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H_∞ Control for Uncertain Neutral Systems via Non-fragile State Feedback Controllers

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Abstract

This paper deals with the problem of non-fragile H_∞ control for neutral systems with time-varying delays. The state feedback gains to be designed are subject to norm-bounded uncertainties. The purpose of the problem is the design of a state feedback controller such that the resulting closed-loop system is asymptotically stable while satisfying a prescribed H_∞ performance level for all admissible uncertainties in the controller gain. Sufficient conditions for the solvability of this problem are obtained for the cases with additive and multiplicative controller uncertainties, respectively. A desired non-fragile state feedback controller can be obtained by solving certain linear matrix inequalities.

1 Introduction

Time delay is frequently a source of instability and poor performance in a control system. Time delays can be encountered in various engineering systems such as chemical processes and long transmission lines in pneumatic systems [2]. Therefore, control of delay systems has received considerable attention and has been one of the most interesting research topics in the past years. Usually, there are many types of delay systems; among them, delay systems with neutral type have been studied since many engineering systems can be modeled by using functional differential equations of the neutral type. Practical examples of neutral delay-differential systems can be found in the distributed networks containing loss-less transmission lines [1], population ecology [4], and other areas [5]. The problems of stability analysis and control of neutral systems have been investigated and many results on these topics have been obtained [5].

On the other hand, in the implementing of a designed controller, perturbations in the controller may arise due to finite word length in digital systems, the imprecision inherent in analog systems, the need for additional tuning of parameters in the final controller implementation and other reasons [3, 8]. Therefore, the non-fragile control problem has been studied over the decades, which is of practical and theoretical importance. In the context of H_∞ control for neutral systems, although the controller's robustness with respect to the system's parameter uncertainties was considered in [6], its robustness with respect to the controller's uncertainties still has not been investigated. This motivates the present study.

In this paper, we are concerned with the problem of non-fragile H_∞ control for neutral systems with time-varying delays. Attention is focused on the design of a state feedback controller, which is subject to norm-bounded uncertainty, such that the resulting closed-loop system is asymptotically stable while satisfying a prescribed H_∞ performance level. Sufficient conditions for the solvability of this problem are given for the cases with additive and multiplicative controller uncertainties, respectively. A desired non-fragile state feedback controllers can be obtained by solving certain linear matrix inequalities (LMIs).

2 Problem Formulation

Consider the following neutral system with time-varying delays:

$$(\Sigma): \quad \begin{aligned} \dot{x}(t) &= Ax(t) + A_1 x(t-\tau_1(t)) + A_2\dot{x}(t-\tau_2(t)) \\ &\quad + Bu(t) + E\omega(t), && (1) \\ z(t) &= Cx(t) + Du(t), && (2) \\ x(\theta) &= \phi(\theta), \quad \theta \in [-\mu, 0], && (3) \end{aligned}$$

where $x(t) \in \mathbb{R}^n$ is the state; $u(t) \in \mathbb{R}^m$ is the control input; $\omega(t) \in \mathbb{R}^q$ is the exogenous input which belongs to $\mathcal{L}_2[0,\infty)$, where $\mathcal{L}_2[0,\infty)$ denotes the space of square-integrable vector functions over $[0, \infty)$; $z(t) \in \mathbb{R}^l$ is the controlled output, A, A_1, A_2, B, C, D and E are known real constant matrices with appropriate dimensions. The scalars $\tau_1(t)$ and $\tau_2(t)$ are the time-varying delays satisfying

$$0 < \tau_1(t) \le \mu_1, \quad \dot{\tau}_1(t) \le h_1 < 1, \qquad (4)$$

$$0 < \tau_2(t) \le \mu_2, \quad \dot{\tau}_2(t) \le h_2 < 1, \qquad (5)$$

where μ_i and h_i, $i = 1, 2$, are real constant scalars, $\mu = \max(\mu_1, \mu_2)$; $\phi(t)$ is a real-valued continuous initial function on $[-\mu, 0]$.

Now, consider the following state feedback controller for system (Σ):

$$u(t) = Kx(t), \tag{6}$$

where $K \in \mathbb{R}^{m\times n}$ is the controller gain to be designed. Since gain perturbations may arise when implementing the controller in (6), the actual controller may be of the following form:

$$u(t) = (K + \Delta K(t))\, x(t), \tag{7}$$

where $\Delta K(t)$ is the controller gain perturbation. In this paper, the following two classes of controller gain perturbations will be considered:

(I) $\Delta K(t)$ is with the norm-bounded additive form

$$\Delta K(t) = \Delta_a(t) = H_a F_a(t) E_a, \tag{8}$$

where H_a and E_a are known matrices, and $F_a(t)$ is an unknown matrix satisfying

$$F_a(t)^T F_a(t) \leq I. \tag{9}$$

(II) $\Delta K(t)$ is with the norm-bounded multiplicative form

$$\Delta K(t) = \Delta_m(t) = H_m F_m(t) E_m K, \tag{10}$$

where H_m and E_m are known matrices, and F_m is an unknown matrix satisfying

$$F_m(t)^T F_m(t) \leq I. \tag{11}$$

The non-fragile H_∞ control problem to be addressed in this paper is the design of a state feedback controller in the form of (6) with perturbations satisfying (8) and (9), or (10) and (11) such that the resulting closed-loop system is asymptotically stable and under the zero initial condition,

$$\|z(t)\|_2 < \gamma \|\omega(t)\|_2, \tag{12}$$

is satisfied for any nonzero $\omega(t) \in \mathcal{L}_2[0, \infty)$. In this case, (6) is said to be a non-fragile state feedback controller.

3 Main Results

The following theorem provides a sufficient condition for the solvability of the non-fragile H_∞ control problem with the controller perturbation Δ_a in (8) and (9).

Theorem 1 *Consider the neutral system (1)-(3) and the controller perturbation Δ_a in (8) and (9). Then the non-fragile H_∞ control problem is solvable if there exist matrices $X > 0$, $Q_1 > 0$, $Q_2 > 0$, Y and a scalar $\epsilon > 0$ such that the following LMI holds:*

$$\begin{bmatrix} \Gamma + \epsilon B H_a H_a^T B^T & A_1 X & A_2 Q_2 & E & W_1^T + \epsilon B H_a H_a^T B^T & W_2^T + \epsilon B H_a H_a^T D^T & X E_a^T \\ X A_1^T & -(1-h_1) Q_1 & 0 & 0 & X A_1^T & 0 & 0 \\ Q_2 A_2^T & 0 & -(1-h_1) Q_2 & 0 & Q_2 A_2^T & 0 & 0 \\ E^T & 0 & 0 & -\gamma^2 I & E^T & 0 & 0 \\ W_1 + \epsilon B H_a H_a^T B^T & A_1 X & A_2 Q_2 & E & \epsilon B H_a H_a^T B^T - Q_2 & \epsilon B H_a H_a^T D^T & 0 \\ W_2 + \epsilon D H_a H_a^T B^T & 0 & 0 & 0 & \epsilon D H_a H_a^T B^T & \epsilon D H_a H_a^T D^T - I & 0 \\ E_a X & 0 & 0 & 0 & 0 & 0 & -\epsilon I \end{bmatrix} < 0, \tag{13}$$

where

$$W_1 = AX + BY, \quad W_2 = CX + DY, \quad \Gamma = W_1 + W_1^T + Q_1. \tag{14}$$

In this case, a desired non-fragile state feedback controller can be chosen as

$$u(t) = Y X^{-1} x(t). \tag{15}$$

Proof. By applying the Schur complement formula to (13), we have

$$\begin{bmatrix} \Gamma & A_1 X & A_2 Q_2 & E & W_1^T & W_2^T \\ X A_1^T & -(1-h_1) Q_1 & 0 & 0 & X A_1^T & 0 \\ Q_2 A_2^T & 0 & -(1-h_1) Q_2 & 0 & Q_2 A_2^T & 0 \\ E^T & 0 & 0 & -\gamma^2 I & E^T & 0 \\ W_1 & A_1 X & A_2 Q_2 & E & -Q_2 & 0 \\ W_2 & 0 & 0 & 0 & 0 & -I \end{bmatrix} + \epsilon^{-1} \begin{bmatrix} X E_a^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} X E_a^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \epsilon \begin{bmatrix} B H_a \\ 0 \\ 0 \\ 0 \\ B H_a \\ D H_a \end{bmatrix} \begin{bmatrix} B H_a \\ 0 \\ 0 \\ 0 \\ B H_a \\ D H_a \end{bmatrix}^T < 0. \tag{16}$$

Note that

$$\begin{bmatrix} B H_a \\ 0 \\ 0 \\ 0 \\ B H_a \\ D H_a \end{bmatrix} F_a(t) \begin{bmatrix} E_a X & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$
+\begin{bmatrix} XE_a^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} F_a(t)^T \begin{bmatrix} H_a^T B^T & 0 & 0 & 0 & H_a^T B^T & H_a^T D^T \end{bmatrix}
$$

$$
\leq \epsilon^{-1} \begin{bmatrix} XE_a^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} XE_a^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \epsilon \begin{bmatrix} BH_a \\ 0 \\ 0 \\ 0 \\ BH_a \\ DH_a \end{bmatrix} \begin{bmatrix} BH_a \\ 0 \\ 0 \\ 0 \\ BH_a \\ DH_a \end{bmatrix}^T .
$$

This together with (16) gives

$$
\begin{bmatrix}
A_c(t)X + XA_c(t)^T + Q_1 & A_1X & A_2Q_2 & E & XA_c(t)^T & XC_c(t)^T \\
XA_1^T & -(1-h_1)Q_1 & 0 & 0 & XA_1^T & 0 \\
Q_2A_2^T & 0 & -(1-h_1)Q_2 & 0 & Q_2A_2^T & 0 \\
E^T & 0 & 0 & -\gamma^2 I & E^T & 0 \\
A_c(t)X & A_1X & A_2Q_2 & E & -Q_2 & 0 \\
C_c(t)X & 0 & 0 & 0 & 0 & -I
\end{bmatrix} < 0 \tag{17}
$$

where

$$
\begin{aligned}
A_c(t) &= A + BYX^{-1} + BH_aF_a(t)E_a, \\
C_c(t) &= C + DYX^{-1} + DH_aF_a(t)E_a.
\end{aligned}
$$

Denote

$$
P = X^{-1}, \quad \tilde{Q}_1 = PQ_1P, \quad \tilde{Q}_2 = Q_2^{-1}.
$$

Then, pre- and post-multiplying (17) by

$$
\operatorname{diag}\left(P, P, \tilde{Q}_2, I, \tilde{Q}_2, I\right)
$$

give

$$
\begin{bmatrix}
PA_c(t) + A_c(t)^T P + \tilde{Q}_1 & PA_1 & PA_2 & PE & A_c(t)^T\tilde{Q}_2 & C_c(t)^T \\
A_1^T P & -(1-h_1)\tilde{Q}_1 & 0 & 0 & A_1^T\tilde{Q}_2 & 0 \\
A_2^T P & 0 & -(1-h_1)\tilde{Q}_2 & 0 & A_2^T\tilde{Q}_2 & 0 \\
E^T P & 0 & 0 & -\gamma^2 I & E^T\tilde{Q}_2 & 0 \\
\tilde{Q}_2A_c(t) & \tilde{Q}_2A_1 & \tilde{Q}_2A_2 & \tilde{Q}_2E & -\tilde{Q}_2 & 0 \\
C_c(t) & 0 & 0 & 0 & 0 & -I
\end{bmatrix} < 0,
$$

which, by the Schur complement formula, implies

$$
\begin{aligned}
\Pi(t) &= \begin{bmatrix}
PA_c(t) + A_c(t)^T P + \tilde{Q}_1 + C_c(t)^T C_c(t) & PA_1 & PA_2 & PE \\
A_1^T P & -(1-h_1)\tilde{Q}_1 & 0 & 0 \\
A_2^T P & 0 & -(1-h_1)\tilde{Q}_2 & 0 \\
E^T P & 0 & 0 & -\gamma^2 I
\end{bmatrix} \\
&\quad + \begin{bmatrix} A_c(t)^T \\ A_1^T \\ A_2^T \\ E^T \end{bmatrix} \tilde{Q}_2 \begin{bmatrix} A_c(t)^T \\ A_1^T \\ A_2^T \\ E^T \end{bmatrix}^T < 0. \qquad (18)
\end{aligned}
$$

and

$$
\begin{aligned}
\Pi_1(t) &= \begin{bmatrix}
PA_c(t) + A_c(t)^T P + \tilde{Q}_1 & PA_1 & PA_2 \\
A_1^T P & -(1-h_1)\tilde{Q}_1 & 0 \\
A_2^T P & 0 & -(1-h_1)\tilde{Q}_2
\end{bmatrix} \\
&\quad + \begin{bmatrix} A_c(t)^T \\ A_1^T \\ A_2^T \end{bmatrix} \tilde{Q}_2 \begin{bmatrix} A_c(t)^T \\ A_1^T \\ A_2^T \end{bmatrix}^T < 0. \qquad (19)
\end{aligned}
$$

Now, applying the state feedback controller in (15) with the norm-bounded additive uncertainty in (8) results in the following closed-loop system:

$$
\begin{aligned}
(\Sigma_c): \qquad \dot{x}(t) &= A_c(t)x(t) + A_1x(t-\tau_1(t)) \\
&\quad + A_2\dot{x}(t-\tau_2(t)) + E\omega(t) \qquad (20) \\
z(t) &= C_c(t)x(t) + Du(t). \qquad (21)
\end{aligned}
$$

We shall show the asymptotic stability of (20). To this end, we consider system (20) with $\omega(t) \equiv 0$; that is,

$$
\dot{x}(t) = A_c(t)x(t) + A_1x(t-\tau_1(t)) + A_2\dot{x}(t-\tau_2(t)). \tag{22}
$$

Define the following Lyapunov functional candidate for system (22):

$$
\begin{aligned}
V(x(t),t) &= x(t)^T Px(t) + \int_{t-\tau_1(t)}^{t} x(s)^T\tilde{Q}_1x(s)ds \\
&\quad + \int_{t-\tau_2(t)}^{t} \dot{x}(s)^T\tilde{Q}_2\dot{x}(s)ds. \qquad (23)
\end{aligned}
$$

Differentiating $V(x(t),t)$ along the solution of (22) results in

$$
\begin{aligned}
\dot{V}(x(t),t) &= 2x(t)^T P\left[A_c(t)x(t) + A_1x(t-\tau_1(t))\right. \\
&\quad \left. + A_2\dot{x}(t-\tau_2(t))\right] + x(t)^T\tilde{Q}_1x(t) \\
&\quad -(1-\dot{\tau}_1(t))x(t-\tau_1(t))^T\tilde{Q}_1x(t-\tau_1(t)) \\
&\quad + \dot{x}(t)^T\tilde{Q}_2\dot{x}(t) \\
&\quad -(1-\dot{\tau}_1(t))\dot{x}(t-\tau_2(t))^T\tilde{Q}_2\dot{x}(t-\tau_2(t)) \\
&\leq \xi(t)^T\Pi_1(t)\xi(t), \qquad (24)
\end{aligned}
$$

where

$$
\xi(t) = \begin{bmatrix} x(t)^T & x(t-\tau_1(t))^T & \dot{x}(t-\tau_2(t))^T \end{bmatrix}^T .
$$

Then, by noting (19), (24), and following a similar line as in the proof of Lemma 1 in [7], we have that (22) is asymptotically stable.

Next, we shall show that system (Σ_c) satisfies (12) for all nonzero $\omega(t) \in \mathcal{L}_2[0, \infty)$. To this end, we assume zero initial condition, that is, $x(t) = 0$ for $t \in [-\mu, 0]$, and introduce

$$J = \int_0^\infty \left[z(t)^T z(t) - \gamma\omega(t)^T\omega(t)\right] dt. \qquad (25)$$

Considering the asymptotic stability of the system and the zero initial condition, we have that for all nonzero $\omega(t) \in \mathcal{L}_2[0, \infty)$,

$$J \le \int_0^\infty \left[z(t)^T z(t) - \gamma\omega(t)^T\omega(t) + \dot{V}(x(t), t)\right] dt. \qquad (26)$$

Along a similar line as in the derivation of (24), it can be shown that

$$z(t)^T z(t) - \gamma\omega(t)^T\omega(t) + \dot{V}(x(t), t) \le \eta(t)^T\Pi(t)\eta(t). \qquad (27)$$

This together with (18) and (25) implies that (12) holds for all nonzero $\omega(t) \in \mathcal{L}_2[0, \infty)$. This completes the proof. □

Based on Theorem 1, we have the following results on the solvability of the non-fragile H_∞ control problem for the neutral delay system (Σ) with the controller perturbation Δ_m in (10) and (11).

Theorem 2 *Consider the neutral system (1)-(3) and the controller perturbation Δ_m in (10) and (11). Then the non-fragile H_∞ control problem is solvable if there exist matrices $X > 0$, $Q_1 > 0$, $Q_2 > 0$, Y and a scalar $\epsilon > 0$ such that the following LMI holds:*

$$\begin{bmatrix} \Gamma + \epsilon BH_mH_m^TB^T & A_1X & A_2Q_2 & E & W_1^T + \epsilon BH_mH_m^TB^T & W_2^T + \epsilon BH_mH_m^TD^T & YE_m^T \\ XA_1^T & -(1-h_1)Q_1 & 0 & 0 & XA_1^T & 0 & 0 \\ Q_2A_2^T & 0 & -(1-h_1)Q_2 & 0 & Q_2A_2^T & 0 & 0 \\ E^T & 0 & 0 & -\gamma^2 I & E^T & 0 & 0 \\ W_1 + \epsilon BH_mH_m^TB^T & A_1X & A_2Q_2 & E & \epsilon BH_mH_m^TB^T - Q_2 & \epsilon BH_mH_m^TD^T & 0 \\ W_2 + \epsilon DH_mH_m^TB^T & 0 & 0 & 0 & \epsilon DH_mH_m^TB^T & \epsilon DH_mH_m^TD^T - I & 0 \\ E_mY & 0 & 0 & 0 & 0 & 0 & -\epsilon I \end{bmatrix} < 0,$$

where W_1, W_2 and Γ are given in (14). In this case, a desired non-fragile state feedback controller can be chosen as

$$u(t) = YX^{-1}x(t).$$

4 Conclusion

In this paper, we have studied the problem of non-fragile H_∞ control for neutral systems with time-varying delays. State feedback controllers, which are subject norm-bounded uncertainties, have been designed to stabilize the given neutral system and reduce the effect of the disturbance input on the controlled output to a prescribed level. Both the cases for controller uncertainties with additive and multiplicative forms have been considered and an LMI approach has been developed.

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References

[1] R. K. Brayton. Bifurcation of periodic solutions in a nonlinear difference-differential equation of neutral type. *Quart. Appl. Math.*, 24:215–224, 1966.

[2] L. Dugard and E. I. Verriest. *Stability and Control of Time-delay Systems.* London: Springer-Verlag, 1998.

[3] A. Jadbabaie, C. T. Abdallah, D. Famularo, and P. Dorato. Robust, non-fragile and optimal controller design via linear matrix inequalities. In *Proc. American Control Conference*, pages 2842–2846, Philadelphia, Pennsylvania, 1998.

[4] Y. Kuang. *Delay Differential Equations with Applications in Population Dynamics.* Math. in Sci. Eng., 191. San Diego: Academic Press, 1993.

[5] S.-I. Niculescu. *Delay Effects on Stability : A Robust Control Approach.* Berlin : Springer, 2001.

[6] S. Xu, J. Lam, and C. Yang. Robust H_∞ control for uncertain linear neutral delay systems. *Optimal Control Application and Methods*, 23:113–123, 2002.

[7] S. Xu, J. Lam, C. Yang, and E. I. Verriest. An LMI approach to guaranteed cost control for uncertain linear neutral delay systems. *Int. J. Robust & Nonlinear Control*, 13:35–53, 2003.

[8] J.-S. Yee, G.-H. Yang, and J. L. Wang. Non-fragile guaranteed cost control for discrete-time uncertain linear systems. *Int. J. Systems Sci.*, 32:845–853, 2001.