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Robust H_∞ Filtering for 2-D Stochastic Systems [1]

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Abstract - This paper investigates the problem of H_∞ □lter design for 2-D stochastic systems. The stochastic perturbation is □rst introduced into the well-known Fornasini-Marchesini local state-space (FMLSS) model. Our attention is focused on the design of full-order and reduced-order □lters, which guarantee the □ltering error system to be mean-square asymptotically stable and has a prescribed H_∞ disturbance attenuation performance. Sufficient conditions for the existence of such □lters are established in terms of linear matrix inequalities (LMIs), and the corresponding □lter design is cast into a convex optimization problem which can be efficiently handled by using available numerical software. In addition, the obtained results are further extended to more general cases where the system matrices also contain uncertain parameters. The most frequently used ways of dealing with parameter uncertainties, including polytopic and norm-bounded characterizations, are taken into consideration.

Keywords: 2-D systems, H_∞ □ltering, linear matrix inequality, stochastic perturbation.

I. Introduction

Many practical systems can be modeled as two-dimensional (2-D) systems, such as those in image data processing and transmission, thermal processes, gas absorption and water stream heating etc. [16], [20]. Therefore, in recent years much attention has been devoted to the analysis and synthesis problems for 2-D discrete systems, and many important results are easily available in the literature. See, for instance, [13], [14], [19] *investigate the stability of 2-D systems through* Lyapunov approaches, [5], [24] are concerned with the controller and □lter design problems, and [6] addresses the model approximation problem for 2-D digital □lters etc. Very recently, owing to the development of linear matrix inequality (LMI) technique, these obtained results have been further extended to systems with deterministic parameter uncertainties (including norm-bounded uncertainty as well as polytopic uncertainty).

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On the other hand, since stochastic modeling has come to play an important role in many branches of science such as biology, economics and engineering applications, systems with stochastic perturbations have drawn much attention from researchers working in related areas. This kind of systems has been called systems with random parametric excitation [1], stochastic bilinear systems [17], [22] and linear stochastic systems with multiplicative noise [11], [12], [23]. Analysis and synthesis for stochastic systems have been investigated extensively and many fundamental results for deterministic systems have been extended to stochastic cases. To mention a few, the analysis of asymptotic behaviour can be found in [18]; the optimal control problems were reported in [12], [23]; and very recently the robust control and □ltering results have also been extended to stochastic systems [7], [15], [22]. However, most of the aforementioned results are concerned with 1-D stochastic systems, and to the best of the authors' knowledge, no effort has been made toward investigating the problems arising in 2-D stochastic systems.

In this paper, we make an attempt to solve the H_∞ □ltering problem for 2-D systems with stochastic perturbations. More speci□cally, the 2-D stochastic system model under investigation is established by introducing stochastic perturbations into the well-known Fornasini-Marchesini local state-space (FMLSS) model. These perturbations are governed by a white-noise signal and cause the system matrices □uctuate above their deterministic nominal values. Our attention is focused on the design of full-order and reduced-order □lters, which guarantee the □ltering error system to be mean-square asymptotically stable and has a prescribed H_∞ disturbance attenuation performance. Sufficient conditions for the existence of such □lters are established in terms of LMIs, and the corresponding □lter design is cast into a convex optimization problem which can be efficiently handled by using available numerical software. In addition, the obtained results are further extended to more general cases where the *system matrices also contain uncertain parameters*. The most frequently used ways of dealing with parameter uncertainties, including polytopic and norm-bounded characterizations, are taken into consideration.

The remainder of this paper is organized as follows. The problem of H_∞ filtering for 2-D stochastic systems is formulated in Section 2. In Section 3, both analysis and synthesis results are presented for systems with exactly known matrix data. These obtained results are further extended in Section 4 to more general cases whose system matrices also contain uncertain parameters, and we conclude this paper in Section 5. For space limitation, all the proofs are omitted which can be found in the full version of this paper.

Notations: The notations used throughout the paper are fairly standard. The superscript "T" stands for matrix transposition; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ is the set of all real matrices of dimension $m \times n$ and the notation $P > 0$ means that P is real symmetric and positive definite; I and 0 represent identity matrix and zero matrix; $|\cdot|$ refers to the Euclidean vector norm; and $\lambda_{\min}(\cdot), \lambda_{\max}(\cdot)$ denote the minimum and the maximum eigenvalue of the corresponding matrix respectively. In symmetric block matrices or long matrix expressions, we use an asterisk $(*)$ to represent a term that is induced by symmetry and diag$\{\ldots\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. In addition, $\mathbb{E}\{x\}$ and $\mathbb{E}\{x|y\}$ will, respectively, mean expectation of x and expectation of x conditional on y.

II. Problem Formulation

Consider the following 2-D stochastic system $\mathcal{S}$:

$$\begin{aligned} \mathcal{S}: x_{i+1,j+1} &= \begin{pmatrix} A_1 x_{i,j+1} + A_2 x_{i+1,j} \\ +B_1\omega_{i,j+1} + B_2\omega_{i+1,j} \end{pmatrix} \\ &+ \begin{pmatrix} M_1 x_{i,j+1} + M_2 x_{i+1,j} \\ +N_1\omega_{i,j+1} + N_2\omega_{i+1,j} \end{pmatrix} v_{i,j} \quad (1) \\ y_{i,j} &= Cx_{i,j} + D\omega_{i,j} \\ z_{i,j} &= Hx_{i,j} \end{aligned}$$

where $x_{i,j} \in \mathbb{R}^n$ is the state vector; $y_{i,j} \in \mathbb{R}^m$ is the measured output; $z_{i,j} \in \mathbb{R}^p$ is the signal to be estimated; $\omega_{i,j} \in \mathbb{R}^l$ is the disturbance input which belongs to $l_2[0,\infty)$; $v_{i,j}$ is a standard random scalar signal satisfying $\mathbb{E}\{v_{i,j}\} = 0$ and

$$\begin{cases} \mathbb{E}\{v_{i,j}v_{m,n}\} = 1 \text{ for } (i,j) = (m,n) \\ \mathbb{E}\{v_{i,j}v_{m,n}\} = 0 \text{ for } (i,j) \neq (m,n) \end{cases}$$

and A_1, A_2, B_1, B_2, M_1, M_2, N_1, N_2, C, D, H are system matrices with compatible dimensions.

Remark 1: Recall the well known 2-D discrete system described by the FMLSS model [8]

$$x_{i+1,j+1} = A_1 x_{i,j+1} + A_2 x_{i+1,j} + B_1\omega_{i,j+1} + B_2\omega_{i+1,j} \quad (2)$$

and rewrite the dynamic equation of system $\mathcal{S}$ in the following form

$$\begin{aligned} x_{i+1,j+1} &= [A_1 + M_1 v_{i,j}]\, x_{i,j+1} + [A_2 + M_2 v_{i,j}]\, x_{i+1,j} \\ &+ [B_1 + N_1 v_{i,j}]\, \omega_{i,j+1} + [B_2 + N_2 v_{i,j}]\, \omega_{i+1,j} \end{aligned}$$

then we can see that system $\mathcal{S}$ is established by introducing stochastic uncertainty into the 2-D FMLSS model, and the terms $M_1 v_{i,j}$, $M_2 v_{i,j}$, $N_1 v_{i,j}$, $N_2 v_{i,j}$ can be seen as stochastic perturbations to the system matrices A_1, A_2, B_1, B_2 respectively. These perturbations are usually caused by some stochastic environment and cause the system matrices to fluctuate above their deterministic nominal values. The FMLSS model in (2) has been well studied in many references, while the 2-D stochastic system $\mathcal{S}$ has not been fully investigated.

Throughout the paper, we make the following assumption on the boundary condition:

Assumption 1: The boundary condition is independent of $v_{i,j}$ and is assumed to satisfy

$$\lim_{N\to\infty} \mathbb{E}\left\{\sum_{k=1}^{N}(|x_{0,k}|^2 + |x_{k,0}|^2)\right\} < \infty$$

Here, we are interested in estimating the signal $z_{i,j}$ by a linear dynamic filter of general structure described by $\mathcal{F}$:

$$\begin{aligned} \mathcal{F}: \tilde{x}_{i+1,j+1} &= G_1\tilde{x}_{i,j+1} + G_2\tilde{x}_{i+1,j} \quad (3) \\ &+ K_1 y_{i,j+1} + K_2 y_{i+1,j} \\ \tilde{z}_{i,j} &= L\tilde{x}_{i,j} \\ \tilde{x}_{i,j} &= 0 \text{ for } i = 0 \text{ or } j = 0 \end{aligned}$$

where $\tilde{x}_{i,j} \in \mathbb{R}^k$ is the filter state vector and (G_1, G_2, K_1, K_2, L) are appropriately dimensioned filter matrices to be determined. It should be pointed out that here we are interested not only in the full-order filtering problem (when $k = n$), but also in the reduced-order filtering problem (when $1 \le k < n$). As can be seen in the following, these two filtering problems are solved in a unified framework.

Augmenting the model of $\mathcal{S}$ to include the states of the filter, we obtain the filtering error system $\mathcal{E}$:

$$\begin{aligned} \mathcal{E}: \xi_{i+1,j+1} &= \begin{pmatrix} \bar{A}_1\xi_{i,j+1} + \bar{A}_2\xi_{i+1,j} \\ +\bar{B}_1\omega_{i,j+1} + \bar{B}_2\omega_{i+1,j} \end{pmatrix} \\ &+ \begin{pmatrix} \bar{M}_1\xi_{i,j+1} + \bar{M}_2\xi_{i+1,j} \\ +\bar{N}_1\omega_{i,j+1} + \bar{N}_2\omega_{i+1,j} \end{pmatrix} v_{i,j} \quad (4) \\ e_{i,j} &= \bar{C}\xi_{i,j} \end{aligned}$$

where $\xi_{i,j} = \begin{bmatrix} x_{i,j}^T & \tilde{x}_{i,j}^T \end{bmatrix}^T$, $e_{i,j} = z_{i,j} - \tilde{z}_{i,j}$ and

$$\begin{aligned} \bar{A}_1 &= \begin{bmatrix} A_1 & 0 \\ K_1 C & G_1 \end{bmatrix}, \quad \bar{A}_2 = \begin{bmatrix} A_2 & 0 \\ K_2 C & G_2 \end{bmatrix}, \\ \bar{B}_1 &= \begin{bmatrix} B_1 \\ K_1 D \end{bmatrix}, \quad \bar{B}_2 = \begin{bmatrix} B_2 \\ K_2 D \end{bmatrix}, \\ \bar{M}_1 &= \begin{bmatrix} M_1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{M}_2 = \begin{bmatrix} M_2 & 0 \\ 0 & 0 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned}\bar{N}_1 &= \begin{bmatrix} N_1 \\ 0 \end{bmatrix}, \quad \bar{N}_2 = \begin{bmatrix} N_2 \\ 0 \end{bmatrix}, \\ \bar{C} &= \begin{bmatrix} H & -L \end{bmatrix} \end{aligned} \tag{5}$$

Before presenting the main objective of this paper, we first introduce the following definitions for the filtering error system $\mathcal{E}$ in (4), which will be essential for our derivation.

Definition 1: The filtering error system $\mathcal{E}$ in (4) with Assumption 1 and $\omega_{i,j} = 0$ is said to be mean-square asymptotically stable if for every initial condition $\mathbb{E}\left\{|\xi_{0,0}|^2\right\} < \infty$

$$\lim_{i+j\to\infty} \mathbb{E}\left\{|\xi_{i,j}|^2\right\} = 0$$

Definition 2: Given a scalar $\gamma > 0$, the filtering error system $\mathcal{E}$ in (4) is said to be mean-square asymptotically stable with an H_∞ disturbance attenuation level γ if it is mean-square asymptotically stable and under zero initial and boundary conditions, $\|e\|_E < \gamma \|\omega\|_2$ for all nonzero $\omega_{i,j} \in l_2[0,\infty)$ where

$$\|e\|_E := \sqrt{\mathbb{E}\left\{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}|e_{i,j}|^2\right\}}, \quad \|\omega\|_2 := \sqrt{\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}|\omega_{i,j}|^2}$$

Our objective is to develop full-order and reduced-order filters of the form $\mathcal{F}$ in (3) such that the filtering error system $\mathcal{E}$ in (4) is mean-square asymptotically stable with an H_∞ disturbance attenuation level γ. Filters guaranteeing such a performance are called H_∞ filters.

Another assumption is made as follows:

Assumption 2: System $\mathcal{S}$ in (1) is mean-square asymptotically stable.

Remark 2: For brevity, we have omitted the known control input terms in $\mathcal{S}$ since it is well known that this does not affect the generality of the discussion on the filter design. Consequently, the original system to be estimated has to be mean-square asymptotically stable, which is a prerequisite for the filtering error system $\mathcal{E}$ to be mean-square asymptotically stable.

III. Filtering for Systems with Exactly Known Matrices

This section is devoted to the H_∞ filtering problem for system $\mathcal{S}$ in (1) with exactly known matrices, that is, there is no uncertain parameter in the system matrices $(A_1, A_2, B_1, B_2, M_1, M_2, N_1, N_2, C, D, H)$.

A. Filter Analysis

This subsection is concerned with the filter analysis problem. More specifically, assuming that the filter matrices (G_1, G_2, K_1, K_2, L) in (3) are exactly known, we shall study the conditions under which the filtering error system $\mathcal{E}$ in (4) is mean-square asymptotically stable with an H_∞ disturbance attenuation level γ. The following theorem shows that the H_∞ performance of the filtering error system can be guaranteed if there exist some positive definite matrices satisfying certain LMIs. This theorem will play an instrumental role in the filter design problems.

Theorem 1: Consider system $\mathcal{S}$ in (1) and suppose the filter matrices (G_1, G_2, K_1, K_2, L) of $\mathcal{F}$ in (3) are given. Then the filtering error system $\mathcal{E}$ in (4) is mean-square asymptotically stable with an H_∞ disturbance attenuation level bound γ if there exist $(n+k)\times(n+k)$ matrices $P > 0$ and $Q > 0$ satisfying

$$\begin{bmatrix} -P & 0 & 0 & 0 & P\bar{M}_1 & P\bar{M}_2 & P\bar{N}_1 & P\bar{N}_2 \\ * & -P & 0 & 0 & P\bar{A}_1 & P\bar{A}_2 & P\bar{B}_1 & P\bar{B}_2 \\ * & * & -I & 0 & \bar{C} & 0 & 0 & 0 \\ * & * & * & -I & 0 & \bar{C} & 0 & 0 \\ * & * & * & * & Q-P & 0 & 0 & 0 \\ * & * & * & * & * & -Q & 0 & 0 \\ * & * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{6}$$

Remark 3: Theorem 1 provides an H_∞ filter analysis result for 2-D stochastic systems, which can be seen as an extension from previous results on H_∞ performance. When we assume $v_{i,j} = 0$, that is, no stochastic uncertainty is present in system $\mathcal{S}$, LMI (6) becomes

$$\begin{bmatrix} -P & 0 & 0 & P\bar{A}_1 & P\bar{A}_2 & P\bar{B}_1 & P\bar{B}_2 \\ * & -I & 0 & \bar{C} & 0 & 0 & 0 \\ * & * & -I & 0 & \bar{C} & 0 & 0 \\ * & * & * & Q-P & 0 & 0 & 0 \\ * & * & * & * & -Q & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{7}$$

LMI (7) is the H_∞ performance condition obtained in [24] for 2-D systems.

In the following, we will present a version of the obtained H_∞ performance condition which is more suitable for systems with polytopic uncertain matrices.

Theorem 2: Consider system $\mathcal{S}$ in (1) and suppose the filter matrices (G_1, G_2, K_1, K_2, L) of $\mathcal{F}$ in (3) are given. Then the filtering error system $\mathcal{E}$ in (4) is mean-square asymptotically stable with an H_∞ disturbance attenuation level bound γ if there exist $(n+k)\times(n+k)$ matrices V, $P > 0$ and $Q > 0$ satisfying

$$\begin{bmatrix} P - V^T - V & 0 & 0 & 0 \\ * & P - V^T - V & 0 & 0 \\ * & * & -I & 0 \\ * & * & * & -I \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}$$

$$\begin{bmatrix} \cdots & V^T\bar{M}_1 & V^T\bar{M}_2 & V^T\bar{N}_1 & V^T\bar{N}_2 \\ \cdots & V^T\bar{A}_1 & V^T\bar{A}_2 & V^T\bar{B}_1 & V^T\bar{B}_2 \\ \cdots & \bar{C} & 0 & 0 & 0 \\ \cdots & 0 & \bar{C} & 0 & 0 \\ \cdots & Q-P & 0 & 0 & 0 \\ \cdots & * & -Q & 0 & 0 \\ \cdots & * & * & -\gamma^2 I & 0 \\ \cdots & * & * & * & -\gamma^2 I \end{bmatrix} < 0$$

Remark 4: It can be seen that the LMI condition presented in Theorem 2 no longer contains product terms between the positive de□nite matrix P and system matrices. This is made possible by the introduction of the slack matrix variable V and enables us to obtain a parameter-dependent performance criterion when extending Theorem 2 to system with polytopic uncertain matrices. This idea stems from [3], following which many works have been reported very recently. In the following subsection, the □lter synthesis problem will be solved based on the improved performance analysis result of Theorem 2.

B. Filter Synthesis

In this subsection, we will focus on the design of full-order and reduced-order H_∞ □lters of the form $\mathcal{F}$ based on Theorem 2. That is, to determine the □lter matrices (G_1, G_2, K_1, K_2, L) which will guarantee the □ltering error system $\mathcal{E}$ to be mean-square asymptotically stable with an H_∞ performance. The following theorem provides a sufficient condition for the existence of such H_∞ □lters for system $\mathcal{S}$.

Theorem 3: Consider system $\mathcal{S}$ in (1). Then an admissible H_∞ □lter of the form $\mathcal{F}$ in (3) exists if there exist $n \times n$ matrices $P_1 > 0$, $Q_1 > 0$, R, $n \times k$ matrices P_2, Q_2, F, $k \times k$ matrices $P_3 > 0$, $Q_3 > 0$, X, $\bar{G}_1$, $\bar{G}_2$, $k \times m$ matrices $\bar{K}_1$, $\bar{K}_2$, and $p \times k$ matrix $\bar{L}$ satisfying

$$\begin{bmatrix} \Upsilon_1 & 0 & 0 & \Upsilon_2 & \Upsilon_3 \\ * & \Upsilon_1 & 0 & \Upsilon_4 & \Upsilon_5 \\ * & * & -I & \Upsilon_6 & 0 \\ * & * & * & \Upsilon_7 & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (8)$$

$$\begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix} > 0 \quad (9)$$

where

$$\Upsilon_1 = \begin{bmatrix} P_1 - R^T - R & P_2 - EX - F \\ * & P_3 - X - X^T \end{bmatrix},$$

$$\Upsilon_2 = \begin{bmatrix} R^T M_1 & 0 & R^T M_2 & 0 \\ F^T M_1 & 0 & F^T M_2 & 0 \end{bmatrix},$$

$$\Upsilon_3 = \begin{bmatrix} R^T N_1 & R^T N_2 \\ F^T N_1 & F^T N_2 \end{bmatrix},$$

$$\Upsilon_4 = \begin{bmatrix} R^T A_1 + E\bar{K}_1 C & E\bar{G}_1 & R^T A_2 + E\bar{K}_2 C & E\bar{G}_2 \\ F^T A_1 + \bar{K}_1 C & \bar{G}_1 & F^T A_2 + \bar{K}_2 C & \bar{G}_2 \end{bmatrix},$$

$$\Upsilon_5 = \begin{bmatrix} R^T B_1 + E\bar{K}_1 D & R^T B_2 + E\bar{K}_2 D \\ F^T B_1 + \bar{K}_1 D & F^T B_2 + \bar{K}_2 D \end{bmatrix},$$

$$\Upsilon_6 = \begin{bmatrix} H & -\bar{L} & 0 & 0 \\ 0 & 0 & H & -\bar{L} \end{bmatrix},$$

$$\Upsilon_7 = \mathrm{diag}\left\{ \begin{bmatrix} Q_1 - P_1 & Q_2 - P_2 \\ * & Q_3 - P_3 \end{bmatrix}, \begin{bmatrix} -Q_1 & -Q_2 \\ * & -Q_3 \end{bmatrix} \right\},$$

$$E = \begin{bmatrix} I_{k\times k} \\ 0_{(n-k)\times k} \end{bmatrix}$$

Moreover, under the above conditions, an admissible □lter can be given by

$$\begin{bmatrix} G_1 & K_1 \\ G_2 & K_2 \\ L & 0 \end{bmatrix} = \begin{bmatrix} X^{-1} & 0 & 0 \\ 0 & X^{-1} & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \bar{G}_1 & \bar{K}_1 \\ \bar{G}_2 & \bar{K}_2 \\ \bar{L} & 0 \end{bmatrix} \quad (10)$$

Remark 5: Theorem 3 presents strict LMI conditions for the existence of desired H_∞ □lters, which can be easily tested by the available LMI toolbox in the Matlab environment [9]. It is worth noting that the matrix E de□ned in Theorem 3 plays an instrumental role in formulating both the full-order and reduced-order □lters in a uni□ed framework. For the full-order □ltering, the matrix E becomes an identity matrix of dimension n, and for the reduced-order case, we have imposed certain structural restriction on the (2,1) block entry of the partitioned matrix V, which introduces some overdesign into the □lter design.

Remark 6: Note that (8), (9) are LMIs not only over the matrix variables, but also over the scalar γ. This implies that the scalar γ can be included as an optimization variable to obtain a reduction of the attenuation level bound. Then the minimum (in terms of the feasibility of (8), (9)) attenuation level of H_∞ □lters can be readily found by solving the following convex optimization problem using LMI toolbox in MATLAB:

Minimize γ subject to (8), (9) over P_1, Q_1, R, P_2, Q_2, F, P_3, Q_3, X, $\bar{G}_1$, $\bar{G}_2$, $\bar{K}_1$, $\bar{K}_2$ and $\bar{L}$.

In the case when there is no stochastic uncertainty in the system $\mathcal{S}$, that is, $v_{i,j}$ is assumed to be zero, Theorem 3 is specialized to the following corollary.

Corollary 1: Consider system $\mathcal{S}$ in (1) with $v_{i,j} = 0$. Then an admissible H_∞ □lter of the form $\mathcal{F}$ in (3) exists if there exist $n \times n$ matrices $P_1 > 0$, $Q_1 > 0$, R, $n \times k$ matrices P_2, Q_2, F, $k \times k$ matrices $P_3 > 0$, $Q_3 > 0$, X, $\bar{G}_1$, $\bar{G}_2$, $k \times m$ matrices $\bar{K}_1$, $\bar{K}_2$, and $p \times k$ matrix $\bar{L}$ satisfying (9) and

$$\begin{bmatrix} \Upsilon_1 & 0 & \Upsilon_4 & \Upsilon_5 \\ * & -I & \Upsilon_6 & 0 \\ * & * & \Upsilon_7 & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (11)$$

IV. Filtering for Systems with Uncertain Matrices

In this section, we consider the H_∞ □ltering problem for system $\mathcal{S}$ in (1) with partially unknown data, that is, uncertain parameters are present in the system matrices (A_1, A_2, B_1, B_2, M_1, M_2, N_1, N_2, C, D, H. In the following, we will consider two types of parameter uncertainties: polytopic uncertainty and norm-bounded uncertainty.

A. Polytopic Uncertain Case

Theorem 3 addresses the H_∞ filtering problem for system $\mathcal{S}$ in (1) where the system matrices are all known. However, since LMIs (8) and (9) are affine in the system matrices, the theorem can be directly used to solve the filtering problem in the case where the system matrices are not exactly known and they reside within a given polytope.

Assumption 3: The matrices A_1, A_2, B_1, B_2, M_1, M_2, N_1, N_2, C, D, H of system $\mathcal{S}$ in (1) contain partially unknown parameters. Assume that

$$\Omega := (A_1, A_2, B_1, B_2, M_1, M_2, N_1, N_2, C, D, H) \in \mathcal{R}$$

where $\mathcal{R}$ is a given convex bounded polyhedral domain described by s vertices:

$$\mathcal{R} := \left\{ \Omega(\lambda) \middle| \Omega(\lambda) = \sum_{i=1}^{s} \lambda_i \Omega_i; \sum_{i=1}^{s} \lambda_i = 1, \lambda_i \geq 0 \right\}$$

where

$$\Omega_i := (A_{1i}, A_{2i}, B_{1i}, B_{2i}, M_{1i}, M_{2i}, N_{1i}, N_{2i}, C_i, D_i, H_i)$$

denotes the vertices of the polytope $\mathcal{R}$.

Theorem 4: Consider system $\mathcal{S}$ in (1) with Assumption 3. Then an admissible robust H_∞ filter of the form $\mathcal{F}$ in (3) exists if there exist $n \times n$ matrices $P_{1i} > 0$, $Q_{1i} > 0$, R, $n \times k$ matrices P_{2i}, Q_{2i}, F, $k \times k$ matrices $P_{3i} > 0$, $Q_{3i} > 0$, X, $\bar{G}_1$, $\bar{G}_2$, $k \times m$ matrices $\bar{K}_1$, $\bar{K}_2$, and $p \times k$ matrix $\bar{L}$ satisfying LMIs (8) and (9) for $i = 1, \ldots, s$, where the matrices P_1, P_2, P_3, Q_1, Q_2, Q_3, A_1, A_2, B_1, B_2, M_1, M_2, N_1, N_2, C, D, H are taken with P_{1i}, P_{2i}, P_{3i}, Q_{1i}, Q_{2i}, Q_{3i}, A_{1i}, A_{2i}, B_{1i}, B_{2i}, M_{1i}, M_{2i}, N_{1i}, N_{2i}, C_i, D_i, H_i respectively.

Remark 7: The parameter uncertainties considered in this subsection are assumed to be of the polytopic-type, entering into all the matrices of the system model. The polytopic uncertainty has been widely used in the problems of robust control and filtering for uncertain systems (see, for instance, [4], [10] and the references therein), and many practical systems possess parameter uncertainties which can be either exactly modeled or overbounded by the polytope $\mathcal{R}$.

Remark 8: Theorem 4 solves the robust H_∞ filtering problem for 2-D stochastic systems with polytopic uncertain matrices. It is worth noting that the obtained result are based on Theorem 2, which is an improved version of the bounded real lemma obtained in Theorem 1. Since Theorem 2 has decoupled the product terms involving the positive definite matrices, the parameter-dependent Lyapunov stability idea has been incorporated into the filter design in the sense that different positive definite matrices $P_i = \begin{bmatrix} P_{1i} & P_{2i} \\ * & P_{3i} \end{bmatrix}$ and $Q_i = \begin{bmatrix} Q_{1i} & Q_{2i} \\ * & Q_{3i} \end{bmatrix}$ are entailed for each vertex of the polytope $\mathcal{R}$. Now, let $\bar{\Omega}(\lambda) = \sum_{i=1}^{s} \lambda_i \Omega_i$ denotes any given point of the polytope $\mathcal{R}$, where $\lambda := (\lambda_1, \ldots, \lambda_s)$ is a vector of dimension s. If the LMIs in Theorem 4 are solvable, then it is not difficult to show that the positive definite matrices for $\bar{\Omega}(\lambda)$ can be recovered by $P(\lambda) = \sum_{i=1}^{s} \lambda_i \begin{bmatrix} P_{1i} & P_{2i} \\ * & P_{3i} \end{bmatrix}$, $Q(\lambda) = \sum_{i=1}^{s} \lambda_i \begin{bmatrix} Q_{1i} & Q_{2i} \\ * & Q_{3i} \end{bmatrix}$, which are dependent of the parameter λ.

B. Norm-Bounded Uncertain Case

An alternative way of dealing with uncertain systems is to assume that the deviation of the system parameters from their nominal values is norm-bounded, which has also been widely used in the robust control and filtering problems (see, for instance, [2], [21] and the references therein). In our case, we make the following assumption.

Assumption 4: Assume that the matrices A_1, A_2, B_1, B_2, M_1, M_2, N_1, N_2, C, D of system $\mathcal{S}$ in (1) have the following form

$$\begin{aligned} A_1 &= A_{10} + \Delta A_1,\ A_2 = A_{20} + \Delta A_2, \\ B_1 &= B_{10} + \Delta B_1,\ B_2 = B_{20} + \Delta B_2, \\ M_1 &= M_{10} + \Delta M_1,\ M_2 = M_{20} + \Delta M_2, \\ N_1 &= N_{10} + \Delta N_1,\ N_2 = N_{20} + \Delta N_2, \\ C &= C_0 + \Delta C,\ D = D_0 + \Delta D \end{aligned} \tag{12}$$

where A_{10}, A_{20}, B_{10}, B_{20}, M_{10}, M_{20}, N_{10}, N_{20}, C_0, D_0 are known constant matrices with appropriate dimensions. ΔA_1, ΔA_2, ΔB_1, ΔB_2, ΔM_1, ΔM_2, ΔN_1, ΔN_2, ΔC, ΔD are real-valued time-varying matrix functions representing norm-bounded parameter uncertainties satisfying

$$\begin{bmatrix} \Delta A_1 & \Delta B_1 \\ \Delta A_2 & \Delta B_2 \\ \Delta M_1 & \Delta N_1 \\ \Delta M_2 & \Delta N_2 \\ \Delta C & \Delta D \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix} \Delta_{i,j} \begin{bmatrix} W_1 & W_2 \end{bmatrix}$$

where $\Delta_{i,j}$ is a real uncertain matrix function with Lebesgue measurable elements satisfying

$$\Delta_{i,j}^T \Delta_{i,j} \leq I$$

and U_1, U_2, U_3, U_4, U_5, W_1, W_2 are known real constant matrices of appropriate dimensions. These matrices specify how the uncertain parameters in $\Delta_{i,j}$ enter the nominal matrices A_{10}, A_{20}, B_{10}, B_{20}, M_{10}, M_{20}, N_{10}, N_{20}, C_0, D_0.

Now we present the robust H_∞ filtering result for system $\mathcal{S}$ with norm-bounded uncertainties in the following theorem.

Theorem 5: Consider system $\mathcal{S}$ in (1) with Assumption 4. Then an admissible robust H_∞ filter of the form $\mathcal{F}$ in (3) exists if there exist $n \times n$ matrices $P_1 > 0$,

$Q_1 > 0$, R, $n \times k$ matrices P_2, Q_2, F, $k \times k$ matrices $P_3 > 0$, $Q_3 > 0$, X, $\bar{G}_1$, $\bar{G}_2$, $k \times m$ matrices $\bar{K}_1$, $\bar{K}_2$, $p \times k$ matrix $\bar{L}$ and scalar $\epsilon > 0$ satisfying LMIs (9) and

$$\begin{bmatrix} \Upsilon_1 & 0 & 0 & \Gamma_2 & \Gamma_3 & \Gamma_{10} \\ * & \Upsilon_1 & 0 & \Gamma_4 & \Gamma_5 & \Gamma_{11} \\ * & * & -I & \Upsilon_6 & 0 & 0 \\ * & * & * & \Gamma_7 & \Gamma_8 & 0 \\ * & * & * & * & \Gamma_9 & 0 \\ * & * & * & * & * & -\epsilon I \end{bmatrix} < 0$$

where Υ_* is the same as in Theorem 3 and

$$\begin{aligned}
\Gamma_2 &= \begin{bmatrix} R^T M_{10} & 0 & R^T M_{20} & 0 \\ F^T M_{10} & 0 & F^T M_{20} & 0 \end{bmatrix}, \\
\Gamma_3 &= \begin{bmatrix} R^T N_{10} & R^T N_{20} \\ F^T N_{10} & F^T N_{20} \end{bmatrix}, \\
\Gamma_4 &= \left[\begin{matrix} R^T A_{10} + E\bar{K}_1 C_0 & E\bar{G}_1 \\ F^T A_{10} + \bar{K}_1 C_0 & \bar{G}_1 \end{matrix} \right. \\
&\qquad \left. \begin{matrix} R^T A_{20} + E\bar{K}_2 C_0 & E\bar{G}_2 \\ F^T A_{20} + \bar{K}_2 C_0 & \bar{G}_2 \end{matrix} \right] \\
\Gamma_5 &= \begin{bmatrix} R^T B_{10} + E\bar{K}_1 D_0 & R^T B_{20} + E\bar{K}_2 D_0 \\ F^T B_{10} + \bar{K}_1 D_0 & F^T B_{20} + \bar{K}_2 D_0 \end{bmatrix}, \\
\Gamma_8 &= \begin{bmatrix} \epsilon W_1^T W_2 & 0 \\ 0 & 0 \\ 0 & \epsilon W_1^T W_2 \\ 0 & 0 \end{bmatrix}, \\
\Gamma_7 &= \mathrm{diag} \left\{ \begin{bmatrix} Q_1 - P_1 + \epsilon W_1^T W_1 & Q_2 - P_2 \\ * & Q_3 - P_3 \end{bmatrix}, \begin{bmatrix} -Q_1 + \epsilon W_1^T W_1 & -Q_2 \\ * & -Q_3 \end{bmatrix} \right\}, \\
\Gamma_9 &= \mathrm{diag} \left\{ \left(-\gamma^2 I + \epsilon W_2^T W_2\right), \left(-\gamma^2 I + \epsilon W_2^T W_2\right) \right\}, \\
\Gamma_{10} &= \begin{bmatrix} R^T U_3 & R^T U_4 \\ F^T U_3 & F^T U_4 \end{bmatrix}, \\
\Gamma_{11} &= \begin{bmatrix} \Xi_1 & \Xi_2 \\ \Xi_3 & \Xi_4 \end{bmatrix} \\
&= \begin{bmatrix} R^T U_1 + E\bar{K}_1 U_5 & R^T U_2 + E\bar{K}_2 U_5 \\ F^T U_1 + \bar{K}_1 U_5 & F^T U_2 + \bar{K}_2 U_5 \end{bmatrix}
\end{aligned}$$

V. Concluding Remarks

The problem of H_∞ filtering for 2-D systems with stochastic perturbations is investigated in this paper. Both full-order and reduced-order filters are designed in a unified framework. Sufficient conditions are obtained for the existence of desired filters in terms of LMIs, and the filter design is cast into a convex optimization problem. These results are further extended to more general cases whose system matrices also contain parameter uncertainties represented by either polytopic or norm-bounded approaches.

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