

Stabilization of Systems with Deadzone Nonlinearity

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Abstract

This paper studies stabilization of control systems with deadband nonlinearity of unknown characteristics. A novel approach to treat the deadband is first proposed using techniques of saturation compensation, assuming crude estimates of gains and bounds for the saturation limiter. Stability of the compensated system is analyzed, revealing that for systems of conditional stability in the presence of deadzone nonlinearity, their stabilization is not possible for small inputs. However, proper stabilization always exists for regulatory control of large enough input magnitudes. Simulated examples illustrate the main results.

1. Introduction

Deadzone nonlinearity comes with mixed blessings in its dynamic effects on control systems. On one end of the spectrum, it leads to chaos [1] while on the other, many robust adaptive/fuzzy/neural control algorithms intentionally introduce deadzones for parameter tunings and system stabilization [e.g. 4,6]. Deadzone occurrence is commonly found in servo valves, whether it is due to poor machining of the valve spools or it is purposely overlapped to prevent leakage. It usually gives rise to 'hunting' movements or limit cycles in the valve positions [1]. Another usual observation is the existence of steady state offsets in regulatory controls.

Sophisticated control algorithms to treat deadzone nonlinearity, using adaptive/fuzzy/neural schemes [e.g. 5,9-12], are reported in the literature. Yet deadzone treatments using simple linear compensators are much less encountered [3]. This paper is to investigate the feasibility of global stabilization of systems with deadzone nonlinearity by simple linear compensators.

Unlike previous works, the treatment proposed here is in vein of saturation compensation techniques, easily found in the literature [7]. It turns out that this grafting process is convenient, efficient and highly productive, as demonstrated below. In [3], a similar framework using direct deadzone compensation is studied; which, however, is less effective in analysis.

In §2, a compensation framework for the deadband nonlinearity via saturation limiter is proposed. Stability of the compensated systems is analyzed in §3 to establish the main results. Design guidelines of deadzone compensators are discussed in §4. Illustrative examples are presented in §5.

2. Compensation Framework

Following a general saturation compensation approach [7-8], for a linear control system with deadzone nonlinearity, it is proposed to amend the linear controller $\{R, S, T\}$ as follows:

$$v = [T/R]w - [S/R]y + P\delta_S \quad (2.1)$$

where y is the system output, w the reference input, v the

controller output; R, S, T are polynomials in Laplace transform variable s . P is the transfer function for the compensator. Output of the ideal deadzone nonlinearity for input v is given by

$$u = \text{DZ}[v] = \begin{cases} m_R(v - b_R) & , v > b_R \\ 0 & , b_L \leq v \leq b_R \\ m_L(v - b_L) & , v < b_L \end{cases} \quad (2.2)$$

where $\{m_R, m_L\}$ and $\{b_R, b_L\}$ are the linear gains and bounds of the deadband parameters. For unknown characteristics, let $\{\hat{m}_R, \hat{m}_L\}$ and $\{\hat{b}_R, \hat{b}_L\}$ be their estimates respectively. It is easily seen that (2.2) can be decomposed into a linear gain and a saturation nonlinearity [2] as

$$\text{DZ}[v] \equiv kv - \text{sat}[v] \quad (2.3a)$$

where k denotes gain m_R or m_L as appropriate, and

$$\text{sat}[v] = \begin{cases} m_R b_R & , v > b_R \\ m_R v & , 0 \leq v \leq b_R \\ m_L v & , b_L \leq v \leq 0 \\ m_L b_L & , v < b_L \end{cases} \quad (2.3b)$$

For unknown characteristics, an estimated model of (2.3b) is obtained using $\{\hat{m}_R, \hat{m}_L\}$ and $\{\hat{b}_R, \hat{b}_L\}$ instead of the exact values. In that case let \hat{k} be the estimate of k accordingly. Despite this approximation, it is shown that significant performance improvements can still be achieved by proper compensators.

There are several possibilities to define the compensation activation δ_S in (2.1). In view of (2.3), it is elected to adopt

$$\delta_S \triangleq \hat{u} - \hat{k}v = \text{sat}[v]_{\text{estimate}} - \hat{k}v \quad (2.4)$$

in which the saturation model is using the parameter estimates. Comparing (2.3)-(2.4), it is seen that

$$\delta_S \equiv -\text{DZ}[v]_{\text{estimate}} \quad (2.5)$$

The block diagram for the compensated system with deadzone nonlinearity is shown in Fig. 1.

For the exact but unknown deadzone nonlinearity, a disturbance δ_D is similarly defined [3]

$$\delta_D \triangleq u - kv \quad (2.6)$$

Using (2.1), (2.6) and for plant G , the closed loop expression for the controller output is derived as

$$v = \frac{T}{R}w - \frac{S}{R}y + P\delta_S = \frac{T}{R}w - \frac{S}{R}Gu + P\delta_S$$

$$\therefore v = \frac{T/R}{1+kG_C}w - \frac{G_C}{1+kG_C}\delta_D + \frac{P}{1+kG_C}\delta_S \quad (2.7)$$

in which $G_C = GS/R$. The system output is given by

$$\left. \begin{aligned} y &= y_0 + \Delta y_D + \Delta y_S \\ y_0 &= Hw, \Delta y_D = H_1\delta_D, \Delta y_S = kH_1P\delta_S \end{aligned} \right\} \quad (2.8)$$

The closed-loop transfer functions in (2.8) are

$$H = \frac{kGT/R}{1+kG_C}, H_1 = \frac{G}{1+kG_C} = \frac{BR}{AR+kBS} \quad (2.9)$$

for $G=B/A$, and $\{A,B\}$ are polynomials in s . From the assumption of acceptable linear system designs, poles of H and H_1 [$=AR+kBS$] are asymptotically stable.

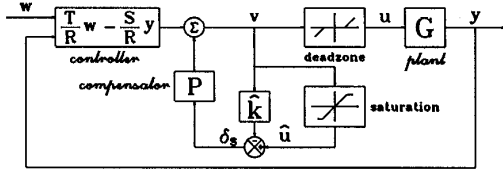


Fig.1 DZ compensation via saturation decomposition

The interpretation of (2.8) is obvious. If there is no deadband nonlinearity but a linear gain k , then the linear system output is identically given by y_0 only. When deadband exists, an output variation Δy_D is incurred. The last term Δy_S is the compensation purposely introduced.

When there is no compensation [$P=0$], there are two general conditions under which no steady state offsets exist, i.e., for $\Delta y_D(t) \rightarrow 0$ as $t \rightarrow \infty$:

- (Z1) either $\delta_D(t \rightarrow \infty) \rightarrow 0$, or
- (Z2) the inverse Laplace transform of $[H_1 \delta_D] \rightarrow 0$.

Since the deadband disturbance $\delta_D \equiv -\text{sat}[v]$, which is non-zero except at the origin ($v=0$), therefore condition (Z1) shall not generally be true. Even for type-1 plants, the only truth is $u_{ss}=u(t \rightarrow \infty) \rightarrow 0$; there is no guarantee that $v_{ss}=v(t \rightarrow \infty) \rightarrow 0$ as well. Rather, according to (2.6), the controller output settles along the line

$$v_{ss} = [u_{ss} - \delta_{Dss}] / k = -\delta_{Dss} / k \quad (2.10)$$

which can be anywhere inside the deadzone with $u_{ss}=0$.

One sufficient condition for (Z2) is (Z2') in (2.9), if $BR = s B_1 R_1$ for some polynomial $B_1 R_1$, and there is a constant δ_{Dss} in steady state, then $\Delta y_D(t) \rightarrow 0$ as $t \rightarrow \infty$.

In summary, (Z1) seeks $\delta_{Dss}=0$ but (Z2') seeks $[d\delta_D/dt]_{t \rightarrow \infty} = 0$ instead. In other words, for a type-0 plant with an integrator in controller, if the deadzone is sufficiently narrow with non-zero steady state output in u , then there will be no offsets.

Unfortunately, both conditions (Z1) and (Z2') are not always met and thus in general, the uncompensated deadband systems retain steady state offsets because there is no mechanism to adjust the output variation Δy_D , as $\{G,R,S,T,k\}$ are predetermined by the linear system design and the deadband characteristics.

However, with the compensation brought about by the compensator P , one of the objectives is to seek the removal of steady state offsets by requiring

$$\Delta y(t) \triangleq \Delta y_D(t) + \Delta y_S(t) \rightarrow 0 \quad \text{as } t \rightarrow \infty \quad (2.11)$$

so that $y(t) \rightarrow y_0(t) \rightarrow w(t)$ as $t \rightarrow \infty$. By writing $\Delta y = H_1 \delta$,

$$\delta \triangleq \delta_D + kP\delta_S \quad (2.12)$$

then via the introduction of P , the conditions to eliminate steady state offsets change from (Z1)-(Z2) to

- (Z3) either $\delta(t \rightarrow \infty) \rightarrow 0$, or
- (Z4) the inverse Laplace transform of $[H_1 \delta] \rightarrow 0$. Or
- (Z4') if $BR = s B_1 R_1$ for some polynomial $B_1 R_1$, and there is a constant δ_{ss} , then $\Delta y(t) \rightarrow 0$ as $t \rightarrow \infty$.

Because Δy_S only appears after activation of the saturation compensator, a limitation of this framework is

that the bound estimates, $\{\hat{b}_R, \hat{b}_L\}$, must not be too large; otherwise there is no saturation activation in the compensation network [$\delta_S=0$] and no compensation takes place. On the other hand, because the bound estimates are rather arbitrary and need not correspond to the actual values of the deadband characteristics (which are unknown anyway), effectively this becomes one additional degree of freedom to gauge the occurrence and magnitude of δ_S . This can be exploited to advantage, see §4 and Ex.3 in §6.

The requirement on the admissibility of P is that it be physically realizable and stable, i.e., $P(s)$ is proper and possibly with one integrator [otherwise Δy_S in (2.8), or δ in (2.12), may become unbounded if $P(s)$ has more than one integrator].

3. Stability analysis of the compensated system

This section discusses stability analysis of the deadzone compensated system via saturation decomposition, based on the framework shown in Fig.1. One basic concern is the existence of compensators to globally stabilize the control system with deadzone nonlinearity, as investigated below.

To derive the equivalent diagram of Fig.1, from (2.1) and substituting δ_S from (2.4), gives

$$v = [T/R]w - [S/R]y + P[\hat{u} - \hat{k}v] \quad (3.1)$$

Using $y=Gu$, (3.1) can be written as

$$v = F_w w - [G_1 u - G_2 \hat{u}] \quad (3.2)$$

$$F_w = \frac{T/R}{1+\hat{k}P}, G_1 = \frac{GS/R}{1+\hat{k}P}, G_2 = \frac{P}{1+\hat{k}P}$$

which is visualized as the system shown in Fig.2.

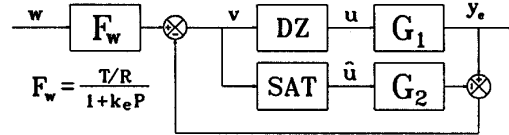


Fig.2 Block Diagram for Stability Analysis

For nonlinear stability analysis, the concept of dynamic gains in harmonic balancing is very useful [2]. Let k_D denote the dynamic gain of the deadzone, and k_S that for the saturation limiter, so that

$$u = k_D v \quad \text{and} \quad \hat{u} = k_S v \quad (3.3)$$

then v in (3.2) becomes

$$v = F_w w - [(k_D G_C - k_S P) / (1 + \hat{k} P)] v \quad (3.4)$$

When Lyapunov stability is being considered, $w=0$ [2]. From (3.4), the characteristic equation is

$$[1 + (k_D G_C - k_S P) / (1 + \hat{k} P)] v = 0 \quad (3.5)$$

which can be written as

$$1 + k_D G_E = 0 \quad (3.6)$$

$$G_E \triangleq G_C / [1 + k_e P], \quad k_e \triangleq \hat{k} - k_S$$

Since $0 \leq k_S \leq \hat{k}$ for dynamic gain k_S and linear gain \hat{k} respectively, thus, $k_e \geq 0, \forall k_S$. In fact, k_e is the dynamic gain of the estimated deadzone [2] and a normalized plot of k_e for symmetric bounds is shown in Fig.3.

The equivalent system G_E in (3.6) establishes the effects of compensator P on the linear system subject to

deadband nonlinearity. Standard techniques, such as Popov criterion and the circle criteria [2], in nonlinear system stability analysis can be immediately applied to G_E , taking into account of the time-varying gain k_e .

When global stability is being considered, G_E must be open-loop asymptotically stable before it can be globally stabilized, therefore another condition on the admissibility of compensator P is that zeros of $1+k_e P=0$ must be asymptotically stable, $\forall 0 \leq k_e \leq \hat{k}$ [2]. This condition, however, is not necessary when only local stability is being considered.

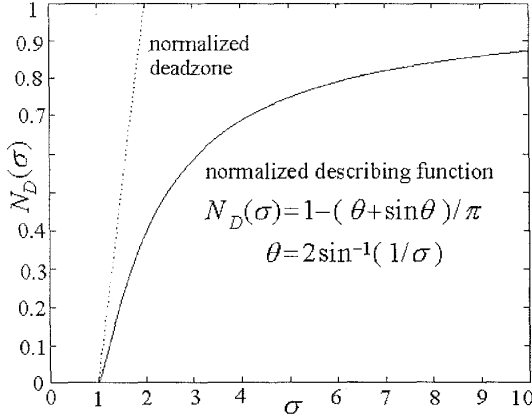


Fig.3 Normalized describing function for symmetric deadzone nonlinearity. $N_D = k_D/k$; $\sigma = a/b$, a =oscillation amplitude, b =symmetric deadzone bound.

Further insight is gained by separating the linear system and the compensator in (3.6), giving

$$G_E(s) = G_C(s) F(s) ; F(s) \triangleq 1/[1+k_e P] \quad (3.7)$$

The interpretation of (3.7) is: the Nyquist plot $G_E(j\omega)$ of the compensated system is that of the linear system $G_C(j\omega)$, modified by the Nyquist curve of the filter $F(j\omega)$. Any effect of the compensator P solely changes the shapes of $F(j\omega)$ alone.

Since the describing function for the deadzone lies entirely on the negative real axis on the $G_E(j\omega)$ -plane beyond the point $-1/k+j0$ [2], by virtue of the circle criterion, if $\text{Re}[G_C(j\omega)] > -1/k$, $\forall 0 \leq \omega \leq \infty$ and $k_e > 0$, then the compensated system shall be asymptotically stable. As always been the case, such sufficient condition is far too conservative in most practical applications.

Theorem

For a linear controlled system G_C , if it is only locally stable in the presence of deadzone nonlinearity, then there does not exist any linear compensator in form of (2.1) which would globally stabilize G_C in the presence of the deadzone nonlinearity. \diamond

Proof If G_C is only locally stable, then for

$$G_C(j\omega) = -X(\omega) + jY(\omega) \quad (3.8)$$

$\exists \omega_0 \in [0, \infty)$ such that $Y(\omega_0) = 0$ and $X(\omega_0) > 1/k$.

With the linear compensator P applied per (2.1), the equivalent system G_E is given by (3.6) and $0 \leq k_e \leq \hat{k}$. For small control signal v [compared with the deadzone bounds], $k_e(v \rightarrow 0) \rightarrow 0$ [Fig.3]. From (3.7), $F(k_e \rightarrow 0) \rightarrow 1$, $\forall P$. This leads to

$$G_E(j\omega) = G_C(j\omega) F(j\omega) \rightarrow G_C(j\omega) \quad (3.9)$$

giving $G_E(j\omega_0) = -X(\omega_0) + j0$. Therefore, the compensated system is only locally stable as well.

For regulatory controls, small reference inputs yield small controller outputs and hence the nonlinear system cannot be compensated to provide global stability if the linear system is only locally stable. \diamond

According to the above result, while there does not exist linear compensator to ensure asymptotic stability of the deadzone system for small setpoint changes, however, for large enough reference inputs so that a finite k_e of sufficient magnitude is reached, then the nonlinear system can always be compensated to ensure asymptotic stability.

A significant conclusion is: the only way to ensure global stability of the controlled system with deadzone nonlinearity is to ensure that the linear controlled system is globally stable in the first place. Under such circumstances, importance of the compensator is on the improvement of transient responses, rather than stabilization of the nonlinear system. This leads to the design of compensators: for stabilization is a somewhat clear-cut issue since there are in general specific criteria to be satisfied. However, it is not clearly defined whether certain behaviours of transient responses are satisfactory. Some guidelines on the general selection of deadzone compensators are thus in order.

4. Compensator Design

One advantage of using estimated parameters of the deadzone model in the formulation of compensation is now clear. According to the result in §3, too small a value of k_e renders all compensators ineffective. Therefore, if the operating condition of the controlled system is known, then however small the setpoint input may be, it is always the designer's discretion to adjust estimates of the deadzone characteristics so that a sufficiently large value of k_e [compared with the deadzone bounds] is reached and maintained. One simple option is to reduce estimates of the deadzone bounds. This shall ensure that the compensator be activated and remain active during the transient stage. Assuming such step has been taken, then the design of compensators is relatively easy and presented below.

From (3.7) and for a $G_C(j\omega)$ system with only conditional stability, it is possible to specify a globally stable $G_E(j\omega)$ -curve and require the phase shifts $[\angle G_E(j\omega) - \angle G_C(j\omega)]$ be effected by $F(j\omega)$. Most often, lead filter designs for $F(s)$ are needed to increase the stability margins of $G_E(s)$ [8]. Once the required $F(s)$ is selected and knowing estimates of the operating condition k_e , the compensator is calculated from

$$P(s) = [1/F(s) - 1]/k_e \quad (4.1)$$

Obviously, if setpoint w is too small so that $k_e \rightarrow 0$, then in (4.1) $P(s) \rightarrow \infty$ and thus it cannot be realized.

Many selections for filter $F(s)$ exist and design methods for simple low order filters are abound. Two compensator choices are nevertheless suggested here, for the two possible types of systems, one requiring phase advances to provide stabilization and the other for phase lags to speed up responses.

(a) Phase lead compensator: choose

$$P(s) = k_p / s(s+p) \quad (4.2)$$

the corresponding filter is, from (3.7),

$$F(s) = \frac{1}{1+k_e p} = \frac{s(s+p)}{s^2+ps+k_a}, \quad k_a = k_e k_p \quad (4.3)$$

$$= \frac{s(s+2\xi\omega_n)}{s^2+2\xi\omega_n s+\omega_n^2}, \quad p=2\xi\omega_n, k_a=\omega_n^2$$

Since $\{p, k_a\} \geq 0$ in order to maintain system stability of G_L , $F(s)$ in (4.3) is identically phase advancing:

$$F(j\omega) = \frac{\omega^2[p^2 - k_a + \omega^2] + j\omega p k_a}{(k_a - \omega^2)^2 + (\omega p)^2} \quad (4.4)$$

therefore $0^\circ \leq \angle F(j\omega) \leq 180^\circ, \forall 0 \leq \omega \leq \infty, k_p > 0$ and $p > 0$. The maximum phase lead $\phi = [\angle F(j\omega)]_{\max}$ for (4.4) is:

(i) if $p^2 \geq k_a$, [i.e., $\xi \geq 0.5$], $\phi = 90^\circ$ at $\omega = \omega_m = 0$.
(ii) if $p^2 < k_a$, [i.e., $\xi < 0.5$], then at $\omega_m = [(k_a - p^2)/3]^{1/2}$,
 $\phi = \pi - \tan^{-1} \left\{ \frac{1}{2} p k_a [3 / (k_a - p^2)]^{3/2} \right\} \quad (4.5)$

Unless stability consideration demands $\phi > 90^\circ$, from the view point of transient responses, it is more favourable to employ case (i) because the equivalent system would be less oscillatory [$\xi \geq 0.5$ in (4.3)] as a result of the compensation. Notice that if a large phase shift is needed for stabilization, then the uncompensated system is very lightly damped and in accordance with the Theorem of §3, the compensator can only achieve stabilization at a price of oscillatory transients.

(b) Phase lead/lag compensator: choose

$$P(s) = k_p / (s+p) \quad (4.6)$$

so
$$F(s) = \frac{s+p}{s+p+k_a} = \frac{s+1/\beta}{s+1/\alpha\beta} \quad (4.7)$$

$$\beta = 1/p, \quad \alpha = 1/[1+k_e(k_p/p)]$$

For system stability, $p \geq 0$ and $p+k_a \geq 0$. Therefore, if $k_p > 0$, $\alpha < 1$, then $0 \leq \angle F(j\omega) \leq 90^\circ$. For $-1/k_a < k_p < 0$, $\alpha > 1$ and then $-90^\circ \leq \angle F(j\omega) \leq 0^\circ$. The maximum phase shift for (4.7) is:

$$\phi = \sin^{-1}[(1-\alpha)/(1+\alpha)] \quad \text{at} \quad \omega_m = 1/\beta\sqrt{\alpha} \quad (4.8)$$

ϕ is phase lead if $0 < \alpha < 1$ and phase lag if $\alpha > 1$.

In both compensator designs of (4.2) and (4.6), only two parameters [feedback gain k_p and pole p] need be determined. Their designs can be calculated [such as from (4.5) or (4.7)-(4.8)] through proper determination of frequency ω_m and specification of the maximum phase shift ϕ , see [8] for similar design procedures developed for saturation compensations. When large phase shifts are required, a cascade of $F(s)$ from (4.3) or (4.7) can be used.

5. Examples

The same system with two controller designs are considered: one is locally stable only and the other is globally stable, so as to illustrate results of the Theorem in §3, and the importance of linear system designs for global stabilization of deadzone nonlinear systems. Finally, the freedom to achieve a large enough dynamic gain k_e in (3.6) is illustrated.

Ex. 1 The plant is described by

$$G(s) = 1/(s+0.1)(s+0.2)(s+2.5) \quad (5.1)$$

and the linear controller is

$$S/R = T/R = 2[(s+0.45)s+0.3]/s(s+2.5) \quad (5.2)$$

which is a realistic PID-controller. The deadband characteristics and their estimates are

$$\left. \begin{aligned} m_R = 5, m_L = 5, b_R = 0.2, b_L = -0.1 \\ \hat{m}_R = \hat{m}_L = 6, \hat{b}_R = 0.3, \hat{b}_L = -0.2 \end{aligned} \right\} \quad (5.3)$$

and thus none of the estimates match the true values of the deadband element. This system is not well designed linearly and $G_C(s)$ is only locally stable, since

$\omega =$	0.1626	0.5126	2.3229
$k G_C(j\omega) =$	-56.36+j0	-2.44+j0	-0.35+j0

Limit cycles therefore exist due to the two interception points with the negative real axis beyond point $-1+j0$. According to the Theorem of §3, there does not exist linear compensator to stabilize the system for small reference inputs.

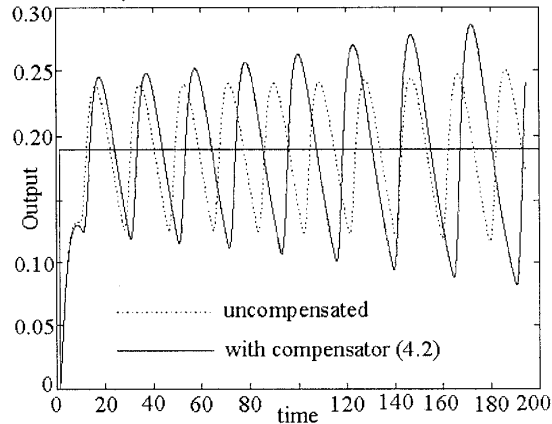


Fig.4 Step response for small input $w=0.19$. System settles to stable limit cycles, whether it is compensated.

From $G_L(j\omega)$ -plots [not shown here] using $P(s)$ of (4.2) with $k_p=p=0.5$ and $k_e=\{0.0, 0.05, 0.15, 1\}$, and for $0 \leq k_e \leq 0.15$, there exist interceptions of $G_L(j\omega; k_e)$ -curves with the negative real axis and therefore the compensated system still contains limit cycles. Step responses in Fig.4 show that both uncompensated and compensated system converge to stable limit cycles for $w=0.19$. Obviously the compensator is not effective for this input size. For larger steps, $P(s)$ becomes effective and system reaches asymptotic stability, Fig.5 for $w=1$.

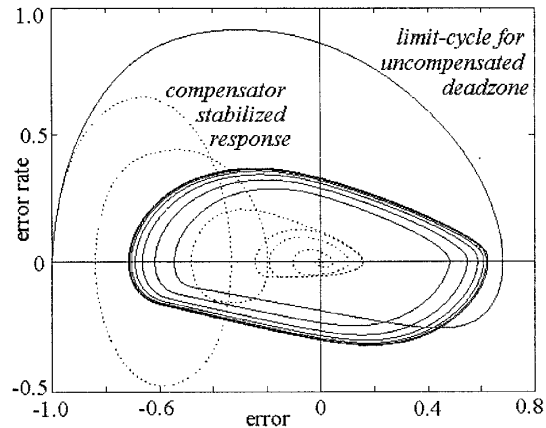


Fig.5 Phase portrait for $w=1.0$ [error $e=y-w$; error rate $=de/dt$]. The same compensator is now effective and stabilizes the deadzone system.

Ex. 2 The above system is globally stabilized by redesigning the linear controller (5.2) into

$$S/R = T/R = 2[(s+0.4)s+0.16]/s(s+2.5) \quad (5.4)$$

The resultant $G_c(j\omega)$ no longer cuts the real axis beyond $-1/k+0$. Using the same setup as Ex.1, step responses for $w=0.19$ are shown in Fig.6. Both compensated and uncompensated system are now stabilized.

Ex. 3 If controller (5.2) remains unchanged, the above nonlinear system can still be stabilized by flexing the extra degree of freedom imbedded in the estimates of the deadzone characteristics. For example, to achieve a dynamic gain k_e of sufficient magnitude, shrink the bounds to

$$\hat{b}_R = 0.1, \quad \hat{b}_L = -0.05 \quad (5.5)$$

but maintain all the other values in (5.3). Choose compensator (4.6). With the same step size $w=0.19$, a phase portrait is shown in Fig.7 for $\{k_p=0.022; p=0.1\}$. Asymptotic stability is now achieved. \diamond

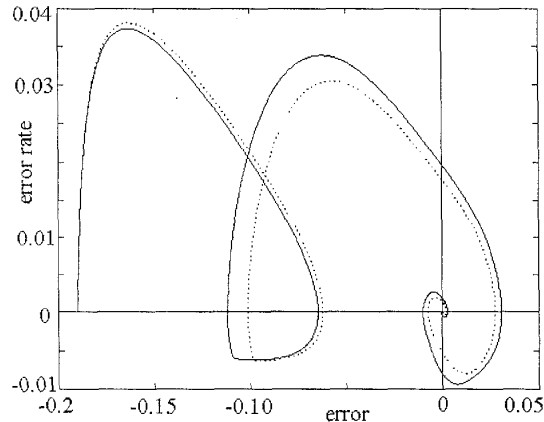


Fig.6 Phase portrait for $w=0.19$ after redesign of the linear controller in Ex.2. Both uncompensated (dotted line) and with P of (4.2) (solid line) are globally stable.

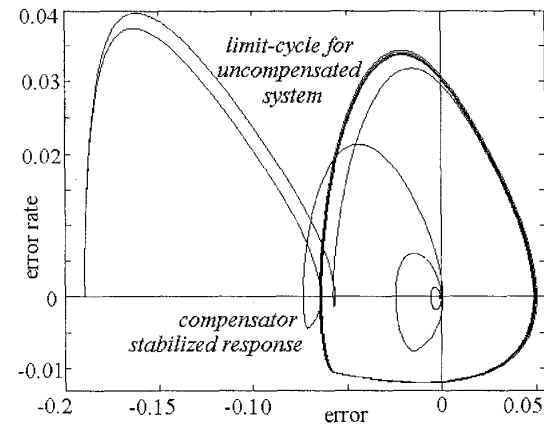


Fig.7 Phase portrait for $w=0.19$ after shrinking the bound estimates to increase k_e . The compensated system with P of (4.6) is stabilized [c.f. Fig.4].

6. Conclusion

This paper presents a novel treatment of the deadband nonlinearity, by first decomposing it into a linear gain and a saturation element. Equivalent system is derived to reveal that compensators, guaranteeing asymptotic stability for systems with only local stability, do not exist for small reference inputs. However, provided the step input is of sufficient magnitude, asymptotic stability for the deadzone system can always be achieved for these locally stable systems. Two specific compensator designs are suggested to cater for systems requiring phase leads or phase lags separately. With proper parameter tunings, these two compensators can provide satisfactory transient responses and system stabilization in the presence of deadzone nonlinearity. A simulated example illustrates the analysis procedures and applications of the two compensator designs.

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