On the validity and identification of long run restrictions for a cointegrated system

by

Sau-Him Paul Lau

Australian National University and Hong Kong Baptist University

October 1999

Abstract:

The identification approach suggested in Blanchard and Quah (1989) and King et al. (1991) makes use of the long run properties of structural disturbances. This paper provides economic underpinning for the use of long run identifying restrictions by showing formally its validity for the class of exogenous growth models under certain conditions. This paper also obtains the minimum number of restrictions, in addition to the long run restrictions, required for the identification of structural disturbances in a cointegrated system.

JEL Classification Numbers: C32, O40

Keywords: long run identifying restrictions; economic underpinning; exogenous growth models

Correspondence to:

S.-H. P. Lau, Department of Economics, Hong Kong Baptist University, Kowloon Tong, Hong Kong.

Phone:	(+852) 2339 5200
Fax:	(+852) 2339 5580
E-mail:	shlau@hkbu.edu.hk

I am grateful to a referee for helpful comments.

1. Introduction

Following Sims' (1980) seminal work, many time series econometric studies are conducted in a vector autoregression (VAR) framework. In these studies, it is well known that identification of the structural disturbances is an important issue if the analysis is used for more than forecasting purposes. One main identification scheme, suggested by Sims, is to impose a recursive ordering on the contemporaneous effects of the disturbances on the variables; and different recursive orderings are usually used to examine the robustness of the estimation results. On the other hand, Bernanke (1986) and Blanchard and Watson (1986) propose a structural approach to identification by using the restrictions suggested by economic theories. In particular, the long run identifying restrictions suggested in Blanchard and Quah (1989) and King et al. (1991) have been widely applied in recent years.

The approach proposed in Blanchard and Quah (1989) and King et al. (1991) makes use of the long run properties of structural disturbances. With the assumption that some structural disturbances produce permanent effects on observed variables and some lead only to temporary effects, the zero long run restrictions of the latter group of disturbances can be used for identification. These identifying restrictions are particularly useful for the empirically relevant cointegrated system in which a vector of variables is integrated of order one but some linear combinations of the components are integrated of order zero.[1,2] For a system of n variables with r $(1 \leq r \leq n-1)$ cointegrating vectors, Stock and Watson (1988) suggest a way to decompose the (reduced-form) system as a linear combination of $n-r$ common stochastic trends and other stationary components. Perhaps because of this common trends interpretation and under

---

[1]A scalar time series is said to be integrated of order d, I(d), if it must be differenced d times before it is stationary. A vector time series is said to be I(d) if at least one element must be differenced d times before it is stationary.

[2]While the long run identifying restrictions can be applied to an I(1) but non-cointegrated system such as in Lastrapes and Selgin (1994), these assumptions have been applied mainly to a cointegrated system such as in Blanchard and Quah (1989), King et al. (1991) or Gali (1992). Note that in Blanchard and Quah (1989), the variables are cointegrated in a trivial way since one variable is I(0) and the other is I(1).

a commonly used convention in empirical studies that there are as many underlying structural disturbances as the number of variables, some researchers further assume that there are r structural disturbances producing temporary effects and $n-r$ disturbances producing permanent effects in this cointegrated system, and use the approach of zero long run restrictions for identification.

To complement the statistical perspective based on integration and cointegration pattern of the data, this paper provides economic underpinning for the use of long run identifying restrictions by showing that this identification scheme is valid for a particular class of economic models. Specifically, it is shown that the long run identifying restrictions are valid for a cointegrated system generated by an exogenous growth model (such as the neoclassical growth model in Solow, 1956; Cass, 1965) in which *sustained growth is caused only by exogenous factors such as technological progress*. This result contrasts sharply with that of the class of endogenous growth models in which sustained growth is explained 'endogenously' in terms of the actions of economic agents (and, therefore, an exogenous growth-generating element is absent in the economic environment described by the model). If a cointegrated system is generated by an endogenous growth model, then no structural disturbance produces zero long run effects on observed variables (unless by pure coincidence); see Lau (1997, 1999). Therefore, *the long run identifying restrictions are not valid for the class of endogenous growth models*.

In light of the difference between endogenous and exogenous growth models in terms of the long run effects of structural disturbances, examining the validity of long run identifying restrictions for various behavioral models is meaningful. The intuition of the different long run effects of structural disturbances between these two classes of growth models can be understood with the help of theoretical results of the cointegration literature. In a system of n variables with r cointegrating vectors, there are only $n-r$ independent columns in the long run impact matrix for the vector moving-average (VMA) representation;[3] see Engle and Granger (1987) or Stock and Watson (1988). A rank of $n-r$ implies that at most r columns of the matrix are zero, or equivalently, at most r structural disturbances produce zero long run effect. This paper shows that if the above cointegrated system is generated by an exogenous growth model, then exactly r columns of the long run impact matrix are zero.[4]

For a cointegrated system generated by an exogenous growth model, this paper also obtains the minimum number of restrictions, in addition to the long run restrictions, required for the identification of structural disturbances. It is shown that in a cointegrated system of three or more variables, other conditions besides the long run restrictions must be imposed in order to achieve identification. For example, King et al. (1991) use conventional recursive ordering for a subset of disturbances, besides the long run restrictions. Similarly, Gali (1992) uses long run restrictions as well as short run restrictions corresponding to theoretical considerations.

The remaining sections of this paper are organized as follows. Section 2 specifies the behavioral relationships among the variables generated by an exogenous growth model in a dynamic simultaneous-equation system. Section 3 shows that the use of long run identifying restrictions is valid for this class of growth models. Section 4 considers the order condition for identification and gives the number of restrictions, besides the long run restrictions, required for the identification of the underlying disturbances in a cointegrated system. The last section provides conclusion.

---

[3]Equivalently, there are $n-r$ independent rows in the long run impact matrix for the (cointegrated) VAR representation, which is the dual of the VMA representation. See Banerjee et al. (1993, p. 151) for a discussion about the duality.

[4]On the other hand, if this cointegrated system is generated by an endogenous growth model, then there are still $n-r$ independent columns in the long run impact matrix but there are no columns of zero (long run effect); see Lau (1999). As a result, the long run identifying restrictions are not valid for an endogenous growth model.

2. A linear structural dynamic simultaneous-equation system

The variables examined in this paper are assumed to be I(0) or I(1) individually; fractional integration is not considered. Most macroeconomic time series, except possibly money supply and price level, appear to satisfy this assumption. As already mentioned, the long run identifying restrictions are mainly applied to a system of I(1) variables. Moreover, this assumption is consistent with a major stylized fact about economic growth that important variables such as real output per capita are growing at fairly constant rates for an extended period of time; see King et al. (1991) for example.

It is also assumed that the evolution of the variables can be captured in a linear framework, which is used in many empirical studies and is adequate for the analysis of growth issues. In economic studies, the use of a linear dynamic system is usually based on the log-linear approximation of the Euler equations around the steady state growth path (as in King et al., 1991) or the solution to a problem with (approximately) quadratic objective function and linear constraints (as in McGrattan, 1994).

Let $X_t = (X_{1t}, \ldots, X_{nt})'$ be a vector of $n\,(n \geq 2)$ variables of interest. For a meaningful growth model, at least one of these variables exhibits sustained growth; moreover, the growing variables are assumed to be expressed in logs. The behavioral relationships among the observed variables generated by an exogenous growth model are assumed to be represented by the following system of linearly independent equations:

$$\Phi(B)X_t = \mu + e_t, \tag{1}$$

where B is the backshift operator $BX_t = X_{t-1}$, $\mu = (\mu_1, \ldots, \mu_n)'$ and $e_t = (e_{1t}, \ldots, e_{nt})'$. The (k,i)-th element of the polynomial matrix $\Phi(B)$ is $\sum_{j=0}^{b} \phi_{ki}^{j} B^j$ where $b \geq 1$ is the number of lag terms included. Parameters ϕ_{ki}^{j} (with ϕ_{kk}^{0} normalized to be 1) and μ_k correspond to the propagation mechanism of a particular growth model. On the other hand, the vector of random variables e_t

represents the external impulses or forcing variables to the system.

Since the variables of interest are either I(0) or I(1), it is natural (and turns out to be necessary too) to assume that the unobserved impulses are I(0) or I(1). Assuming further that each forcing variable is a stationary autoregressive moving-average process either in level or in first difference, then the forcing variables e_t can be collectively represented by:

$$diag[(1-B)I_f, I_{n-f}]\, W(B)\, e_t = \gamma + V(B)\epsilon_t, \tag{2}$$

where (a) $diag[(1-B)I_f, I_{n-f}]$ is a n-dimensional diagonal matrix with all the first f $(0 \leq f \leq n)$ elements being $(1-B)$ and all the last $n-f$ elements being 1, (b) $W(B)$ and $V(B)$ are diagonal polynomial matrices of dimension n such that each diagonal element of $W(B)$ or $V(B)$ has all roots outside the unit circle, (c) $\gamma = (\gamma_1, \ldots, \gamma_n)'$, and (d) ϵ_t is a vector of structural disturbances such that its components are serially and mutually uncorrelated. That is, the equations are ordered such that the first f external impulses are I(1) and the others are I(0). The short run dynamics are not restricted in the above specification, as economic theories in general and growth models in particular do not have much to offer on these issues.

It should be emphasized that the above system of equations is *structural*, capturing the dynamic relationships of the variables according to a particular exogenous growth model. Equation (1) describes how the exogenous impulses e_t affect the observed variable X_t through the propagation mechanism of the model, and equation (2) expresses the impulses in terms of the structural disturbances. Unlike the *reduced form* system (which is obtained by pre-multiplying equation (1) by the inverse of Φ_0, the leading structural VAR parameter matrix) specified in a statistical framework which sometimes suppresses the constant or trend terms for simplicity (as in Engle and Granger, 1987, p. 254), an important point of the above specification is that the constant terms are explicitly included and all the variables involved are without transformation. This complete specification is crucial to the analysis of this paper which focuses on the interaction between a sustained growth requirement and the parameters of the growth model.

The above specification of (1) and (2) is commonly used in analyzing a system of difference stationary variables, which is the focus of this paper.[5] A similar but simpler specification, with first order dynamics only and with the constant terms suppressed, has been used in Canova et al. (1994).

3. Validity of the use of long run restrictions for the class of exogenous growth models

In an exogenous growth model (such as the constant saving rate version in Solow (1956) or the optimizing version in Cass (1965)), sustained growth is generated only by exogenous factors. In the (log-) linear and stochastic framework used in this paper, the exogenous growth-generating element is represented by the vector γ.[6]

To incorporate the possibility of unit root cancellation, define

$$|\Phi(B)| = (1-B)^m g(B), \tag{3}$$

$$adj[\Phi(B)] = (1-B)^h Q(B), \tag{4}$$

where $|\Phi(B)|$ and $adj[\Phi(B)]$ are respectively the determinant and adjoint of the polynomial matrix $\Phi(B)$, m is a positive integer or zero, $g(B)$ contains only roots strictly outside the unit circle (since only integrated processes are considered), and h is a non-negative integer such that there is no common factor of $(1-B)$ for the non-zero elements of the polynomial matrix $Q(B)$. It can be shown, for example in Lemma 1 of Lau (1997) that, parameters m and h are related by:

---

[5]However, trend stationary variables cannot be addressed in this framework, but can be addressed in a slightly different specification of $\Phi(B)(X_t - \gamma t) = \mu + e_t$ and $diag[(1-B)I_f, I_{n-f}]\, W(B)\, e_t = V(B)\,\epsilon_t$ used in Lau (1997), which is motivated by the format generated in theoretical growth models.

[6]Exact conditions for exogenous steady state growth will be given in Proposition 1 below. On the other hand, steady state growth is generated by the propagation mechanism, instead of the exogenous impulses, for the class of endogenous growth models.

$$m(n-1) \geq hn. \tag{5}$$

Solving simultaneously the system of equations in (1) and using (3) and (4) gives the following univariate time series representation of the variables in vector form:

$$(1-B)^{m-h} g(B) X_t = Q(B)(\mu + e_t),$$

which, together with (2), may be rewritten as:

$$(1-B)X_t = (1-B)^{1-(m-h)} g^{-1}(1) Q(1)\mu + (1-B)^{-(m-h)} g^{-1}(1) Q(1)\lambda + (1-B)^{-(m-h)} g^{-1}(B) Q(B) R(B) V(B) \epsilon_t, \tag{6}$$

where

$$R(B) = diag[I_f, (1-B)I_{n-f}] W^{-1}(B); \quad \lambda = R(1)\gamma = R(1) \begin{bmatrix} \bar{\gamma} \\ \Upsilon \end{bmatrix}, \tag{7}$$

and $\bar{\gamma} = (\gamma_1, \ldots, \gamma_f)'$ and $\Upsilon = (\gamma_{f+1}, \ldots, \gamma_n)'$. Equation (6) gives the first difference of the vector of variables, and a particular component is interpreted as the growth rate of the variable if it is expressed in log. Proposition 1 gives the conditions for steady state growth to be generated exogenously in a model represented by equations (1) and (2). Prior to that, two assumptions about the propagation and impulse parameters are given.

Assumption [A1]: If $\lambda \neq 0$, then $Q(1)\lambda \neq 0$.

Assumption [A2]: If $\mu \neq 0$, then $Q(1)\mu \neq 0$.

Since the coefficients in μ and $\Phi(1)$ represent propagation (such as preference and technology) parameters and the coefficients in λ represents impulse parameters, these two assumptions exclude fortuitous cancellations which have no apparent economic significance.

Proposition 1. For a growth model represented by (1) and (2), positive steady state growth is generated by exogenous factors when the following conditions are satisfied:

$$1 \leq f \leq n; \;\; \bar{\gamma} \neq 0; \tag{8}$$

$$m = h = 0. \tag{9}$$

Proof. It is observed from equations (6) and (7) that if either (a) $f=0$ or (b) $1 \leq f \leq n$ and $\bar{\gamma}=0$, then $\lambda=0$, and therefore positive steady state growth is achieved when $m-h=1$.[7] However, in this case, sustained growth is not generated by the exogenous forcing variables, but generated endogenously by the propagation mechanism of the model. On the other hand, when condition (8) is satisfied, then λ is non-zero and thus $Q(1)\lambda$ is non-zero according to assumption [A1]. In this case, it can be observed from equation (6) that positive steady state growth is achieved when $m-h=0$. Substituting $m-h=0$ into (5) and using $m \geq 0$ gives (9).[8] Under conditions (8) and (9), the average growth rate is $g^{-1}(1)Q(1)\lambda$. □

The intuition of Proposition 1 is as follows. As mentioned above, equation (1) specifies the internal dynamics of the model and equation (2) describes the external impulses. Under the maintained assumptions that the external impulses are I(0) or I(1), it is clear from (2) that there is no deterministic trend in the external impulses when either (a) $f=0$ or (b) $1 \leq f \leq n$ and $\bar{\gamma}=0$. In the former case, all impulses are I(0); in the latter, the I(1) impulses are drift-free. Therefore,

---

[7]When $m-h=1$, the (vector of) average growth rate is $g^{-1}Q(1)\mu$, which is non-zero under the assumption that $\mu \neq 0$ and assumption [A2].

[8]The absence of root(s) of $|\Phi(B)| = 0$ inside the unit circle can similarly be shown, so long as the assumption of steady state growth is maintained. A sketch of the proof is as follows. Add $(1-\rho B)^q$ to the right-hand side of (3) and $(1-\rho B)^w$ to that of (4) where $|\rho| > 1$, $q \geq 0$ and $w \geq 0$. It can be shown that (a) $q(n-1) \geq wn$, and (b) steady state growth implies that $q \leq w$. Therefore, $q = w = 0$.

condition (8), which means that at least one external impulse is I(1) with drift, is necessary in order for sustained growth to be generated exogenously. Proposition 1 further shows that in order to generate non-explosive growth, some feature about the propagation mechanism is also required. This is given by condition (9).

A consequence of Proposition 1 is that the long run identifying restrictions are valid for the class of exogenous growth models. This is given by:

Proposition 2. For a system of n difference stationary (with drift) variables generated by an exogenous steady state growth model, the use of long run restrictions suggested in Blanchard and Quah (1989) and King et al. (1991) is valid when

$$1 \leq f \leq n-1. \tag{10}$$

In this case, the vector of variables X_t is cointegrated with cointegrating rank $n-f$ which is at least one and at most $n-1$.

Proof. Pre-multiplying the system of equations (1) by $V^{-1}(B) diag[(1-B)I_f, I_{n-f}] W(B)$ leads to $S(B) X_t = V^{-1}(1)\left[diag[0_f, I_{n-f}] W(1)\mu + \gamma\right] + \epsilon_t$, where $S(B) = V^{-1}(B) diag[(1-B)I_f, I_{n-f}] W(B)\Phi(B)$. The vector error correction representation of this system is given by:

$$\Pi(B)(1-B)X_t + S(1)X_{t-1} = V^{-1}(1)\left[diag[0_f, I_{n-f}] W(1)\mu + \gamma\right] + \epsilon_t, \tag{11}$$

where $S(1)$ is the long run impact matrix for the structural (cointegrated) VAR representation of X_t, and $\Pi(B)$ is a function of $S(B)$.

It is well known that the multivariate time series properties of X_t depend on $rank[S(1)]$; see Banerjee et al. (1993) or Hamilton (1994) for example. First of all, equation (3) and

Proposition 1 imply that $\Phi(1)$ is of full rank.[9] Therefore,

$$rank[S(1)] = rank\left[V^{-1}(1)\,diag[0_f, I_{n-f}]\,W(1)\,\Phi(1)\right] = rank\left[diag[0_f, I_{n-f}]\right] = n-f, \tag{12}$$

where the second equality holds since $V(1)$, $W(1)$ and $\Phi(1)$ are of full rank n, and the third equality is obvious.

When there is at least one I(1) with drift external impulse in an exogenous growth model, $S(1)$ is of reduced rank and the vector X_t is difference stationary. Furthermore, X_t is cointegrated with $n-f$ cointegrating vectors when condition (10) is satisfied, but is not cointegrated when $f=n$. In the former case, some (f) exogenous impulses are I(1) and the remaining $(n-f)$ impulses are I(0). In an exogenous growth model, condition (9) is satisfied and $|\Phi(1)|$ is of full rank; thus, I(0) impulses produce temporary effects on observed variables and I(1) impulses produce permanent effects. In this case, the long run identifying restrictions are valid. On the other hand, these identifying restrictions are not useful when all n exogenous impulses are I(1). □

To summarize, Proposition 1 shows that if steady state growth is generated by exogenous factors only, then the factor $(1-B)$ is absent in the autoregressive (propagation) polynomial $|\Phi(B)|$. Under this condition, Proposition 2 shows that I(1) external impulses give rise to difference stationarity and cointegration. The idea that I(1) forcing variables generate integration and cointegration in observed variables of exogenous growth real business cycle models has been mentioned in King et al. (1991) and Canova et al. (1994). The above analysis not only formalizes this idea, but also shows that a condition for the 'I(1) in, I(1) out' result is the full rank of $\Phi(1)$, and this condition is satisfied for exogenous growth models but not for endogenous growth

---

[9]Note that condition (9), instead of just $m-h=0$, is required to have $rank[\Phi(1)]=n$.

models. The source of this difference between endogenous and exogenous growth models is that sustained growth is generated by external impulses in exogenous growth models, but by the propagation mechanism in endogenous growth models. In order to generate steady state growth in the absence of exogenous growth-generating element, there has to be one factor of $(1-B)$ in $|\Phi(B)|$. As a result, all I(0) external impulses produce permanent effects in an endogenous growth model and therefore the use of long run restrictions suggested in Blanchard and Quah (1989) and King et al. (1991) is not valid; see also footnote 4. On the other hand, according to Proposition 2, the Blanchard-Quah approach is valid for a system of variables generated by an exogenous growth model when the number of independent I(1) impulses is at least one and is smaller than the number of variables. The zero long run effects of the remaining I(0) impulses impose useful restrictions.

4. Order condition for identification of long run restrictions in a cointegrated system

The analysis in section 3 suggests that for a cointegrated system generated by an exogenous growth model, conditions (8) to (10) are satisfied and it is meaningful to classify the structural disturbances into those producing permanent effects (i.e. the first f components of ϵ_t) and those producing temporary effects respectively. Under these conditions, this section examines the order condition under which a cointegrated VAR system is identified by long run restrictions (at least partially).[10]

The order condition for identification of a VAR system is that the number of restrictions implied by a particular identification scheme is at least equal to $n(n-1)/2$. This is because an

[10]While the order condition for identification of general VAR models are well-known in the literature (see, Hamilton (1994) and Robertson and Wickens (1994), for example), this paper focuses on a cointegrated system specifically and obtains sharper results. Robertson and Wickens (1994) mentioned how the cointegrating rank could affect the identification of the VAR system. This paper further points out the difference between $n-f$ *columns of zero elements* and *a reduced rank of* f in the long run impact matrix of the underlying structural model of n variables. The first condition is related to whether the long run identifying restrictions are valid, and the second condition is related to the cointegrating rank of the system.

identification scheme needs to recover the n^2 parameters in the leading VAR parameter matrix (or equivalently, the parameters in the leading VMA parameter matrix) of the structural model but there are only $n(n+1)/2$ estimated parameters in the symmetric variance-covariance matrix of the innovations (i.e. the disturbances to the reduced form model); see Blanchard and Quah (1989, p. 657) and Hamilton (1994, p. 332).

In a cointegrated system of n variables with f $(1 \leq f \leq n-1)$ independent structural disturbances that produce permanent effects as described in section 3, the relationship $S(1) = V^{-1}(1)\, diag[0_f, I_{n-f}]\, W(1)\Phi(1)$ implies that the first f rows of $S(1)$ are zero. Since the rank of $S(1)$ is $(n-f)$, there are only $(n-f)$ independent columns and hence the number of independent zero restrictions is

$$f(n-f). \tag{13}$$

A more intuitive explanation of the number of independent long run restrictions is given by looking at the VMA representation. In this cointegrated system, the long run effect of each of the last $n-f$ structural disturbances on the level of each of the n variables is zero. Therefore, $(n-f)$ columns of the long run impact matrix for the VMA representation are zero. However, the presence of $(n-f)$ cointegrating vectors means that there are only f independent rows for the long run impact matrix for the VMA representation. Therefore, there are only $f(n-f)$ independent restrictions of zero long run effect in this cointegrated system.

Under the conditions that a system is cointegrated and the long run identification approach is valid, the following Proposition gives the number of additional restrictions required for identifying the underlying disturbances.

Proposition 3. For a cointegrated system of n $(n \geq 2)$ variables with f (which satisfies condition (10)) independent structural disturbances that produce permanent effects on observed variables, the minimum number of restrictions, in addition to the long run restrictions, required for the identification of structural disturbances is:

$$\frac{n(n-1)}{2} - f(n-f), \tag{14}$$

which is zero when $n=2$ and is positive when $n \geq 3$.

Proof. For an integer f satisfying (10), it is straightforward to show that (a) if n is even, the function (13) is maximized at $f=n/2$, and the maximum value of this function is $n^2/4$; and (b) if n is odd, the maximum value of (13) is $(n-1)(n+1)/4$, which is attained when $f=(n-1)/2$ or $f=(n+1)/2$.

If n $(n \geq 2)$ is even, then

$$\frac{n(n-1)}{2} - f(n-f) \geq \frac{n(n-1)}{2} - \frac{n^2}{4} = \frac{n(n-2)}{4} \geq 0,$$

with equality holds only for $n=2$. Note than when $n=2$, the only value of f satisfying (10) is one. On the other hand, if n $(n \geq 2)$ is odd, write $n = 2s+1$ where s is an integer greater than or equal to one. Therefore,

$$\frac{n(n-1)}{2} - f(n-f) \geq (2s+1)s - s(s+1) = s^2 > 0. \qquad \square$$

Proposition 3 implies that the only case in which long run identifying restrictions alone will satisfy the order condition is when $n=2$ (and $f=1$). In a bivariate cointegrated system with one long run restriction (such as in Blanchard and Quah, 1989), both the order and rank conditions for identification are satisfied, and the structural disturbances are just-identified. On the other hand, the necessary order condition for identification in a cointegrated system is not satisfied by long run restrictions alone when $n \geq 3$. As an example, in the four-variable system in Gali (1992), permanent effects are produced by only one structural disturbance. According to Proposition 3, three more restrictions are needed (as $n = 4$ and $f = 1$) besides the long run restrictions.

5. Conclusion

This paper shows that in order to have steady state growth generated by exogenous factors only, the determinant of $\Phi(1)$ for the propagation mechanism, as defined in equation (1), has to be of full rank. As a consequence, if a cointegrated system is generated by an exogenous growth model, then there must be at least one structural disturbance producing temporary effects on observed variables and at least one disturbance producing permanent effects. These results are consistent with those in Davidson and Hall (1991, Section II). They show, in a more general framework, that integration and cointegration may arise because of I(1) driving variables, and describe it as the "stable case". On the other hand, the "unstable case" (in Section III of their paper) corresponds to the presence of autoregressive roots of unity in the dynamic processes generating the variables, and an example of this case is the class of stochastic endogenous growth models (such as Lau, 1999). Davidson and Hall (1991, p. 249) conclude that "the phenomena of integration and cointegration are associated with certain restrictions on the coefficients of these system" and suggest that "it remains a considerable challenge to devise well-founded theories of economic behaviour which embody these restriction." The analysis of this paper and Lau (1999) could be regarded as a response to this challenge. Moreover, the results of this paper provide economic underpinning for the use of zero long run restrictions (Blanchard and Quah, 1989; King et al., 1991) in the class of exogenous growth models.

Based on the class of exogenous growth models for which the use of long run identifying restrictions is valid, this paper also discusses the order condition for identification, and shows that this condition is met in a bivariate cointegrated system. A by-product of the analysis in this paper, which is currently pursued in another paper, is the possibility of deriving an empirically verifiable condition for a bivariate cointegrated system under which the recursive ordering approach and the long run identifying restrictions deliver statistically similar results.

References

Banerjee, A., J. Dolado, J. W. Galbraith and D. F. Hendry (1993), Co-integration, and the Econometric Analysis of Non-stationary Data, Oxford University Press.

Bernanke, B. S. (1986), "Alternative explanations of the money-income correlation," Carnegie-Rochester Conference Series on Public Policy, 25:49-100.

Blanchard, O. J. and D. Quah (1989), "The dynamic effects of aggregate supply and demand disturbances," American Economic Review 79:655-673.

Blanchard, O. J. and M. Watson (1986), "Are business cycles all alike?" in Robert Gordon (ed.), The American Business Cycle: Continuity and Change, NBER and University of Chicago Press, 123-156.

Canova, F., M. Finn and A. R. Pagan (1994), "Evaluating a real business cycle model," in Colin Hargreaves (ed.), Nonstationary Time Series Analysis and Cointegration, 225-255, Oxford University Press.

Cass, D. (1965), "Optimum growth in an aggregative model of capital accumulation," Review of Economic Studies 32:233-240.

Davidson, J. and S. Hall (1991), "Cointegration in recurvise systems," Economic Journal 101:239-251.

Engle, R. F. and C. W. J. Granger (1987), "Co-integration and error correction: Representation, estimation, and testing," Econometrica 55:251-276.

Gali, J. (1992), "How well does the IS-LM model fit postwar U.S. data?" Quarterly Journal of Economics, 107:709-738.

Hamilton, J. D. (1994), Time Series Analysis, Princeton University Press.

King, R. G, C. I. Plosser, J. H. Stock and M. W. Watson (1991), "Stochastic trends and economic fluctuations," American Economic Review 81:819-840.

Lastrapes, W. D. and G. Selgin (1994), "Buffer-stock money: Interpreting short-run dynamics using long-run restrictions," Journal of Money, Credit and Banking 26:34-54.

Lau, S.-H. P. (1997), "Using stochastic growth models to understand unit roots and breaking trends," Journal of Economic Dynamics and Control 21:1645-1667.

Lau, S.-H. P. (1999), "I(0) in, integration and cointegration out: Time series properties of endogenous growth models," Journal of Econometrics 93:1-24.

McGrattan, E. R. (1994), "The macroeconomic effects of distortionary taxation," Journal of Monetary Economics, 33:573-601.

Robertson, D. and M. Wickens (1994), "VAR modelling," in S. Hall (ed.), Applied Economic Forecasting Techniques, 29-47, Harvester Wheatsheaf.

Sims, C. A. (1980), "Macroeconomics and reality," Econometrica 47:1-48.

Solow, R. M. (1956), "A contribution to the theory of economic growth," Quarterly Journal of Economics 70:65-94.

Stock, J. H. and M. W. Watson (1988), "Testing for common trends," Journal of the American Statistical Association 83:1097-1107.