THEORY AND DESIGN OF A CLASS OF COSINE-MODULATED NON-UNIFORM FILTER BANKS

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ABSTRACT

In this paper, the theory and design of a class of PR cosine-modulated nonuniform filter bank is proposed. It is based on a structure previously proposed by Cox, where the outputs of a uniform filter bank is combined or merged by means of the synthesis section of another filter bank with smaller channel number. Simplifications are imposed on this structure so that the design procedure can be considerably simplified. Due to the use of CMFB as the original and recombination filter banks, excellent filter quality and low design and implementation complexities can be achieved. Problems with these merging techniques such as spectrum inversion, equivalent filter representations and protrusion cancellation are also addressed. As the merging is performed after the decimation, the arithmetic complexity is lower than other conventional approaches. Design examples show that PR nonuniform filter banks with high stopband attenuation and low design and implementation complexities can be obtained by the proposed method.

I. INTRODUCTION

M-channel uniform filter banks with perfect reconstruction (PR) property have been extensively studied [1]. In some applications such as audio coding and subband adaptive filtering, a nonuniform frequency partitioning may be preferred. Efficient structure and design procedure for general nonuniform PR filter banks are therefore desirable. PR nonuniform filter bank has been studied in [3]-[8], [10-12]. Interested readers are referred to [11] for an excellent review of the topic. The PR condition of nonuniform filterbank was first studied by Hoang and Vaidyanathan [3], where a structure for P-band nonuniform QMF filter bank was proposed (Figure 1). Unlike the uniform filterbanks, the decimation ratios M_k 's are in general nonidentical. For critical sampling, they have to satisfy the condition $\sum_{k=0}^{K-1}(1/M_k)=1$. Due to the structure in Figure 1, the analysis filters in the P-channel uniform filter banks are dependent and are somewhat constrained. Therefore, it is not always possible to construct a PR system [3]. More recently, design methods based on the cosine-modulated filter banks (CMFB) were proposed [8,10,12]. The pseudo PR nonuniform filterbank in [8] is obtained by combining *directly* some of the decimated branches of the CMFB. Merging of filter bank outputs to obtain nonuniform filter banks has been known for some time. In [6], Cox has proposed a two-stage structure for nonuniform filter bank, where certain channels in an M-channel uniform filter bank are combined or merged together using the synthesis filters of another filterbank with smaller channel number. Here we shall call this method the indirect or recombination (merging) method, and the resulting filter shown in Figure 2 illustrates the concept of this method. In this particular example, the first m_k channels of an M-channel filterbank, $H_k(z)$, are combined using the synthesis filters of an m_k -channel uniform filterbank, $G_{o,i}(z)$. The sampling rate after merging is reduced by a factor of m_k/M. In the synthesis filters of the nonuniform filter bank, the analysis section of the m_k -channel uniform filter bank, $G'_{o,i}(z)$, is used to produce the m_k subband coefficients for the M-channel uniform filter bank. In Cox's work, the analysis and synthesis filter banks were derived from the pseudo-quadrature mirror filters similar to the CMFB, but they are not PR. It can be seen that if $G_{o,i}(z)$ and $G'_{o,i}(z)$ form an m_k -channel perfect reconstruction filter bank, then the merging and decomposing parts in Figure 2, which are enclosed with dotted lines, constitute a perfect reconstruction transmultiplexer. This is equivalent to introducing a certain delay, due to the transmultiplexer, in the first m_k channels of the original M-channel filter bank. If this delay is compensated in the other branches of the M-channel filter bank, the entire system is PR. Therefore, it is relatively simple to maintain the PR condition in such system. Another advantage of this structure is that it is possible to employ efficient filter structure such as the CMFB, as the original uniform filter bank as well as the recombination filter bank, as we shall see later in this paper. Also, as the merging is done after the decimators of the M-channel filter bank, the additional computational complexity of the merging process is greatly reduced. This is different from the recent work of Kok et al [10] and [12] where the outputs of the analysis filters *before decimation* are combined. As the combining is performed before the decimation, higher arithmetic complexity is expected. Also, fast implementation of conventional CMFB is usually performed in the decimated domain, it is not clear whether such fast algorithm can still be applied. Some of the disadvantages of Cox's original structure are their relatively large system delay and the lack of an equivalent filter representation as in the M-channel uniform filter bank. In fact, the system is a linear-time periodic varying system satisfying PR condition. Therefore, it is very difficult to setup a proper objective function to perform the minimization. Because of this reason, the two stages of the filter banks are usually designed separately. In [4], Kovacevic and Vetterli have proposed another structure, called the direct structure, or nonuniform filter bank. The advantage of this direct structure is that it has an equivalent filter representation. Therefore, it has more control over the quality of the filters.

In this paper, we present the theory and design of a class of PR nonuniform filter bank using the CMFB. It is based on Cox's structure with some simplification so that the design procedure can be considerably simplified. Due to the use of CMFB as the original and recombination filter banks, excellent filter quality, efficient design and low implementation complexity can be achieved.

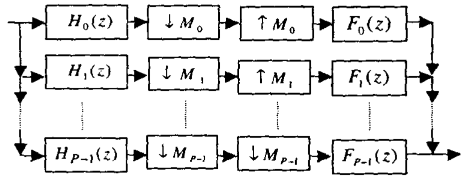


Figure 1: Nonuniform filter bank structure in [1]

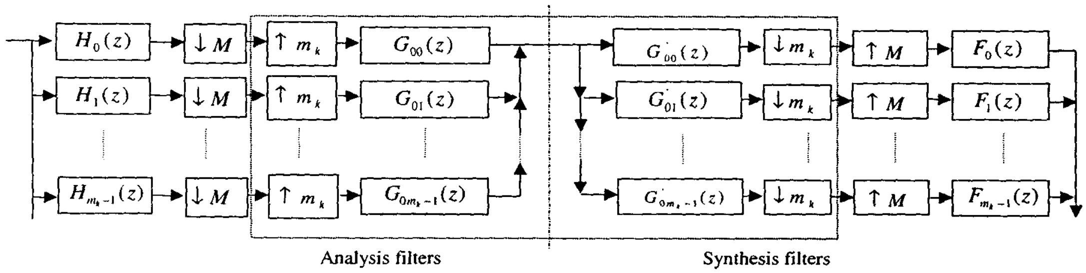


Figure 2: Structure of Recombination Nonuniform PR filter bank (only the first m_k branches are plotted)

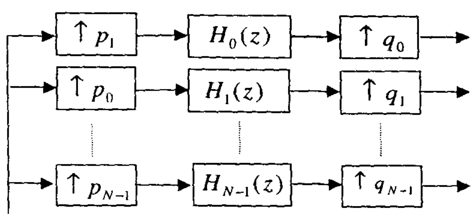


Figure 3: Direct structure in [4]. (p_i, q_i) are coprime, and $(\sum_{k=0}^{N-1}(p_k/q_k)=1)$.

The structure and theory of the proposed nonuniform filter banks will be described in Section II. Section III is devoted to the problem of protrusion cancellation that is encountered in designing such filter banks. Design procedure and several design examples are given in Section IV to illustrate the usefulness of the proposed method. Finally, we summarize the results in the conclusion.

II. COSINE-MODULATED NONUNIFORM FB

As mentioned in Section I, the recombination nonuniform filter banks do not generally have an equivalent filter representation like the direct structure shown in Figure 3. However, it can be shown that [4] if M is coprime to m_k, then the recombination structure admits an equivalent filter representation as in Figure 4. For simplicity, the first subscript of the recombination filters $G_{o,i}(z)$ is dropped. Actually, if M is coprime to m_k, then the decimators and the interpolators can be interchanged.

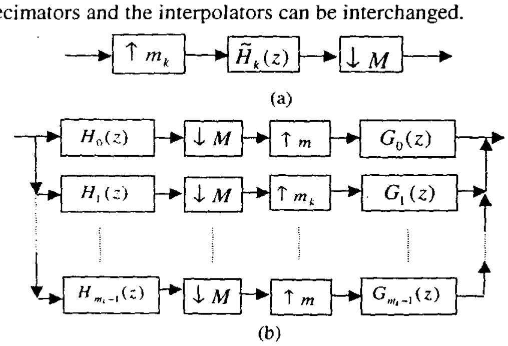


Figure 4: Equivalent structures when M and m_k are coprime.

Using the Noble identity [1], one gets,

$$\tilde{H}_k(z)=\sum_{i=0}^{m_k-1}H_i(z^{m_k})G_i(z^M). \qquad (1)$$

It should be noted that if M is a power of two and all the m_k are odd numbers, then the coprime assumption is automatically satisfied. The structure so obtained is not so restrictive as M can be chosen to be sufficiently large to approximate a given band splitting in practice. The remaining problem that we need to solve is the possible spectrum inversion found in the merged subbands. Supposed that we are going to merge the m_k subbands starting from the ℓ-th ($\ell=0,1,\ldots,M-1$) channel of the M-channel uniform filter bank. It can be shown that even for ideal filters, there will be spectrum inversion in the merged subbands, if ℓ is not an even numbers [4]. This seems to be a very restrictive requirement. But careful examination reveals that if ℓ is odd, then the output spectrum is merely inverted. This can easily be corrected by multiplying the merged output with the sequence $(-1)^n$ before the upsamplers [6]. The resulting structure is shown in Figure 5. Therefore, it is possible to start the merging at any channel of the M-channel uniform filter bank. From (1), it can be noted that the filter quality of $\tilde{H}_k(z)$ depends on the frequency responses of $H_i(z)$ and $G_i(z)$. In general, a joint optimization of $H_i(z)$ and $G_i(z)$ has to be performed so that $\tilde{H}_k(z)$ will have good frequency characteristics. Due to the good performance, low design and implementation complexities of CMFB, it is employed in the proposed nonuniform filter banks. In the following section, the theory of CMFB and a problem called protrusion cancellation which we have encountered in designing such cosine-modulated nonuniform filter banks are described.

III. THEORY OF CMFB AND PROTRUSION CANCELLATION

In CMFB, the analysis and synthesis filter banks $h_k(n)$ and $f_k(n)$ are obtained respectively by modulating the prototype filter $h(n)$ with a cosine modulation as follows

$$h_k(n)=2h(n)\cos\left[\frac{(2k+1)\pi}{2M}\left(n-\frac{N-1}{2}\right)+(-1)^k\frac{\pi}{4}\right], \qquad (2)$$

$$f_k(n)=2h(n)\cos\left[\frac{(2k+1)\pi}{2M}\left(n-\frac{N-1}{2}\right)-(-1)^k\frac{\pi}{4}\right], \qquad (3)$$

$$k=0,1,\ldots,M-1,\ n=0,1,\ldots,N-1,$$

where N is the length of the filters. Let $H(z)=\sum_{q=0}^{2M-1}z^{-q}P_q(z^{2M})$ be the type-I polyphase decomposition [1] of the prototype filter, it can be shown [2] that the PR conditions are given by:

$$P_k(z)P_{2M-k-1}(z)+P_{M+k}(z)P_{M-k-1}(z)=\beta\cdot z^{-\alpha} \qquad (4)$$

$$k=0,1,\ldots,M-1.$$

Since $H_k(z)$ is frequency shifted version of the prototype filter $H(z)$, it is only necessary to minimize $H(z)$ in the stopband, when the CMFB is orthogonal. In this case, $h(n)$ will be linear-phase. The problem can be formulated as the following constrained optimization

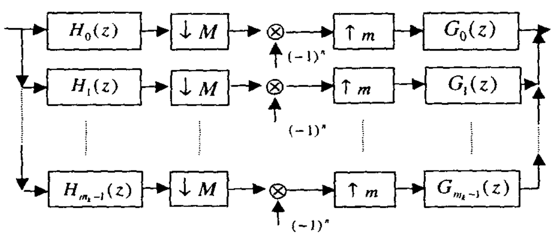


Figure 5: A solution to the spectrum inversion problem

$$\min_h \Phi = \int_{\omega_s}^{\pi} \left|H(e^{j\omega})\right|^2 d\omega \qquad (5)$$

subjected to the PR constraint in (4). The value of the cut-off frequency ω_s depends on the desired transition bandwidth. It should be between $\pi/2M$ and π/M. Since the number of design variables is much less than the general M-channel uniform filter bank, the design complexity is greatly reduced. In this work, both the original M-channel uniform filter bank and the recombination filter bank are implemented using the CMFB.

Intuitively, the parameters of $H_i(z)$ and $G_i(z)$ have to be jointly optimized so that optimal performance of $\tilde{H}_k(z)$ can be achieved. If two independent CMFB with good stopband attenuation are merged together, protrusions or dumps will appear in the stopband of $\tilde{H}_k(z)$. This is illustrated in Figure 6 for a two-channel nonuniform filter bank with decimation factors ($2/5, 3/5$). Here, the first two channels of a 5-channel CMFB with a filter length of 50 are merged by a 2-channel filter bank with filter length 20. The remaining three channels are merged by a 3-channel filter bank with filter length 30. The solid lines in Figures 6(a) and 6(b) show the frequency responses of the interpolated filters $H_0(z^2)$ and $H_1(z^2)$. The frequency responses of the interpolated filters $G_0(z^5)$ and $G_1(z^5)$ are also shown in dotted lines. Due to the mismatch in the transition bands of the filters, (note the overlaps of $H_0(z^2)$ and $G_0(z^5)$, and that of $H_1(z^2)$ and $G_1(z^5)$ in Figures 6(a) and 6(b)) $\tilde{H}_k(z)$ will exhibit protrusion in the transition bands of the interpolated filters, Figure 6(c). We found that the protrusion can be reduced by adjusting the stopband cutoff frequencies of the interpolated filters $H_i(z)$ and $G_i(z)$. By adjusting their stopband cutoff frequencies, it can be seen that the protrusion can be suppressed and good stop band attenuation can be obtained. Details of the design procedure and more design examples will be given in the next section.

IV. DESIGN PROCEDURES

As mentioned earlier, a joint optimization of the coefficients of the prototype filter $H(z)$ and those of the $G_{i,j}(z)$ can give the best performance. It is, however, very complicated. Here, we shall design $H(z)$ separately by minimizing its stopband attenuation subjected to the PR condition in (4). The recombination filters are then designed to minimize the stopband attenuation of the equivalent filters $\tilde{H}_k(z)$. To minimize the dump in the stopband, the cut-off frequency of the prototype filter $g_i(n)$ associated with $G_{i,j}(z)$ are chosen properly to match that of $H(z)$.

Let the prototype filters be matched to each other in the process of designing the uniform filter banks. The problem of designing $g_i(n)$ can be formulated as the following constrained optimization

$$\min_{g_i} \Phi = \int_{stopband} \left|\tilde{H}_i(e^{j\omega})\right|^2 d\omega \qquad (6)$$

subject to the PR condition (4) for $g_i(n)$.

Here $\mathbf{g}_i$ is the vector containing the impulse response of the prototype filter $g_i(n)$. In practice, we found that it is also possible to minimize the stopband attenuation of $g_i(n)$, if their cut-off frequencies are properly chosen as suggested by the following procedure.

Given the decimation ratios $\{m_k/M\}$, $k = 0,1,\ldots,K-1$, with m_k coprime to M and $\sum_{k=0}^{K-1} m_k = M$.

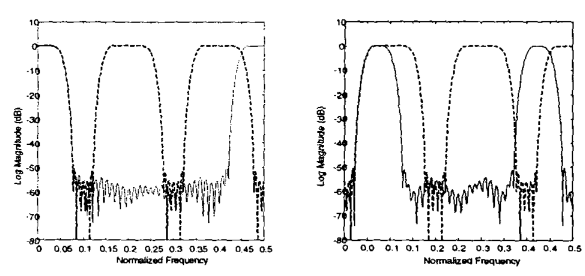
Design an M-channel uniform CMFB with stopband cutoff frequency ω_s, using equation 5.

(1) Design an m_k-channel CMFB using equation (6) or (5). The transition band of the m_k-channel uniform CMFB, after interpolation, should be equal to that of the M-channel uniform CMFB, after interpolation as suggested by equation (1). The M-channel CMFB is interpolated by m_k, while the m_k-channel CMFB is interpolated by M.

(2) If all starting indices l are even, the CMFB can be merged by the synthesis filter banks.

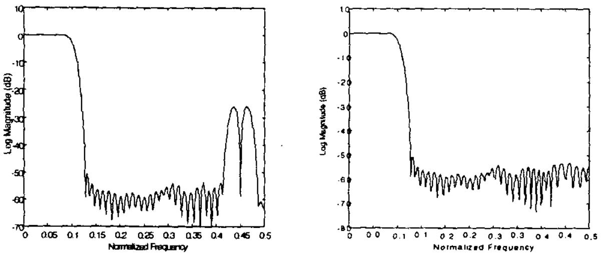
(3) If any ℓ is odd, the sequence $(-1)^n$ should be multiplied to the corresponding channels before the subband merging.

The performance of the proposed method is evaluated using several design examples. Figure 7 shows a 2-channel nonuniform filter bank with rational sampling factors of (4/7,3/7). The lowpass filter is obtained by merging the first 4 channels of a 7-channel uniform CMFB using another 4-channel CMFB. While the highpass filter is obtained by merging the remaining 3 channels of the 7-channel uniform CMFB. Figure 7(a) show the frequency responses of the analysis filters before interpolation. The frequency responses of the nonuniform filter banks are shown in Figure 7(b). The filter length of the 7-channel, 4-channel, and 3-channel uniform CMFB are 70, 40, and 30 respectively. Figure 8 and Figure 9 show two other nonuniform filter banks with sampling factors (4/9, 5/9) and (6/11, 5/11) designed by the proposed method. The filter length of the 4-channel, 5-channel, 6-channel, 9-channel, and 11-channel are respectively 40, 50, 60, 90, and 110. It can be seen that perfect reconstruction nonuniform filter banks with very high stopband attenuation and low design and implementation complexities can be obtained by the proposed method.



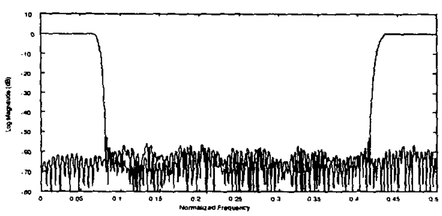
(a) $H_0(z^2)$ -solid line $G_0(z^5)$ - dotted line.

(b) $H_1(z^2)$ -solid line $G_1(z^5)$ - dotted line.

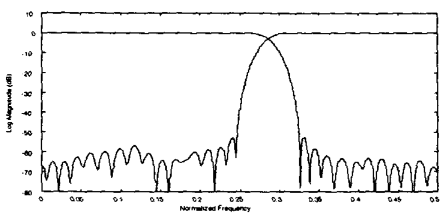


(c) $\tilde{H}_0(z)$ (d) $\tilde{H}_0(z)$

Figure 6: The protrusion cancellation in an (2/5,3/5) nonuniform filter bank. Frequency responses of (a) $H_0(z)$ and $G_0(z)$ after interpolation, (b) $H_1(z)$ and $G_1(z)$ after interpolation; Frequency responses of (c) $\tilde{H}_0(z)$ before adjusting cutoff frequencies, (d) after adjusting cutoff frequency.

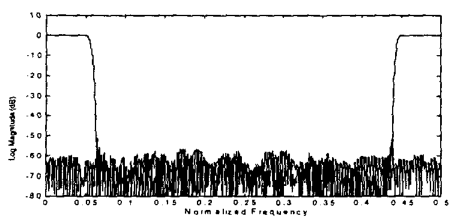


(a)

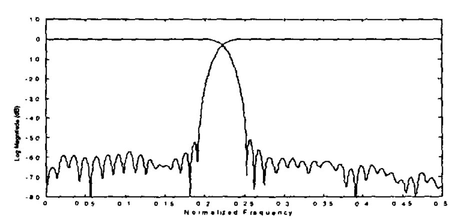


(b)

Figure 7: Nonuniform filter bank with rational sampling factors (4/7,3/7). (a) frequency responses of filters before interpolation, (b) frequency response of the nonuniform filter bank (after interpolation).

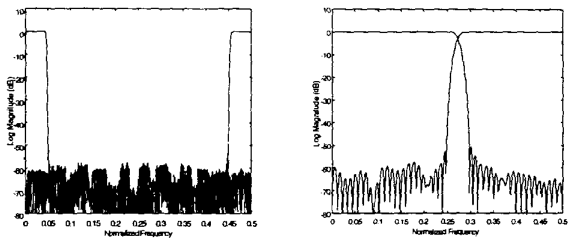


(a)



(b)

Figure 8: Nonuniform filter bank with rational sampling factors (4/9,5/9). (a) frequency response of filters before interpolation, (b) frequency response of the nonuniform filter bank (after interpolation).



(a) (b)

Figure 9: Nonuniform filter bank with rational sampling factors (6/11,5/11). (a) frequency response of filters before interpolation (b) frequency response of the nonuniform filter bank (after intepolation).

V. CONCLUSION

In this paper, the theory and design of a class of PR cosine-modulated nonuniform filter bank is presented. It is based on a structure previously proposed by Cox, where the outputs of a uniform filter bank is combined or merged by means of the synthesis section of another filter bank with smaller channel number. Simplifications are imposed on this structure so that the design procedure can be considerably simplified. Due to the use of CMFB as the original and recombination filter banks, excellent filter quality and low design and implementation complexities can be achieved. Problems with these merging techniques such as spectrum inversion, equivalent filter representations and protrusion cancellation are also addressed. As the merging is performed after the decimation, the arithmetic complexity is lower than other conventional approaches. Design examples show that PR nonuniform filter banks with high stopband attenuation and low design and implementation complexities can be obtained by the proposed method.

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