A brief note submitted to the International Journal of Fracture, September 2004.

A BRIEF NOTE ON ELASTIC *T*-STRESS FOR CENTRED CRACK IN ANISOTROPIC PLATE

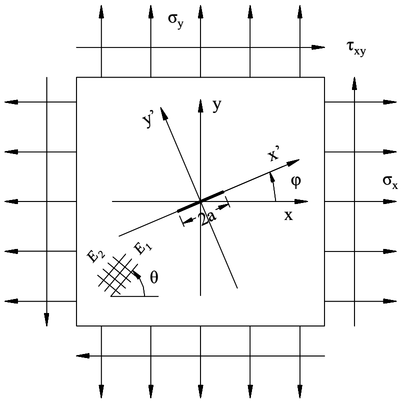
R.K.L. Su[a,1] and H.Y. Sun[b]
[a] *Assistant Professor, Department of Civil Engineering, The University of Hong Kong, Pokfulam Road, Hong Kong, PRC*
[b] *PhD Research Student, Department of Building and Construction, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong, PRC*
E-mail: klsu@hkucc.hku.hk

Abstract. The stress intensity factors (SIFs) and the *T*-stress for a planar crack with anisotropic materials are evaluated by the fractal finite element method (FFEM). The FFEM combines an exterior finite element model and a localized inner model near the crack tip. The mesh geometry of the latter is self-similar in radial layers around the tip. A higher order displacement series derived from Laurent series and Goursat functions is used to condense the large numbers of nodal displacements at the inner model near the crack tip into a small set of unknown coefficients. In this study, the variations of the SIFs and the *T*-stress with material properties and orientations of a crack are presented. The separation of the analytical displacement series into four fundamental cases is necessary to cover all the material variations and the orientations of a crack in the plate with general rectilinear anisotropic materials.

Keywords: Anisotropic, planar crack, higher order terms, stress intensity factor, *T*-stress, fractal finite element.

1. Introduction. The *T*-stress which acts parallel to a crack corresponds to the second, non-singular, term of the asymptotic stress field at the crack tip. The experimental tests conducted by Williams and Ewing (1972) on mixed mode fracture showed that the inclusion of this term could improve the accuracy of the theoretical predictions of the crack initiation angle and the critical stress intensity factors (SIFs). Other studies indicated that *T*-stress could influence the fracture toughness (Smith *et al*. 2001), the size and shape of the crack-tip plastic zone (Larsson and Carlsson 1973) and the crack path stability in isotropic materials (Cotterell and Rice 1980). Various numbers of analytical and numerical methods (Leevers and Radon 1982, Cardew *et al*. 1984, Kfouri 1986, Sham 1991, Karihaloo and Xiao 2001, Chen *et al*. 2001, Tan and Wang 2003, Chen and Tian 2000) have been developed to evaluate the *T*-stress for planar cracks in isotropic materials; however, only Chen and Tian (2000) have dealt with anisotropic materials. Recently, Su and Sun (2003) derived an asymptotic stress and displacement fields at the crack tip in an anisotropic elastic plate. By using the fractal finite element method (FFEM), the SIFs of cracked geometries in anisotropic materials were solved. In this study, the FFEM is applied to solve the *T*-stress of crack in the anisotropic plates and a new set of numerical solutions for centred crack in anisotropic plates are obtained.

2. Evaluation of SIFs and T-stress. Fig. 1 shows a through cracked composite lamina which can be assumed to be in a state of generalized plane stress which models as a homogeneous anisotropic material subjected to in-plane loading. The eigenfunction solutions for a crack in 2-D anisotropic plate can be separated into four fundamental cases that are based upon its material properties in which the first 3 cases correspond to orthotropic cases and the last case associated with rectilinear anisotropic case (Su and Sun 2003). The detailed classifications for the four cases are shown in Figure 1. The parameters $A = E_1/2G_{12} - \nu_{12}$ and $B = E_1/E_2$ are dependent on the material properties of the plate.



Case	Orientation of elasticity axes	Material properties
I	$\lvert\theta-\varphi\rvert =$ 0° or 90°	$A^2 - B > 0$ Orthotropic
II		$A^2 - B = 0$ Orthotropic/Isotropic
III		$A^2 - B < 0$ Orthotropic
IV	$\lvert\theta-\varphi\rvert \neq 0°$	Rectilinear anisotropic

Fig 1. Classifications of 2-D anisotropic plate with crack.

The FFEM (Su and Sun 2003) is used to determine the *T*-stress as well as the SIFs. The principle of the FFEM is that, while the local interpolating shape function can reduce an infinite number of degrees of freedom (DOF) within a finite element to finite number of the nodal displacements, the global interpolation functions derived from Laurent series and Goursat functions can further reduce the number of nodal displacements to a small set of unknown coefficients. By generating a self-similar mesh at the crack tip region with infinite number of DOF around the singular point, a higher order displacement series is used to condense the large numbers of nodal displacements near the crack tip to a small set of unknown coefficients. The SIF and the *T*-stress can be obtained directly from the generalized coordinates of the global interpolation functions without any post-processing technique.

3. Numerical study for centred crack in anisotropic plate. A centred crack in an anisotropic plate as shown in Figure 2 with the crack length to plate width ratio (*a/W*) of 0.5 and the aspect ratio of the plate (*H/W*) of 1 is used in the analysis.

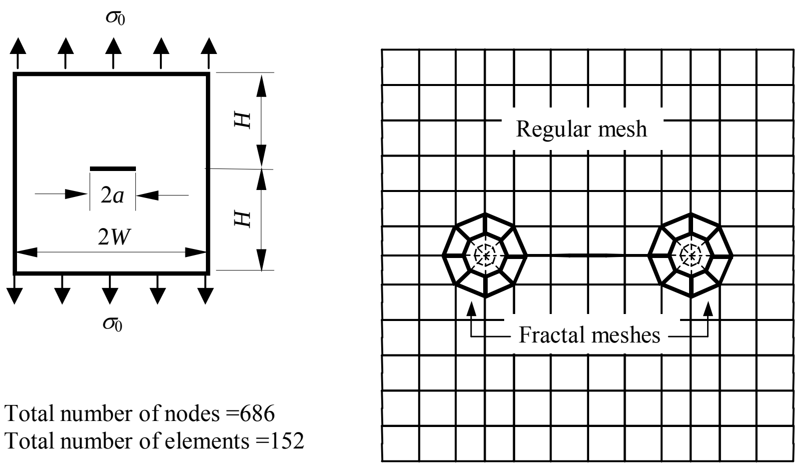


Fig. 2. The finite element mesh for cracked plate: *H/W*=1 and *a/W*=0.5.

The finite element mesh used to model the centred crack is shown in Figure 2. The total number of nodes used is 686 and the total numbers of regular finite elements and fractal finite elements required are 136 and 16 respectively. Four numerical studies have been preformed.

The first and second examples aim at showing the convergence of the fracture parameters (the SIFs and the *T*-stress) from the orthotropic situation to the isotropic situation. The third and fourth examples illustrate the possible transition of the fracture parameters from case I to the other cases (II, III and IV). The results are summarized in Tables 1 to 4 for the four examples respectively.

4. Discussions. By means of the FFEM, the new solutions of the SIFs and the *T*-stress of a centred crack in the plate with general rectilinear anisotropic materials are presented. The results from Tables 1 and 2 clearly indicate a smooth transition from Cases I and II orthotropic solutions to the corresponding isotropic solution. The results from Table 3 show that the SIFs and the *T*-stress are not so sensitive to the change in shear modulus. Lastly, a very smooth transition from Case IV anisotropic problem to Case I orthotropic problem is observed from Table 4. The study clearly demonstrated that the separation of the eigenfunction solutions into four fundamental cases is of necessity to cover all the material variations and the orientations of a crack in the plate with general rectilinear anisotropic materials.

6. Acknowledgement. The work described in this paper was fully supported by the grant from the University Research Grants (URG) through small project funding 2004-05, The University of Hong Kong.

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Table 1. Variations of the SIF and *T*-stress from Case I to isotropic case. (E_1 = 30.0 and ν_{12}=0.3)

Case	E_2	G_{12}	$\frac{K_I}{\sqrt{\pi a}}$		T	
			Present	Bowie & Freese (1972)	Present	Cardew et al. (1984)
I	300	17.6471	1.8167	1.85	-0.42543	
	150	16.6667	1.6634		-0.65344	
	100	15.7895	1.5668	1.57	-0.80578	
	75	15.0000	1.5021		-0.92399	
	60	14.2857	1.4551	1.46	-1.0226	
	50	13.6364	1.4193		-1.1085	
	42.8571	13.0435	1.3910	1.39	-1.1856	
	37.5	12.5000	1.3681		-1.2560	
	33.3333	12.0000	1.3492	1.35	-1.3212	
Isotropic	30	11.5385	1.3334		-1.3823	-1.384
I	27.2727	11.1111	1.3199	1.32	-1.4400	
	25	10.7143	1.3083		-1.4949	
	20	9.6774	1.2816	1.28	-1.6459	
	12	7.3171	1.2358	1.24	-2.0596	
	8.5714	5.8824	1.2153	1.22	-2.3969	
	6.6667	4.9180	1.2038	1.20	-2.6905	

Table 2. Variations of the SIF and *T*-stress from Case II to isotropic case. (E_1 = 30.0 and ν_{12}=0.3)

Case	E_2	G_{12}	$\frac{K_I}{\sqrt{\pi a}}$	T	
				present	Cardew et al. (1984)
II	14.4676	8.6207	1.2500	-1.8961	
	15.4737	8.8632	1.2558	-1.8409	
	16.5687	9.1152	1.2622	-1.7865	
	17.7624	9.3773	1.2689	-1.7331	
	19.0655	9.6500	1.2763	-1.6805	
	20.4904	9.9338	1.2842	-1.6288	
	22.0509	10.2291	1.2927	-1.5779	
	23.7628	10.5367	1.3018	-1.5279	
	25.6441	10.8570	1.3116	-1.4786	
	27.7154	11.1907	1.3221	-1.4301	
	28.8294	11.3628	1.3277	-1.4061	
Isotropic	30.0	11.5385	1.3334	-1.3823	-1.384

Table 3. Variations of the SIF and *T*-stress from Case I to Case II and then to Case III. (E_1 = 30.0, E_2 = 300.0 and ν_{12}=0.3)

Case	G_{12}	K_I	T
I	17	7.2008	-0.42296
	19	7.2003	-0.42998
	21	7.2032	-0.43541
	23	7.2086	-0.43967
II	24.34165	7.2133	-0.44201
III	26	7.2278	-0.44233
	28	7.2373	-0.44476
	30	7.2478	-0.44669
	32	7.2590	-0.44822

Table 4. Variations of the SIFs and *T*-stress from Case IV to Case I. (E_1 = 30.0, E_2 = 300.0, G_{12} = 17.64706 and ν_{12}=0.3)

Case	θ (°)	K_I	K_{II}	T
I	0	7.1986	0	-0.42587
IV	5	7.1642	-0.11892	-0.43112
	10	7.0707	-0.22711	-0.44696
	15	6.9367	-0.31773	-0.47302
	20	6.7736	-0.38776	-0.50840
	25	6.5843	-0.43662	-0.55430
	30	6.3726	-0.46596	-0.61554
	35	6.1457	-0.47776	-0.69716
	40	5.9102	-0.47224	-0.80119
	45	5.6747	-0.45035	-0.92870
	50	5.4523	-0.41680	-1.0835
	55	5.2545	-0.37681	-1.2726
	60	5.0823	-0.33041	-1.5044
	65	4.9314	-0.27736	-1.7886
	70	4.8027	-0.22265	-2.1416
	75	4.7059	-0.17449	-2.5886
	80	4.6537	-0.13440	-3.1328
	85	4.6364	-0.07885	-3.6531
I	90	4.6335	0	-3.8810