled via a stochastic approach:

$$y_{fk} = e(k)y_k$$

where $\{e(k)\}$ is Bernoulli process. $\{e(k)\}$ models the intermittent nature of the link from the plant to the filter. More specifically, $e(k) = 0$ when the link fails (that is, data is lost), and $e(k) = 1$ means successful transmission. A natural assumption on $\{e(k)\}$ can be made as

$$\text{Prob}\{e(k)=1\} = \mathbb{E}\{e(k)\} = \bar{e}, \quad \text{Prob}\{e(k)=0\} = 1-\bar{e}.$$

Based on this, we have

$$\begin{aligned}\hat{x}_{k+1} &= A_f(h)\hat{x}_k + B_f(h)e(k)y_k\\ \hat{z}_k &= L_f(h)\hat{x}_k.\end{aligned}\tag{7}$$

D. Filter Error System

From (3) and (7), the filter error system is given by

$$\begin{aligned}\bar{x}_{k+1} &= A_1(h)\bar{x}_k + \tilde{e}(k)A_2(h)\bar{x}_k + B_1(h)w_k + \tilde{e}(k)B_2(h)w_k\\ \bar{z}_k &= \bar{L}(h)\bar{x}_k\end{aligned}\tag{8}$$

where

$$\bar{x}_k = \begin{bmatrix} x_k\\ \hat{x}_k\end{bmatrix}, \quad \bar{z}_k = z_k - \hat{z}_k \tag{9}$$

$$\begin{aligned}A_1(h) &= \begin{bmatrix} A(h) & 0\\ \bar{e}B_f(h)C(h) & A_f(h)\end{bmatrix}, \quad B_1(h) = \begin{bmatrix} B(h)\\ \bar{e}B_f(h)D(h)\end{bmatrix}\\ A_2(h) &= \begin{bmatrix} 0 & 0\\ B_f(h)C(h) & 0\end{bmatrix}, \quad B_2(h) = \begin{bmatrix} 0\\ B_f(h)D(h)\end{bmatrix}\\ L(h) &= [\,L(h) \quad -L_f(h)\,]\end{aligned}\tag{10}$$

and $\tilde{e}(k) = e(k) - \bar{e}$. It is clear that $E\{\tilde{e}(k)\} = 0$ and that $E\{\tilde{e}(k)\tilde{e}(k)\} = \bar{e}(1-\bar{e})$.

Before proceeding further, we first introduce the following definition.

Definition 1: The filter error system in (8) is said to be stochastically stable in the mean square when $w(k) \equiv 0$ for any initial condition x_0 if there exists a finite $W > 0$ such that

$$E\left\{\sum_{k=0}^{\infty} |x_k|^2 \,\middle|\, x_0\right\} < x_0^T W x_0.$$

Then, the problem to be addressed in this paper is expressed as follows.

Problem H_∞ filtering with intermittent measurements (HFIM): Consider the filtering problem shown in Fig. 1, and suppose the intermittent transmission parameter $\bar{e}$ is known. Given a scalar $\gamma > 0$, design a fuzzy filter in the form of (7) such that

1) (stochastic stability) the filter error system in (8) is stochastically stable in the sense of Definition 1;
2) (H_∞ performance) under zero initial condition, the error output $\bar{z}_k$ satisfies

$$||\bar{z}||_E \leq \gamma ||w||_2 \tag{11}$$

where

$$||\bar{z}||_E \triangleq E\left\{\sqrt{\sum_{k=0}^{\infty} \bar{z}_k^T \bar{z}_k}\right\}.$$

If the previous two conditions are satisfied, the filter error system is called stochastically stable with a guaranteed H_∞ performance γ.

III. Filtering Performance Analysis

In this section, the filtering analysis problem is concerned. More specifically, we assume that the filter matrices in (6) are known, and we will study the condition under which the filter error system in (8) is stochastically stable in the mean square with a given H_∞ performance γ. The following theorem shows that the H_∞ performance of the filter error system can be guaranteed if there exist some fuzzy-basis-dependent matrices and additional matrices satisfying a certain linear matrix inequality (LMI).

Theorem 1: Consider the fuzzy system in (3), and suppose that the filter in (6) is given. The filter error system in (8) is stochastically stable with a given H_∞ performance γ, if there exist fuzzy-basis-dependent matrices $P(h) > 0$, $\Omega(h)$, for any $h \in \rho$, $h^+ \triangleq (h_1(\theta_{k+1}), h_2(\theta_{k+1}), \ldots, h_r(\theta_{k+1})) \in \rho$, satisfying

$$\begin{bmatrix} \Theta & 0 & 0 & \Omega^T(h^+)A_1(h) & \Omega^T(h^+)B_1(h)\\ * & \Theta & 0 & f\Omega^T(h^+)A_2(h) & f\Omega^T(h^+)B_2(h)\\ * & * & -I & \bar{L}(h) & 0\\ * & * & * & -P(h) & 0\\ * & * & * & * & -\gamma^2 I\end{bmatrix} < 0 \tag{12}$$

where

$$\Theta = P(h^+) - \Omega(h^+) - \Omega^T(h^+)$$

$f = \sqrt{\bar{e}(1-\bar{e})}$.

Proof: We first prove the stochastic stability of the filter error system in (8). To this end, assume $w_k \equiv 0$, and choose a Lyapunov function as

$$V_k = \bar{x}_k^T \left[P(h)\right] \bar{x}_k. \quad (13)$$

When $w_k \equiv 0$, (8) becomes

$$\begin{aligned}\bar{x}_{k+1} &= A_1(h)\bar{x}_k + \tilde{e}(k)A_2(h)\bar{x}_k\\ \bar{z}_k &= \bar{L}(h)\bar{x}_k.\end{aligned}$$

Then, we have

$$\begin{aligned}\Delta V_k &= E\{V_{k+1} \,|\, \bar{x}_k\} - V_k\\ &= E\big\{\bar{x}_k^T \big(A_1^T(h) + \tilde{e}(k)A_2^T(h)\big)P(h^+)\\ &\quad \times (A_1(h) + \tilde{e}(k)A_2(h))\bar{x}_k|\bar{x}_k\big\} - \bar{x}_k^T P(h)\bar{x}_k\\ &= \bar{x}_k^T \big(A_1^T(h)P(h^+)A_1(h) + f^2 A_2^T(h)P(h^+)A_2(h)\\ &\quad - P(h)\big)\bar{x}_k.\end{aligned}$$

Note that the inequality

$$[P(h^+) - \Omega(h^+)]^T P^{-1}(h^+)[P(h^+) - \Omega(h^+)] \geq 0$$

implies that

$$P(h^+) - (\Omega(h^+) + \Omega^T(h^+)) \geq -\Omega^T(h^+)P^{-1}(h^+)\Omega(h^+)$$

which together with (12) yields

$$\begin{bmatrix} \tilde{\Theta} & 0 & 0 & \Omega^T(h^+)A_1(h) & \Omega^T(h^+)B_1(h)\\ * & \tilde{\Theta} & 0 & f\Omega^T(h^+)A_2(h) & f\Omega^T(h^+)B_2(h)\\ * & * & -I & \bar{L}(h) & 0\\ * & * & * & -P(h) & 0\\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (14)$$

where $\tilde{\Theta} = -\Omega^T(h^+)P^{-1}(h^+)\Omega(h^+)$. Clearly, $\Omega(h^+)$ is invertible. Diag $\{\Omega^{-T}(h^+), \Omega^{-T}(h^+), I, I, I\}$ and postmultiplying diag $\{\Omega^{-1}(h^+), \Omega^{-1}(h^+), I, I, I\}$ on the left and right sides of (14), we obtained the following inequality:

$$\begin{bmatrix} -P^{-1}(h^+) & 0 & 0 & A_1(h) & B_1(h)\\ * & -P^{-1}(h^+) & 0 & fA_2(h) & fB_2(h)\\ * & * & -I & \bar{L}(h) & 0\\ * & * & * & -P(h) & 0\\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0$$

by Schur complement, which leads to

$$\begin{bmatrix} A_1^T(h) & fA_2^T(h) & \bar{L}^T(h)\\ B_1^T(h) & fB_2^T(h) & 0 \end{bmatrix} \begin{bmatrix} P(h^+) & 0 & 0\\ 0 & P(h^+) & 0\\ 0 & 0 & I \end{bmatrix} \times \begin{bmatrix} A_1(h) & B_1(h)\\ fA_2(h) & fB_2(h)\\ \bar{L}(h) & 0 \end{bmatrix} - \begin{bmatrix} P(h) & 0\\ 0 & \gamma^2 I \end{bmatrix} < 0. \quad (15)$$

Here, (15) implies

$$A_1^T(h)P(h^+)A_1(h) + f^2 A_2^T(h)P(h^+)A_2(h) - P(h) < 0$$

and thus, we have

$$\Delta V_k < 0.$$

Define

$$\Phi \triangleq A_1^T(h)P(h^+)A_1(h) + f^2 A_2^T(h)P(h^+)A_2(h) - P(h)$$

and we get

$$E\{V_{k+1} \,|\, \bar{x}_k\} - V_k \leq -\lambda_{\min}(-\Phi)\bar{x}_k^T \bar{x}_k.$$

Taking mathematical expectation of both sides, for any $T \geq 1$, and summing up the inequality on both sides from $k = 0, \ldots, T$, we have

$$E\{V_{T+1}\} - V_0 \leq -\lambda_{\min}(-\Phi)E\{|\bar{x}_k|^2\}$$

which implies

$$E\{|\bar{x}_k|^2\} \leq (\lambda_{\min}(-\Phi))^{-1}(V_0 - E\{V_{T+1}\}).$$

Considering $E\{V(k)\} \geq 0$ for all $k \geq 0$, we have

$$\begin{aligned} E\left\{\sum_{k=0}^{\infty} |\bar{x}_k|^2 \middle| \bar{x}_0\right\} &\leq (\lambda_{\min}(-\Phi))^{-1}\bar{x}_0^T \max(P(h))\bar{x}_0\\ &= \bar{x}_0^T(\lambda_{\min}(-\Phi))^{-1}\max(P(h))\bar{x}_0\\ &= \bar{x}_0^T W \bar{x}_0 \end{aligned}$$

where x_0 is the initial condition, and $W \triangleq (\lambda_{\min}(-\Phi))^{-1} \max(P(h))$. According to Definition 1, the filter error system is stochastically stable in the mean square.

Next, the H_∞ performance criteria for the filter error system in (8) will be established. To this end, assume zero initial conditions. An index is introduced as

$$\bar{J} = E\{V_{k+1} \,|\, \xi_k\} + \bar{z}_k^T \bar{z}_k - \gamma^2 w_k^T w_k - \bar{x}_k^T P(h)\bar{x}_k$$

where

$$\xi_k = \begin{bmatrix} \bar{x}_k\\ w_k \end{bmatrix}.$$

Since

$$\begin{aligned} &E\{V_{k+1}|\xi_k\}\\ &= E\left\{\xi_k^T \begin{bmatrix} A_1^T(h) + \tilde{e}(k)A_2^T(h)\\ B_1^T(h) + \tilde{e}(k)B_2^T(h) \end{bmatrix} \begin{bmatrix} P(h^+) & 0\\ 0 & P(h^+) \end{bmatrix}\right.\\ &\quad \times [\,A_1(h) + \tilde{e}(k)A_2(h) \quad B_1(h) + \tilde{e}(k)B_2(h)\,]\,\xi_k \,|\, \xi_k\}\\ &= E\left\{\xi_k^T \left(\begin{bmatrix} A_1^T(h)\\ B_1^T(h) \end{bmatrix} P(h^+)\,[\,A_1(h) \quad B_1(h)\,]\right.\right.\\ &\quad \left.\left. + f^2 \begin{bmatrix} A_2^T(h)\\ B_2^T(h) \end{bmatrix} P(h^+)\,[\,A_2(h) \quad B_2(h)\,]\right)\xi_k \middle| \xi_k\right\} \end{aligned}$$

and

$$\bar{z}_k^T \bar{z}_k = \xi_k^T \begin{bmatrix} \bar{L}^T(h)\\ 0 \end{bmatrix} [\,\bar{L}(h) \quad 0\,]\,\xi_k$$

we have

$$\bar{J} = \xi_k^T \left(\begin{bmatrix} A_1^T(h) \\ B_1^T(h) \end{bmatrix} P(h^+) \left[A_1(h) \quad B_1(h) \right] + f^2 \begin{bmatrix} A_2^T(h) \\ B_2^T(h) \end{bmatrix} P(h^+) \left[A_2(h) \quad B_2(h) \right] + \begin{bmatrix} \bar{L}^T(h) \\ 0 \end{bmatrix} \left[\bar{L}(h) \quad 0 \right] - \begin{bmatrix} P(h) & 0 \\ 0 & \gamma^2 I \end{bmatrix} \right) \xi_k.$$

From inequality (15), we know that

$$\bar{J} \le 0$$

that is

$$E\left\{ V_{k+1} | \, \xi_k \right\} + \bar{z}_k^T \bar{z}_k - \gamma^2 w_k^T w_k - \bar{x}_k^T P(h) \bar{x}_k \le 0.$$

Take mathematical expectation on both sides, we have

$$E\{V_{k+1}\} - E\{V_k\} + E\left\{ \bar{z}_k^T \bar{z}_k \right\} - \gamma^2 w_k^T w_k \le 0.$$

For $k = 0, 1, 2, \ldots,$ summing up both sides, considering $E\{V(k)\} \ge 0$ for all $k \ge 0$, under zero initial condition, we obtain

$$E\left\{ \sum_{k=0}^{\infty} \bar{z}_k^T \bar{z}_k \right\} - \sum_{k=0}^{\infty} \gamma^2 w_k^T w_k \le 0$$

which is equivalent to (11). The proof is completed. ■

Remark 1: If there is no data dropout in the channel between the physical plant and the filter, that is, perfect communication links exist between the plant and the filter, then we have the following corollary, which can be proved by following similar arguments, as in the proof of Theorem 1.

Corollary 1: Consider the fuzzy system in (3) and suppose that the filter in (6) is given. When $\bar{e} = 1$, the filter error system in (8) is stochastically stable with a given H_∞ performance γ, if there exist fuzzy-basis-dependent matrices $P(h) > 0$, $\Omega(h)$, for any h, $h^+ \in \rho$, satisfying

$$\begin{bmatrix} \Theta & 0 & \Omega^T(h^+) A_1(h) & \Omega^T(h^+) B_1(h) \\ * & -I & \bar{L}(h) & 0 \\ * & * & -P(h) & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \qquad (16)$$

where h^+ and Θ are defined in (12).

IV. Filter Design

In this section, we will design a fuzzy filter in the form of (6) based on Theorem 1, that is, to determine the filter matrices in (6) such that the filter error system in (8) is stochastically stable with a guaranteed H_∞ performance. Since the condition in (12) cannot be utilized to obtain the filter directly, we introduce some slack matrices, which will simplify the filter design procedure.

Theorem 2: Consider the fuzzy system in (3). For a given positive constant γ, if there exist fuzzy-basis-dependent matrices

$$Q(h) = \begin{bmatrix} Q_1(h) & Q_2(h) \\ Q_2^T(h) & Q_3(h) \end{bmatrix} > 0$$

and $R, S, W, \bar{A}_f(h), \bar{B}_f(h), \bar{L}_f(h)$, for any $h, h^+ \in \rho$ satisfying

$$\begin{bmatrix} \varphi_{11} & \varphi_{12} \\ * & \varphi_{22} \end{bmatrix} < 0 \qquad (17)$$

where

$$\Xi = \begin{bmatrix} Q_1(h^+) & Q_2(h^+) \\ Q_2^T(h^+) & Q_3(h^+) \end{bmatrix} - \begin{bmatrix} R + R^T & S + W \\ W^T + S^T & W + W^T \end{bmatrix} \qquad (18)$$

$$\varphi_{11} = \begin{bmatrix} \Xi & 0 & 0 \\ * & \Xi & 0 \\ * & * & -I \end{bmatrix}, \quad \varphi_{12} = \begin{bmatrix} \varphi_{12}^{(11)} & \varphi_{12}^{(12)} \\ \varphi_{12}^{(21)} & \varphi_{12}^{(22)} \\ \varphi_{12}^{(31)} & 0 \end{bmatrix}$$

$$\varphi_{22} = \begin{bmatrix} \begin{bmatrix} -Q_1(h) & -Q_2(h) \\ -Q_2^T(h) & -Q_3(h) \end{bmatrix} & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \qquad (19)$$

and

$$\varphi_{12}^{(11)} = \begin{bmatrix} R^T A(h) + \bar{e} \bar{B}_f(h) C(h) & \bar{A}_f(h) \\ S^T A(h) + \bar{e} \bar{B}_f(h) C(h) & \bar{A}_f(h) \end{bmatrix}$$

$$\varphi_{12}^{(12)} = \begin{bmatrix} R^T B(h) + \bar{e} \bar{B}_f(h) D(h) \\ S^T B(h) + \bar{e} \bar{B}_f(h) D(h) \end{bmatrix}$$

$$\varphi_{12}^{(21)} = f \begin{bmatrix} \bar{B}_f(h) C(h) & 0 \\ \bar{B}_f(h) C(h) & 0 \end{bmatrix}, \quad \varphi_{12}^{(22)} = f \begin{bmatrix} \bar{B}_f(h) D(h) \\ \bar{B}_f(h) D(h) \end{bmatrix}$$

$$\varphi_{12}^{(31)} = \left[L(h) \quad -\bar{L}_f(h) \right] \qquad (20)$$

then there exists a fuzzy filter in the form of (6) such that the filtering error system in (8) is stochastically stable with a prescribed H_∞ norm bound γ. Moreover, if the aforementioned condition is satisfied, the matrices for the filter in (6) are given by

$$\begin{bmatrix} A_f(h) & B_f(h) \\ L_f(h) & 0 \end{bmatrix} = \begin{bmatrix} \Omega_4^{-T} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{A}_f(h) & \bar{B}_f(h) \\ \bar{L}_f(h) & 0 \end{bmatrix} \times \begin{bmatrix} \Omega_4^{-1} \Omega_3 & 0 \\ 0 & I \end{bmatrix} \qquad (21)$$

where Ω_3 and Ω_4 can be obtained by the decomposition on W.

Proof: Suppose that there exist matrices $Q(h) > 0$, R, S, W, $\bar{A}_f(h)$, $\bar{B}_f(h)$, and $\bar{L}_f(h)$ satisfying (17). From (17), we know that $W > 0$. One can always find square and nonsingular matrices Ω_3 and Ω_4 that $W = \Omega_4^T \Omega_3^{-1} \Omega_4$. Let

$$R = \Omega_1, \quad S = \Omega_2 \Omega_3^{-1} \Omega_4$$

$$\Omega = \begin{bmatrix} \Omega_1 & \Omega_2 \\ \Omega_4 & \Omega_3 \end{bmatrix}, \quad T = \begin{bmatrix} I & 0 \\ 0 & \Omega_3^{-1} \Omega_4 \end{bmatrix} \qquad (22)$$

and

$$T^{-T} \begin{bmatrix} Q_1(h^+) & Q_2(h^+) \\ Q_2^T(h^+) & Q_3(h^+) \end{bmatrix} T^{-1} = \begin{bmatrix} P_1(h^+) & P_2(h^+) \\ P_2^T(h^+) & P_3(h^+) \end{bmatrix}$$

$$\begin{bmatrix} A_f(h) & B_f(h) \\ L_f(h) & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \Omega_4^{-T} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{A}_f(h) & \bar{B}_f(h) \\ \bar{L}_f(h) & 0 \end{bmatrix} \begin{bmatrix} \Omega_4^{-1}\Omega_3 & 0 \\ 0 & I \end{bmatrix}. \quad (23)$$

By (22) and (23), one has

$$T^{-T}\Xi T^{-1} = \begin{bmatrix} P_1(h^+) & P_2(h^+) \\ P_2^T(h^+) & P_3(h^+) \end{bmatrix} - \Omega - \Omega^T. \quad (24)$$

With the support of (10), (22), and (23), it can be verified that

$$\begin{aligned} &T^T\Omega^T A_1(h)T \\ &= \begin{bmatrix} \Omega_1^T A(h) + \bar{e}\Omega_4^T B_f(h)C(h) & \Omega_4^T A_f(h)\Omega_3^{-1}\Omega_4 \\ \Omega_4^T\Omega_3^{-T}\Omega_2^T A(h) + \bar{e}\Omega_4^T B_f(h)C(h) & \Omega_4^T A_f(h)\Omega_3^{-1}\Omega_4 \end{bmatrix} \\ &= \begin{bmatrix} R^T A(h) + \bar{e}\bar{B}_f(h)C(h) & \bar{A}_f(h) \\ S^T A(h) + \bar{e}\bar{B}_f(h)C(h) & \bar{A}_f(h) \end{bmatrix} \\ &fT^T\Omega^T A_2(h)T \\ &= f\begin{bmatrix} \Omega_4^T B_f(h)C(h) & 0 \\ \Omega_4^T B_f(h)C(h) & 0 \end{bmatrix} = f\begin{bmatrix} \bar{B}_f(h)C(h) & 0 \\ \bar{B}_f(h)C(h) & 0 \end{bmatrix} \\ &T^T\Omega^T B_1(h) \\ &= \begin{bmatrix} \Omega_1^T B(h) + \bar{e}\Omega_4^T B_f(h)D(h) \\ \Omega_4^T\Omega_3^{-T}\Omega_2^T B(h) + \bar{e}\Omega_4^T B_f(h)D(h) \end{bmatrix} \\ &= \begin{bmatrix} R^T B(h) + \bar{e}\bar{B}_f(h)D(h) \\ S^T B(h) + \bar{e}\bar{B}_f(h)D(h) \end{bmatrix} \\ &fT^T\Omega^T B_2(h) \\ &= f\begin{bmatrix} \Omega_4^T B_f(h)D(h) \\ \Omega_4^T B_f(h)D(h) \end{bmatrix} = f\begin{bmatrix} \bar{B}_f(h)D(h) \\ \bar{B}_f(h)D(h) \end{bmatrix} \\ &\bar{L}(h)T \\ &= [\,L(h) \quad -L_f(h)\Omega_3^{-1}\Omega_4\,] = [\,L(h) \quad -\bar{L}_f(h)\,]. \end{aligned} \quad (25)$$

Letting

$$\Omega(h) = \Omega, \quad P(h) = T^{-T}\begin{bmatrix} Q_1(h) & Q_2(h) \\ Q_2^T(h) & Q_3(h) \end{bmatrix} T^{-1} \quad (26)$$

one can readily obtain from (24)–(26) that (17) is equivalent to

$$\begin{bmatrix} T^T & 0 & 0 & 0 & 0 \\ 0 & T^T & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & T^T & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix} \times \begin{bmatrix} \Theta & 0 & 0 & \Omega^T(h^+)A_1(h) & \Omega^T(h^+)B_1(h) \\ * & \Theta & 0 & f\Omega^T(h^+)A_2(h) & f\Omega^T(h^+)B_2(h) \\ * & * & -I & \bar{L}(h) & 0 \\ * & * & * & -P(h) & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} \times \begin{bmatrix} T & 0 & 0 & 0 & 0 \\ 0 & T & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & T & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix} < 0$$

where Θ is defined in (12), which together with (17) implies that, for any $h, h^+ \in \rho$, (12) holds.

The proof is completed. ■

The condition in (17) cannot be directly employed for filter design. One way to facilitate Theorem 2 for the construction of a fuzzy filter is to convert (17) into a finite set of LMI constraints. To this end, one must further restrict the choice of the fuzzy-basis-dependent Lyapunov functions. The following theorem gives a possible way to achieve this.

Theorem 3: Consider the fuzzy system in (3). For a given positive constant γ, if there exist matrices $Q_i = \begin{bmatrix} Q_{1i} & Q_{2i} \\ Q_{2i}^T & Q_{3i} \end{bmatrix} > 0$, and R, S, W, $\bar{A}_{fi}$, $\bar{B}_{fi}$, $\bar{L}_{fi}$, for all $i, j, l \in \{1, \ldots, r\}$ satisfying

$$\begin{bmatrix} \psi_{11} & \psi_{12} \\ * & \psi_{22} \end{bmatrix} < 0 \quad (27)$$

where

$$\bar{\Xi} = \begin{bmatrix} Q_{1l} & Q_{2l} \\ Q_{2l}^T & Q_{3l} \end{bmatrix} - \begin{bmatrix} R + R^T & S + W \\ W^T + S^T & W + W^T \end{bmatrix} \quad (28)$$

$$\psi_{11} = \begin{bmatrix} \bar{\Xi} & 0 & 0 \\ * & \bar{\Xi} & 0 \\ * & * & -I \end{bmatrix},$$

$$\psi_{22} = \begin{bmatrix} \begin{bmatrix} -Q_{1i} & -Q_{2i} \\ -Q_{2i}^T & -Q_{3i} \end{bmatrix} & 0 \\ 0 & -\gamma^2 I \end{bmatrix}$$

$$\psi_{12} = \begin{bmatrix} \begin{bmatrix} R^T A_i + \bar{e}\bar{B}_{fi}C_j & \bar{A}_{fi} \\ S^T A_i + \bar{e}\bar{B}_{fi}C_j & \bar{A}_{fi} \end{bmatrix} & \begin{bmatrix} R^T B_i + \bar{e}\bar{B}_{fi}D_j \\ S^T B_i + \bar{e}\bar{B}_{fi}D_j \end{bmatrix} \\ f\begin{bmatrix} \bar{B}_{fi}C_j & 0 \\ \bar{B}_{fi}C_j & 0 \end{bmatrix} & f\begin{bmatrix} \bar{B}_{fi}D_j \\ \bar{B}_{fi}D_j \end{bmatrix} \\ [\,L_i \quad -\bar{L}_{fi}\,] & 0 \end{bmatrix} \quad (29)$$

then there exists a fuzzy filter in (6) such that the filter error system in (8) is stochastically stable with a prescribed H_∞ norm bound γ. Moreover, if the earlier condition is satisfied, the matrices for the filter in (6) are given by

$$\begin{bmatrix} A_f(h) & B_f(h) \\ L_f(h) & 0 \end{bmatrix} = \sum_{i=1}^{r} h_i \begin{bmatrix} \Omega_4^{-T} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{A}_{fi} & \bar{B}_{fi} \\ \bar{L}_{fi} & 0 \end{bmatrix} \times \begin{bmatrix} \Omega_4^{-1}\Omega_3 & 0 \\ 0 & I \end{bmatrix} \quad (30)$$

where Ω_3 and Ω_4 can be obtained by the decomposition on W.

Proof: Suppose that there exist matrices R, S, W, $\bar{A}_{fi}$, $\bar{B}_{fi}$, $\bar{L}_{fi}$, and $Q_i = \begin{bmatrix} Q_{1i} & Q_{2i} \\ Q_{2i}^T & Q_{3i} \end{bmatrix} > 0$, for all i, j, $l \in \{1, \ldots, r\}$ satisfying (17). Then, we use these matrices and the fuzzy basis function $h \in \rho$ to define the following functions:

$$Q(h) = \sum_{l=1}^{r} \left\{ h_l \begin{bmatrix} Q_{1l} & Q_{2l} \\ Q_{2l}^T & Q_{3l} \end{bmatrix} \right\}, \quad \bar{A}_f(h) = \sum_{i=1}^{r} h_i \bar{A}_{fi}$$

$$\bar{B}_f(h) = \sum_{i=1}^{r} h_i \bar{B}_{fi}, \quad \bar{L}_f(h) = \sum_{i=1}^{r} h_i \bar{L}_{fi}$$

which together with (4) imply that

$$\begin{bmatrix} \varphi_{11} & * \\ \varphi_{21} & \varphi_{22} \end{bmatrix} = \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{l=1}^{r} h_i h_j h_l^+ \begin{bmatrix} \psi_{11} & * \\ \psi_{21} & \psi_{22} \end{bmatrix}$$

and (17) is clearly verified, where h_l^+ is $h_l(\theta_{k+1})$, as is defined in (12), and φ_{11}, φ_{21}, φ_{22}, ψ_{11}, ψ_{21}, and ψ_{22} are defined as in (17), (19), (27), and (29). ■

Corollary 2: Consider the fuzzy system in (3). When $\bar{e} = 1$, for a given positive constant γ, if there exist matrices $Q_i = \begin{bmatrix} Q_{1i} & Q_{2i} \\ Q_{2i}^T & Q_{3i} \end{bmatrix} > 0$, and R, S, W, $\bar{A}_{fi}$, $\bar{B}_{fi}$, and $\bar{L}_{fi,}$ for all i, j, $l \in \{1, \ldots, r\}$ satisfying

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix} < 0 \tag{31}$$

where

$$\Pi_{11} = \begin{bmatrix} \bar{\Xi} & 0 \\ 0 & -I \end{bmatrix}, \quad \Pi_{22} = \begin{bmatrix} \begin{bmatrix} -Q_{1i} & -Q_{2i} \\ -Q_{2i}^T & -Q_{3i} \end{bmatrix} & 0 \\ 0 & -\gamma^2 I \end{bmatrix}$$

$$\Pi_{12} = \begin{bmatrix} \begin{bmatrix} R^T A_i + \bar{B}_{fi} C_j & \bar{A}_{fi} \\ S^T A_i + \bar{B}_{fi} C_j & \bar{A}_{fi} \end{bmatrix} & \begin{bmatrix} R^T B_i + \bar{B}_{fi} D_j \\ S^T B_i + \bar{B}_{fi} D_j \end{bmatrix} \\ [L_i \quad -\bar{L}_{fi}] & 0 \end{bmatrix} \tag{32}$$

and $\bar{\Xi}$ is defined in (28), then there exists a fuzzy filter in the form of (6), and the filter error system in (8) is stochastically stable with a prescribed H_∞ norm bound γ. Moreover, if the previous condition is satisfied, the matrices for the filter in (6) are given by (30).

Remark 2: Theorem 3 is obtained by using the basis-dependent Lyapunov function. It is clear that when $Q_i = Q$ for any $i \in \{1, \ldots, r\}$, (13) becomes the quadratic Lyapunov function that has been widely used in the literature. Then, the following corollary based on the quadratic approach is obtained.

Corollary 3: Consider the fuzzy system in (3). For a given positive constant γ, if there exist matrices $Q = \begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix} > 0, R, S, W, \bar{A}_{fi}, \bar{B}_{fi}$, and $\bar{L}_{fi}$, for all $i, j \in \{1, \ldots, r\}$ satisfying

$$\begin{bmatrix} \tilde{\psi}_{11} & \psi_{12} \\ * & \tilde{\psi}_{22} \end{bmatrix} < 0 \tag{33}$$

where

$$\tilde{\Xi} = \begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix} - \begin{bmatrix} R + R^T & S + W \\ W^T + S^T & W + W^T \end{bmatrix}$$

$$\tilde{\psi}_{11} = \begin{bmatrix} \tilde{\Xi} & 0 & 0 \\ * & \tilde{\Xi} & 0 \\ * & * & -I \end{bmatrix}, \quad \tilde{\psi}_{22} = \begin{bmatrix} \begin{bmatrix} -Q_1 & -Q_2 \\ -Q_2^T & -Q_3 \end{bmatrix} & 0 \\ 0 & -\gamma^2 I \end{bmatrix}$$

and ψ_{12} is defined in (29), then there exists a fuzzy filter in the form of (6) such that the filter error system in (8) is stochastically stable with a prescribed H_∞ norm bound γ. Moreover, if the earlier condition is satisfied, the matrices for the filter in (6) are given by (30).

Remark 3: Theorem 3 is obtained by restricting the fuzzy-basis-dependent Lyapunov functions. The expression of fuzzy-basis-dependent Lyapunov functions adopted here is consistent with the compact presentation of system matrices in (3), which is adopted by mostc of the literature. This fuzzy-basis-dependent Lyapunov approach has been recognized to be less conservative. However, in this basis-dependent framework, the restriction on the Lyapunov function still introduces some overdesign. How to further reduce this conservatism still needs further investigation.

Remark 4: The number of inequalities in Theorem 3 will increase with the number of fuzzy rules of the model, thus; a computational problem might arise for high-order nonlinear systems. One effective way to solve this problem is to try to reduce the number of fuzzy rules when modeling the nonlinear system based on fuzzy logic, which can be found in [28].

V. Illustrative Example

In this section, we use an example to illustrate the effectiveness of the theoretical results developed before.

Consider a tunnel diode circuit shown in Fig. 2, whose fuzzy modeling was done in [2], where $x_1(t) = v_C(t)$, $x_2(t) = i_L(t)$, $w(t)$ is the disturbance noise input, $y(t)$ is the measurement output, and $z(t)$ is the controlled output. With a sampling time $T = 0.02$, the discrete-time model is obtained as

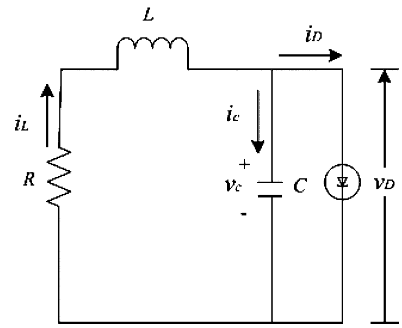

Fig. 2. Tunnel diode circuit.

$$\begin{aligned} x_{k+1} &= A(h)x_k + B(h)w_k \\ y_k &= C(h)x_k + D(h)w_k \\ z_k &= L(h)x_k \end{aligned} \tag{34}$$

where

$$A_1 = \begin{bmatrix} 0.9887 & 0.9024 \\ -0.0180 & 0.8100 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.0093 \\ 0.0181 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 0.90337 & 0.8617 \\ -0.0172 & 0.8103 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.0091 \\ 0.0181 \end{bmatrix}$$
$$C_1 = [1 \quad 0], \quad C_2 = [1 \quad 0]$$
$$D_1 = 1, \quad D_2 = 1, \quad L_1 = [1 \quad 0], \quad L_2 = [1 \quad 0].$$

To show the effectiveness of the obtained results, we assume the membership function to be

$$h_1 = \begin{cases} \dfrac{x_k^{(1)} + 3}{3}, & -3 \leq x_k^{(1)} \leq 0 \\ 0, & x_k^{(1)} < -3 \\ \dfrac{3 - x_k^{(1)}}{3}, & 0 \leq x_k^{(1)} \leq 3 \\ 0, & x_k^{(1)} > 3 \end{cases}$$
$$h_2 = 1 - h_1. \tag{35}$$

The purpose here is to design a filter in the form of (5) such that the system in (34) is stochastically stable with a guaranteed H_∞ norm bound γ.

Suppose $\bar{e} = 0.8$. By solving LMI (27), the minimum H_∞ performance $\gamma^* = 0.1463$ is obtained, and the filter matrices are obtained:

$$\bar{A}_{f1} = \begin{bmatrix} 4.9722 & 14.2575 \\ 9.1080 & 69.4648 \end{bmatrix}, \quad \bar{B}_{f1} = \begin{bmatrix} -0.3451 \\ -1.4883 \end{bmatrix}$$
$$\bar{A}_{f2} = \begin{bmatrix} 4.6054 & 13.9920 \\ 8.4237 & 67.9560 \end{bmatrix}, \quad \bar{B}_{f2} = \begin{bmatrix} -0.2005 \\ -0.8318 \end{bmatrix} \tag{36}$$
$$\bar{L}_{f1} = [-1.0000 \quad -0.0002] \tag{37}$$
$$\bar{L}_{f2} = [-0.9955 \quad 0.0152] \tag{38}$$
$$W = \begin{bmatrix} 5.5335 & 11.5195 \\ 11.5162 & 71.9180 \end{bmatrix}. \tag{39}$$

By (30), we have

$$A_{f1} = \begin{bmatrix} 0.9524 & 0.8488 \\ -0.0259 & 0.8300 \end{bmatrix}, \quad B_{f1} = \begin{bmatrix} -0.0289 \\ -0.0161 \end{bmatrix}$$
$$A_{f2} = \begin{bmatrix} 0.8827 & 0.8423 \\ -0.0242 & 0.8100 \end{bmatrix}, \quad B_{f2} = \begin{bmatrix} -0.0182 \\ -0.0086 \end{bmatrix}$$
$$L_{f1} = [-1.0000 \quad -0.0002]$$
$$L_{f2} = [-0.9955 \quad 0.0152].$$

First, we assume that $w_k \equiv 0$ and the that initial condition $x_0 = [0.2 \quad -0.8], \hat{x}_0 = [0 \quad 0]$. Fig. 3 shows that the estimation error response converges to zero.

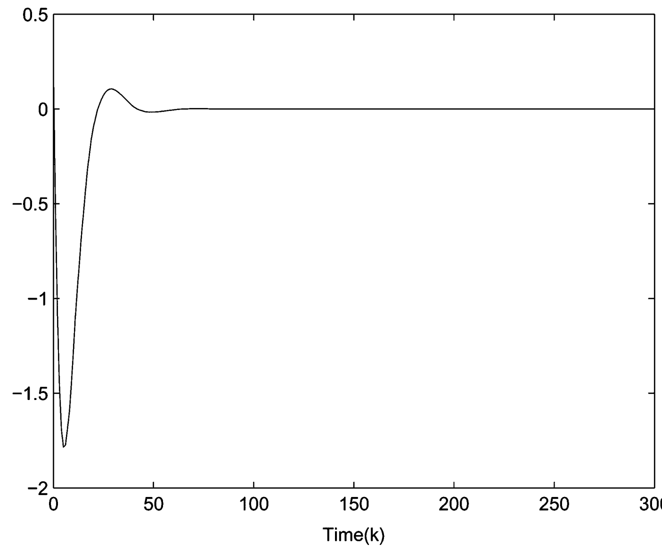

Fig. 3. Estimation error when $w_k \equiv 0$.

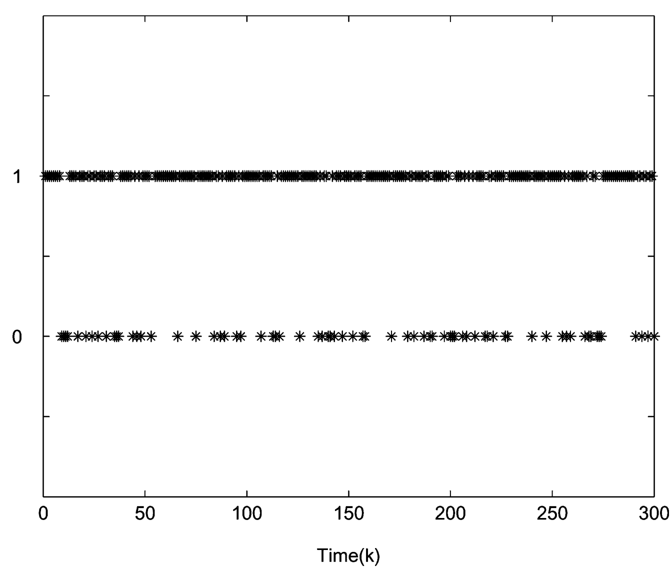

Fig. 4. Data packet dropout.

To illustrate the performance of the designed filter, we assume the initial conditions and the external disturbance $w(k)$ to be

$$w(k) = \begin{cases} 2, & 30 \leq k \leq 50 \\ -2, & 70 \leq k \leq 100 \\ 0, & \text{elsewhere.} \end{cases} \tag{40}$$

In the simulation, the data packet dropouts are generated randomly according to $\bar{e} = 0.8$, which is shown in Fig. 4. Fig. 5 shows the response of signal $\bar{z}(k)$. Fig. 6 shows the simulation results of z_k and $\hat{z}_k$. By calculation, we obtain that $||\bar{z}||_2^2 = 1.0881$ and $||w||_2^2 = 208$, which yields $\gamma = 0.0723$ (below the minimum $\gamma^* = 0.1463$), showing the effectiveness of the filter design.

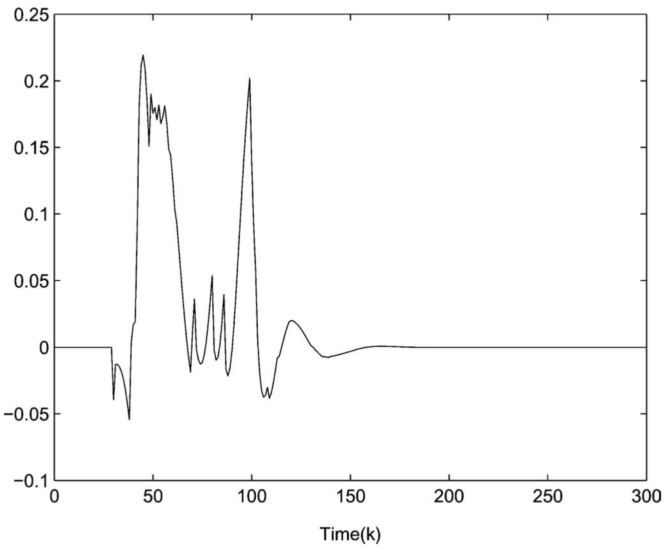

Fig. 5. Estimation error.

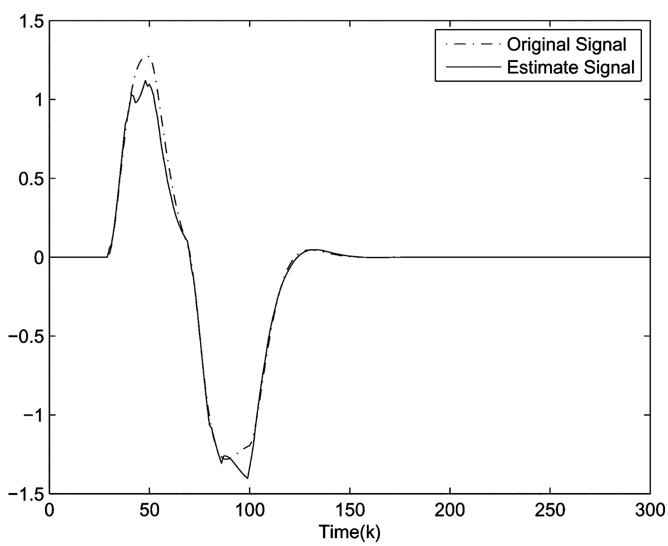

Fig. 6. Estimation signals.

VI. Concluding Remarks

In this paper, the problem of H_∞ fuzzy filtering of nonlinear systems under unreliable communication links has been investigated. The T–S fuzzy system is utilized to model the nonlinear plant, and the communication link failure is modeled via a stochastic variable satisfying the Bernoulli random binary distribution. The basis-dependent Lyapunov function has been used to design an H_∞ filter such that the filter error system is stochastically stable and preserves a guaranteed H_∞ performance. Some slack matrices have been introduced to facilitate the H_∞ filter design. An example has been given to illustrate the effectiveness of the proposed approach.

It should be noted that in practical networked control systems, in addition to data missing, the phenomenon of transmission delay often occurs. It can also degrade the performance of the systems and even cause system instability. In this paper, we have only considered data missing, but the study of networked control of fuzzy systems with simultaneous consideration of packet dropout and signal delay deserves further investigation.

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