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DESIGN CRITERIA FOR UNIFIED STRUT AND TIE MODELS

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Summary

In the past two decades, the concept of strut and tie models is being used as one of the most popular and rational approach for the design of non-flexural members of reinforced concrete structures. Design guidelines mainly based on past decade technology were given in many national codes such as Eurocode (ENV 1992-1-1:1992), the Canadian Standard (CSA Standard A23.3-94), the Australian Standard (AS3600-1994) and New Zealand Standard (NZS3101:Part2:1995) as well as the international standard Model Code (CEB-FIP: 1990). The review of recent advancement in strut and tie modeling in this paper enable a new set of design formulae and design tables for the strength of strut, node and bearing to be derived and presented. The design formulae proposed for strut and node in this paper are in form of product of two partial safety factors which taken into account (i) the orientation of strut-tie, (ii) the brittle effects as the strength of concrete increases, (iii) the strain state of both concrete and steel and (iv) the stress state of the boundary of node. The design values proposed for plain concrete with bearing plate ensure that the node would not crack at service conditions and possesses sufficient strength under ultimate load conditions. To enhance the worldwide use of such design tables, both the concrete cylinder strength and the concrete cube strength were used to define the strength of concrete.

Keywords

Strength, Struts, Ties, Nodes, Bearings, Design Code, Cube Strength, Cylinder Strength

Introduction

Nonflexural members are common in reinforced concrete structures and include such elements as deep beams, corbels, pile caps, brackets, and connections. Compared to flexural elements such as beams and slabs, relatively little guidance is given in codes of practice for the design of nonflexural elements. Design codes having the strut-tie design criteria include Eurocode (ENV 1992-1-1:1992), the Canadian Standard (CSA Standard A23.3-94), the Australian Standard (AS3600-1994) and New Zealand Standard (NZS3101:Part2:1995) and the Model Code (CEB-FIP: 1990). However, since those design codes have their own system of partial safety factors for materials and loads, designers from other countries would find difficulty in using those codes directly. In this paper, the strength of struts, nodes and bearing specified in different codes and proposed by different researchers are reviewed. The appropriate design formulae which take into account of the types of stress fields, crack in strut and the brittle effects as the strength of concrete increases are proposed. Design tables based on both cube and cylinder concrete strength are worked out for use in design applications.

In the early development of practical design procedures for reinforced concrete at the end of the 19^{th} century it was rapidly recognized that the simple theories of flexure were inadequate to handle regions which were subjected to high shear. A rational design approach was developed, primarily by Ritter (1899) and Mörsh (1902) based on an analogy with the way a steel truss carries loads. The truss analogy promoted the subsequent use of transverse reinforcement as a means for increasing the shear capacity of beams. Rausch(1929) extended the plane-truss analogy to a space-truss and thereby proposed the torsion resisting mechanism of reinforced concrete beams. Slater(1927) and Richart (1927), proposed more sophisticated truss models where the inclined stirrups and the compressive struts were oriented at angles other than 45^{o}. The method was further refined and expanded by Rüsch(1964), Kupfer(1964) and

Leonhardt(1965). Only in the past two decades, after the works by Marti (1985), Collins and Mitchell (1986), Rogowsky and Macgregor (1986), and Schlaich *et al*. (1987), has the design procedure been systematically derived and been successfully applied to solve various reinforced concrete problems. The work by Schlaich *et al*.(1987) extended the beam truss model to allow application to nearly all parts of the structure in the form of strut-tie systems. Schlaich suggested a load-path approach aided by the principal stress trajectories based on a linear elastic analysis of the structure. The principal compressive stress trajectories can be used to select the orientation of the strut members of the model. The strut-tie system is completed by placing the tie members so as to furnish a stable load-carrying structure. Adebar *et al*. (1990) and Adebar and Zhou (1996) designed pile caps by a strut-and-tie model. The models were found to describe more accurately the behavior of deep pile caps than the ACI Building Code. Alshegeir and Ramirez (1992), Siao(1993), Tan *et al*. (1997) used the strut-and-tie models to design deep beams. Experimental studies by Tan et al. indicated that the strut-and-tie model is able to predict the ultimate strengths of reinforced concrete deep beams, which may be subjected to top, bottom or combined loading. In general, the strength predictions are conservative and consistent. The approach is more rational than the other empirical or semi-empirical approaches from CIRIA guide 2 (1977), and gives engineers an insight into the flow of internal forces in the structural members. MacGregor(1997) recommended design strengths of nodes and struts which are compatible with the load and resistance factors in the ACI code. Hwang *et al*. (2001) and (2000) used the strut and tie model to predict the shear strength capacity of squat walls and the interface shear capacity of reinforced concrete.

Strength of struts

The design of nonflexural members using strut-and-tie models incorporates lower-bound plasticity theory, assuming the concrete and steel to be elastoplastic. Concrete, however, does not behave as a perfectly plastic material and full internal stress redistribution does not occur. The major factors affecting the compressive strength of a strut are (i) the cylinder concrete compressive strength f'_c (or cube concrete compressive strength f_{cu}), (ii) the orientation of cracks in the strut, (iii) the width and the extent of cracks, and (iv) the degree of lateral confinement. To account for the above factors, the effective compressive strength may be written as

$$f'_{cd} = \nu f'_c \tag{1}$$

where f'_c is the specified compressive strength of concrete and ν is the efficiency factor for the strut ($\nu \leqq 1.0$). The design compressive strength is usually expressed as

$$f_{cd} = \phi f'_{cd} \tag{2}$$

where ϕ is the partial safety factor of the material.

Based on plasticity analysis of shallow beams, Nielsen *et al.*(1978) proposed an empirical relationship for the efficiency factor

$$\nu = 0.7 - f'_c / 200 \;; \qquad f'_c \leq 60\text{MPa} \tag{3}$$

The proposed values of ν depend on the strength of concrete and range from 0.6 to 0.4 for f_c' of 20MPa to 60MPa, respectively, with a typical value of 0.5. A similar expression is adopted by the current Australian Standard for determination of the strength of a strut. The equation implies that the efficiency factor is simply a function of concrete strength and does not account for the effect of cracks in the strut. Foster and Gilbert(1996) reviewed this relationship and found that the observed compression failures of non-flexural members with normal strength concrete do not correlate well equation(3). The level of agreement is even worse for high strength concrete. They recommended not to employ this relationship for design of strut-and-tie models.

Ramirez and Breen (1983) studied the shear and torsional strength of beams and expressed the maximum diagonal compression stress of beams and beam-type members to be

$$\nu = 2.5/\sqrt{f_c'} \,. \qquad (4)$$

Typical efficiency factor predicted by the equation (4) for normal strength concrete range from 0.65 to 0.37. Ramirez and Breen (1991) checked the accuracy of the proposed formula against load tests of reinforced concrete beams with f'_c ranging from 15 to 45 MPa. The results indicated that equation(4), on average, over-estimated the strength of the reinforced concrete beams and prestressed concrete beams by 18% and 144%, respectively. All the beams had shear span *a* to effective depth *d* ratio greater than 2.0, which indicates that all beams were relatively slender. Furthermore, the angle of main diagonal compressive strut to tension reinforcement was quite shallow and was approximately equal to 30^o. As a result, skewed cracks formed in the main struts with a severe crack width. These factors may explain the relatively conservative prediction of the compressive stress of beams by the proposed efficiency factor.

Marti (1985) based on experimental results and proposed an average value of $\nu = 0.6$ for general use. The proposed value was in general higher than those predicted from equations (3) and (4). Marti further stated that the value might be increased depending on the presence of distribution bars or lateral confinement. Rogowsky and MacGregor (1986) took into account the fact that the truss selected may differ significantly from the actual elastic compressive stress trajectories and that; significant cracks may form in the strut, and they suggested an average value of ν=0.6 for use. However, if the compressive strut could be selected within 15^o of the slope of the elastic compressive stress trajectories, a higher value of ν up to 0.85 was recommended.

Schlaich et al. (1987) and Alshegeir (1992a,b) independently proposed similar values of the efficiency factors for struts under different orientation and width of cracks. The proposed values along with the recommended values by other researchers are listed in Table 1. For the ease of comparison, the angle θ=60^o between the strut and the yielded tie is assumed, corresponding to the case of a strut with parallel cracks and with normal crack width. Angle θ equal to 45^o is assumed to correspond to the case of a strut with skewed cracks and with a severe crack width. Angle θ less than 30^o is associated with the minimum strength of a strut. It is noted that strain incompatibility is likely to occur when the angle between the compressive strut and tie is less than 30^o. It is therefore taken that angle θ should be assumed greater than 30^o for typical strut-tie systems. The typical values of ν shown in Table.1 vary between 0.85 for an uncracked strut with uniaxial compressive stress, to 0.55 for a skewed cracked strut with severe crack width. The minimum value of ν is around 0.35.

Based on extensive panel tests of normal strength concrete (f'_c from 12MPa to 35MPa), Vecchio and Collins (1986) showed that the maximum compressive strength might be considerably reduced by the presence of transverse strains and cracks. A rational relationship for the efficiency factor, which is a function of the orientation of strut as well as the strains of both concrete and steel, was proposed as follows

$$\nu = 1/\left(0.8 + 170\varepsilon_1\right) \leq 1.0 \qquad (5a)$$

and $$\varepsilon_1 = \varepsilon_x + \left(\varepsilon_x - \varepsilon_2\right)\cot^2\theta\,, \qquad (5b)$$

where ε_1 and ε_2 are the major and minor principal strains of concrete respectively, and θ is the angle of the strut to the horizontal tie.

Foster and Gilbert (1996) proposed that at the ultimate state, the yield strain of horizontal reinforcing steel may be taken as ε_x=0.002 and the peak strains of concrete may be equal to –0.002 and –0.003 for grade 20MPa and 100MPa concrete, respectively. The efficiency factor of equation (5a) can then be rewritten as

$$\nu = \frac{1}{1.14 + \left(0.64 + f'_c / 470\right)\left(a/d\right)^2} \leq 0.85 \qquad (6)$$

As the relationship is not sensitive to f'_c, Foster and Gilbert further simplified this relationship to derive the modified Collins and Mitchell relationship which is expressed as

$$\nu = \frac{1}{1.14 + 0.75\left(a/d\right)^2} \leq 0.85. \qquad (7)$$

By carrying out a series of nonlinear finite element analyses, Warwick and Foster (1993) proposed the following efficiency factor for concrete strength up to 100MPa:

$$\nu = 1.25 - \frac{f_c'}{500} - 0.72\left(\frac{a}{d}\right) + 0.18\left(\frac{a}{d}\right)^2 \leq 0.85 \text{ for } a/d<2 \tag{8a}$$

$$\nu = 0.53 - \frac{f_c'}{500} \text{ for } a/d \geq 2 \tag{8b}$$

The equations from the modified Collins and Mitchell relationship (7) and from Warwick and Foster (8) give similar results for high strength concrete, but for lower strength concrete Warwick and Foster's equations give higher values of the efficiency factor. The equations were reviewed by Foster and Gilbert (1996), and both equations (7) and (8) were found to give a fair correlation against experimental data for non-flexural members where the failure mode is governed by the strength of the concrete struts.

MacGregor (1997) introduced a new form of the efficiency factor in which the factor is given as the product $\nu_1 \nu_2$. The first partial efficiency factor ν_1 accounts for the types of stress fields, cracks in the strut and the presence of transverse reinforcement. The second partial efficiency factor ν_2 accounts for brittle effects as the strength of concrete increases. The partial safety factor has been embedded in the product of partial efficiency factors. Therefore,

$$f_{cd} = \nu_1 \nu_2 f_c' \tag{9a}$$

and $$\nu_2 = 0.55 + \frac{1.25}{\sqrt{f_c'}} \tag{9b}$$

where ν_1 is shown in Table 4 and ν_2 as shown in equation (9b) is originally from Bergmeister *et al.* (1991). Table 2 presents the normalized efficiency values for ease of comparison.

Table 3 compares the partial safety factor of dead and live loads amongst various design standards including the British Standard BS8110: 1997 and the Chinese Standard GBJ 10-89. The equivalent design standard to ACI 318-1995 was derived by MacGregor (1997). Since for typical structures, live load is usually in the order of 20% to 30% (with average of 25%) of the dead load, the equivalent load factors that combine the live load with the dead load of different codes are shown in Table 3. The load adjustment factors μ are determined by dividing 1.725 (which is the combined load factor of CEB-FIP: 1990) by each combination of the load factor. The result indicates that the ACI code, with partial load factors for dead and live loads of 1.4 and 1.7 respectively, is the most conservative code in terms of loading amongst all the selected codes. The Chinese code, on the other hand, with partial load factors for dead and live loads of 1.2 and 1.4 respectively, is the most lenient code. In general, the ultimate design load is higher than the service load by 30-40%.

Table 4 presented the codified strength for struts. The design strength of a strut is modified by the load adjustment factor μ, as shown in Table 3, to allow for the difference in the definitions of partial safety factor of loads. When comparing the adjusted design strength of a strut, the Canadian Standard, New Zealand Standard and the equivalent American Standard, all give similar values except that the equivalent ACI standard allows relatively high efficiency values of $0.71f'_c$ and $0.57\ f'_c$ for the uncracked strut and the cracked strut with transverse reinforcement, respectively. Those codified values generally have a safety margin of approximately 1.5 times when compared with the unfactored values shown in Table 1. The maximum experimental strength of strut, $0.85f'_c$, is sufficiently higher than the typical maximum codified design strength of $0.55\ f'_c$, by 50%. The minimum residual strength of a strut allowed by the codes is around $0.2\ f'_c$. When compared with the typical minimum value of $0.35\ f'_c$ as suggested by most of the researchers in Table 1, a sufficient factor of safety of 1.75 is indicated. The suggested design strength of 0.48 f'_c for an uncracked strut by the Model Code 90 and 0.40 f'_c for uniaxial loaded strut by Eurocode 92 is considered to be relatively conservative, as the factor of safety against compressive failure is around 1.9. The design formulae by the Australian Code, similar to equation (3), do not take into account the orientation and width of cracks in strut and are not recommended for use due to the inherent inaccuracy for predicting the strength of a strut [Foster and Gilbert(1996)].

Strength of nodes

The strength of concrete in the nodal zones depends on a number of factors such as (1) the confinement of the zones by reactions, compression struts, anchorage plates for prestressing, reinforcement from the adjoining members, and hoop reinforcement; (2) the effects of strain discontinuities within the nodal zone when ties strained in tension are anchored in, or cross, a compressed nodal zone; and (3) the splitting stresses and hook-bearing stresses resulting from the anchorage of the reinforcing bars of a tension tie in or immediately behind a nodal zone. The effective strength of a node may be expressed as

$$f'_{cd} = \eta f'_c \qquad (10)$$

where f'_c is the specified compressive strength of concrete and η is the efficiency factor for a node ($\eta \leqq 1.0$). The expression of the design strength of a node is similar to equation (2).

By using the Mohr's circle technique, Marti (1985) described a procedure to transform the unequal stresses from struts or ties intersected at nodal zones to the equivalent equal intensity stresses. The node joined with one compressive strut together with 2 tension ties required a proper lateral confinement to provide sufficient lateral support to the compressive shell behind the node being highlighted. Marti proposed that the average stress of nodal zones should be $0.6f'_c$ for general use. The value may be increased when lateral confinement is provided.

Collins *et al*. (1986) introduced different design values for the efficiency factor η under various boundary conditions of nodes such as CCC, CCT and CTT, where C and T denote the node met with compressive strut and tension tie, respectively. By following the suggestion of Marti (1985) that the node met with ties required additional lateral confinement to provide the same level of strength for the node, lower efficiency factors were adopted for a node met with an increasing number of ties. This concept had considerable impact on other researchers and national standards as it has been adopted by MacGregor (1988), the Canadian Standard (A23.3-94) Eurocode (ENV 1992, 1-1:1992) and the New Zealand Standard (NZS3101: Part2:1995). On the other hand, Schlaich *et al*. (1987) and other standards such as the Model Code (CEB-FIP: 1990) adopted other rules; these only distinguished between nodes joined with or without tension ties, and associated different efficiency factors to the respective nodes.

The proposed efficiency factors given by Collins *et al*. (1986), Schlaich *et al*. (1987, 1991), MacGregor (1988), Bergmeister *et al*. (1991) and Jirsa *et al*.(1991) are summarized in Table 5. For ease of comparison, the normalized efficiency values for nodes are presented in Table 6. It can be observed that only a small variation of η values exists for different types of nodes. The typical η values of CCC, CCT and CTT nodes are 0.85, 0.68 and 0.6 respectively. Schlaich *et al*. (1991) slightly increased η from 0.85 to 0.94 for CCC node under 2- or 3- dimensional state of compressive stresses in nodal region. Experimental study of concrete nodes by Jirsa *et al*. (1991) reported that the minimum strength of CCT and CTT nodes is $0.8f'_c$.

MacGregor (1997) introduced a similar product form ($\eta_1\eta_2$) of the efficiency factor for both struts and nodes. The first partial efficiency factor η_1 accounted for the type of node such as CCC, CCT and CTT, as shown in Table 7. The second partial efficiency factor η_2 accounted for the brittle effects as the strength of concrete increases and was given in equation (9b). The partial safety factor has been embedded in the product of partial efficiency factors.

Table 7 presents the codified strength for nodes. The design strength of a node is multiplied with the load adjustment factor μ, as shown in Table 3, to give the adjusted design strength of the node.

Comparing the adjusted design strength of nodes, it is found that the Canadian Standard, New Zealand Standard and Eurocode, all give similar values. The nodes of types CCC, CCT, and CTT are of typical strength 0.56 f'_c, 0.48 f'_c, and 0.40 f'_c, respectively. When the factor of safety of 1.5 is included in those codified values, very good agreement can be found when compared with the unfactored values shown in Table 5. Eurocode suggests maximum strength of node of 0.67 f'_c under triaxial stress state and a minimum strength of $0.5\phi f'_c$ under CTT stress state. The suggested design strength of 0.48 f'_c for CCC node and $0.34f'_c$ for C&T node by the Model Code 90 is considered to be relatively conservative when compared with the other standards such as Eurocode. The design nodal strength, ϕ $(0.8-f'_c/200)f'_c$ suggested by the Australian Code, may be unconservative for CTT node and is not recommended for use. The equivalent ACI nodal strength is found to be consistently higher than the values suggested by Eurocode or the Canadian Code.

Strength of ties and minimum reinforcement

The strength of ties specified in different codes is given in Table 8. The partial safety factor for ties are generally equal to 0.87, except that the suggested value of 0.70 from the Australian Code is substantially conservative.

Schlaich *et al*.(1987) observed that the shape of the compressive strut is bowed and, as a result, transverse tensile forces exist within the strut. It is important that a minimum quantity of reinforcement is provided to avoid cracking of the compressive strut due to the induced tensile forces so as to maintain the efficiency level for the strut as shown in Tables 1 and 4. This reinforcement contributes significantly to the ability of a deep beam to redistribute the internal forces after cracking, as suggested by Marti(1985). Finite element experiments by Foster (1992) have shown that deep beams exhibit almost linear elastic behavior before cracking. In order to maintain wide compression struts developed beyond the cracking point, sufficient tension tie steel should be provided to ensure that the beam does not fail prematurely by diagonal splitting.

Foster and Gilbert (1996) further pointed out that when sufficient distribution bars are added, diagonal cracking would be distributed more evenly across the compressive strut. Moreover, the provision of distribution bars reduces transverse strains and hence increases the efficiency of the strut. Foster and Gilbert(1997) assessed the web splitting failure mode by a strut-tie system. They found that for an increase in the concrete compressive strength, there is a corresponding increase in the minimum distribution bars. This is because members with higher strength concrete are generally stressed to higher levels in the compression struts and thus are subject to greater bursting forces. By assuming cracked concrete maintains residual 30% of tensile strength, the minimum recommended distribution bars varied from 0.2% to 0.4%, for concrete grade f'_c from 25MPa to 80MPa, respectively.

Strength of bearing

The bottle-shaped stress field with its bulging stress trajectories develops considerable transverse stresses; comprising compression in the bottleneck and tension further away. The transverse tension can cause longitudinal cracks and initiate an early failure of the member. It is therefore necessary to consider the transverse tension or to reinforce the stress field in the transverse direction, when determining the failure load of the strut.

Hawkins (1968), based on 230 load bearing tests on concrete with 22MPa$<f'_c<$50MPa, suggested the following expression for unfactored bearing strength of concrete f_b

$$f_b \le \left[1 + \frac{4.15}{\sqrt{f'_c}}\left(\sqrt{\frac{A}{A_b}} - 1\right)\right] f'_c \text{ ; in MPa} \qquad (11)$$

Where A and A_b represents the area of supporting surface and the area of bearing plate, respectively.

Schlaich *et al*. (1987) suggested that the concrete compressive stresses within an entire disturbed region can be considered safe if the maximum bearing stress in all nodal zones is limited to $0.6f'_c$, or in unusual cases $0.4\ f'_c$, for design purposes.

Bergmeister *et al*. (1991) recommended that for an unconfined node with bearing plate, the factored bearing strength can be determined by

$$f_b \leq (0.5 + 1.25/\sqrt{f'_c})(A/A_b)^{0.5} f'_c \qquad (12)$$

Adebar and Zhou (1993) suggested an equation and values of the bearing strength of concrete compressive struts confined by plain concrete, based on the results of analytical and experimental studies. The maximum bearing stress when designing deep members without sufficient reinforcement and without internal cracks is limited to

$$f_b \leq 0.6(1 + 2\alpha\beta) f'_c \qquad (13a)$$

where

$$\alpha = 0.33\left(\sqrt{A/A_b} - 1\right) \leq 1.0 \qquad (13b)$$

$$\beta = 0.33\left(h/b - 1\right) \leq 1.0 \qquad (13c)$$

The ratio *h*/*b* represents the aspect ratio (height/width) of the compressive strut. The parameter α accounts for the amount of confinement, while the parameter β accounts for the geometry of the compression stress field. The lower bearing stress limit of $0.6f'_c$ was suggested if there is no confinement, regardless of the height of the compression strut, as well as when the compression strut is relatively short, regardless of the amount of confinement. The upper limit of $1.8\,f'_c$ was suggested. If the concrete compressive strength is significantly greater than 34.5MPa, a limit for the bearing stress was suggested of

$$f_b \leq 0.6\left(1+\frac{10\alpha\beta}{\sqrt{f_c'}}\right)f_c' \text{; MPa.} \tag{14}$$

The ultimate bearing load is found to be 1.83 times that of the uncracked bearing load, as given in equations(13) and (14). Table 9 summarized the bearing stress level determined from equations (11) to (14). It is found that the expressions suggested by Adebar and Zhou (1993), which preclude shear failure due to transverse splitting of a compression strut, are relatively conservative. When comparing the ultimate bearing stress level of Adebar and Zhou (1993) with Hawkins (1968), it is found that the bearing stress levels are similar to each other for the lower strength concrete strength and are smaller then the equations of Adebar and Zhou for higher strength concrete. Experimental tests of pile cap by Adebar *et al*.(1990), indicated that the average values of the critical bearing stress at failure was 1.2 f'_c.

Based on the experimental test results of two-dimensional plain concrete under biaxial stresses, Kupfer and Hilsdorf(1969) determined the maximum effective stress level of concrete strut of 1.0f'_c and 1.22f'_c under uniaxial compression, and under biaxial compression respectively. Yun and Ramirez (1996) used those stress levels to define the strength of concrete struts in their numerical model and found good agreement with the experimental results. Bergmeister *et al*.(1991) suggested a higher value of 2.5 when the node is subjected to the triaxial confinement state. The strength of a node may be further increased up to 5-20% depending on the confinement provided by reinforcement or any anchorage or bearing plate (Yun 1994).

Anchorage

Safe anchorage of ties in the node has to be assured: minimum radii of bent bars and anchorage lengths of bars are selected following the code recommendations. The tension tie reinforcement must be uniformly distributed over an effective area of concrete at least equal to the tie force divided by the concrete stress limits for the node. The anchorage must be located within and 'behind' the nodes. The anchorage begins where the transverse compression stress trajectories meet the bars and are deviated. The bar must extend to the other end of the node region. If this length is less than required by the code, the bar may be extended beyond the node region. The tensile forces introduce behind the node can resist the remaining forces developed within the nodal regions.

Suggested design formula for strut-tie models

From the above study, we find that the Canadian Code recommended the design formula of strut [equation (5)] which is a function of the orientation of the strut as well as the strains of both concrete and steel. This is considered to be the most rational approach. However, this formula did not take into account the brittle effects as the strength of concrete increases. In this paper, we adopt the approach from MacGregor(1997) assuming the efficiency factor of struts as a product of two partial safety factors, as shown below

$$f_{cd} = \phi \nu_1 \nu_2 f_c' \tag{15a}$$

where ϕ=0.67 (15b)

$$\nu_1 = \frac{1}{1.14 + 0.75\cot^2\theta} \tag{15c}$$

and $$\nu_2 = 1.15(1 - f_c'/250) \tag{15d}$$

The first partial safety factor ν_1 originates from the modified Collins and Mitchell relationship, taking into account the orientation and the extent of cracks. The second partial safety factor ν_2 from the Model Code 90, incorporates the brittle effects as the strength of concrete increases. The comparison of the proposed equations with the Canadian Code as well as equation (8) of Warwick and Foster (1993) is shown in Figure 1. The proposed strength of strut is in general conservative compared with that from Warwick and Foster (1993). For lower strength concrete, f'_c<40MPa, the proposed strength of strut is slightly higher than that from the Canadian Code. However for higher strength concrete, f'_c>40MPa, the proposed strength of strut predicts lower values, as the brittle effects of high strength concrete have been considered.

To relate the concrete cube strength with concrete cylinder strength, we may use the relationship by L'Hermite (1955), namely

$$f_c' = \left(0.76 + 0.2\log_{10}\frac{f_{cu}}{19.582}\right)f_{cu} \tag{16}$$

The design strength of strut, assuming the partial safety factor ϕ to be 0.67, has been evaluated in Table 10.

By adopting the similar approach (product form) of efficiency factor for the strength of node, the strength of node may be determined by the following formula

$$f_{cd} = \phi \eta_1 \eta_2 f_c' \qquad (17)$$

where, ϕ=0.67, $\eta_2 = 1.15(1 - f_c' / 250)$ and η_1 = 1.0, 0.85, 0.75, 0.65 and 0.5 for nodes with triaxial stress state, CCC, CCT, CTT stress states and most adverse stress state, respectively. The proposed partial safety factor η_1 is generally in line with the values shown in Table 5 according to various researchers and in Table 7 for Eurocode, the Canadian Code and the New Zealand Code. The design strength of a node expressed in concrete cylinder strength and the cube strength is shown in Table 11a and Table 11b, respectively.

The bearing strength of unconfined concrete suggested by Adebar and Zhou(1993) in equations(13) and (14), which precludes shear failure due to transverse splitting of a compression strut, is considered to be appropriate for the service load condition. As the ultimate loads are usually higher than the service loads by roughly 30%, whereas the experimental result from Adebar and Zhou indicated that the ultimate bearing stress is higher than the uncracked bearing stress by 80%, the design ultimate strength could be determined conservatively by multiply equations (13) and (14) by 0.87(=1.3×0.67), where 0.67 is the partial safety factor for concrete. The design bearing strength expressed in concrete cylinder strength and cube strength are shown in Table 12a and Table 12b, respectively. Design values shown in Table 12a and 12b ensure that the un-reinforced concrete node supported by a steel bearing plate would not crack under service conditions.

Conclusions

The strength of struts, ties and nodes of a strut-tie system has been reviewed in this paper. The design formula proposed for strut has been taken into account explicitly the orientation of strut-tie, the brittle effects as the strength of concrete increases, as well as implicitly the strains of both concrete and steel. The design formula proposed for a node adopted the efficiency factor of nodes as a product of two partial safety factors. Due consideration has been given to the brittle effects as the strength of concrete increases, and to the stress state of the boundary of node. The design values proposed for plain concrete with bearing plate ensure that the node would not crack at service conditions and possesses sufficient strength under ultimate load conditions. To enhance the worldwide use of such design tables, both the concrete cylinder strength and the concrete cube strength were used to define the strength of concrete.

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Table 1. Effective stress level for concrete strut

Sources	Efficiency Factor ν for Strut	
Uncracked strut with uniaxial state of compressive stress		
Nielsen et al.(1978)	0.50	$(0.7-f'_c/200)$; $f'_c<60$MPa
Rogowsky and MacGregor(1986)	0.85	
Schlaich et al. (1987)	0.85	
Alshegeir (1992a,b)	0.80-0.95	
Warwick and Foster (1993)	0.85	
Foster and Gilbert(1996)	0.85	
Cracks parallel to the strut with normal crack width. (assuming θ=60°)		
Schlaich et al. (1987)	0.68	
Alshegeir(1992a,b)	0.75	
Warwick and Foster (1993)	0.81	$1.25-\frac{f'_c}{500}-0.72\cot\theta+0.18\cot^2\theta$
Foster and Gilbert(1996)	0.72	$1/\{1.14+0.75\cot^2\theta\}$
Cracks skewed to the strut with severe crack width. (assuming θ=45°)		
Schlaich et al. (1987)	0.51	
Alshegeir(1992a,b)	0.50	
Warwick and Foster (1993)	0.63	$1.25-\frac{f'_c}{500}-0.72\cot\theta+0.18\cot^2\theta$
Foster and Gilbert(1996)	0.53	$1/\{1.14+0.75\cot^2\theta\}$
Minimum strength of strut (assuming $\theta\leqq 30°$)		
Schlaich et al. (1987)	0.34	
Alshegeir(1992a,b)	0.2-0.25	
Warwick and Foster (1993)	0.45	$0.53-f'_c/500$
Foster and Gilbert(1996)	≅0.25	$1/\{1.14+0.75\cot^2\theta\}$

Note: f'_c assumed to be 40MPa
Ratio of $a/d=\cot\theta$, where θ represents the angle between the strut and tie

Table 2. Normalized efficiency factors of strut against the extent and the orientation of cracks

Sources	Normalized Efficiency Factors of Struts			
	Uncracked strut	Cracks paralled to the strut	Cracks skewed to the strut	Minimum strength of strut
Schlaich et al. (1987)	1.0	0.80	0.60	0.40
Alshegeir(1992a,b)	1.0	0.88	0.59	0.26
Warwick and Foster (1993)	1.0	0.94	0.72	0.50
Foster and Gilbert(1996)	1.0	0.84	0.63	0.30
MacGregor(1997)	1.0	0.80	0.55	0.28

The efficiency factors in the table are normalized by the factor of 0.85

Table 3. Partial Safety Factors for Loads

Design standards	**Load factors D+L**	**Combined Load factors***	**Load Adjustment Factor μ**
CEB-FIP: 1990	1.35D+1.5L	1.725	1.000
ENV 1992-1-1:1992	1.35D+1.5L	1.725	1.000
CSA Standard A23.3-94	1.25D+1.5L	1.625	1.062
NZS3101:Part2:1995	1.20D+1.6L	1.600	1.081
AS3600-1994	1.25D+1.5L	1.625	1.062
ACI 318-1995	1.40D+1.7L	1.825	0.945
BS8110-1997	1.40D+1.6L	1.800	0.958
GBJ 10-89	1.20D+1.4L	1.550	1.113

* Assuming live load to dead load ratio is 0.25

Table 4. Codified Stress level in Concrete Strut

Design Standards	**Partial Safety Factor**	**Codified Strength of Strut**	**Design Strength^{+} f_{cd}**	**Adjusted Design Strength$^{\&}$**
CEB-FIP: 1990	ϕ=0.67 $f'_c\leq$80MPa	$\phi\,0.85(1-f'_c/250)f'_c$ uncracked strut $\phi\,0.60(1-f'_c/250)f'_c$ cracked strut	0.48 f'_c 0.34 f'_c	0.48 f'_c 0.34 f'_c
ENV 1992-1-1:1992	ϕ=0.67 $f'_c\leq$50MPa	$\phi f'_c$ triaxial load $0.6\phi f'_c$ uniaxial load	0.67 f'_c 0.40 f'_c	0.67 f'_c 0.40 f'_c
CSA Standard A23.3-94	ϕ=0.6 $f'_c\leq$80MPa	$\phi f'_c/(0.8+170\varepsilon_1)$ $<0.85\phi f'_c$ $\varepsilon_1=\varepsilon_s+(\varepsilon_s+0.002)\cot^2\theta$	0.51 f'_c, θ=90^o $^{\#}$ 0.44 f'_c, θ=60^o 0.33 f'_c, θ=45^o 0.19 f'_c, θ=30^o	0.54 f'_c 0.47 f'_c 0.35 f'_c 0.20 f'_c
NZS3101: Part2:1995	ϕ=0.8 $f'_c\leq$70MPa	$0.65\phi f'_c$; CCC $0.55\phi f'_c$; CCT $0.45\phi f'_c$; CTT	0.52 f'_c 0.44 f'_c 0.36f'_c	0.56 f'_c 0.46 f'_c 0.38f'_c
AS3600-1994	ϕ=0.7 $f'_c\leq$50MPa	$\phi(0.8-f'_c/200)f'_c$	0.42 f'_c	0.45 f'_c
Equivalent to ACI-318-1995*	$f'_c\leq$55MPa	$\nu_2 f'_c$ uncracked strut $0.80\nu_2 f'_c$ cracked strut with transverse rebars $0.65\nu_2 f'_c$ cracked strut without transverse rebars $0.55\nu_2 f'_c$ severely cracked slender beam, θ=45^o $0.30\nu_2 f'_c$ severely cracked slender beam, θ=30^o $\nu_2=(0.55+1.25/\sqrt{f'_c})$	0.75 f'_c 0.60 f'_c 0.49 f'_c 0.41 f'_c 0.22 f'_c	0.71 f'_c 0.57 f'_c 0.46 f'_c 0.39 f'_c 0.21 f'_c

Note: $^{+}f'_c$ assumed to be 40MPa
*Referred to MacGregor (1997)
$^{\#}30^o<\theta_s<90^o$; $0<\varepsilon_s<0.002$
$^{\&}$Adjusted design strength = μf_{cd}

Table 5. Effective Stress Level for Concrete Node

Sources		**Efficiency Factor η for Node**
CCC node		
Collins *et al*. (1986)	0.85	
Schlaich *et al*. (1987)	0.85	
MacGregor (1988)	0.85	
Schlaich *et al*. (1991)	0.94	
Bergmeister *et al*. (1991)	2.50	Triaxially confined nodes
	0.76	Unconfined nodes
		0.8, $f'_c \leqq 27.6$MPa
		(0.9-0.25 f'_c/69), $27.6 \leqq f'_c \leqq 69$MPa
		0.65, $f'_c \geqq 69$MPa
CCT node		
Collins *et al*. (1986)	0.75	
Schlaich *et al*. (1987)	0.68	
MacGregor (1988)	0.65	
Schlaich *et al*. (1991)	0.68	
Jirsa *et al*. (1991)	0.80	
CTT node		
Collins *et al*. (1986)	0.60	
Schlaich *et al*. (1987)	0.68	
MacGregor (1988)	0.50	
Schlaich *et al*. (1991)	0.68	
Jirsa *et al*. (1991)	0.80	

Note: f'_c assumed to be 40MPa

Table 6. Normalized efficiency factors of nodes under different boundary conditions

Sources	**Normalized Efficiency Factors of Nodes**		
	CCC	CCT	CTT
Collins *et al*. (1986)	1.0	0.88	0.70
Schlaich *et al*. (1987)	1.0	0.80	0.80
MacGregor (1988)	1.0	0.76	0.59
Schlaich *et al*. (1991)	1.1	0.80	0.80
Jirsa *et al*. (1991)	--	0.94	0.94

The efficiency factors in the table are normalized by the factor of 0.85

Table 7. Codified Stress Level for Concrete Node

Design Standards	Partial Safety Factor	Codified Strength of Nodes	Design Strength[+] f_{cd}	Adjusted Design Strength[&]
CEB-FIP: 1990	ϕ=0.67 $f'_c \le$80MPa	$\phi\,0.85(1-f'_c/250)f'_c$ CCC $\phi\,0.60(1-f'_c/250)f'_c$ C&T	$0.48\,f'_c$ $0.34\,f'_c$	$0.48\,f'_c$ $0.34\,f'_c$
ENV 1992-1-1:1992	ϕ=0.67 $f'_c \le$50MPa	$\phi\eta\,f'_c$ η=1.0 triaxial η=0.85 CCC η=0.7 CCT η= (0.7-f'_c/200) >0.5 CTT	 $0.67\,f'_c$ $0.56\,f'_c$ $0.46\,f'_c$ $0.34\,f'_c$	 $0.67\,f'_c$ $0.56\,f'_c$ $0.46\,f'_c$ $0.34\,f'_c$
CSA Standard A23.3-94	ϕ=0.6 $f'_c \le$80MPa	$0.85\phi f'_c$ CCC $0.75\phi f'_c$ CCT $0.65\phi f'_c$ CTT	$0.51\,f'_c$ $0.45\,f'_c$ $0.39\,f'_c$	$0.54\,f'_c$ $0.48\,f'_c$ $0.41\,f'_c$
NZS3101: Part2:1995	ϕ=0.8 $f'_c \le$70MPa	$0.65\phi f'_c$ CCC $0.55\phi f'_c$ CCT $0.45\phi f'_c$ CTT	$0.52\,f'_c$ $0.44\,f'_c$ $0.36\,f'_c$	$0.56\,f'_c$ $0.48\,f'_c$ $0.39\,f'_c$
AS3600-1994	ϕ=0.7 $f'_c \le$50MPa	$\phi\,(0.8-f'_c/200)f'_c$	$0.42\,f'_c$	$0.45\,f'_c$
Equivalent to ACI-318-1995[*]	$f'_c \le$55MPa	$1.00\,\eta_2\,f'_c$, CCC $0.85\,\eta_2\,f'_c$, CCT $0.75\,\eta_2\,f'_c$, CTT $\eta_2=(0.55+1.25/\sqrt{f'_c})$	$0.75\,f'_c$ $0.63\,f'_c$ $0.56\,f'_c$	$0.71\,f'_c$ $0.60\,f'_c$ $0.53\,f'_c$

Note: [+] f'_c assumed to be 40MPa
[*] Referred to MacGregor (1997)
[&] Adjusted design strength = μf_{cd}

Table 8. Codified Stress Level for Concrete Node

Design Standards	Partial safety factor	Codified Strength of Strut
CEB-FIP: 1990	ϕ=0.87	$0.87f_y$
ENV 1992-1-1:1992	ϕ=0.87	$0.87f_y$
CSA Standard A23.3-94	ϕ=0.85	$0.85f_y$
NZS3101:Part2:1995	ϕ=0.87	$0.87f_y$
AS3600-1994	ϕ=0.70	$0.70f_y$

Table 9. Effective Stress Level for Bearing Strength of Concrete Node

***A*/*A*$_b$**	**Effective Stress Level of Bearing Stress**		
	Hawkins (1968)[#]	Bergmeister et al.(1991)[*]	Adebar and Zhou (1993)[+]
f'$_c$=30MPa			
9.0	2.52	2.18	1.12 (2.05)
4.0	1.76	1.46	0.86 (1.58)
2.5	1.44	1.15	0.79 (1.38)
1.0	1.00	0.73	0.60 (1.10)
f'$_c$=40MPa			
9.0	2.31	2.09	1.01 (1.85)
4.0	1.66	1.40	0.81 (1.48)
2.5	1.38	1.10	0.72 (1.32)
1.0	1.00	0.70	0.60 (1.10)
f'$_c$=60MPa			
9.0	2.07	1.98	0.94 (1.72)
4.0	1.54	1.32	0.77 (1.41)
2.5	1.31	1.05	0.70 (1.28)
1.0	1.00	0.66	0.60 (1.10)

[#] unfactored stress level
[*] factored stress level
[+] unfactored stress level and *h*/*b* assumed to be 3.0, values in the parenthesis represent the ultimate effective bearing stress (1.83 × uncracked effective bearing stress)

Table 10. Design Strength of Strut in Cube Strength

Angle	**Cube Strength of Concrete f_{cu} (MPa)**					
θ	30	35	40	45	60	80
90.0°	0.45	0.45	0.45	0.45	0.45	0.43
75.0°	0.45	0.45	0.45	0.45	0.44	0.41
60.0°	0.40	0.40	0.40	0.39	0.38	0.35
52.5°	0.35	0.35	0.35	0.35	0.33	0.31
45.0°	0.29	0.29	0.29	0.29	0.28	0.26
37.5°	0.23	0.23	0.23	0.23	0.22	0.20
30.0°	0.16	0.16	0.16	0.16	0.16	0.14

(1) Maximum strength of 0.45fcu is assumed
(2) Partial safety factor of 0.67 is allowed

Table 11a. Design Strength of Node in Cylinder Strength

Conditions of Node	**Cylinder Strength of Concrete *f'*$_c$ (MPa)**			
	30	40	60	80
Triaxial CCC	0.68	0.65	0.59	0.52
Uniaxial CCC	0.58	0.55	0.50	0.45
CCT	0.51	0.49	0.44	0.39
CTT	0.44	0.42	0.38	0.34
Minimum	0.34	0.32	0.29	0.26

Partial safety factor of 0.67 is allowed

Table 11b. Design Strength of Node in Cube Strength

Conditions of Node	**Cube Strength of Concrete f_{cu} (MPa)**					
	30	35	40	45	60	80
Triaxial CCC	0.56	0.55	0.55	0.55	0.52	0.49
Uniaxial CCC	0.47	0.47	0.47	0.46	0.45	0.41
CCT	0.42	0.42	0.41	0.41	0.39	0.37
CTT	0.36	0.36	0.36	0.35	0.34	0.32
Minimum	0.28	0.28	0.28	0.27	0.26	0.24

Partial safety factor of 0.67 is allowed

Table 12a. Design Strength of Plain Concrete Node with Bearing Plate Expressed in Cylinder Strength

	Cylinder Strength of Concrete f'_c (MPa)			
A/A$_b$	30	40	60	80
9.0	0.98	0.88	0.82	0.78
4.0	0.75	0.70	0.67	0.65
2.5	0.65	0.63	0.61	0.60
1.0	0.52	0.52	0.52	0.52

Partial safety factor of 0.67 is allowed

Table 12b. Design Strength of Plain Concrete Node with Bearing Plate Expressed in Cube Strength

	Cube Strength of Concrete f_{cu} (MPa)					
A/A$_b$	30	35	40	45	60	80
9.0	0.80	0.80	0.80	0.74	0.72	0.70
4.0	0.60	0.60	0.60	0.59	0.58	0.58
2.5	0.52	0.52	0.52	0.52	0.51	0.50
1.0	0.45	0.45	0.45	0.45	0.45	0.45

Partial safety factor of 0.67 was allowed

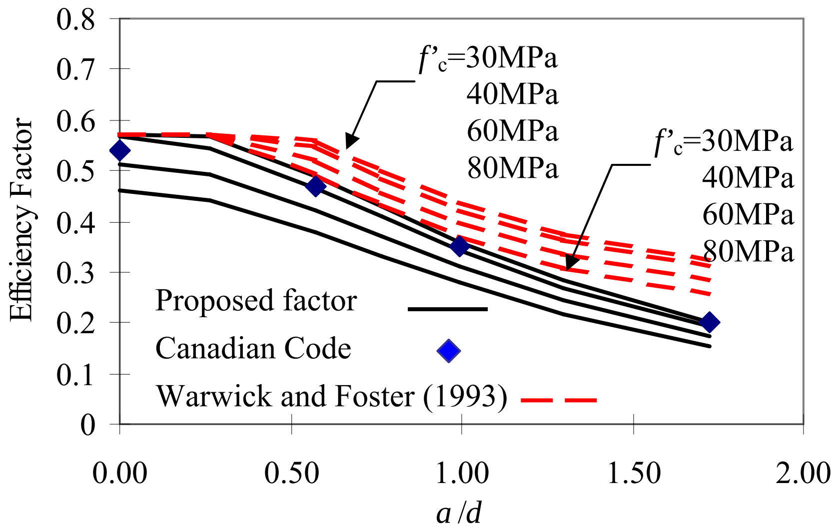

Figure 1. Proposed efficiency factor of strut