

Fixed Prices versus Predetermined Prices and the Equilibrium Probability of Price

Adjustment

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Abstract

Elsewhere, papers comparing fixed prices with predetermined prices have assumed that the frequency of re-setting price contracts is equal in either case. This note demonstrates that in equilibrium, the frequency of re-setting price contracts is greater with fixed prices than predetermined prices.

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Section 1. Introduction

In standard sticky-price models, firms face menu costs, and therefore set a fixed price (FP) for all periods of a price contract. Recent papers by Devereux and Yetman (2001) and Mankiw and Reis (2001) have introduced an alternative framework in which prices are predetermined (PP), but a price contract may specify different prices for each future period, as in Fischer (1977). This alternative is consistent with the idea that sticky prices stem from the cost of drawing up a contract, rather than menu costs.¹ In both frameworks, firms face a constant, exogenous probability of readjusting their price contracts each period, as in Calvo (1983).

In this existing literature, the probability of setting a new price contract each period is exogenous, and common to both the FP and PP models. However, there is no intuitive rationale for why this should be the case. In this paper, the probability of readjusting price contracts each period is treated as endogenous as in Romer (1990) and Devereux and Yetman (2002). We show that the Nash equilibrium frequency of price adjustment is less with PP than FP, and the difference between them is increasing in the level of real rigidity.

The next section develops the model. Section 3 illustrates the results. A conclusion then follows.

Section 2: A model of sticky prices

The quantity theory equation is written in log terms as

$$(1) \quad y_t = m_t - p_t,$$

¹ Mankiw and Reis (2001) refer to these as “Sticky Prices” and “Sticky Information” respectively.

where y_t is aggregate output and $m_t - p_t$ represents real balances. The nominal marginal cost facing each firm is

$$(2) \quad w_t = p_t + \nu y_t,$$

where ν measures the elasticity of the real wage (marginal cost) to output. Therefore the desired price of any firm is just the marginal cost in any period (ignoring the constant mark-up term), and can be written as

$$(3) \quad p_t^* = (1 - \nu)p_t + \nu m_t.$$

The lower ν is, the less willing individual firms are to adjust their desired price, relative to the aggregate price level. This is the case where there is *real rigidity* [Ball and Romer (1990), Romer (1996)], or *strategic complementarity* [Woodford (2001)].

Consider the pricing decision of a representative firm. A firm that must set its price in advance experiences a loss in expected profits, relative to a situation where price adjustment is instantaneous, approximately equal to the squared deviation of the log price from the desired log price [Walsh (1998)]. Thus any firm i faces an expected loss of

$$(4) \quad L_t(i) = F + E_t \sum_{j=0}^{\infty} (\beta\kappa)^j (\hat{p}_{t+j,t}(i) - p_{t+j}^*)^2 + \frac{1-\kappa}{\kappa} \sum_{j=1}^{\infty} (\beta\kappa)^j L_{t+j}(i),$$

where β is the discount factor, F is the cost of updating a price contract, κ is the probability that the firm does not change its price in any given period, and $\hat{p}_{t+j,t}(i)$ is the price set by firm i in period t for period $t+j$.

Monetary process

The money stock follows an AR(1) in first differences with drift (empirical support for this is provided later). Thus,

$$(5) \quad m_t - m_{t-1} = \rho(m_{t-1} - m_{t-2}) + \bar{m} + u_t,$$

where $u_t \sim iid(0, \sigma_u^2)$.

Fixed Prices (FP)

Suppose that changing a nominal price is costly. Then, when setting a price contract at time t , a firm will set a fixed nominal price that holds for the life of the contract (that is, $\hat{p}_{t+j,t}(i) = \hat{p}_t(i) \forall j$). The firm sets its price to minimize an expected loss of

$$(6) \quad L_t(i) = E_t \sum_{j=0}^{\infty} (\beta\kappa)^j (\hat{p}_t(i) - p_{t+j}^*)^2,$$

or

$$(7) \quad \hat{p}_t(i) = (1 - \beta\kappa) E_t \sum_{j=0}^{\infty} (\beta\kappa)^j p_{t+j}^*.$$

The aggregate price level for the economy is then given by

$$(8) \quad p_t = (1 - \kappa) \hat{p}_t + \kappa p_{t-1}.$$

Combining (3), (5), (7) and (8) the price level is given by

$$(9) \quad p_t = \mu p_{t-1} + (1 - \mu) m_t + \alpha_1 (m_t - m_{t-1}) + \alpha_2 \bar{m},$$

$$\text{where } \mu = \frac{1}{2} \left(1 - \nu + \kappa\nu + \frac{\kappa(1-\nu) + \nu}{\beta\kappa} - \sqrt{\left(1 - \nu + \kappa\nu + \frac{\kappa(1-\nu) + \nu}{\beta\kappa} \right)^2 - \frac{4}{\beta}} \right),$$

$$\alpha_1 = \frac{\rho\beta\mu(1-\mu)}{1-\rho\beta\mu}, \text{ and } \alpha_2 = \frac{\beta\kappa(1-\mu)^2}{(1-\kappa)(1-\beta\kappa)(1-\rho\beta\mu)\nu}.$$

The equilibrium time-dependent κ may be approximated numerically.²

² See the appendix for details.

Predetermined Prices (PP)

Now consider the alternative model where there is a fixed cost to drawing up a contract, which may entail a different price for each future period. Beginning the next period, the firm will again face a constant probability of adjusting its price contract.

Clearly the optimal price sequence for firm i is

$$(10) \quad \hat{p}_{t+j,t}(i) = E_t p_{t+j}^*,$$

while the aggregate price level for the economy is given by

$$(11) \quad p_t = (1-\kappa) \sum_{j=0}^{\infty} (\kappa)^j E_{t-j} p_t^*.$$

The price level is the weighted sum of the *desired* price this period and previous periods' expectations of this desired price. Using equations (3), (5), (10), and (11),

$$(12) \quad p_t = m_t - \sum_{j=0}^{\infty} \theta(j) u_{t-j},$$

$$\text{where } \theta(j) = \frac{\kappa^{j+1}(1-\rho^{j+1})}{(1-\rho)[1-(1-\kappa^{j+1})(1-\nu)]}.$$

Substituting into (4), the expected loss with PP for firm l who chooses frequency of price adjustment κ_l is given by

$$(13) \quad L^{CC} = (1-\beta\kappa_l)F + \frac{\beta\kappa_l\nu^2\sigma_u^2}{(1-\rho)^2} \left\{ \sum_{j=0}^{\infty} (\beta\kappa_l)^j \left[\frac{1-\rho^{j+1}}{1-(1-\kappa^{j+1})(1-\nu)} \right]^2 \right\}.$$

The equilibrium time-dependent κ may then be solved numerically.^{3,4}

³ This loss function contains an infinite sum, where later terms are more heavily discounted. As a result, a finite approximation is adequate. For example, the value of κ_l that minimizes the first 100 terms is identical to the one that minimizes the first 1000 terms to 4 decimal places.

⁴ The Nash equilibrium κ is given by $\kappa = \kappa_l$, where κ_l minimizes (13). This is determined iteratively, resetting $\kappa = \kappa_l$ each iteration, until they converge. Note that the Nash equilibrium may be very different from the "optimal" value of κ (that is, the value if the behavior of other price setters was not taken as

Section 3. A comparison of the FP and PP specifications.

The optimal contract length may be solved numerically under both FP and PP. The monetary process is calibrated to the U. S. economy using non-borrowed reserves, a widely used measure of an exogenous policy-determined monetary aggregate. Results over the 1959-2000 period are given in Table 1.

Table 1: Estimated Non-borrowed Reserves			
Dependent variable: $m_t - m_{t-1}$			
Variable	Coefficient	Standard Error	Significance
Constant	0.0058	0.0027	0.0352
$(m_{t-1} - m_{t-2})$	0.2732	0.0750	0.0004
σ_u^2	0.0012		

Note that the constant is statistically significant. \bar{m} , ρ , and σ_u^2 are calibrated to the point estimates, and the optimal probability of price adjustment is then solved numerically for $\beta = 0.985$. The results are given in Figure 1, for different ν and F .

In all cases, the equilibrium κ is less with FP than PP, implying longer average contract length in the latter case. This is not surprising, given that with PP, a price setter may adjust their prices over the life of a contract, while with FP, they cannot. Note also that as the level of real rigidity increases (ν declines), the difference between the equilibrium κ 's increases.

given). For example, for $F = 0.002$ and $\nu = 0.1$, the Nash equilibrium is $\kappa_t = 0.483$, while the optimal value is $\kappa_t = 0.694$.

Section 4. Conclusion

Elsewhere, papers comparing fixed prices with predetermined prices have assumed that the frequency of re-setting price contracts is equal in either case. This note demonstrates that in equilibrium, the frequency of re-setting price contracts is greater with fixed prices than predetermined prices, for realistic parameter values.

Appendix: Solving for κ with Fixed Prices

For a given κ , (3), (5), (8), and (9) are jointly simulated. The optimal price set by firm l with price adjustment probability κ_l is

$$(A1) \quad \hat{p}_{t,l} = \phi_1 p_{t-1} + (1 - \phi_1) m_t + \phi_2 (m_t - m_{t-1}) + \phi_3 \bar{m},$$

$$\text{where } \phi_1 = \frac{(1 - \beta\kappa_l)(1 - \nu)\mu}{1 - \beta\kappa_l\mu}, \quad \phi_2 = \frac{(1 - \beta\kappa_l)(1 - \nu)\alpha_1 + \beta\kappa_l[\phi_1\alpha_1 + (1 - \phi_1)\rho]}{1 - \beta\kappa_l\rho},$$

$$\phi_3 = (1 - \nu)\alpha_2 + \frac{\beta\kappa_l[\phi_1\alpha_2 + (1 - \phi_1) + \phi_2]}{1 - \beta\kappa_l},$$

and the realized price in any given period may be computed as

$$(A2) \quad \bar{p}_{t,l} = \phi_{t,l} \hat{p}_{t,l} + (1 - \phi_{t,l}) \bar{p}_{t-1,l},$$

where $\phi_{t,l}$ is a random variable that takes the value 0 with probability κ_l and 1 otherwise.

Average loss over many possible shocks is given by

$$(A3) \quad L^{MC} = \frac{1}{L} \sum_{l=1}^L \sum_{t=t_0}^T \beta^{t-t_0} \left[\phi_{t,l} F + (\bar{p}_{t,l} - p_{t,l}^*)^2 \right].$$

For suitably large t_0 (so that initial conditions do not matter), T , and L , a grid search may be used to determine the value of κ_l that minimizes (A3). The simulation is then repeated using the estimated κ_l as the new value of κ , until they converge (to the Nash equilibrium). The results presented here are for $t_0 = 100$, $T = 200$, and $L = 1000$.

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Figure 1: Equilibrium Frequency of Price Adjustment

