Predetermined prices and the persistent effects of money on output

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Abstract

This note illustrates a model of predetermined pricing, where firms set a fixed schedule of nominal prices at the time of price readjustment, based on the work of Fischer (1977). This contrasts with the model of fixed pricing, the specification underlying most recent dynamic sticky-price models. It is well known that predetermined pricing cannot generate substantial persistence in the real effects of monetary shocks when prices are set via fixed duration contracts, unless the contracts are of long duration. However, we show that with a probabilistic model of price adjustment, a predetermined pricing specification can produce almost as much persistence as the more conventional model of fixed prices, without the assumption of long average contract duration.

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Section 1. Introduction

This paper develops a simple dynamic model of aggregate price and output adjustment under predetermined prices (PP), where firms may set different prices for each future period, as in Fischer (1977). We contrast this with the standard specification, in which a single price is set for all future periods (fixed prices, or FP). In both frameworks, firms face a constant, exogenous probability of readjusting their prices each period, so that some prices are only readjusted asymptotically.¹ Conventionally, it is argued that the PP model does not allow for substantial persistence in the real effects of money shocks without assuming that price contracts are fixed for very long durations. In contrast, it is well known (e.g. Taylor 1979, Ball and Romer 1990, Romer 1996, Erceg 1997, Walsh 1998, and Jeanne 1998) that the FP model can allow for substantial persistence in the presence of real rigidities, even when price contracts are adjusted quite frequently.² Our results, however, show that the same property holds for the PP model. In the presence of real rigidities, the response to money shocks can display substantial persistence, even though firms may set different prices for each future period of the life of the price contract. The critical difference between our results and previous versions of the PP model lies in the use of a probabilistic specification of price adjustment. Intuitively, even a small fraction of firms that take a long time to readjust their prices in response to a monetary shock can result in resistance on the part of other firms to change their prices. This, in turn, can generate substantial persistence, even if the average contract length is short.

Our results show that for a special case in which the elasticity of real marginal cost to output is unity (and money follows a random walk), the two pricing specifications
are exactly equivalent. More generally, in the response of the price level and output to money shocks, the two specifications are quantitatively very similar. When the degree of real rigidity is very high, both specifications display substantial persistence in the real effects of money shocks. In this case, we find that the PP model implies a greater impact effect of a money shock, while the FP model implies a more persistent effect of a money shock.

The underlying rationale for the FP model is the presence of menu costs. Taken literally, these represent costs of having nominal prices different between periods. But if we take a broader interpretation of the costs of changing prices as related to the costs of gathering information and managerial decision-making (as suggested by empirical studies e.g. Bergen et al. 1997, 1999), then the PP model would seem more appropriate. In the PP model, these informational and managerial costs, which we might describe as contracting costs (as in the original Fischer (1977) model), will lead to prices to be predetermined, but different for future periods, reflecting the expected future marginal costs facing the firm.

The next section develops the model. Section 3 illustrates our results. A conclusion then follows.

**Section 2: A model of predetermined prices**

The main elements of dynamic sticky-price economies are very familiar (see Walsh 1998 for many references). Here we set out the minimum structure necessary to compare the two different price setting specifications discussed in the introduction. An appendix (available at http://www.arts.ubc.ca/econ/devereux/appendix.pdf) sketches out
how the model can be derived from an underlying dynamic general equilibrium environment.

Under each pricing specification, firms set prices in advance based on desired or target prices. Desired prices depend on expected marginal cost, which itself depends upon both current output (or the output gap), and the prices of all other firms (or the price level). A simple quantity theory equation (or aggregate demand equation) relates output to the economy-wide price level.

The quantity theory equation is written in log terms as

\[ y_t = m_t - p_t, \]

where \( y_t \) is aggregate output and \( m_t - p_t \) represents real balances. The nominal marginal cost facing each firm is also a function of the aggregate price level and output. It can be written as

\[ w_t = p_t + \nu y_t. \]

The parameter \( \nu \) measures the elasticity of the real wage (marginal cost) to output.

The desired price of any firm is just the marginal cost in any period (ignoring the constant mark-up term). Using (1) and (2), the desired price level can be written as

\[ p^*_t = (1 - \nu)p_t + \nu m_t. \]

Equation (3) says that the desired price level is equal to an average of the economy-wide price level and nominal aggregate demand. The higher \( \nu \) is, the more sensitive marginal cost is to movements in output (or the output gap), and the more willing individual firms are to adjust their desired price, relative to the aggregate price level (the average prices of all firms). But when \( \nu \) is very small, marginal cost is very insensitive to output, and firms’ desired prices are very close to the aggregate price level. In this case, firms are
extremely reluctant to set prices that differ from the average prices of other firms in the economy. This is the case where there is significant real rigidity, in the terminology of Ball and Romer (1990) and Romer (1996).

We now focus on the pricing decision for the representative firm. Let firms face the constant discount factor, $\beta < 1$. Then a firm that must set its price in advance experiences a loss in expected profits, relative to a situation where price adjustment is instantaneous. Following Walsh (1998), it may be shown that the loss in profits is approximately given by the squared deviation of the log price from the desired log price. Thus, any firm $i$ faces an expected loss of

$$L_t(i) = E_t \sum_{j=0}^{\infty} \beta^j \Phi (p_{t+j}^i(i) - p_{t+j}^*)^2,$$

where $\Phi$ is a constant. Loss function (4) applies, irrespective of the pricing regime.

**Fixed prices**

We now assume that each firm’s nominal price is set in advance, and is fixed for the duration of the price contract. We denote this specification as one of fixed prices (FP).

Price setting is staggered across firms, as in Taylor (1979), Calvo (1983), Yun (1996), and many others. In the Taylor (1979) model, firms set overlapping price contracts for fixed durations, while in the models of Calvo (1983) and Yun (1996), price contracts are of random duration for each firm, but in aggregate, a constant fraction of firms readjust their prices each period. Roberts (1995) shows that both the Taylor and Calvo models produce a similar forward-looking ‘Phillip’s curve.’ We employ the Calvo assumption here, because the distinction between fixed duration and random duration price contracts is important in the alternative model of predetermined prices (see below).
A firm can revise its price in each period with probability \((1 - \kappa)\), irrespective of how long its price has been fixed for in the past. When adjusting its price at time \(t\), the firm must set a fixed nominal price \(\hat{p}_t(i)\) that then holds for all future periods until it faces an opportunity to revise its price again. The firm therefore faces an expected loss of

\[
L_t(i) = \sum_{j=0}^{\infty} (\beta \kappa)^j \Phi(\hat{p}_t(i) - p_{t+j}^*)^2.
\]

It is easy to establish that the optimal price for firm \(i\) is

\[
\hat{p}_t(i) = (1 - \beta \kappa) \sum_{j=0}^{\infty} (\beta \kappa)^j p_{t+j}^*.
\]

Each period, a fraction \((1 - \kappa)\) of firms will readjust their prices. Since all firms are alike, they each set their price equal to the right hand side of (6). The aggregate price level for the economy is then given by

\[
p_t = (1 - \kappa) \hat{p}_t + \kappa p_{t-1},
\]

and from (6), the newly set price \(\hat{p}_t\) satisfies

\[
\hat{p}_t = (1 - \beta \kappa) p_{t-1}^* + E_t \beta \kappa \hat{p}_{t+1}.
\]

We may combine (3), (7) and (8) to solve for the dynamics of \(p_t, \hat{p}_t,\) and \(p_t^*\) for an economy with fixed prices. The solution requires an assumption on the stochastic process determining nominal aggregate demand.

**Predetermined prices**

While the fixed price model has a very appealing structure, it relies on the assumption that firms who are resetting prices set the same nominal price over the life of the contract. This model is consistent with menu-costs, so that it is costly for a firm to have prices that differ from one period to the next. Direct empirical evidence on menu-
costs of price changes is difficult to obtain. Bergen et al (1997, 1999) in studies of US retailing, suggest that menu-costs have to be interpreted broadly, both as literal costs of changing sticker prices, as well as informational and managerial decision making costs.

Interpreted in this broader sense, however, it is more difficult to defend the notion that the main costs of price adjustment arise from having prices different from one period to the next. Suppose, instead, that there was a fixed cost to drawing up a price contract, perhaps arising from the cost of obtaining information. This would not necessarily involve any costs in having prices at different levels in different periods, so price contracts could involve a different price for each future period. But the critical feature is that each price contract would be based on information available to the firm when setting the price contract.

In light of this, we now assume that each firm faces the same constant probability $1 - \kappa$ of revising its price contract, and when it does, it may set a sequence of prices for all future periods, $\{\hat{p}_{t+j}\}_{j=0}^{\infty}$. Beginning the next period, it will again face a constant probability of adjusting its price contract. We denote this specification as one of predetermined prices (PP). These assumptions accord with the price setting model of Fischer (1977) (see Romer (1996) for a discussion), but with the important distinction that Fischer assumes that price contracts are of fixed duration.

Under this price setting arrangement, when setting a price sequence, the expected loss function of the firm is given by

$$L_t(i) = E_t \sum_{j=0}^{\infty} (\beta \kappa)^j \Phi(\hat{p}_{t+j}(i) - \bar{p}_{t+j})^2,$$

(9)
where \( \hat{p}_{t+j,i}(i) \) is defined as the price set by firm \( i \), at time \( t \), pertaining to future time period \( t+j \). The optimal price sequence for firm \( i \) is

\[
\hat{p}_{t+j,i}(i) = E_t p_{t+j}^*.
\]

At time period \( t \), \( 1 - \kappa \) firms readjust their prices, and each set the same price sequence, given by the right hand side of (10). The aggregate price level for the economy is therefore given by

\[
p_t = (1 - \kappa) \sum_{j=0}^{\infty} (\kappa)^j E_{t-j} p_{t+j}^*.
\]

Equation (11) indicates that the price at any time \( t \) is the weighted sum of the desired price this period and previous periods’ expectations of this desired price, where expectations are based on the information available at the time of price adjustment.

Using equations (3) and (11) we may obtain the solution for actual and desired aggregate prices for the economy with predetermined prices.

**Monetary process**

In order to compare the effects of the two pricing regimes, we must make an assumption about the stochastic process for the money stock. Assume that the money stock exhibits an AR(1) process in growth rates. Thus,

\[
m_t - m_{t-1} = \rho (m_{t-1} - m_{t-2}) + u_t,
\]

where \( u_t \sim iid(0, \sigma^2) \). There is no drift in the money stock.

**Section 3. A comparison of the PP and FP specifications.**

**Solution: fixed prices**

Under the fixed pricing regime, we may solve equations (3), (7), (8), and (12) to obtain

\[
p_t = \mu p_{t-1} + (1 - \mu) m_t + \frac{\rho \beta \mu (1 - \mu)}{(1 - \rho \beta \mu)} (m_t - m_{t-1}),
\]
where $\mu$ is the stable root of the implied dynamic system in $\tilde{p}$ and $p$. Then, using (1), we can write output as

$$y_t = \mu y_{t-1} + \frac{\mu(1-\rho\beta)}{1-\rho\beta\mu}(m_t - m_{t-1}).$$

**Solution: predetermined prices**

Under the predetermined pricing regime, we may write the expression for the aggregate price level as

$$p_t = (1-\kappa)\sum_{j=0}^{\infty} \kappa^j E_{t-j}((1-\nu)p_t + \nu m_t).$$

The general solution to equation (15), using (12), is given by

$$p_t = \sum_{j=0}^{\infty} \theta(j)u_{t-j}$$

where

$$\theta(j) = \frac{(1-\rho^{j+1})\nu(1-\kappa^{j+1})}{(1-\rho)(1-(1-\kappa^{j+1})(1-\nu))}.$$ 

Then, the level of output may be obtained from equation (1) together with (16).

**Equivalence of the two pricing schemes**

The first result to note is that when $\nu = 1$ and $\rho = 0$, the solutions for (13) and (16) are equivalent: we obtain $\mu = \kappa$, so from (13), we have

$$p_t = \kappa p_{t-1} + (1-\kappa)m_t$$

and

$$y_t = \kappa y_{t-1} + \kappa u_t.$$ 

From (16), when $\rho = 0$, $m_t = \sum_{j=0}^{\infty} u_{t-j}$, and (17) and (18) also follow. Thus, when the elasticity of marginal cost to output is unity and the money stock follows a random walk,
both pricing regimes result in the same aggregate price dynamics, and therefore the same behavior for aggregate output. In this case, the dynamics of the price level and output are driven purely by the probability of price adjustment. A shock to the money stock at time $t, u_t$, is absorbed into the aggregate price level up to the proportion $1 - \kappa^T$ after $T$ periods. Since $\kappa^T$ is just the proportion of firms who have not yet readjusted their prices following the shock, there is no persistence beyond that imparted by the structure of price adjustment itself.$^4$

**Random Walk Specification for Money Growth**

When $\nu \neq 1$, the two pricing regimes have different implications for the dynamics of prices and output. We continue to assume that $\rho = 0$ for now. The dynamics of the price level and output under fixed prices are well known (see Romer (1996), Walsh (1998), Chari, Kehoe, and McGrattan (2000)). In particular it is easy to show that $\mu > \kappa (\mu < \kappa)$ as $\nu < 1 (\nu > 1)$. $^5$ In the first case, prices converge at a rate slower than that dictated by the fraction of firms who are adjusting. As a consequence, there is more persistence in output than that imparted by the exogenous price adjustment process. This excess persistence is driven by the presence of real rigidity.

In the conventional predetermined pricing model, as developed by Fischer (1977) (see Romer (1996)), firms adjust their price contracts at intervals of fixed duration, and the timing of readjustment is staggered across firms. In response to an unanticipated money shock, the price level will again adjust by less than (greater than) the rate implied by the exogenous fraction of firms who are adjusting, as $\nu < 1 (\nu > 1)$. But once all price contracts have been re-adjusted, the aggregate price level must fully reflect the money shock, and there can be no continuing real effects of money at all. For instance, if price
contracts are adjusted every $N$ periods, then there can be a substantial delay in price adjustment to a monetary shock for $\nu < 1$, but full adjustment must occur after $N$ periods have elapsed. In order to obtain substantial persistence in the real effects of money shocks when price contracts of fixed duration, it is necessary to assume that contract length ($N$) is large, along with real rigidity ($\nu < 1$).

But it is possible that some firms adjust prices quite quickly, while other firms are quite slow to adjust. Average contract length may then be relatively short, but the presence of some non-adjusting firms may have a substantial dampening effect on price level adjustment, if $\nu < 1$. If this is the case, then in general the price level will adjust by less than the fraction of firms who have adjusted. Thus, the presence of real rigidity may generate substantial output persistence in the PP model, even without a very large average contract length.

The Calvo model of price adjustment provides an example of such a situation. While most firms may have adjusted after a relatively short time, there are still some firms who take a long time to adjust. In the above model, a permanent shock to the money stock at time $t$ will increase the price level by $\frac{\nu(1-\kappa^T)}{1-(1-\kappa^T)(1-\nu)}$ after $T$ periods. This is less than (greater than) the share of all firms that have adjusted, $(1-\kappa^T)$, as $\nu < 1 (\nu > 1)$. When the average contract length $\frac{1}{1-\kappa}$ is low, most firms adjust fairly quickly. But the remaining unadjusted firms may have a disproportionate impact on the price level. This will be the case when the adjusting firms are unwilling to allow their prices to differ from those of all other firms (i.e. when $\nu$ is small). Thus, the presence of real rigidity can generate substantial persistence in output, even in the PP model, when
contracts are readjusted in the manner described here. Relative to the model of Fischer (1977), persistence does not require that all price contracts last a long time – there can be substantial persistence even though the average contract is adjusted relatively frequently.

Table 1 illustrates the properties of price adjustment following a 1 percent money shock, for the random walk money supply case, and $\kappa = 0.75$ (for the other parameter values, see the next section). The immediate price response is less under the PP case than the FP case. This is intuitive, as in the PP model, adjusting firms will set their price equal to the current marginal cost, which depends on the current aggregate price level. In the FP model, on the other hand, firms will set the price as an average of current and expected future marginal costs, so that the future rise in marginal costs due to aggregate price adjustment is taken into account. Thus, the output effects of money shocks are generically larger in the PP model. Both models display substantial persistence. For instance, the Table indicates that after 8 quarters, 90 percent of firms have adjusted their price, but the price level has risen by less than 50 percent of the money shock in both cases. While the initial price adjustment is less in the PP model, at some point, the price adjustment in the PP model exceeds that in the FP model, since in the latter model, the lagged price level is an important constraint on price adjustment, while it is not relevant in the PP model.

Elsewhere, Kiley (1999) has noted that sticky price models that follow partial adjustment rules, such as the Calvo model employed here, display aggregate price and output dynamics with more inherent persistence than the staggered price setting models of Taylor (1979), for most parameterizations of the elasticity of real marginal cost. The results here are in accordance with Kiley. A model of price setting that requires long
contract length to generate substantial persistence under staggered pricing may result in
significant persistence under the partial adjustment specification, even with short average
contract duration.

Quantitative analysis

Here we illustrate the response to a money shock when money growth rates are
persistent. We use two different values for \( \nu \). The setting \( \nu = 1.2 \) reflects the
parameterization used in Chari et al. (2000), based on a dynamic general equilibrium
version of the Taylor overlapping contracts model. With \( \nu = 0.1 \) there is a much higher
elasticity of marginal cost to output, and a higher degree of real rigidity.\(^8\)

| Table 1: Calibrated parameter values |
|-------------|-------------|-------------|------------|
| \( \beta \)  | \( \kappa \) | \( \nu \)     | \( \rho \)   |
| 0.985       | 0.75        | 1.2, 0.1    | 0.23        |

The rationale for the other parameter values chosen is as follows. A value of \( \beta \) of 0.985
implies an annual real interest rate of 6 percent, and \( \kappa \) equal to 0.75 implies an average
length of price adjustment of four quarters. Finally, to choose a value of \( \rho \), we directly
estimated equation (12) on US Federal Reserve non-borrowed reserve data over the 1959-
2000 period. Non-borrowed reserves represent a widely used measure of an exogenous
policy-determined monetary aggregate for the US economy.

The figures illustrate that in general, the response of the price level and output is
quite similar for the two different pricing schemes. When \( \nu = 1.2 \), the two specifications
display almost identical price and output responses. But when \( \nu = 0.1 \), we find that, as in
Table 1, the immediate price impact of a money shock is less under PP model than under
FP. As a result, the immediate impact on output is larger under PP, although PP displays less persistence. But note that both specifications display considerable excess persistence in the case of $\nu = 0.1$. After 8 quarters, output under PP falls below that under FP, and adjusts towards its steady state at a faster pace subsequently.

Section 5. Conclusion

We have introduced a model of predetermined pricing, where firms set a fixed schedule of nominal prices at the time of price readjustment, based on the model of Fischer (1977). This contrasts with the model of fixed pricing, the specification underlying most recent dynamic sticky-price models. It is well known that predetermined pricing cannot generate substantial persistence in the real effects of monetary shocks when prices are set via fixed duration contracts, unless the contracts are of long duration. However, we show that with a probabilistic model of price adjustment, a predetermined pricing specification can produce almost as much persistence as the more conventional model of fixed prices, without the assumption of long average contract duration. Moreover, fixed price models rely heavily on the literal assumption of menu costs of having prices differ from one period to the next. By contrast, a model of predetermined pricing may be justified by a broader notion of the costs involved in re-contracting price schedules, including managerial and informational costs.

Finally, although we have not emphasized the distinction in this paper, there is another important difference between the two models, not emphasized in the presentation above. As noted by McCallum (1994) and Walsh (1998), the FP model does not satisfy the natural rate hypothesis, under which the unconditional mean of output is independent of monetary policy. There is a positively sloped long run Philips curve; and therefore
there exists a long run trade-off between inflation and output. In contrast, in the PP model price setters can adjust their price schedules in advance for perfectly forecastable trend inflation. Hence the unconditional mean of output is independent of monetary growth; the natural rate hypothesis applies. Although this distinction did not arise in the analysis above, given the absence of a trend in money growth, it might be seen as a reason to favor the PP model over the FP model.
Footnotes

1. This follows the Calvo (1983) specification.

2. Following Ball and Romer (1990), we define a real rigidity to be any mechanism that causes firms to be reluctant to adjust their price relative to the average prices of all other firms in the market.

3. The expression for $\mu$

$$\mu = \frac{1}{2} \left( 1 - v + \kappa v + \frac{\kappa (1 - v) + v}{\beta \kappa} - \sqrt{\left( 1 - v + \kappa v + \frac{\kappa (1 - v) + v}{\beta \kappa} \right)^2 - 4 \frac{1}{\beta}} \right).$$

4. This result is quite general. When $v = 1$, expected marginal costs depend only on money, and not on other firm’s prices, and when $\rho = 1$, expected marginal costs for all future dates are constant. Thus, whatever price adjustment rule is used, the PP and FP models will deliver equivalent aggregate outcomes.

5. To see this, let $a = 1 - v + \kappa v + \frac{\kappa (1 - v) + v}{\beta \kappa}$. It follows from footnote 3 that

$$\mu(v) = \frac{1}{2} \left( a - \sqrt{a^2 - 4 \frac{1}{\beta}} \right),$$

where $\mu(v)$ reflects the dependence of the root on $v$.

Note that $\mu(1) = \kappa$, and $\mu'(v) < 0$.

6. If price contracts are re-adjusted every $N$ periods, and in each period $1/N$ firms readjust their contracts, then the price level is given by

$$p_t = m_{t-N} + \sum_{j=1}^{N} \frac{v j u_{t-(j-1)}}{N - j (1 - v)}.$$

For $v < 1$, the adjustment rate of a monetary shock at time $t - (j - 1)$ is less than $(N - j) / N$, the adjustment rate implied by the exogenous price adjustment frequency.
7. A referee has pointed out that the disproportionate effect of non-adjusting agents on the equilibrium outcome has a parallel in the model of Haltiwanger and Waldman (1991).

8. Ball and Romer (1990) and Chari et al. (2000) show that values of $\nu < 1$ are difficult to reconcile with conventional general equilibrium models under standard assumptions about labor supply. But other papers have argued that additional distortions in the labor market, such as efficiency wages (e.g. Ball and Romer 1990, Jeanne 1997) or sticky nominal wages (Erceg 1997, Christiano, Eichenbaum, and Evans 2001) can substantially reduce the response of the firms marginal cost to output, rationalizing values of $\nu$ in the range of 0.1.

9. Of course, many FP models circumvent this issue by assuming that firms can add a deterministic trend to their newly set price, based on expected trend inflation (e.g. Yun (1996)). In this case, however, it is more difficult to apply the literal interpretation of menu costs of changing the nominal price from period to period, and the contracting cost alternative would seem more relevant. If that is the case, then the PP model would seem more appropriate for such an environment.
References


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