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On-line Fault Detection and Isolation of Nonlinear Systems

C.W. Chan, K.C. Cheung, Y. Wang and W.C. Chan

Department of Mechanical Engineering, University of Hong Kong
Pokfulam Road, Hong Kong, China. E-mail: mechan@hkucc.hku.hk

Abstract: This paper describes an on-line fault detection scheme for a class of nonlinear dynamic systems with modelling uncertainty and inaccessible states. Only the inputs and outputs of the system can be measured. The faults are assumed to be functions of the state, instead of the output, and the input of the system. A nonlinear on-line approximator using dynamic recurrent neural network (DRNN) is utilised to monitor the faults in the system. The construction and the learning algorithm of the on-line approximator are presented. The stability, robustness and sensitivity of the fault detection scheme under certain assumptions are analysed. An example demonstrates the efficiency of the proposed fault detection scheme.

Keywords: nonlinear system, fault detection, recurrent neural network, observer.

1. Introduction

The growing need for effective fault detection and isolation (FDI) in complex systems have attracted a lot of attention. In recent years, model based FDI methodology has been investigated in many aspects and several kinds of FDI schemes have been proposed. These include unknown input observer [1, 2], eigenstructure assignment [5], and others. Almost all the fault detection studies mentioned above assume that the nominal model of the plant is linear and the failures are modelled as external additive inputs that are functions of time. These assumptions allow the use of linear system theory to design and analyse FDI architectures. However, most practical systems are nonlinear in nature and most failures are better described as nonlinear functions of the state and the input variables. Yang and Saif [13] investigated FDI for a class of nonlinear systems using adaptive observer. The approach involves considerable computational complexity. Zhou and Bennett [15] utilised off-line trained neural network as approximator to design a robust adaptive observer for FDI. Vemuri and Polycarpou [10] proposed a FDI scheme for a general class of nonlinear systems that uses an on-line approximator. Compared with off-line approximators, the on-line approximator does not require training and is more adaptive. However, in [10] it is necessary to assume the existence of a local diffeomorphism to transform the system into a new coordinate system. In the new coordinate system, the failures can be modelled as function of the output, instead of the state, and the input of the system. Even though necessary and sufficient conditions for the existence of such transformation are usually satisfied in practice, it is difficult to find such one.

This paper presents a FDI scheme for nonlinear systems that uses a dynamic recurrent neural network (DRNN) based on-line approximator. Faults of the system can be detected by the on-line approximator even in the presence of modelling uncertainties. The on-line approximator can also be used for fault isolation and identification. When there is no fault in the system, the output of the on-line approximator is very close to zero. If faults emerge, the output becomes large enough to indicate the existence of faults. The learning algorithm of the on-line approximator is proposed in this paper. Also, the stability, robustness, sensitivity and other properties of the proposed FDI scheme are analysed.

Section 2 of this paper describes the problem to be addressed. In section 3, Lyapunov theory is used to prove the stability of the proposed on-line approximator. Robustness and sensitivity of the approximator are also analysed. A practical fault detection scheme is then introduced. In section 4, an example is used to show the effectiveness of the proposed FDI scheme.

2. Problem Formulation

A nonlinear dynamic system is described as:

$$\left.\begin{aligned} \dot{x} &= Ax + f(x,u) + \varphi(x,u) + B\gamma(t-T)\phi(x,u) \\ y &= Cx \end{aligned}\right\} \qquad (1)$$

where $x \in R^n$ is the state of the system, $u \in R^m$ is the input of the system, $y \in R^l$ is the measurable output of the system, $A \in R^{n\times n}$, $B \in R^{n\times p}$ and $C \in R^{l\times n}$ are constant matrices. $f, \varphi : R^n \times R^m \to R^n$, $\phi : R^n \times R^m \to R^p$ are smooth vector fields with f representing the nonlinearity of the system, φ representing the bounded modelling uncertainty, and ϕ representing the failure of the system. $\gamma : R \to R$ is a function representing the time profile of failures. For an abrupt failure at time T, the function γ takes the form of a step function. For incipient failures, γ is a ramp-type function. Many practical dynamic systems can be represented by equation (1) which describes a particular kind of nonlinear system. In equation (1), the failures are assumed to be functions of the system's state and input. This assumption is valid for most cases. In [10], the nonlinearity f and the failure ϕ are both functions of the input and output variables. Under this condition, it is unnecessary to estimate the state of the system, so the problem becomes simpler. Actuator fault and component fault can be easily modelled by the failure representation formulation given in equation (1). For sensor faults the method provided by [2] can be used to transform the faults so that they can be represented by equation (1). A nonlinear robust fault detection scheme for the dynamic system model described by equation (1) is required. Before continuing with the analysis, the following assumptions are proposed for the nonlinear system represented by equation (1).

Assumption 1: *The system states remain bounded after the occurrence of a failure, i.e.,* $x(t) \in L_\infty$.

Assumption 2: *The input of the system* u *is bounded, i.e.,* $\|u\| \le u_d$.

Assumption 3: *The matrix pair* (A, C) *is observable.*

Assumption 4: *The nonlinear function f is Lipschitz in x with Lipschitz constant η, i.e., $\|f(x,u)-f(\hat{x},u)\| < \eta\|x-\hat{x}\|$.*

The normal model of a system represented by equation (1) is:

$$\left.\begin{aligned} \dot{x} &= Ax + f(x,u) + \varphi(x,u) \\ y &= Cx \end{aligned}\right\} \tag{1a}$$

If the uncertain term, $\varphi(x,u)$, produced by parameter error and other disturbances, is bounded by a known value, the following assumption can be made.

Assumption 5: *The uncertainty φ is bounded, i.e.,*

$$|\varphi(x,u)| \le \varphi_0, \qquad \forall (x,u) \in R^n \times R^m$$

where φ_0 is a known constant.

It is sometimes difficult to have *a priori* knowledge of φ_0. In this case, $\varphi(x,u)$ have to be modelled using empirical input-output data of the normal system. Zhang *et al.* [14] proposed an approach to estimate the unknown nonlinear term using artificial neural networks. Tu and Stein [9] suggested a method for model error compensation.

3. Fault Detection Scheme

3.1 Definitions

The following definitions will be used in this paper. The 2-norm of a vector $x \in R^n$ is defined as $\|x\|_2 = \sqrt{x_1^2 + \cdots + x_n^2}$ and the 2-norm of a matrix $A \in R^{n\times m}$ as $\|A\|_2 = \sqrt{\lambda_{\max}[A^T A]} = \sigma_{\max}[A]$, where $\lambda_{\max}[.]$ and $\lambda_{\min}[.]$ are the largest and smallest eigenvalues of a matrix respectively and $\sigma_{\max}$ is the maximum singular value. Given $A = [a_{ij}]$, the Frobenius norm is defined as $\|A\|_F = \sqrt{tr(A^T A)} = \sqrt{\sum a_{ij}^2}$ with $tr(.)$ the trace operator. The Frobenius norm is compatible with the 2-norm, therefore $\|Ax\|_2 \le \|A\|_F \|x\|_2$ with $A \in R^{n\times m}$ and $x \in R^m$. The following convention shall be adopted for the vector and matrix norms unless specified otherwise: $\|x\| := \|x\|_2$, and $\|A\| := \|A\|_2$.

3.2 Nonlinear state estimation and error equation

Equation (2) below represents an observer that estimates states for the system given in equation (1).

$$\left.\begin{aligned} \dot{\hat{x}} &= A\hat{x} + f(\hat{x},u) + B[\gamma(t-T)\hat{\phi}(\hat{x},u) - v(t)] + K(y - C\hat{x}) \\ \hat{y} &= C\hat{x} \end{aligned}\right\} \tag{2}$$

$\hat{x}$ and $\hat{y}$ denote the estimates of the state x and output y. $K \in R^{n\times l}$ is the observer gain vector, so chosen that the characteristic polynomial $A - KC$ is strictly Hurwitz. $\hat{\phi}(.)$ is the estimate of $\phi(.)$ in terms of $\hat{x}$. The robustifying vector $v(t)$, yet to be defined, is a function that provides robustness in the face of bounded disturbances [4]. The bounded modelling uncertainty $\varphi(.)$ is not included in equation (2).

Denote the state and output estimation errors as $\tilde{x} = x - \hat{x}$ and $\tilde{y} = y - \hat{y}$ respectively and let $A_c = A - KC$, the error dynamic equation can be derived from equations (1) and (2) as:

$$\left.\begin{aligned} \dot{\tilde{x}} &= A_c\tilde{x} + f(x,u) - f(\hat{x},u) + \varphi(x,u) + B[\gamma(t-T)(\phi(x,u) - \hat{\phi}(\hat{x},u)) + v(t)] \\ \tilde{y} &= C\tilde{x} \end{aligned}\right\} \tag{3}$$

In equations (2) and (3), $\hat{\phi}(\hat{x},u)$ is the output of an on-line approximator that uses an artificial neural network to model the fault function $\phi(x,u)$.

$\phi(x,u)$ can be modelled using neural network as:

$$\phi_i(x,u) = W_i^T \sigma_i(x,u) + \varepsilon_i(x,u) \quad i = 1,\cdots,p \tag{4}$$

where W_i and σ_i are the optimal weight vector and the activation function vector of the i^{th} neural network with proper dimension respectively. The W_i's are bounded by a known value, *i.e.*, $\|W_i\|_F \le W_i^M$ [4]. $\|\varepsilon_i(x,u)\| \le \varepsilon_i^N$ is the bounded error. It has been proved theoretically that $\varepsilon_i(.)$ can be as small as possible for a sufficiently large neural network. For radial basis function neural network, equation (4) is accurate, but for multi-layer sigmoidal neural network there is a high-order error [6].

In the following discussion, unless stated otherwise, $i = 1,\cdots,p$.

The neural network weights are tuned on-line, with no off-line learning required. The output of the on-line approximator $\hat{\phi}_i(\hat{x},u)$ is $\hat{W}_i^T \sigma_i(\hat{x},u)$ where $\hat{W}$ is an estimate of the optimal network weight matrix W in equation (4). Note that the neural network in the approximator is a recurrent one since one of the input variable to the network, $\hat{x}$, is obtained from the output of the network. Equation (5) below is obtained from equation (4) and the output of the on-line approximator.

$$\phi_i(x,u) - \hat{\phi}_i(\hat{x},u) = W_i^T \sigma_i(x,u) - \hat{W}_i^T \sigma_i(\hat{x},u) + \varepsilon_i(x,u) \tag{5}$$

Adding and subtracting $W_i^T \sigma_i(\hat{x},u)$ in equation (5) gives:

$$\phi_i(x,u) - \hat{\phi}_i(\hat{x},u) = \tilde{W}_i^T \sigma_i(\hat{x},u) + w_i(t) + \varepsilon_i(x,u) \tag{6}$$

where $\tilde{W}_i = W_i - \hat{W}_i$, $w_i(t) = W_i^T(\sigma_i(x,u) - \sigma_i(\hat{x},u))$. Note that W_i is bounded and the activation function matrix σ_i is also bounded, therefore $w_i(t)$ is bounded, *i.e.*, $\|w_i(t)\| \le \beta_i$ where β_i is a constant. Substituting equation (6) into equation (3) yields:

$$\left.\begin{aligned} \dot{\tilde{x}} &= A_c\tilde{x} + f(x,u) - f(\hat{x},u) + \varphi(x,u) + \\ & B[\gamma(t-T)\begin{bmatrix} \tilde{W}_1^T \sigma_1(\hat{x},u) + w_1(t) + \varepsilon_1(x,u) \\ \vdots \\ \tilde{W}_p^T \sigma_p(\hat{x},u) + w_p(t) + \varepsilon_p(x,u) \end{bmatrix} + v(t)] \\ \tilde{y} &= C\tilde{x} \end{aligned}\right\} \tag{7}$$

3.3 On-line learning algorithm and stability of on-line fault approximator

The on-line neural network approximator in equation (7) uses a dynamic recurrent neural network. Theorem 1 below gives the training algorithm for this network.

Theorem 1. *Suppose the class of nonlinear dynamic systems described by equation (1) satisfies assumptions 1-5, and suppose also there is a positive definite symmetric matrix P such that*

$$PB - (TC)^T = 0 \tag{8}$$

where $T \in R^{p\times l}$ is an arbitrary matrix and P satisfies the Lyapunov equation

$$A_c^T P + PA_c = -Q \tag{9}$$

where Q is a positive definite symmetric matrix.

If the following equation is satisfied

$$\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} > \eta \tag{10}$$

then the state estimation error $\tilde{x}$ *and the neural network weight estimate error* $\tilde{W}$ *in equation (7) are uniformly ultimately bounded (UUB) [4] if the robustifying term* $v(t)$ *is given by:*

$$v_i(t) = -\beta_i \cdot \frac{|\bar{y}_i|}{\bar{y}_i} \quad i = 1, \cdots, p \tag{11}$$

and the neural network weight tuning is given by

$$\dot{\hat{W}}_i = \Gamma_i \sigma_i(\hat{x}, u)\bar{y}_i^T - k_i|\bar{y}_i|\Gamma_i \hat{W}_i \quad i = 1, \cdots, p \tag{12}$$

where $\bar{y} = T\tilde{y}$ *and* $\bar{y}_i$ *are the entries of* $\bar{y}$, Γ_i *and* k_i *are a positive definite symmetric matrix and a scalar to be selected respectively.*

Proof: At first suppose that the uncertainty $\varphi(x,u)$ is zero, i.e., $\varphi_0 = 0$. Without loss of generality, $\gamma(t-T)$ is supposed to be 1. Consider the Lyapunov function candidate

$$V = \tilde{x}^T P \tilde{x} + \sum_{i=1}^{p} tr(\tilde{W}_i^T \Gamma_i^{-1} \tilde{W}_i) \tag{13}$$

$P = P^T > 0$ satisfies equations (8) and (9), Q is a positive definite symmetric matrix. The time derivative $\dot{V}$ then becomes:

$$\dot{V} = -\tilde{x}^T Q \tilde{x} + +2\tilde{x}^T PB[\begin{bmatrix} \tilde{W}_1^T \sigma_1(\hat{x},u) + w_1(t) + \varepsilon_1(x,u) \\ \vdots \\ \tilde{W}_p^T \sigma_p(\hat{x},u) + w_p(t) + \varepsilon_p(x,u) \end{bmatrix} + v(t)] \tag{14}$$

$$+ 2\tilde{x}^T P[f(x,u) - f(\hat{x},u)] + 2\sum_{i=1}^{p} tr(\tilde{W}_i^T \Gamma_i^{-1} \dot{\tilde{W}}_i)$$

Using assumptions 4 and 5 and equation (8),

$$\dot{V} \le -(\lambda_{\min}(Q) - 2\eta\lambda_{\max}(P))\|\tilde{x}\|^2 + 2\sum_{i=1}^{p} \bar{y}_i^T [\tilde{W}_i^T \sigma_i(\hat{x},u) + w_i(t) + \varepsilon_i(x,u) + v_i(t)]$$
$$+ 2\sum_{i=1}^{p} tr(\tilde{W}_i^T \Gamma_i^{-1} \dot{\tilde{W}}_i) \tag{15}$$

Evaluating equation (15) along the trajectories of equation (11) and (12) gives

$$\dot{V} \le -(\lambda_{\min}(Q) - 2\eta\lambda_{\max}(P))\|\tilde{x}\|^2 + 2\sum_{i=1}^{p} |\bar{y}_i|[\varepsilon_i^N + k_i tr(\tilde{W}_i^T (W_i - \tilde{W}_i))] \tag{16}$$

Since $\|\bar{y}\| \le \|TC\|_F \|\tilde{x}\|$, from equation (10),

$$-(\lambda_{\min}(Q) - 2\eta\lambda_{\max}(P))\|\tilde{x}\|^2 \le -\frac{(\lambda_{\min}(Q) - 2\eta\lambda_{\max}(P))}{\|TC\|_F}\|\bar{y}\|^2 = -\alpha\|\bar{y}\|^2 \tag{17}$$

where $\alpha = \dfrac{(\lambda_{\min}(Q) - 2\eta\lambda_{\max}(P))}{\|TC\|_F}$.

Also

$$tr[\tilde{W}_i^T (W_i - \tilde{W}_i)] \le W_i^M \|\tilde{W}\|_F - \|\tilde{W}\|_F^2 \tag{18}$$

Substituting equations (17) and (18) into (16) gives:

$$\dot{V} \le -\alpha\|\bar{y}\|^2 + 2\sum_{i=1}^{p} |\bar{y}_i|(\varepsilon_i^N + k_i W_i^M \|\tilde{W}\|_F - k_i \|\tilde{W}_i\|_F^2)$$
$$= -2\sum_{i=1}^{p} |\bar{y}_i|(k_i\|\tilde{W}_i\|_F^2 - k_i W_i^M \|\tilde{W}_i\|_F - \varepsilon_i^N + \frac{\alpha}{2}|\bar{y}_i|) \tag{19}$$

From equation (19), $\dot{V}$ will be negative if

$$|\bar{y}_i| \ge \frac{4\varepsilon_i^N + k_i (W_i^M)^2}{2\alpha} \tag{20}$$

or

$$\|\tilde{W}_i\|_F \ge \frac{W_i^M}{2} + \sqrt{\frac{(W_i^M)^2}{4} + \frac{\varepsilon_i^N}{k_i}} \tag{21}$$

According to standard Lyapunov theorem, this demonstrates that $\|\tilde{x}\|$ and $\|W_i\|_F$ are UUB.

In the above proof if the modelling uncertainty $\varphi(x,u)$ is not zero, $\dot{V}$ can be derived as:

$$\dot{V} \le -(\lambda_{\min}(Q) - 2\eta\lambda_{\max}(P))\|\tilde{x}\|^2 +$$
$$2\sum_{i=1}^{p} |\bar{y}_i|[\varepsilon_i^N + k_i tr(\tilde{W}_i^T (W_i - \hat{W}_i))] + 2\lambda_{\max}(P)\varphi_0\|\tilde{x}\|$$
$$= -\|\tilde{x}\|((\lambda_{\min}(Q) - 2\eta\lambda_{\max}(P))\|\tilde{x}\| - 2\lambda_{\max}(P)\varphi_0) +$$
$$2\sum_{i=1}^{p} |\bar{y}_i|[\varepsilon_i^N + k_i tr(\tilde{W}_i^T (W_i - \hat{W}_i))] \tag{22}$$

So if $(\lambda_{\min}(Q) - 2\eta\lambda_{\max}(P))\|\tilde{x}\| - 2\lambda_{\max}(P)\varphi_0 \ge 0$, i.e.,

$$\|\tilde{x}\| \ge \frac{2\lambda_{\max}(P)\varphi_0}{\lambda_{\min}(Q) - 2\eta\lambda_{\max}(P)} \tag{23}$$

and equation (21), $\dot{V}$ can still be guaranteed to be negative. Hence $\|\tilde{x}\|$ and $\|W_i\|_F$ are still UUB.

From the above proof, the boundedness of the state estimation error is given by equations (20) or (23). If the value of $\lambda_{\min}(Q)$ is sufficiently large, the estimation error will be small. This can be achieved by a suitable choice of observer gain K. From theorem 1, the on-line learning algorithm for the dynamic recurrent neural network can be expressed as:

$$\Delta W_i = \xi(\Gamma_i \sigma_i(\hat{x},u)\bar{y}_i^T - k_i|\bar{y}_i|\Gamma_i \hat{W}_i) \quad i = 1, \cdots, p \tag{24}$$

where ΔW_i is the weight adjustment and ξ is the learning rate to be specified. The initial value of $\hat{W}_i$ can be chosen as zero or any small value. Note that equation (24) is simply the discrete form of equation (12), a properly chosen ξ can guarantee the stability of the approximator.

3.4 Robustness and sensitivity of the on-line fault approximator

Robustness analysis investigates the behaviour of the on-line approximator in the presence of modelling uncertainties prior to the occurrence of any faults. Sensitivity analysis examines the behaviour of the on-line approximator after the occurrence of a fault and characterises the class of faults that can be detected by the fault diagnosis scheme [10].

Without loss of generality, suppose the output matrix C is equal to $[I_{l\times l} \quad O_{l\times(n-l)}]$, where $I_{l\times l}$ and $O_{l\times(n-l)}$ are unit and zero matrices respectively. As described in [10], the learning algorithm of equation (24) can be changed to equation (25) below in order to keep the robustness and sensitivity of the on-line approximator.

$$\Delta W_i = \begin{cases} 0 & \text{if } |\bar{y}| < \dfrac{\mu\varphi_0}{\lambda} \\ \xi(\Gamma_i \sigma_i(\hat{x},u)\bar{y}_i^T - k_i|\bar{y}_i|\Gamma_i \hat{W}_i) & \text{otherwise} \end{cases} \quad (25)$$

In equation (25), μ and λ are positive constants such that $\|\exp(A_c t)\|_2 \le \mu \exp(-\lambda t)$. By using the learning algorithm depicted in equation (25) the output of the approximator will be non-zero only after the occurrence of faults. This decreases the false fault detection ratio.

3.5 A practical fault detection scheme

In order to guarantee the stability of the on-line learning algorithm, equations (8) and (10) must be satisfied. Equation (10) is derived from the Lipschitz nonlinear function $f(x,u)$. In fact, equation (10) guarantees the stability of the observer for the following nonlinear system.

$$\left.\begin{aligned} \dot{x} &= Ax + f(x,u) \\ y &= Cx \end{aligned}\right\} \quad (26)$$

$f(x,u)$ is supposed to be a Lipschitz nonlinearity with Lipschitz constant η as in Assumption 4. Equation (10) is a well-known result introduced in [8] and has been used and extended by some other scholars [7][11]. However, equation (10) is very difficult to testify after equation (8) is satisfied, especially when the Lipschitz constant η is big. Moreover, this equation is only a sufficient condition, not a necessary one. From the proof for theorem 1, it can be seen that equations (8), (11) and (12) are used to guarantee the stability of the on-line approximator, and equation (10), the stability of the Lipschitz function $f(x,u)$. Since equation (10) is difficult to check, the following design steps are proposed for a practical fault detection scheme.

(1) Check whether K, P, Q and T can be found to satisfy equation (8) and (9). If there is no suitable K, P, Q and T, the proposed fault detection scheme cannot be used.

(2) Use matrix K to design the observer. From one group of input and output datum (under normal condition without faults) test whether the observer is stable. If not, choose a new K and start over from step (1).

(3) Design the nonlinear observer described in equation (2) for the system. The observer gain matrix K is the same as that in step (2).

(4) Use the on-line dynamic recurrent neural network approximator to approximate faults. The on-line learning algorithm is given by equation (24).

Although the above method cannot guarantee the existence of an on-line approximator, it avoids the complexity of testing mathematically whether equation (10) is satisfied.

4. Example

Consider a DC motor model given in [12].

$$\begin{bmatrix} \dot{i}_f \\ \dot{i}_a \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} -\dfrac{R_f}{L_f} & 0 & 0 \\ 0 & -\dfrac{R_a}{L_a} & 0 \\ 0 & 0 & -\dfrac{D}{J} \end{bmatrix} \begin{bmatrix} i_f \\ i_a \\ \omega \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L_f} \\ \dfrac{1}{L_a} \\ 0 \end{bmatrix} V + \begin{bmatrix} 0 \\ -\dfrac{M_f}{L_a} i_f \omega \\ \dfrac{M_f}{J} i_f i_a \end{bmatrix} \quad (27)$$

where R_f =50 Ω, L_f =20 H R_a =3.8 Ω L_a =0.5 H, D =0.042 *Nms/rad*, and J =0.4 kgm^2. The system measurement equation is:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_f \\ i_a \\ \omega \end{bmatrix} \quad (28)$$

Suppose the input voltage $V = 100 + 10 * \sin(\frac{t}{15}\pi)$. For this motor model, the following three simulations were carried out.

(1) *Verification of stability and accuracy*

A nonlinear observer was designed for the motor model. Substituting the values of the parameters into equation (27), matrices $A = \begin{bmatrix} -2.5 & 0 & 0 \\ 0 & -7.6 & 0 \\ 0 & 0 & -0.105 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and nonlinearity $f(x) = \begin{bmatrix} 0 \\ -0.442\, i_a \omega \\ 0.5525\, i_f i_a \end{bmatrix}$ were obtained. The observer gain matrix is chosen as $K = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 9.895 \end{bmatrix}$. Simulation results showed that the observer is stable and accurate. (Because of space limited, we do not show the simulated results).

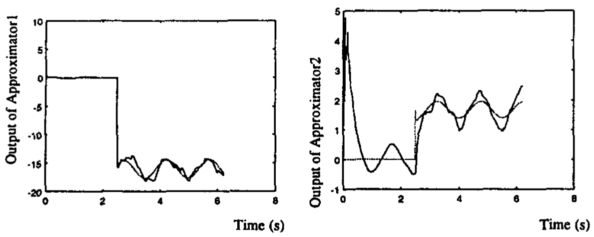
(2) *Fault with no system uncertainty*

Suppose that there is a 15% increase in the parameter M_f but no uncertainty in the system. Since there are no faults in the first entry of equation (27), there are two non-zero terms in the fault vector $\varphi(.)$ and $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$. Observer gain matrix K is the same as that in the last simulation. The matrices P and Q are chosen as $P = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $Q = \begin{bmatrix} 20 & 10.1 & 0 \\ 10.1 & 15.2 & 0 \\ 0 & 0 & 20 \end{bmatrix}$, and $PB = C^T$. Note that $f(.)$ is local Lipschitz for this example. Also, using the proposed design method, there is no need to check whether equation (10) is satisfied. In this simulation B-spline neurofuzzy network is used to approximate the faults. Ten second-order B-spline basis functions are used for each approximator. The faults are supposed to be suddenly happening at $t = 2.5s$. The on-line learning algorithm described by equation (12), with $\Gamma_1 = diag(0.45)$, $\Gamma_2 = diag(0.39)$, $k_1 = 0.02$ and $k_2 = 0.02$, is used to train the network and the observer. Figure 1 displays the outputs of the two on-line approximators (solid line) and the simulated fault nonlinearities (dotted line). From figure 1 it can be seen that the outputs of the on-line approximators can approximate the fault nonlinearities very well and the fault can be detected easily by using a proper threshold.

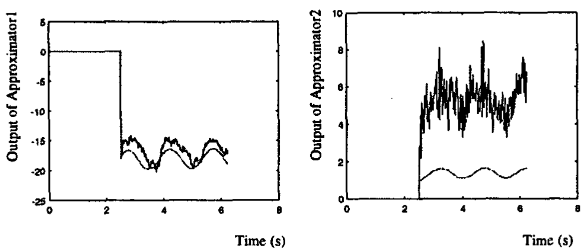
(3) *Fault with bounded system uncertainty*

In the previous simulation, the uncertainty term $\varphi(x,u)$ is assumed to be zero. In this simulation, the modelling uncertainty vector in the system is random but bounded by $\varphi_0 = \begin{bmatrix} 0.5 \\ 2.5 \\ 8.0 \end{bmatrix}$. The on-line learning algorithm represented by equation (25) is used. μ and λ are chosen as 1.0 and 2.5 respectively. Although matrix C is not in the form used to derive equation (25), the two output errors can be denoted as $\frac{u}{\lambda}(\varphi_0(1) + \varphi_0(2))$ and $\frac{\mu}{\lambda}\varphi_0(3)$, where $\varphi_0(i), i = 1,2,3$

is the i^{th} entry of φ_0. The parameters used in the previous simulation are used to train the network. In figure 2, the outputs of the on-line approximators (solid line) and the faults are shown. From figure 2, it is observed that before the occurrence of the faults, the outputs of the on-line approximators are zero though the uncertainty of the system is non-zero. This indicates the robustness of the on-line approximator and also decreases the false fault detection ratio. Compared with the previous simulation, the outputs of the approximators do not match the faults very well. This is because the on-line approximator has to trace not only the faults but also the uncertainty.



Solid line: approximator; Dotted line: fault
Figure 1 Fault and its estimation with no system uncertainty



Solid line: approximator; Dotted line: fault
Figure 2 Fault and its estimation with system uncertainty

6. Conclusion

A fault detection scheme for nonlinear systems with inaccessible state has been proposed. This scheme only makes use of the input and output of the system. An on-line approximator based on dynamic recurrent neural network predicts the faults in the system. Stability analysis, learning algorithm and other properties of the approximator are described in the paper. The proposed fault detection scheme is different from that presented in [10] which used a local diffeomorphism to transfer the nonlinear system into another, simpler, system. This kind of diffeomorphism does not always exist and, even if it exists, is difficult to find. The proposed scheme avoids using such a local diffeomorphism. Of course some other assumptions must be existed for the systems.

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