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Concentric Panorama Geometry

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Abstract— **In this paper, the geometric relationship between mosaic images for concentric panorama acquired by slit cameras along circular paths is considered. A concentric epipolar constraint is derived for mosaic images. It is shown how the concentric epipolar constraint can be used to calibrate the slit camera using point correspondences in two mosaic images. The concentric epipolar constraint is further extended to trifocal geometric relationships for three mosaic views.**

Keywords— **concentric panoramas; mosaic images; epipolar geometry; fundamental matrix; scene navigation**

I. Introduction

Concentric panorama has been shown to be an effective basis for rendering scenes in real-time scene navigation [1]. With concentric panorama, it is possible to synthesize the scene realistically while moving the viewpoint freely within a circular region on a plane. In this paper, we will discuss the problem of representations of points and lines in concentric panorama and consider some novel concentric panorama geometries between mosaic images.

In concentric panorama, a slit camera is mounted on a rotary table at a known radius with view axis tangential to the circular path. The slit camera captures a column 1-D image at equally spaced points along the circular path. After a complete revolution, a panoramic image can be formed by gluing all the column images together. This is called a mosaic image. Repeating the above process for different radii, we can collect a set of mosaic images.

The relationship of corresponding points in two 2-D planar images can be described by well-established epipolar geometry. A point in an image can be back-projected into 3-D space as a ray passing through the camera origin and the point. This ray is then projected on the other image producing the epipolar line. Similarly, the corresponding point on the second image can be projected as an epipolar line on the first image. This mutual relationship can be described by a 3×3 matrix called the fundamental matrix [2], which relates corresponding points on both images directly without any prior knowledge about the parameters of the camera.

In this paper, we will define the slit-camera model to project a point and a line from 3-D space to a mosaic image. The projection of a 3-D point on a mosaic image is still a point. However, the projection of a line from 3-D space on a mosaic image becomes a curve [3], which will be referred to as a 'mosaic line'. We will obtain the nonlinear equation for this curve.

To extend epipolar geometry to mosaic images, a ray is back-projected from the camera origin through a point on the mosaic image. This ray lies on a vertical plane that is tangential to the circular path and it touches the circular path at the camera origin. The projection of this ray on another mosaic image is a curve which can be represented by the equation of a mosaic line. This is called an epipolar curve. With the properties of epipolar curves, the idea of the fundamental matrix can be extended to mosaic images. We will derive the mutual relationship between any two mosaic images in a way similar to the role of the fundamental matrix for 2-D planar images. Two mosaic images are said to be weakly calibrated [4] if this relationship is known.

Furthermore, given two corresponding points on two mosaic images M_1 and M_2, which are weakly calibrated with a third mosaic images M_3, we will show that it is possible to locate the corresponding point on the third mosaic image. Projecting the given corresponding points on M_1 and M_2 to two epipolar curves on M_3, the intersection of these two epipolar curves will give the required point. This is similar to the use of trifocal tensors for relating a triple of views in 2-D planar images. There are two trifocal constraints for mosaic images. One relates all three vertical elevation values y of the corresponding points. Another relates all the three rotated angles θ. A partial 3-D reconstruction (X-Z plane only) can be performed by three views. We will also consider how to calibrate the slit camera with fixed intrinsic parameters using point correspondences in mosaic images.

II. Image Acquisition System

Fig.1 shows the top-view of the acquisition system for concentric panorama. A slit camera is mounted on a rotary table at a known distance r from the centre with view axis tangential to the circular path. The slit camera captures a column 1-D image at equally spaced points along the circular path. After a complete revolution, a panoramic image can be formed by gluing all the column images together. This is called a mosaic image as shown in Fig. 2. At any angular position θ, let the image coordinate of the 1-D image captured by the slit camera

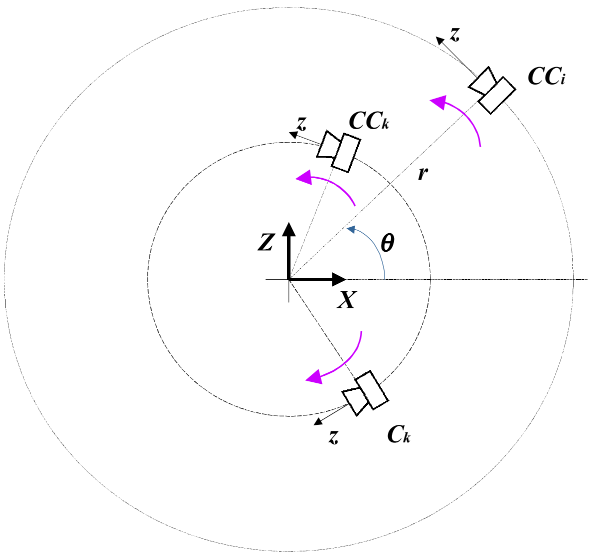

Fig. 1. Aquisition System Configuration

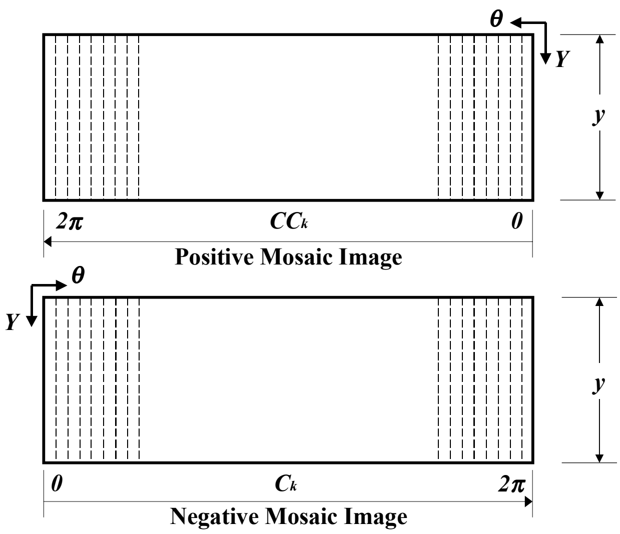

Fig. 2. Two different orientations of Mosaic Images

be denoted by y. Then, image coordinate of the mosaic image is given by $[\theta, y]^T$, where $\theta \in [0, 2\pi)$ and y lies in the vertical image range of the slit camera. The radius r of the camera is fixed for a mosaic image. Repeating the 1-D image gathering process for different radii, we can collect a set of mosaic images, each labelled by its value of r.

We define the world coordinate system with the origin set at the centre of the rotary table, the X-Z plane taken as the horizontal plane and the vertical axis as the Y-axis with positive direction pointing downward. The angular position θ of the camera is measured from the X-axis with the positive sense in the counter-clockwise direction.

Suppose the view direction of the camera is defined to be the same as the rotating direction. For each circular

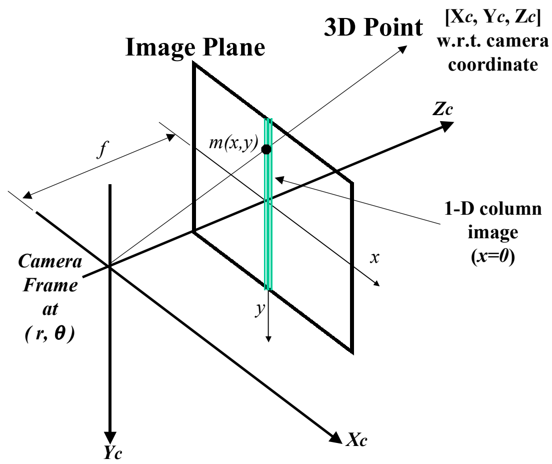

Fig. 3. Slit Camera Model

path, there are two possible view directions, corresponding to counter-clockwise or clockwise rotation of the rotary table. When the rotation is counter-clockwise, the mosaic image is called a positive mosaic image. Clockwise rotation produces negative mosaic images. We will only consider positive mosaic images in this paper.

A. Slit Camera Model

The extrinsic parameters of a camera describe the location of the camera in the 3-D world coordinate system. The extrinsic parameters of the slit camera are given by the angle θ and radius r since they completely determine the location and orientation of the camera.

Given a 3-D point $\begin{bmatrix} X & Y & Z \end{bmatrix}^T$. This point will be seen by the slit camera at only one angular position θ on a circular path. To obtain the projection of the 3-D point onto the slit camera placed at the appropriate θ, consider a general pin-hole camera in stead of a slit camera placed at position (r, θ). The advantage of using with a pin-hole camera model to start with rather than the simpler slit camera model will become apparent below. We can then turn the pin-hole camera into a slit camera by restricting the image to the vertical axis. Let x-y be a coordinate system defined on the image plane and let the intrinsic parameter matrix of the pin-hole camera be given by

$$A = \begin{bmatrix} s_x & 0 & c_x \\ 0 & s_y & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

where s_x and s_y are the horizontal and vertical scale factors (dependent on the pixel size and the focal length f) and (c_x, c_y) is the principle point on the image plane. Since the image of the slit camera will be confined to the y-axis, we may assume without loss of generality that $s_x = 1$ and $c_x = 0$.

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When the camera is at the position (r, θ), the transformation from the camera frame back to the world coordinate system is

$$R = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}, t = \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix}$$

where R is the rotation and t is the translation of the camera. The projection matrix P of the pin-hole camera is given by

$$\begin{aligned} P &= A \begin{bmatrix} R^{-1} & -t \end{bmatrix}_{3\times 4} \\ P &= \begin{bmatrix} \cos\theta & 0 & \sin\theta & -r \\ -c_y \sin\theta & s_y & c_y \cos\theta & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \end{bmatrix} \end{aligned} \quad (1)$$

B. Projections of 3D point on mosaic image

A 3D point $M = \begin{bmatrix} X & Y & Z & 1 \end{bmatrix}^T$ (expressed in homogeneous coordinates) can be projected onto a point m_s on the image plane by the projection matrix P. Since the imaging device is a slit camera, the point m_s will appear on the slit camera image only if it has the form $\begin{bmatrix} 0 & y & 1 \end{bmatrix}^T$. Hence, we have

$$\begin{bmatrix} 0 \\ y \\ 1 \end{bmatrix} \approx \begin{bmatrix} \cos\theta & 0 & \sin\theta & -r \\ -c_y \sin\theta & s_y & c_y \cos\theta & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \quad (2)$$

where $\approx$ means equal up to a scaling factor. Note that this equation expresses not only how the 3D point $\begin{bmatrix} X & Y & Z \end{bmatrix}^T$ projects onto the vertical coordinate y of the 1D image, but also where the slit camera should be placed in order for the image point to be seen. Eliminating the scaling factor from (2), we have

$$r = X\cos\theta + Z\sin\theta \quad (3)$$

$$\frac{y - c_y}{s_y} = \frac{Y}{Z\cos\theta - X\sin\theta} \quad (4)$$

We can regard (3) and (4) as the equations for projecting $\begin{bmatrix} X & Y & Z \end{bmatrix}^T$ onto the mosaic image coordinates $[\theta, y]^T$ for a given radius r - the first equation is for locating θ and the second one for solving y. In (3), there are two possible solutions for θ corresponding to the positive and the negative mosaic images. However, the value of y is the same for both of these mosaic images.

III. Projections of a straight line in 3D space on mosaic image

A straight line in 3D space is completely determined by two given points, $M_1 = \begin{bmatrix} X_1 & Y_1 & Z_1 & 1 \end{bmatrix}^T$ and $M_2 = \begin{bmatrix} X_2 & Y_2 & Z_2 & 1 \end{bmatrix}^T$ on the line. Any point M_l lying on the line can be represented as $M_l = (1-\gamma) M_1 + \gamma M_2$, where $\gamma \in (-\infty, +\infty)$. By projecting M_l onto a mosaic image using (3) and (4) and then eliminating γ, we get the equation of the projected line, which will be referred to as a mosaic line:

$$y = c_y - s_y \frac{\alpha_0 + \gamma_1 \cos(\theta - \phi_1)}{\beta_0 + \gamma_2 \cos(\theta - \phi_2)} \quad (5)$$

where

$$\begin{aligned} \alpha_0 &= (Y_1 - Y_2)\, r \\ \beta_0 &= X_2 Z_1 - X_1 Z_2 \\ \gamma_1 &= \sqrt{(Y_2 X_1 - Y_1 X_2)^2 + (Y_2 Z_1 - Y_1 Z_2)^2} \\ \gamma_2 &= r\sqrt{(Z_2 - Z_1)^2 + (X_2 - X_1)^2} \\ \phi_1 &= \arctan\frac{Y_2 Z_1 - Y_1 Z_2}{Y_2 X_1 - Y_1 X_2} \\ \phi_2 &= -\arctan\frac{X_2 - X_1}{Z_2 - Z_1} \end{aligned}$$

In general, the projection of a line in 3-D space is no longer a straight line on the mosaic image.

IV. Epipolar Geometry

Based on the idea of epipolar geometries for 2-D planar images, we will derive in this section the epipolar constraint for concentric panoramas. A method will be proposed for determining the coefficients of the Concentric Epipolar Constraint (CEC) using point correspondences, similar to the approach of the seven-point algorithm for determining the fundamental matrix for a pair of planar images. If all the parameters for the CEC are known, the pair of mosaic images are said to be weakly calibrated.

A. Epipolar Geometry for 2-D Planar Images

Epipolar geometry for 2-D planar images describes the geometrical relationship between the image pair. The epipolar constraint says that given a point on an image, its corresponding point in another image must lie on the epipolar line, which can be found by back-projecting a ray from the camera centre passing through the given point on the first image into 3-D space, and then projecting the ray onto the second image. The epipolar constraint can be expressed in the form: $f(m, m') = m'^T F m = 0$, where F is a 3×3 matrix (called the Fundamental Matrix), and m and m' are two corresponding points on the two 2-D planar images.

B. Epipolar Geometry for Concentric Panoramas

We will now extend the idea of epipolar constraint for planar images to the case of concentric panoramas. Given an image point on an mosaic image, its correspondence in another mosaic image is a mosaic line which can be determined using the same principle as the planar

case. Let $u_j = \begin{bmatrix} \theta_j & y_j & r_j \end{bmatrix}^T$ be a point on an mosaic image j with radius r_j. A ray can be back-projected from the camera origin and passing through the point u_j into 3-D space as a straight line.
The origin of the camera is $O_k = Rt = \begin{bmatrix} r_j \cos\theta_j & 0 & r_j \sin\theta_j \end{bmatrix}^T$ w.r.t. the world coordinate system. Suppose the point u_j on the mosaic image expressed w.r.t. the image coordinate system is $M_{jc} = \begin{bmatrix} 0 & y_j & 1 \end{bmatrix}^T$. This point expressed w.r.t. world coordinate system is $M_{jk} = RA^{-1}M_{jc} + Rt$. Hence, all points lying on the back-projected ray are given by $M_{ol} = \lambda O_k + (1-\lambda) M_{jk}$, $\lambda \in R$. It can be shown that O_k and M_{jk} has coordinates given respectively by

$$O_k = \begin{bmatrix} r_j \cos\theta_j & 0 & r_j \sin\theta_j \end{bmatrix}^T, \quad (6)$$

$$M_{jk} = \begin{bmatrix} r_j \cos\theta_j - \sin\theta_j \\ \frac{y_j - c_y}{s_y} \\ r_j \sin\theta_j + \cos\theta_j \end{bmatrix} \quad (7)$$

B.1 Uncalibrated Epipolar Curve

Let $u_l = \begin{bmatrix} \theta_l & y_l & r_l \end{bmatrix}^T$ be a possible candidate on the mosaic image labelled by l corresponding to u_j. u_l must lie on the projection of the ray $O_k M_{jk}$ on the mosaic image l. This projection can be obtained by substituting (6) and (7) (as points M_1 and M_2) into (5), which yields

$$\frac{y_l - c_{yl}}{s_{yl}} = -\frac{y_j - c_{yj}}{s_{yj}} \frac{r_l - r_j \cos(\theta_l - \theta_j)}{r_j - r_l \cos(\theta_l - \theta_j)} \quad (8)$$

This equation represents the epipolar constraint for concentric panorama. Note that the constraint, unlike the planar case, is no longer linear in the image coordinates (r_j, θ_j) and (r_l, θ_l). Given (r_j, θ_j), we can regard (8) as an equation defining a curve on the (r_l, θ_l)-plane, which will be called an Uncalibrated Epipolar Curve (with c_{yl}, s_{yl}, c_{yj}, s_{yj} as unknown parameters). We will consider how the parameters in (8) can be calibrated by means of point correspondences in the sequel.

B.1.a Range of Epipolar Curve. Given a point on one mosaic image j together with known intrinsic parameters (i.e. r_l, c_{yl}, s_{yj}, etc.), we can plot the epipolar curve (8) on another mosaic image l. The epipolar curve has bounded ranges in both θ and y directions. Clearly, a straight line in 3-D space has a mosaic image which can only extend over a maximum range of π on the horizontal θ-axis. The critical points of an epipolar curve in the y-direction can be found by setting the partial derivative of y_l w.r.t. θ_l to zero:

$$\frac{\partial y_l}{\partial \theta_l} = -s_{yl} \frac{y_j - c_{yj}}{s_{yj}} \frac{\left(r_l^2 - r_j^2\right) \sin(\theta_l - \theta_j)}{\left(r_j - r_l \cos(\theta_l - \theta_j)\right)^2} = 0$$

Solving for the maximum and minimum points, we get

$$y_l \in \left(c_{yl} - (y_j - c_{yj}) \frac{s_{yl}}{s_{yj}}, c_{yl} + (y_j - c_{yj}) \frac{s_{yl}}{s_{yj}} \right)$$

B.2 Concentric Epipolar Constraint

Equation (8) shows the mutual relationship between corresponding points on a mosaic image pair, and is symmetric w.r.t. the labelling j and l of the two images. We can rewrite (8) as the Concentric Epipolar Constraint:

$$\begin{aligned} & f_c(u_l, u_j) \\ = \; & s_{yj}(y_l - c_{yl})(r_j - r_l \cos(\theta_l - \theta_j)) \\ & + s_{yl}(y_j - c_{yj})(r_l - r_j \cos(\theta_l - \theta_j)) \\ = \; & 0 \end{aligned} \quad (9)$$

(9) is the extension of the relationship $f(m, m') = 0$ for planar images to the case of concentric panorama. The coefficients of concentric epipolar constraint can be evaluated from a set of point correspondences. Define the coefficients in the constraint equation to be

$$\begin{aligned} a_0 &= -s_{yj} c_{yl} r_j - s_{yl} c_{yj} r_l && (10) \\ &= -c_{yl} a_1 - c_{yj} a_2 \\ a_1 &= s_{yj} r_j && (11) \\ a_2 &= s_{yl} r_l && (12) \\ a_3 &= s_{yj} c_{yl} r_l + s_{yl} c_{yj} r_j && (13) \\ &= -c_{yl} a_4 - c_{yj} a_5 \\ a_4 &= -r_l s_{yj} && (14) \\ a_5 &= -r_j s_{yl} && (15) \\ a_1, a_2 &> 0, \text{ and } a_4, a_5 < 0 \end{aligned}$$

By these definitions, the coefficients satisfy the conditions:

$$\begin{aligned} 0 &= a_1 a_2 - a_4 a_5 && (16) \\ 0 &\neq a_1 a_5 - a_2 a_4, \forall r_j \neq r_l && (17) \end{aligned}$$

In terms of the coefficients a_i, (9) becomes

$$\begin{aligned} 0 = \; & a_0 + a_1 y_l + a_2 y_j + a_3 \cos(\theta_l - \theta_j) && (18) \\ & + (a_4 y_l + a_5 y_j) \cos(\theta_l - \theta_j) \end{aligned}$$

Because of (16), only five among the six unknown coefficients a_i are independent. The minimum number of corresponding points for determining the coefficients (up to scale) is 4. Given four such pairs of corresponding points, we can write (18) as

$$Af = -Y_l \quad (19)$$

where

$$f = \begin{bmatrix} a_0/a_1 \\ a_2/a_1 \\ a_3/a_1 \\ a_4/a_1 \\ a_5/a_1 \end{bmatrix}, \; Y_l = \begin{bmatrix} y_l \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}_{4\times1}$$

$$A = \begin{bmatrix} 1 & y_j & \omega_{lj} & y_l \omega_{lj} & y_j \omega_{lj} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix}_{4\times5}$$

$$\omega_{lj} = \cos(\theta_l - \theta_j)$$

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in which each row of the matrix A and the vector Y_l contains the data from one pair of corresponding points. We can write the general solution to (19) as

$$f \approx f_{ls} + \alpha f_{ns}$$

where f_{ls} is the least-squares solution to (19), f_{ns} is a vector spanning the null space of A (assumed to be of rank 4) and α is an undetermined parameter. Substituting f into (16) gives a quadratic equation in α from which two possible solutions can be solved. (17) may help to eliminate one of the two solutions. Another way is to substitute other corresponding points into (18) to verify which solution is correct.

B.3 Calibration from Concentric Epipolar Constraint

We now assume that all the coefficients a_i $(i = 0, 1, ..., 5)$ have been found. From (10) and (13), the intrinsic parameters c_{yl} and c_{yj} for the two mosaic images can be determined as

$$\begin{bmatrix} c_{yl} \\ c_{yj} \end{bmatrix} = -\begin{bmatrix} a_1 & a_2 \\ a_4 & a_5 \end{bmatrix}^{-1} \begin{bmatrix} a_0 \\ a_3 \end{bmatrix} \qquad (20)$$

As $0 \neq a_1 a_5 - a_2 a_4$, there is always a unique solution. Furthermore, the ratios, r_l/r_j and s_{yl}/s_{yj} can be solved as

$$\frac{r_l}{r_j} = -\frac{a_4}{a_1}, \quad \frac{s_{yl}}{s_{yj}} = -\frac{a_5}{a_1} \qquad (21)$$

However, it is not possible to solve for the individual values using (9) alone.

B.4 Cosine Form of Concentric Epipolar Constraint

We shall need another form of the concentric epipolar constraint. After some manipulations, (9) can be expressed as

$$\cos(\theta_j - \theta_l) = \frac{\eta_y + \eta_r \eta_s}{\eta_y \eta_r + \eta_s} \qquad (22)$$

where

$$\eta_y = \frac{y_l - c_{yl}}{y_j - c_{yj}}, \quad \eta_r = \frac{r_l}{r_j}, \quad \eta_s = \frac{s_{yl}}{s_{yj}}$$

(22) will be referred to as the cosine form of the epipolar constraint.

V. Three-view Geometry

In the previous sections, the relationship between a pair of mosaic images have been studied. We will now consider the geometric relationship between three views. Suppose the third view is labelled by k with intrinsic parameters are c_{yk} and s_{yk}. Points on the third view k (corresponding to u_j and u_l in the first two views) will be denoted $u_k = [\theta_k, y_k, r_k]^T$. The mutual relationship between any possible combined pairs from the three mosaic images will be assumed to be weakly calibrated in this section.

A. Trifocal Constraints for mosaic images

In 2-D planar images, the trifocal Constraint relates three views by a trifocal tensor which is a matrix of dimension $3 \times 3 \times 3$. For mosaic images, the trifocal constraint is a nonlinear relationship, but the constraint on y and θ are decoupled so that the constraint on y does not depend on θ, and vice versa.

A.1 Trifocal Constraint for y

For simplicity of notation, we will assume that all the cameras are already calibrated and that $s_{yj} = s_{yl} = s_{yk} = 1$, $c_{yj} = c_{yl} = c_{yk} = 0$. Given three views, we have three epipolar constraints of the cosine form (22) for all combinations of two views. We seek to eliminate the θ_j, θ_k and θ_l from all the equations to relate the vertical elevations y_j, y_k and y_l. By the cosine and the sine law, we can combine the three equations to give

$$0 = \left(\sum_{i=\{j,k,l\}} \alpha_i y_i^2 \right) \left(\sum_{m \neq n \in \{j,k,l\}} \beta_{m,n} y_m^2 y_n^2 \right) \qquad (23)$$

$$\alpha_j = \left(r_l^2 - r_k^2\right) r_j^2, \; \alpha_k = \left(r_j^2 - r_l^2\right) r_k^2,$$
$$\alpha_l = \left(r_k^2 - r_j^2\right) r_l^2, \; \beta_{kl} = \beta_{lk} = \left(r_l^2 - r_k^2\right)/2$$
$$\beta_{jk} = \beta_{kj} = \left(r_k^2 - r_j^2\right)/2, \; \beta_{jl} = \beta_{lj} = \left(r_j^2 - r_l^2\right)/2$$

(23) can be expanded as

$$0 = \gamma y_l^2 y_j^2 y_k^2 + \sum_{m \neq n \in \{j,k,l\}} \lambda_{m,n} y_m^4 y_n^2 \qquad (24)$$

where

$$\lambda_{jk} = r_l^2 r_j^2 r_k^2 + r_j^4 r_k^2 - r_k^4 r_j^2 - r_j^4 r_l^2$$
$$\lambda_{jl} = r_l^2 r_j^2 r_k^2 + r_j^4 r_l^2 - r_l^4 r_j^2 - r_j^4 r_k^2$$
$$\lambda_{kj} = r_l^2 r_j^2 r_k^2 + r_k^4 r_j^2 - r_j^4 r_k^2 - r_k^4 r_l^2$$
$$\lambda_{kl} = r_l^2 r_j^2 r_k^2 + r_k^4 r_l^2 - r_l^4 r_k^2 - r_k^4 r_j^2$$
$$\lambda_{lj} = r_l^2 r_j^2 r_k^2 + r_l^4 r_j^2 - r_j^4 r_l^2 - r_l^4 r_k^2$$
$$\lambda_{lk} = r_l^2 r_j^2 r_k^2 + r_l^4 r_k^2 - r_k^4 r_l^2 - r_l^4 r_j^2$$

$$\gamma = -6 r_l^2 r_j^2 r_k^2 + r_j^4 r_k^2 + r_j^4 r_l^2 + r_l^4 r_j^2 + r_k^4 r_j^2 + r_k^4 r_l^2 + r_l^4 r_k^2$$

The mutual relationship (24) between the three mosaic images j, k and l is such that the equation remains unchanged by exchanging any two labels among j, k and l. Furthermore, the coefficients satisfy

$$0 = \gamma + \sum_{m \neq n \in \{j,k,l\}} \lambda_{m,n} \qquad (25)$$

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A.2 Trifocal Constraint for θ-axis

To derive the trifocal constraint for θ, observe that three constraints (8) can be obtained for the three combinations of pairs of views. The three equations can be combined into one by multiplication to eliminate the y terms, giving the trifocal constraint for the θ-axis:

$$\begin{aligned} 0 \; = \; & -r_j r_k r_l \left(-3+\cos(2\theta_j-2\theta_k)\right. \\ & \left. +\cos(2\theta_k-2\theta_l)+\cos(2\theta_j-2\theta_l)\right) \\ & +r_j\left(r_k^2+r_l^2\right)\left(\cos(\theta_l-2\theta_j+\theta_k)-\cos(\theta_k-\theta_l)\right) \\ & +r_k\left(r_l^2+r_j^2\right)\left(\cos(\theta_j-2\theta_k+\theta_l)-\cos(\theta_j-\theta_l)\right) \\ & +r_l\left(r_j^2+r_k^2\right)\left(\cos(\theta_j-2\theta_l+\theta_k)-\cos(\theta_j-\theta_k)\right) \end{aligned}$$

A.3 Prediction by Trifocal Constraints

From (24) and given y_j, y_l, r_j, r_l and r_k, there are two possible solutions for y_k^2:

$$y_k^2 = \frac{\left(r_j^2-r_k^2\right) r_l^2 y_l^2 + \left(r_k^2-r_l^2\right) r_j^2 y_j^2}{r_k^2\left(r_j^2-r_l^2\right)}$$

$$\text{or} \quad y_k^2 = \frac{\left(r_j^2-r_l^2\right) y_l^2 y_j^2}{\left(r_k^2-r_j^2\right) y_j^2 + \left(r_l^2-r_k^2\right) y_l^2}$$

Hence, there are a total of 4 possible solutions in y_k. Although we have derived (24) for calibrated cameras. The results can be easily adopted for uncalibrated camera by substituting $\dfrac{y-c_y}{s_y}$ for y in the above formulas.

VI. 3-D reconstruction

We will consider reconstructing only the X and Z coordinates of the 3-D point from point correspondences in three views. From (3), we can construct a set of equation for each 3-D point seen in three images j, k and l as:

$$\begin{bmatrix} \cos\theta_j & \sin\theta_j & -1 & 0 & 0 \\ \cos\theta_k & \sin\theta_k & 0 & -1 & 0 \\ \cos\theta_l & \sin\theta_l & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} X \\ Z \\ r_j \\ r_k \\ r_l \end{bmatrix} = \underline{0} \quad (26)$$

Note that only the θ values of the point correspondences are used in (26). Given n corresponding points in three mosaic images, the total number of equations is $3n$. The total number of unknowns is $2n+3$ (including the (X, Z) coordinates of the n points and 3 unknown radii). Hence, the condition for solvability is

$$\begin{aligned} 3+2n & \leq 3n \\ \text{i.e. } 3 & \leq n \end{aligned}$$

With 3 or more corresponding points, the (X, Z) components of the points as well as r_j, r_k and r_l and be solved up to a scaling factor by applying a Singular Value Decomposition (SVD) to (26). In practice, the radii of the mosaic images are likely to be known. As long as one such r is given, X and Z for each 3-D point can be reconstructed. For the depth correction [1] of real-time rendering of concentric mosaics, the positions of those 3-D points on the X-Z plane (top-view) are more important than the value of Y.

VII. Conclusion

In this paper, we have obtained the geometrical relationship of corresponding points between a pair of concentric mosaic image by extending the epipolar constraint for planar images. It is shown that this relationship can be expressed as a concentric epipolar constraint equation which can be regarded as the counterpart of the Fundamental matrix relationship for planar image geometry. A method is proposed to calibrate the camera using concentric epipolar constraint. A method for identifying the parameters of the CEC has also been proposed similar to the seven-point algorithm for identifying the fundamental matrix in the case of planar images. The results of this paper can be used to perform 3-D reconstruction from concentric mosaic images. Depth information recovered from mosaic images can be used for the purpose of horizontal depth correction in the rendering process.

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