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Control of Discrete Linear Repetitive Processes with $\mathcal{H}_\infty$ and ℓ_2 - ℓ_∞ Performance

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***Abstract*— This paper considers the control of discrete linear repetitive processes, which are a distinct class of two-dimensional (2-D) discrete linear systems, i.e. information propagation in two independent directions, which are of both system-theoretic and applications interest. In this paper we report new results on the design of control laws with guaranteed levels of performance. In particular, develop algorithms for the design of an $\mathcal{H}_\infty$ and ℓ_2-ℓ_∞ dynamic output feedback controller which guarantees that the resulting controlled process is stable and has prescribed disturbance attenuation performance as measured by $\mathcal{H}_\infty$ and ℓ_2-ℓ_∞ norms.**

I. Introduction

The unique characteristic of a repetitive, or multipass, process is a series of sweeps, termed passes, through a set of dynamics defined over a fixed finite duration known as the pass length. On each pass an output, termed the pass profile, is produced which acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile. This, in turn, leads to the unique control problem for these processes in that the output sequence of pass profiles generated can contain oscillations that increase in amplitude in the pass-to-pass direction.

To introduce a formal definition, let $\alpha < +\infty$ denote the pass length (assumed constant). Then in a repetitive process the pass profile $y_k(p)$, $0 \leq p \leq \alpha - 1$, generated on pass k acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile $y_{k+1}(p)$, $0 \leq p \leq \alpha - 1$, $k \geq 0$. The source of the unique control problem then appears (if at all) in the output sequence generated in the form of the collection of pass profile vectors $\{y_k\}_k$.

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Physical examples of repetitive processes include long-wall coal cutting and metal rolling operations (see, for example, the references cited in [10]). Also in recent years applications have arisen where adopting a repetitive process setting for analysis has distinct advantages over alternatives. Examples of these so-called algorithmic applications include classes of iterative learning control (ILC) schemes (see, for example, [4]) and iterative algorithms for solving nonlinear dynamic optimal control problems based on the maximum principle [9]. In the case of iterative learning control for the linear dynamics case, the stability theory for differential (and discrete) linear repetitive processes is one method which can be used to undertake a stability/convergence analysis of a powerful class of such algorithms and thereby produce vital design information concerning the trade-offs required between convergence and transient performance (see, for example, [5]).

Attempts to control these processes using standard (or 1D) systems theory/algorithms fail (except in a few very restrictive special cases) precisely because such an approach ignores their inherent 2D systems structure, i.e. information propagation occurs from pass-to-pass and along a given pass. Also the initial conditions are reset before the start of each new pass and the structure of these can be somewhat complex. For example, if they are an explicit function of points on the the previous pass profile then this alone can destroy stability. In seeking a rigorous foundation on which to develop a control theory for these processes, it is natural to attempt to exploit structural links which exist between these processes and other classes of 2D linear systems.

The case of 2D discrete linear systems recursive in the positive quadrant $(i, j) : i \geq 0,\ j \geq 0$ (where i and j denote the directions of information propagation) has been the subject of much research effort over the years using, in the main, the well known Roesser and Fornasini Marchesini state-space models. More recently, productive research has been reported on $\mathcal{H}_\infty$ and $\mathcal{H}_2$ approaches to analysis and controller design — see, for example, [12] and [1]. In general, this theory is not applicable to repetitive processes due to the fact that information propagation in one of the independent directions, along the pass, only occurs over a finite duration — the pass length. Also the boundary conditions are reset

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before the start of each new pass and, as noted above, the structure of these is crucial in terms of stability (and hence control law design).

In terms of control laws for repetitive processes, it is possible to use feedback control action on the current pass and/or feedforward control from the previous pass (or passes). The critical role of the previous pass profile dynamics means that current pass feedback control alone is not enough and it must be augmented by feedforward control. This approach has been the subject of significant research effort and results are beginning to emerge on how to undertake control law design in the presence of uncertainty. For example, [6], [8] give results on control law design in an $\mathcal{H}_\infty$ setting. The control laws used in some of this work are based on the use of feedback of the current state vector which, of course, requires that all elements of this vector can be measured to allow control law implementation. Often, however, this assumption is not valid, since some of these elements cannot be measured for various reasons.

There are two commonly used methods to deal with the control design problem when state components are not accessible. One is to design a state observer to estimate the immeasurable state components, and synthesize an observer-based control law, the other is to design a controller which is only activated by pass profile (or output) information where such controllers are usually classified as either static or dynamic respectively.

Generally speaking, dynamic output feedback is the more flexible since the controller introduces additional dynamics. Also it is known that the problem of designing such control laws can be formulated as a convex optimization problem over LMIs [7], [11] and hence can be effectively computed using numerical optimization packages. This work also shows that there are two complementary approaches to problem formulation. These are the well known variables elimination procedure and using linearizing variables transforms, respectively.

This latter approach provides a general framework to formulate synthesis problems as a convex optimization problem involving LMIs. It is based on applying specific invertible transforms of the controller parameters to achieve LMI conditions in terms of the new set of variables. When the resulted LMIs have a solution, the controller parameters can be computed by applying inverse transforms. This approach becomes less computationally effective as the number of decision variables increases and hence elimination of some decision variables can be still required, but these can only be applied to the specific structures of underlying matrix inequalities. The known results on designing a so-called $\mathcal{H}_\infty$ dynamic pass profile controller are based on variables elimination method, see [7].

Based on the above, it can be concluded that there is a clear need to investigate alternative design algorithms based on linearizing variables transform method, with the overall aim of providing a general set of control law/controller design tools for the designer to chose the one most appropriate to the the particular application under consideration. Moreover, to-date only H_∞ and H_2 (and mixed H_2/H_{zinfty} settings have been considered but there are alternatives to deal, for example, with the case when there is only partial information available on the noise corruption present. Here we develop significant new results in this direction with ℓ_2-ℓ_∞ performance included. In particular, we extend the results of [7], to the design of $\mathcal{H}_\infty$ and ℓ_2-ℓ_∞ dynamic output feedback control for discrete linear repetitive processes to guarantee stability and have $\mathcal{H}_\infty$ and an ℓ_2-ℓ_∞ disturbance attenuation respectively. Sufficient conditions for the existence of such dynamic output feedback controllers are established in the strict LMI form , which can be readily solved using standard numerical software [2].

Throughout this paper, the null matrix and the identity matrix with appropriate dimensions are denoted by 0 and I, respectively. Moreover, $M > 0$ (respectively, ≥ 0) denotes a real symmetric positive definite (respectively, semi-definite) matrix. Similarly, $M < 0$ (respectively, ≤ 0) denotes a real symmetric negative definite (respectively, semi-definite) matrix. We also require the signal space $\ell_2\{[0,\infty),[0,\infty)\}$, i.e. the space of square summable sequences on $\{[0,\infty),[0,\infty)\}$ with values in $\mathbb{R}^n$.

II. $\mathcal{H}_\infty$ AND ℓ_2-ℓ_∞ PERFORMANCE

A. Process Description and Preliminaries

As essential background for the rest of this paper, we in this section what is meant by $\mathcal{H}_\infty$ and ℓ_2-ℓ_∞ performances for discrete linear repetitive processes described by the following state-space model over $0 \leq p \leq \alpha$, $k \geq 0$,

$$\begin{aligned} x_{k+1}(p+1) &= Ax_{k+1}(p) + B_0 y_k(p) + B_1 \omega_{k+1}(p) \\ y_{k+1}(p) &= Cx_{k+1}(p) + D_0 y_k(p) + D_1 \omega_{k+1}(p) \end{aligned} \qquad (1)$$

where on pass k, $x_k(p) \in \mathbb{R}^n$ is the state vector; $y_k(p) \in \mathbb{R}^m$ is the pass profile vector; $\omega_k(p) \in \mathbb{R}^l$ is the disturbance vector which belongs to $\ell_2\{[0,\infty),[0,\infty)\}$; A, B_0, B_1, C, D_0 and D_1 are real constant matrices. The current pass state ($x_k(p)$) and pass profile ($y_k(p)$) may not be fully accessible. Hence consider the control (and estimation) in such cases based on the assumed available measured output signal vector given by

$$v_{k+1}(p) = Ex_{k+1}(p) + F_0 y_k(p) + F_1 \omega_{k+1}(p) \qquad (2)$$

where $v_k(p) \in \mathbb{R}^r$ and E, F_0 and F_1 are real constant matrices. The controlled output signal is given by

$$z_{k+1}(p) = Gx_{k+1}(p) + H_0 y_k(p) \qquad (3)$$

where $z_k(p) \in \mathbb{R}^q$, G and H_0 are real constant matrices.

To complete the process description, it is necessary to specify the boundary conditions, that is, the state initial vector on each pass and the initial pass profile (that is, on pass 0). Here we consider the case when

$$\begin{aligned} x_{k+1}(0) =& d_{k+1}, \quad \forall \ k \geq 0 \\ y_0(p) =& f(p) \end{aligned} \tag{4}$$

where $d_{k+1} \in \mathbb{R}^n$ has known constant entries and $f(p) \in \mathbb{R}^m$ is an vector whose entries are known functions of p over $[0, \alpha]$.

This state space model allows for disturbances which affect both the state and pass profile dynamics on each pass. The stability theory [10] for linear repetitive processes is based on an abstract model in a Banach space setting which includes a wide range of such processes as special cases, including those described by (1). In terms of their dynamics it is the pass-to-pass coupling (noting again the unique control problem for them) which is critical. This is of the form $y_{k+1} = L_\alpha y_k$, where $y_k \in E_\alpha$ (E_α a Banach space with norm $||\cdot||$) and L_α is a bounded linear operator mapping E_α into itself. Two concepts of stability can be defined but in general it is the stronger of these, so-called stability along the pass which is required. This holds if, and only if there exist numbers $M_\infty > 0$ and $\lambda_\infty \in (0,1)$ independent of α such that $||L_\alpha^k|| \leq M_\infty \lambda_\infty^k$, $k \geq 0$ (where $||\cdot||$ also denotes the induced operator norm) and can be interpreted as bounded-input bounded-output stability independent of the pass length. Note also that stability along the pass can be analyzed mathematically by letting $\alpha \to \infty$ and we make no further explicit reference to this fact for the remainder of this paper.

For the processes considered here, there are a wide range of stability along the pass tests which could be employed. Here, however, we use the following LMI based condition [3] since, see also below, it leads immediately to algorithms for control law design — a feature which is not present in alternatives.

Lemma 1: A discrete linear repetitive process described by (1) and (4) is stable along the pass if there $\exists$ matrix $W = \mathrm{diag}\{\mathrm{W}_1, \mathrm{W}_2\} > 0$ such that the following LMI holds

$$\begin{bmatrix} -W & M^T W \\ * & -W \end{bmatrix} < 0 \tag{5}$$

where $M = \begin{bmatrix} A & B_0 \\ C & D_0 \end{bmatrix}$.

To assess the performance of the controlled process under the $\mathcal{H}_\infty$ and ℓ_2-ℓ_∞ measures, we introduce the following definition. From this point onwards assume zero boundary conditions and make no further explicit reference to them.

Definition 1: A discrete linear repetitive process described by (1) and (4) is said to have $\mathcal{H}_\infty$ (or ℓ_2-ℓ_∞) performance level $\gamma_{2,2} > 0$ (or $\gamma_{2,\infty} > 0$), and for all nonzero $\omega_{k+1}(p) \in l_2\{[0,\infty),[0,\infty)\}$, we have

$$\|z_{k+1}(p)\|_{2,\infty} < \gamma_{2,2} \|\omega_{k+1}(p)\|_{2,\infty} \quad (\gamma_{2,2} > 0) \tag{6}$$

$\mathcal{H}_\infty$ performance, and for ℓ_2-ℓ_∞ performance, we have

$$\|z_{k+1}(p)\|_{\infty,\infty} < \gamma_{2,\infty} \|\omega_{k+1}(p)\|_{2,\infty} \quad (\gamma_{2,\infty} > 0) \tag{7}$$

where

$$\begin{aligned} \|f_k(p)\|_{2,\infty} \ &:= \ \sqrt{\sum_{k=0}^{\infty} \sum_{p=0}^{\infty} f_k^T(p) f_k(p)} \\ \|f_k(p)\|_{\infty,\infty} \ &:= \ \sqrt{\sup_{\forall \ k,p \in [0,\infty]} f_k^T(p) f_k(p)} \end{aligned}$$

and $\gamma_{2,2}$, $\gamma_{2,\infty}$ are given real positive scalars.

B. $\mathcal{H}_\infty$ Performance

Here we consider $\mathcal{H}_\infty$ performance for discrete linear repetitive process, for which the following result is developed.

Theorem 1: A discrete linear repetitive process described by (1) is stable along the pass with an $\mathcal{H}_\infty$ performance level $\gamma_{2,2} > 0$ if there exist matrices $P > 0$ and $Q > 0$ such that the following LMI holds

$$\begin{bmatrix} -P & 0 & 0 & A^T P & C^T Q & G^T \\ * & -Q & 0 & B_0^T P & D_0^T Q & H_0^T \\ * & * & -\gamma_{2,2}^2 I & B_1^T P & D_1^T Q & 0 \\ * & * & * & -P & 0 & 0 \\ * & * & * & * & -Q & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{8}$$

Proof: To establish the stability along the pass, consider the following candidate Lyapunov function

$$\begin{aligned} V(k,p) =& V_1(p,k) + V_2(k,p) \\ =& x_{k+1}^T(p) P x_{k+1}(p) + y_k^T(p) Q y_k(p) \end{aligned} \tag{9}$$

where $P > 0$ and $Q > 0$ with increment

$$\begin{aligned} \Delta V(k,p) \ &= \ \Delta V_1(p,k) + \Delta V_2(k,p) \\ &= \ x_{k+1}^T(p+1) P x_{k+1}(p+1) \\ &- \ x_{k+1}^T(p) P x_{k+1}(p) \\ &+ \ y_{k+1}^T(p) Q y_{k+1}(p) - y_k^T(p) Q y_k(p) \end{aligned} \tag{10}$$

Substitution from the plant state-space model (noting that stability along the pass is independent of the disturbance terms and hence they can be set equal to zero) following by some routine manipulation and an application of the Schur's formula then yields the LMI condition of (8).

To establish the $\mathcal{H}_\infty$ performance, the route is to consider the following performance index

$$\mathcal{J} := \|z_{k+1}(p)\|_{2,\alpha}^2 - \gamma_{2,2}^2 \|\omega_{k+1}(p)\|_{2,\alpha}^2 \tag{11}$$

It can be shown that

$$\mathcal{J} < \sum_{k=0}^{\infty}\sum_{p=0}^{\alpha} \eta_k^T(p)\Pi\eta_k(p) \qquad (12)$$

where $\eta_k(p) \triangleq \begin{bmatrix} x_{k+1}^T(p) & y_k^T(p) & \omega_{k+1}^T(p) \end{bmatrix}^T$, and

$$\Pi \triangleq \begin{bmatrix} -P & 0 & 0 \\ * & -Q & 0 \\ * & * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} A^T \\ B_0^T \\ B_1^T \end{bmatrix} P \begin{bmatrix} A^T \\ B_0^T \\ B_1^T \end{bmatrix}^T + \begin{bmatrix} C^T \\ D_0^T \\ D_1^T \end{bmatrix} Q \begin{bmatrix} C^T \\ D_0^T \\ D_1^T \end{bmatrix}^T + \begin{bmatrix} G^T \\ H_0^T \\ 0 \end{bmatrix}\begin{bmatrix} G^T \\ H_0^T \\ 0 \end{bmatrix}^T < 0.$$

Further transformation complete the proof. ■

C. ℓ_2-ℓ_∞ Performance

Here we consider ℓ_2-ℓ_∞ performance for discrete linear repetitive process, for which the following result is developed.

Theorem 2: A discrete linear repetitive process described by (1) is stable along the pass with an ℓ_2-ℓ_∞ performance level $\gamma_{2,\infty} > 0$ if there exist matrices $P > 0$ and $Q > 0$ such that the following LMIs hold

$$\begin{bmatrix} -P & 0 & 0 & A^TP & C^TQ \\ * & -Q & 0 & B_0^TP & D_0^TQ \\ * & * & -I & B_1^TP & D_1^TQ \\ * & * & * & -P & 0 \\ * & * & * & * & -Q \end{bmatrix} < 0 \qquad (13)$$

$$\begin{bmatrix} P & 0 & G^T \\ * & Q & H_0^T \\ * & * & \gamma_{2,\infty}^2 I \end{bmatrix} > 0 \qquad (14)$$

Proof: Stability along the pass is established as in the previous result. The performance level is established by considering the performance index

$$\mathcal{I} = V(k,p) - \sum_{s=0}^{\infty}\sum_{\beta=0}^{\infty} \omega_{s+1}^T(\beta)\omega_{s+1}(\beta) \qquad (15)$$

■

Note 1: Repetitive processes are defined over the finite pass length α, and in practice the process will only complete a finite number of passes, say, N. Hence the corresponding cost function in this last result should be evaluated as

$$\mathcal{I} = V(k,p) - \sum_{s=0}^{k-1}\sum_{\beta=0}^{p-1} \omega_{s+1}^T(\beta)\omega_{s+1}(\beta) \qquad (16)$$

However, it is routine to argue that the signals involved can be extended from $[0,\alpha]$ to the infinite interval in such a way that projection of the infinite interval solution onto the finite interval is possible. Likewise from the infinite set to $[0,N]$, and hence we will work with (15).

III. Dynamic Output Feedback Control

A. Problem Formulation

The processes considered here are described by the model of (1) augmented by control input terms, i.e.

$$\begin{aligned} x_{k+1}(p+1) &= Ax_{k+1}(p) + Bu_{k+1}(p) \\ &\quad + B_0y_k(p) + B_1\omega_{k+1}(p) \\ y_{k+1}(p) &= Cx_{k+1}(p) + Du_{k+1}(p) \\ &\quad + D_0y_k(p) + D_1\omega_{k+1}(p) \end{aligned} \qquad (17)$$

where on pass k, $u_k(p) \in \mathbb{R}^s$ is the control input vector and B and D are real constant matrices.

Here, we are interested in designing a full-order dynamic output feedback controller of general structure described by

$$\begin{aligned} \varphi_{k+1}(p+1) &= A_c\varphi_{k+1}(p) + B_{0c}\phi_k(p) + B_cz_{k+1}(p) \\ \phi_{k+1}(p) &= C_c\varphi_{k+1}(p) + D_{0c}\phi_k(p) + D_cz_{k+1}(p) \\ u_{k+1}(p) &= G_c\varphi_{k+1}(p) + H_{0c}\phi_k(p) + H_cz_{k+1}(p) \end{aligned} \qquad (18)$$

where on pass k, $\varphi_k(p) \in \mathbb{R}^n$ is the state vector of controller, $\phi_k(p) \in \mathbb{R}^m$ is the pass profile vector, and A_c, B_{0c}, B_c, C_c, D_{0c}, D_c, G_c, H_{0c} and H_c are appropriately dimensioned constant matrices to be determined. Now, augmenting the model of (17) to include the states of dynamic output feedback controller of (18) and considering (2)–(3), we obtain the following closed-loop process

$$\begin{aligned} \xi_{k+1}(p+1) &= \tilde{A}\xi_{k+1}(p) + \tilde{B}_0\zeta_k(p) + \tilde{B}_1\omega_{k+1}(p) \\ \zeta_{k+1}(p) &= \tilde{C}\xi_{k+1}(p) + \tilde{D}_0\zeta_k(p) + \tilde{D}_1\omega_{k+1}(p) \\ v_{k+1}(p) &= \tilde{G}\xi_{k+1}(p) + \tilde{H}_0\zeta_k(p) \end{aligned} \qquad (19)$$

where $\xi_{k+1}(p) \triangleq \begin{bmatrix} x_{k+1}^T(p) & \varphi_{k+1}^T(p) \end{bmatrix}^T$ and

$$\tilde{A} \triangleq \begin{bmatrix} A+BH_cE & BG_c \\ B_cE & A_c \end{bmatrix}, \tilde{C} \triangleq \begin{bmatrix} C+DH_cE & DG_c \\ D_cE & C_c \end{bmatrix}$$

$$\tilde{B}_0 \triangleq \begin{bmatrix} B_0+BH_cF_0 & BH_{0c} \\ B_cF_0 & B_{0c} \end{bmatrix}, \tilde{B}_1 \triangleq \begin{bmatrix} B_1+BH_cF_1 \\ B_cF_1 \end{bmatrix}$$

$$\tilde{D}_0 \triangleq \begin{bmatrix} D_0+DH_cF_0 & DH_{0c} \\ D_cF_0 & D_{0c} \end{bmatrix}, \tilde{D}_1 \triangleq \begin{bmatrix} D_1+DH_cF_1 \\ D_cF_1 \end{bmatrix}$$

$$\tilde{G} \triangleq \begin{bmatrix} G & 0 \end{bmatrix}, \tilde{H}_0 \triangleq \begin{bmatrix} H_0 & 0 \end{bmatrix},$$

$$\zeta_k(p) \triangleq \begin{bmatrix} y_k^T(p) & \phi_k^T(p) \end{bmatrix}^T$$

The problem considered in this section now is: design a controller of the form (18) with either $\mathcal{H}_\infty$ or ℓ_2-ℓ_∞) performance subject to the following two requirements:

1) The controlled process is stable along the pass;
2) The controlled process has disturbance attenuation level $\gamma_{2,2}$ in an $\mathcal{H}_\infty$ (or level $\gamma_{2,\infty}$ in an ℓ_2-ℓ_∞) sense. In particular, for all nonzero $\omega_{k+1}(p) \in \ell_2\{[0,\infty),[0,\infty)\}$, (6) holds for the $\mathcal{H}_\infty$ case and (7) for ℓ_2-ℓ_∞.

B. $\mathcal{H}_\infty$ Dynamic Output Feedback Control Design

The following result for stability along the pass of the controlled process is obtained from interpreting Theorem 1 in terms of (19).

Theorem 3: Consider the discrete linear repetitive processes in (1), and let $\gamma_{2,2} > 0$ be a prescribed scalar. There exists a full-order dynamic output feedback controller in the form of (18) such that the closed-loop process (19) is stable along the pass and (6) is satisfied if there exist matrices $\mathcal{P} > 0$, $\mathcal{R} > 0$, $\mathcal{Q} > 0$, $\mathcal{S} > 0$, $\mathcal{A}_c$, $\mathcal{B}_{0c}$, $\mathcal{B}_c$, $\mathcal{C}_c$, $\mathcal{D}_{0c}$, $\mathcal{D}_c$, $\mathcal{G}_c$, $\mathcal{H}_{0c}$ and $\mathcal{H}_c$ such that the following LMI holds:

$$\begin{bmatrix} -\mathcal{P} & -I & 0 & 0 & 0 & (\mathcal{P}A+\mathcal{B}_cE)^T & (A+B\mathcal{H}_cE)^T & (\mathcal{Q}C+\mathcal{D}_cE)^T & \Upsilon_{19} & G^T \\ * & -\mathcal{R} & 0 & 0 & 0 & \mathcal{A}_c^T & (A\mathcal{R}+B\mathcal{G}_c)^T & \mathcal{C}_c^T & \Upsilon_{29} & \mathcal{R}G^T \\ * & * & -\mathcal{Q} & -I & 0 & (\mathcal{P}B_0+\mathcal{B}_cF_0)^T & (B_0+B\mathcal{H}_cF_0)^T & (\mathcal{Q}D_0+\mathcal{D}_cF_0)^T & \Upsilon_{39} & H_0^T \\ * & * & * & -\mathcal{S} & 0 & \mathcal{B}_{0c}^T & (B_0\mathcal{S}+B\mathcal{H}_{0c})^T & \mathcal{D}_{0c}^T & \Upsilon_{49} & \mathcal{S}H_0^T \\ * & * & * & * & -\gamma_{2,2}^2 I & (\mathcal{P}B_1+\mathcal{B}_cF_1)^T & (B_1+B\mathcal{H}_cF_1)^T & (\mathcal{Q}D_1+\mathcal{D}_cF_1)^T & \Upsilon_{59} & 0 \\ * & * & * & * & * & -\mathcal{P} & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & -\mathcal{R} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\mathcal{Q} & -I & 0 \\ * & * & * & * & * & * & * & * & -\mathcal{S} & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \quad (20)$$

where

$$\Upsilon_{19} = (C+D\mathcal{H}_cE)^T, \ \Upsilon_{29} = (C\mathcal{R}+D\mathcal{G}_c)^T,$$
$$\Upsilon_{39} = (D_0+D\mathcal{H}_cF_0)^T, \ \Upsilon_{49} = (D_0\mathcal{S}+D\mathcal{H}_{0c})^T,$$
$$\Upsilon_{59} = (D_1+D\mathcal{H}_cF_1)^T \quad (21)$$

Moreover, a desired $\mathcal{H}_\infty$ dynamic output feedback controller can be found by solving the following equations:

$$\begin{aligned} \mathcal{H}_c &\triangleq H_c, \\ \mathcal{H}_{0c} &\triangleq H_cF_0\mathcal{S} + H_{0c}S_{12}^T, \\ \mathcal{G}_c &\triangleq H_cE\mathcal{R} + G_cR_{12}^T, \\ \mathcal{D}_c &\triangleq \mathcal{Q}DH_c + Q_{12}D_c, \\ \mathcal{B}_c &\triangleq \mathcal{P}BH_c + P_{12}B_c, \\ \mathcal{D}_{0c} &\triangleq \mathcal{Q}(D_0 + DH_cF_0)\mathcal{S} + Q_{12}D_cF_0\mathcal{S} \\ &+ \mathcal{Q}DH_{0c}S_{12}^T + Q_{12}D_{0c}S_{12}^T, \\ \mathcal{C}_c &\triangleq \mathcal{Q}(C + DH_cE)\mathcal{R} + Q_{12}D_cE\mathcal{R} \\ &+ \mathcal{Q}DG_cR_{12}^T + Q_{12}C_cR_{12}^T, \\ \mathcal{B}_{0c} &\triangleq \mathcal{P}(B_0 + BH_cF_0)\mathcal{S} + P_{12}B_cF_0\mathcal{S} \\ &+ \mathcal{P}BH_{0c}S_{12}^T + P_{12}B_{0c}S_{12}^T, \\ \mathcal{A}_c &\triangleq \mathcal{P}(A + BH_cE)\mathcal{R} + P_{12}B_cE\mathcal{R} \\ &+ \mathcal{P}BG_cR_{12}^T + P_{12}A_cR_{12}^T \end{aligned} \quad (22)$$

Proof: The proof is based on the stability along the pass with an $\mathcal{H}_\infty$ performance for the closed-loop process. ■

C. ℓ_2-ℓ_∞ Dynamic Output Feedback Control

In a similar manner to the $\mathcal{H}_\infty$ case, the following result can be established using, in effect, the arguments required in the proof of Theorem 2.

Theorem 4: Consider the discrete linear repetitive processes in (1), and let $\gamma_{2,\infty} > 0$ be a prescribed scalar. There exists a full-order dynamic output feedback controller in the form of (18) such that the closed-loop process (19) is stable along the pass and (7) is satisfied if there exist matrices $\mathcal{P} > 0$, $\mathcal{R} > 0$, $\mathcal{Q} > 0$, $\mathcal{S} > 0$, $\mathcal{A}_c$, $\mathcal{B}_{0c}$, $\mathcal{B}_c$, $\mathcal{C}_c$, $\mathcal{D}_{0c}$, $\mathcal{D}_c$, $\mathcal{G}_c$, $\mathcal{H}_{0c}$ and $\mathcal{H}_c$ such that the following LMIs hold:

$$\begin{bmatrix} -\mathcal{P} & -I & 0 & 0 & 0 & (\mathcal{P}A+\mathcal{B}_cE)^T & (A+B\mathcal{H}_cE)^T & (\mathcal{Q}C+\mathcal{D}_cE)^T & \Upsilon_{19} \\ * & -\mathcal{R} & 0 & 0 & 0 & \mathcal{A}_c^T & (A\mathcal{R}+B\mathcal{G}_c)^T & \mathcal{C}_c^T & \Upsilon_{29} \\ * & * & -\mathcal{Q} & -I & 0 & (\mathcal{P}B_0+\mathcal{B}_cF_0)^T & (B_0+B\mathcal{H}_cF_0)^T & (\mathcal{Q}D_0+\mathcal{D}_cF_0)^T & \Upsilon_{39} \\ * & * & * & -\mathcal{S} & 0 & \mathcal{B}_{0c}^T & (B_0\mathcal{S}+B\mathcal{H}_{0c})^T & \mathcal{D}_{0c}^T & \Upsilon_{49} \\ * & * & * & * & -I & (\mathcal{P}B_1+\mathcal{B}_cF_1)^T & (B_1+B\mathcal{H}_cF_1)^T & (\mathcal{Q}D_1+\mathcal{D}_cF_1)^T & \Upsilon_{59} \\ * & * & * & * & * & -\mathcal{P} & -I & 0 & 0 \\ * & * & * & * & * & * & -\mathcal{R} & 0 & 0 \\ * & * & * & * & * & * & * & -\mathcal{Q} & -I \\ * & * & * & * & * & * & * & * & -\mathcal{S} \end{bmatrix} < 0 \quad (23)$$

$$\begin{bmatrix} -\mathcal{P} & -I & 0 & 0 & G^T \\ * & -\mathcal{R} & 0 & 0 & \mathcal{R}G^T \\ * & * & -\mathcal{Q} & -I & H_0^T \\ * & * & * & -\mathcal{S} & \mathcal{S}H_0^T \\ * & * & * & * & -\gamma_{2,\infty}^2 I \end{bmatrix} < 0 \quad (24)$$

where Υ_{19}, Υ_{29}, Υ_{39}, Υ_{49} and Υ_{59} have been defined in (21). Moreover, a desired ℓ_2-ℓ_∞ dynamic output feedback controller can be computed from (22).

Proof. The proof again is based on the exploiting the requirement that the closed-loop process in (19) is stable along the pass with an ℓ_2-ℓ_∞ performance level $\gamma_{2,\infty} > 0$.

IV. Conclusion

In this paper, the problems of robust $\mathcal{H}_\infty$ and ℓ_2- ℓ_∞ based controllers for discrete linear repetitive processes have been investigated. Sufficient conditions have first been developed for the existence of such controllers to guarantee stability along the pass and prescribed $\mathcal{H}_\infty$ and ℓ_2-ℓ_∞ performance.

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