GENETIC DESIGN OF UNCONSTRAINED DIGITAL PID CONTROLLERS

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ABSTRACT

In previous genetic design procedures, the equations for the digital PID controllers were incorporated into the genetic algorithm in order to obtain optimally tuned values of various controller parameters for finite sampling frequencies. The performance of PID controllers constrained by such design equations may be sub-optimal and so this paper illustrates the use of genetic algorithms in selecting controller matrices for PID controllers without using controller design equations. This unconstrained genetic design methodology is illustrated in this paper by the design of model-following flight-control systems for the F-16 aircraft.

1. INTRODUCTION

The design of non-adaptive or adaptive model-following systems incorporating digital PID controllers is greatly facilitated by the methodologies of Porter et al [1][2]. However, in this design process, it is necessary to optimise the performance of such controllers by choosing values of certain parameters in the appropriate design equations. These design equations arise from the underlying singular perturbation theory of Porter et al [1][2], which also provides asymptotically optimal parameter settings for the PID controllers as the associated sampling frequencies become very large. However, since practical sampling frequencies are obviously finite, genetic algorithms have been used by Porter and Hicks [3][4][5] to tune such digital PID controllers for finite sampling frequencies. It was shown [3][4][5] that this use of genetic algorithms provides superior model-following behaviour to that obtainable using asymptotically optimal tuning of the controllers.

However, in all these previous genetic designs of Porter and Hicks [3][4][5], the design equations for the PID controllers were incorporated into the genetic algorithms in order to obtain the optimally tuned values of the various controller parameters. But the performance of PID controllers constrained by such design equations may be condemned to sub-optimality and therefore be inferior to that of unconstrained controllers. However, the design of such unconstrained digital PID controllers for complex multivariable plants constitutes a formidable high-dimensional optimisation problem.

It is nevertheless shown in this paper that genetic algorithms can be readily used to design such PID controllers without using controller design equations. Such unconstrained genetic controller design does not incorporate controller design equations of any form, but simply selects the values of the controller matrices that yield the best model-following behaviour. This unconstrained genetic design methodology is illustrated for the F-16 aircraft so that direct comparisons can then be made with the constrained genetic design of Porter and Hicks [6].

2. GENETIC DESIGN PROCEDURE

The closed-loop digital model-following systems under investigation incorporate the following two principal components, as shown in Figure 1:

(i) an explicit multivariable dynamical model that generates desired model output vectors, $w(t)$, in response to command input vectors, $v(t)$;

(ii) a multivariable digital PID controller that generates appropriate control input vectors, $u(t)$, in response to errors between model output vectors, $w(t)$, and plant output vectors, $y(t)$.

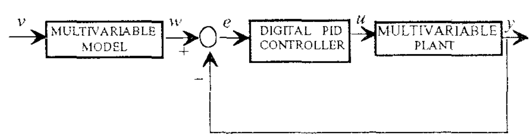

Figure 1: Block diagram of digital model-following system.

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It is assumed that the linear multivariable plants under consideration are governed on the continuous-time set $T=[0,+\infty)$ by state and output equations of the respective forms

$$\dot{x}(t)=Ax(t)+Bu(t) \tag{1}$$

and

$$y(t)=Cx(t) \; . \tag{2}$$

Such model-following systems are controlled by fast-sampling digital PID controllers governed by control-law equations of the form

$$u(kT)=K_1(T)r(kT)+K_2(T)z(kT) \tag{3}$$

where $T \in R^+$ is the sampling period. These controllers are designed so as to cause the plant output vector, $y(kT)$, to track any model output vector, $w(kT)$, in the sense that

$$\lim_{k\to\infty} e(kT)=\lim_{k\to\infty}\{w(kT)-y(kT)\}=0 \tag{4}$$

for arbitrary initial conditions, where $r(kT) \in R^l$ and $z(kT) \in R^l$ are generated in accordance with the equations

$$s\{(k+1)T\}=-\alpha I_l s(kT)+e(kT), \tag{5}$$

$$r(kT)=-\tfrac{2}{T}(1+\alpha)Ds(kT)+(I_l+\tfrac{2}{T}D)e(kT), \tag{6}$$

and

$$z\{(k+1)T\}=z(kT)+Tr(kT) \; . \tag{7}$$

In these equations $K_1, K_2 \in R^{l\times l}$, $D \in R^{l\times l}$, and $\alpha \in (-1,+1)$.

In order to demonstrate the use of genetic algorithms in the unconstrained design of fast-sampling digital controllers, the controller matrices K_1, K_2, and D can be determined together with the controller parameter α. Indeed, if minimum maximum multivariable generalised model-following error is regarded as the ultimate design requirement, genetic algorithms can be readily used to select the optimal controller parameter set $\{K_1,K_2,D,\alpha\}$ such that the measure of generalised model-following error

$$\varepsilon=\sum_{i=1}^{l}\sum_{j=1}^{l}\{\omega_{ij}\left|e_j^{(i)}(t)\right|+\sum_{i=1}^{l}\sum_{j=1}^{l}\{\mu_{ij}\left|\Delta u_j^{(i)}(t)\right|\} \tag{8}$$

is minimised. In this measure of generalised model-following error, $e_j^{(i)}(t)$ is the model-following error in the jth channel when a command is applied to the ith channel, $\Delta u_j^{(i)}(t)$ is the corresponding change in the jth control input (over a sampling period), and ω_{ij} and μ_{ij} are weighting parameters.

3. ILLUSTRATIVE EXAMPLE

The procedure for the genetic design of unconstrained digital PID controllers can be conveniently illustrated by considering the F-16 aircraft for which a digital PID controller was previously designed non-genetically [1] and genetically [6] (using constrained as opposed to unconstrained methods).

It is desired to design a digital PID controller that minimises the maximum multivariable generalised tracking error when the F-16 aircraft performs pitch-pointing and vertical-translation manoeuvres for the F-16 flying at Mach 0.9 at an altitude of 15,000 ft. In these manoeuvres, it is known that practical position and rate limits [7] are comfortably satisfied by selecting models with transfer function [1]

$$g(s)=\frac{50}{(s+2)(s^2+8s+25)}. \tag{9}$$

in both the pitch-angle and flight-path-angle channels.

In formulating this genetic design problem, a population size N=50, a crossover probability p_c=0.6, and a mutation probability p_m=0.01 were specified. Furthermore, the weighting parameters in equation (8) were assigned the values $\omega_{ij}=1$ and $\mu_{ij}=0.01$ throughout.

The results of solving this unconstrained model-following design problem by means of a genetic algorithm are shown in Figures 2, 3, 4, and 5 over 200 generations. In Figures 2(a) and (b), the best-of-generation performance measure and the controller parameter α are plotted against generation number whilst, in Figures 3(a), (b), (c), and (d), and 4(a), (b), (c), and (d), the best-of-generation controller matrix elements K_{11}, K_{12}, K_{13}, K_{14}, K_{21}, K_{22}, K_{23}, and K_{24} are respectively plotted, where

$$K_1=\begin{bmatrix} K_{11} & , & K_{12} \\ K_{13} & , & K_{14} \end{bmatrix} \tag{10}$$

and

$$K_2=\begin{bmatrix} K_{21} & , & K_{22} \\ K_{23} & , & K_{24} \end{bmatrix} \; . \tag{11}$$

Similarly, in Figures 5(a), (b), (c), and (d), the best-of-generation derivative matrix elements D_1, D_2, D_3, and D_4 are plotted against generation number, where

$$D=\begin{bmatrix} D_1 & , & D_2 \\ D_3 & , & D_4 \end{bmatrix} \; . \tag{12}$$

The optimal genetically designed unconstrained digital PID controller for a sampling period of T=0.01*sec* is thus found to be governed by the control-law equation

$$\begin{bmatrix} u_1(kT) \\ u_2(kT) \end{bmatrix} = \begin{bmatrix} -108.338 & , & -6.525 \\ 515.054 & , & -3.842 \end{bmatrix} \begin{bmatrix} r_1(kT) \\ r_2(kT) \end{bmatrix}$$

$$+ \begin{bmatrix} -1930.65 & , & -107.009 \\ 8086.02 & , & -52.405 \end{bmatrix} \begin{bmatrix} z_1(kT) \\ z_2(kT) \end{bmatrix}, \quad (13)$$

where

$$s\{(k+1)T\} = -0.5164 I_l s(kT) + e(kT) \quad , \quad (14)$$

$$\begin{bmatrix} r_1(kT) \\ r_2(kT) \end{bmatrix} = -\begin{bmatrix} 0.0026 & , & 1.8295 \\ -0.0024 & , & 87.546 \end{bmatrix} \begin{bmatrix} s_1(kT) \\ s_2(kT) \end{bmatrix}$$

$$+ \begin{bmatrix} 1.0016 & , & 1.2065 \\ -0.00173 & , & 58.732 \end{bmatrix} \begin{bmatrix} e_1(kT) \\ e_2(kT) \end{bmatrix}, \quad (15)$$

and

$$\begin{bmatrix} z_1\{(k+1)T\} \\ z_2\{(k+1)T\} \end{bmatrix} = \begin{bmatrix} z_1(kT) \\ z_2(kT) \end{bmatrix} + 0.01 \begin{bmatrix} r_1(kT) \\ r_2(kT) \end{bmatrix} . \quad (16)$$

The time-domain behaviour corresponding to this genetically designed unconstrained controller is shown in Figure 6.

It is clear from Figure 6 that the actual responses (denoted by the solid lines) of the F-16 aircraft in the case of the genetically designed unconstrained controller closely approximate the desired responses (denoted by the dashed lines). In fact, the actual responses shown in Figure 6 exactly match the desired responses in the channels which are being activated, and so the dashed lines are indistinguishable from the solid lines in these channels. The minimal value of the generalised model-following error in this unconstrained design case is $\varepsilon = 0.3033$.

The equivalent constrained genetic optimisation of the controller parameter set $\{\sigma_1, \sigma_2, \alpha, \rho, \delta\}$ has previously been presented by Porter and Hicks [6]. It was found that superior model-following behaviour was achieved using such constrained genetic tuning when compared to the responses obtained from asymptotically tuned digital PID controllers. However, the generalised model-following error corresponding to this constrained genetic design case (for a sampling period of T=0.01*sec)* is $\varepsilon = 0.379$. This indicates that the present unconstrained genetic design yields superior model-following behaviour when compared to the previous constrained genetic design.

4. CONCLUSION

It has been shown in this paper that genetic algorithms can be used in unconstrained controller design where genetic algorithms select optimal controllers for multivariable model-following systems without using controller design equations. In this way, the controller matrices can be chosen so that the generalised model-following error is minimised.

This genetic design procedure has been illustrated by the design of a model-following flight control system for the F-16 aircraft for which a digital PID controller was previously designed both asymptotically [1] and genetically [6]. It has thus been shown that such unconstrained genetic tuning yields improvements in model-following behaviour when compared with the results obtained from previous asymptotic and constrained genetic tuning.

However, it should be noted that the improvements in model-following behaviour obtained by using such unconstrained genetic tuning are minor when compared with the results obtained from constrained genetic tuning [6]. In addition, these small improvements in model-following behaviour have been procured at a 'cost' to the genetic algorithm in that an increased number of controller parameters, an enlarged search space, and an increased number of generations are needed to optimise the unconstrained controller design.

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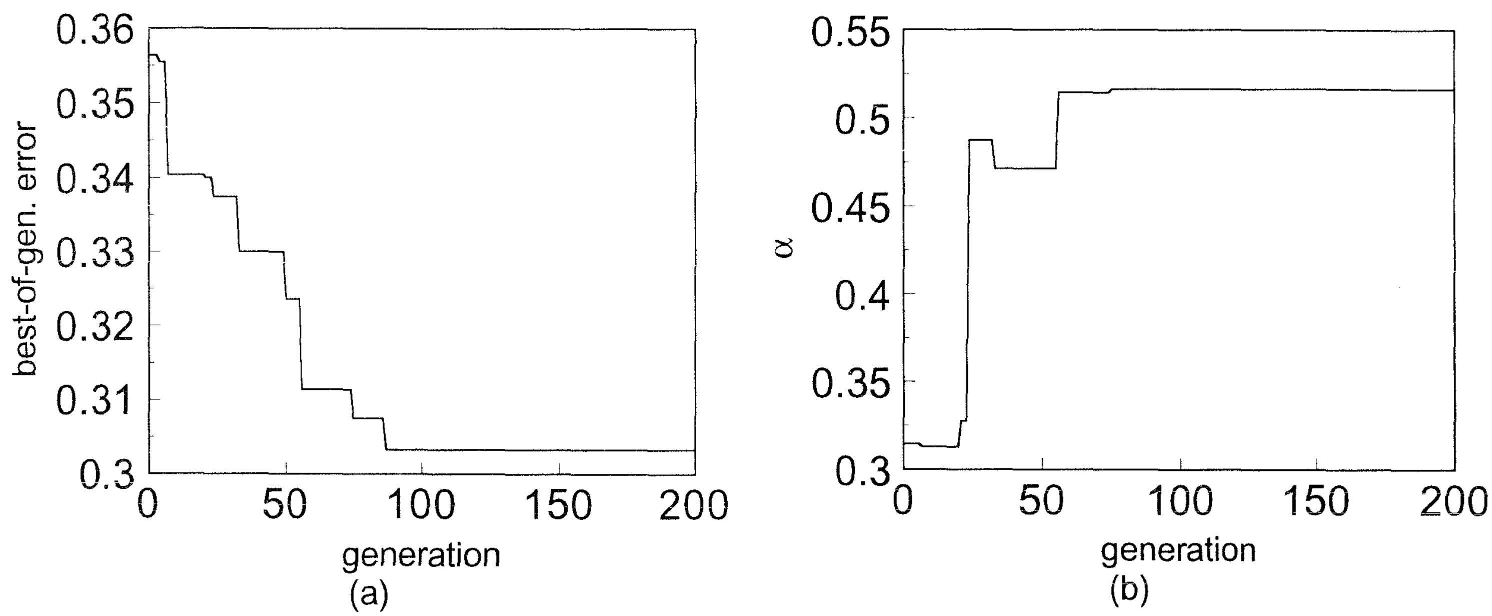

Figure 2: Best-of-generation generalised model-following error and controller parameter α

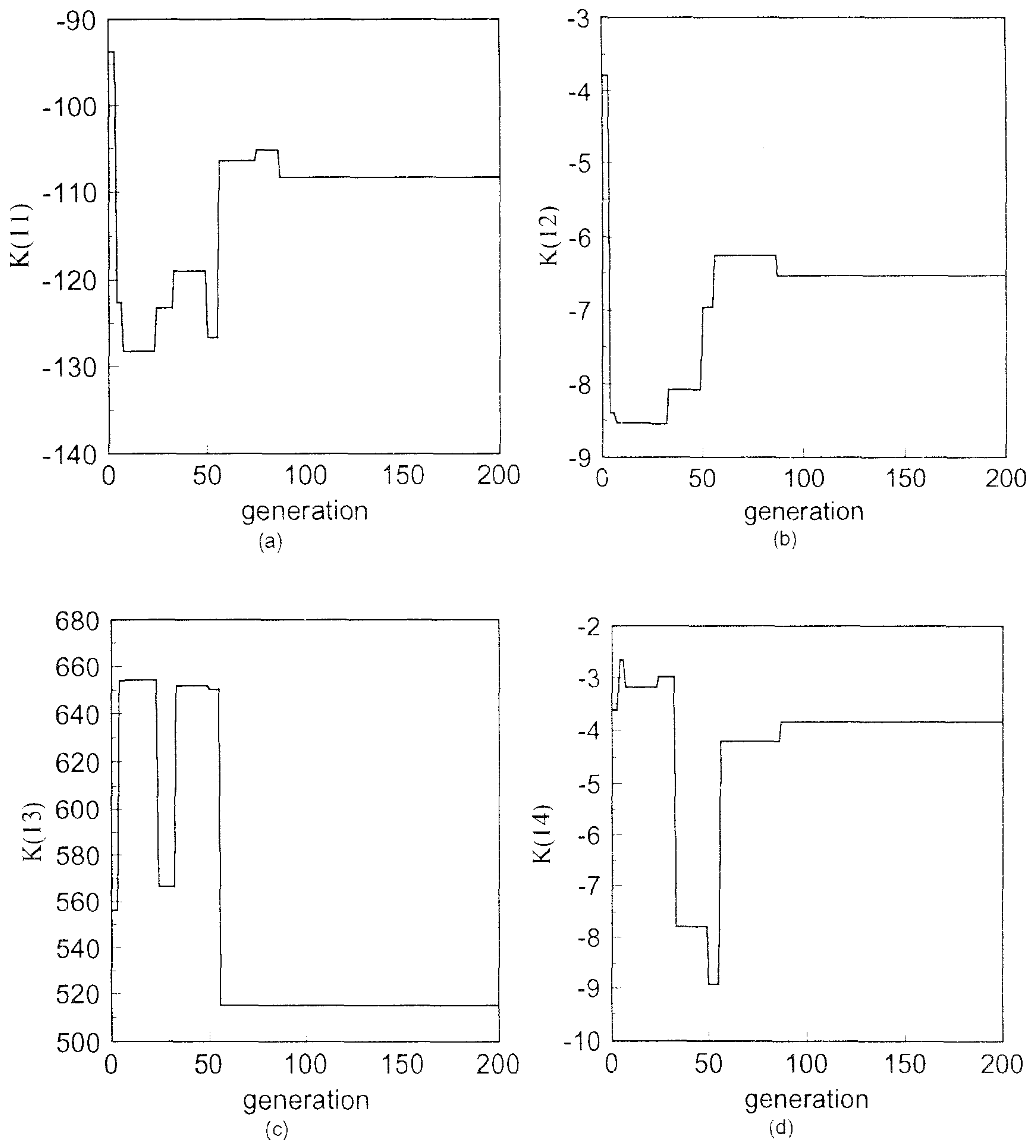

Figure 3: Best-of-generation unconstrained controller parameters

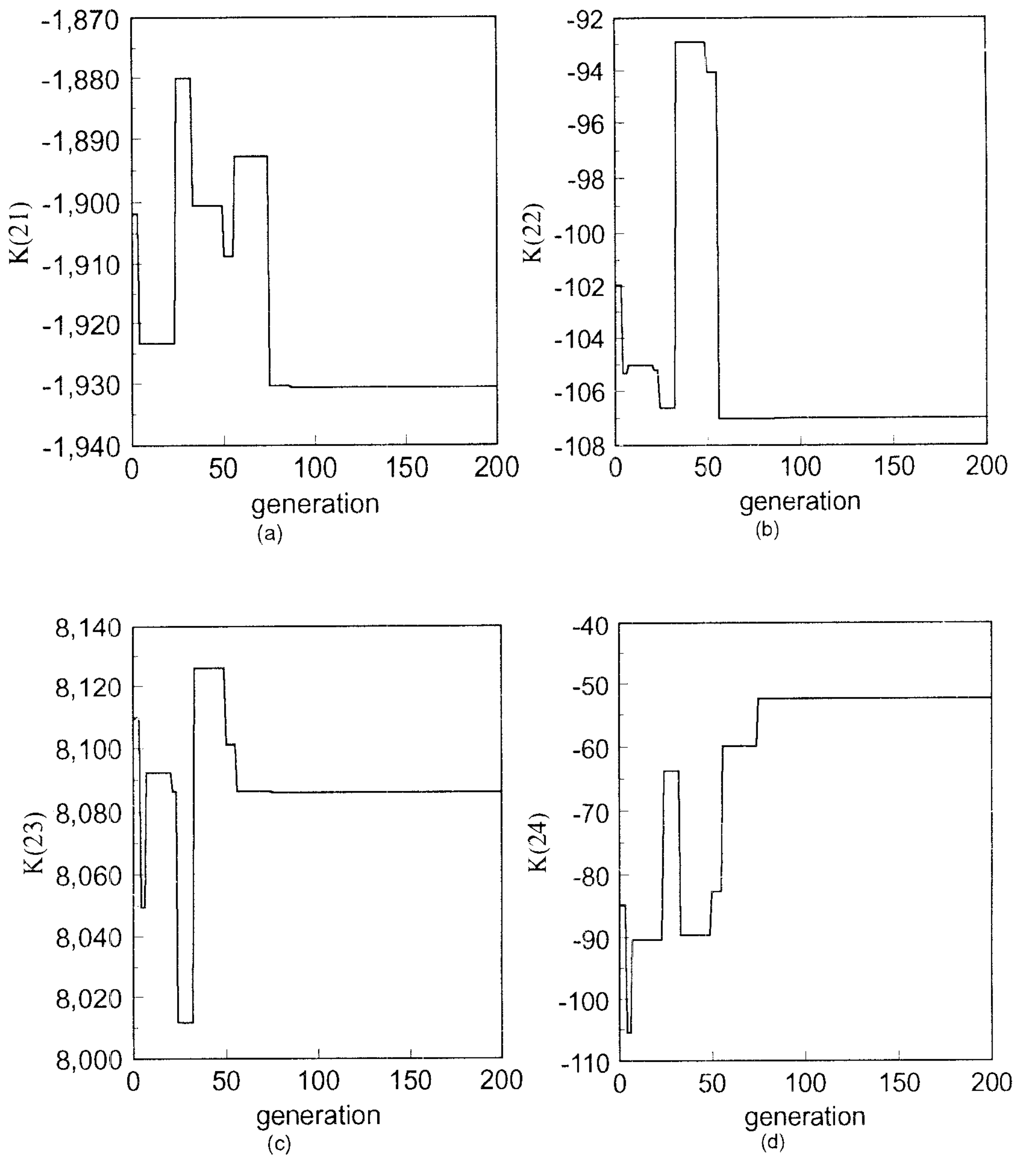

Figure 4: Best-of-generation unconstrained controller parameters

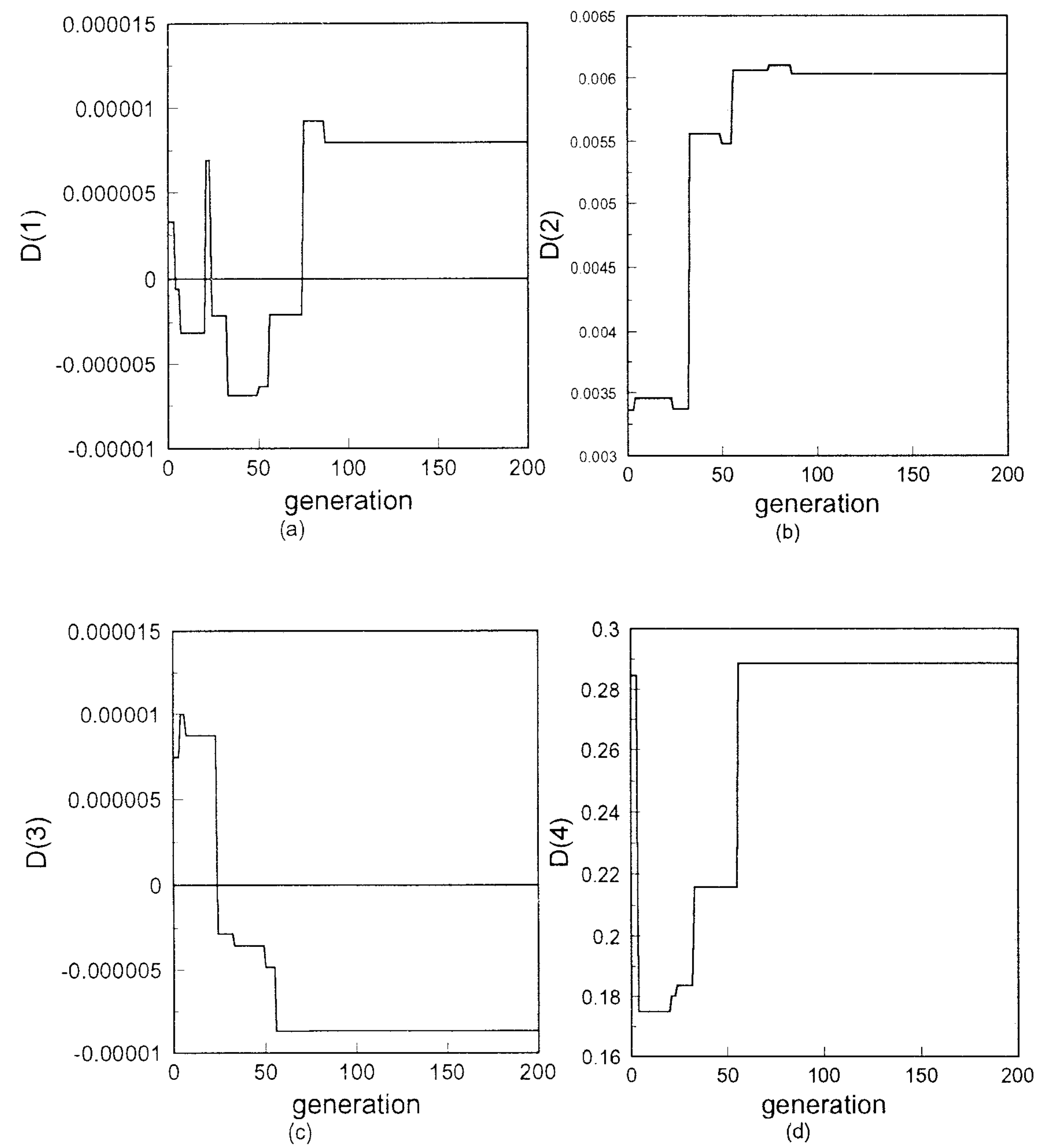

Figure 5: Best-of-generation unconstrained controller parameters

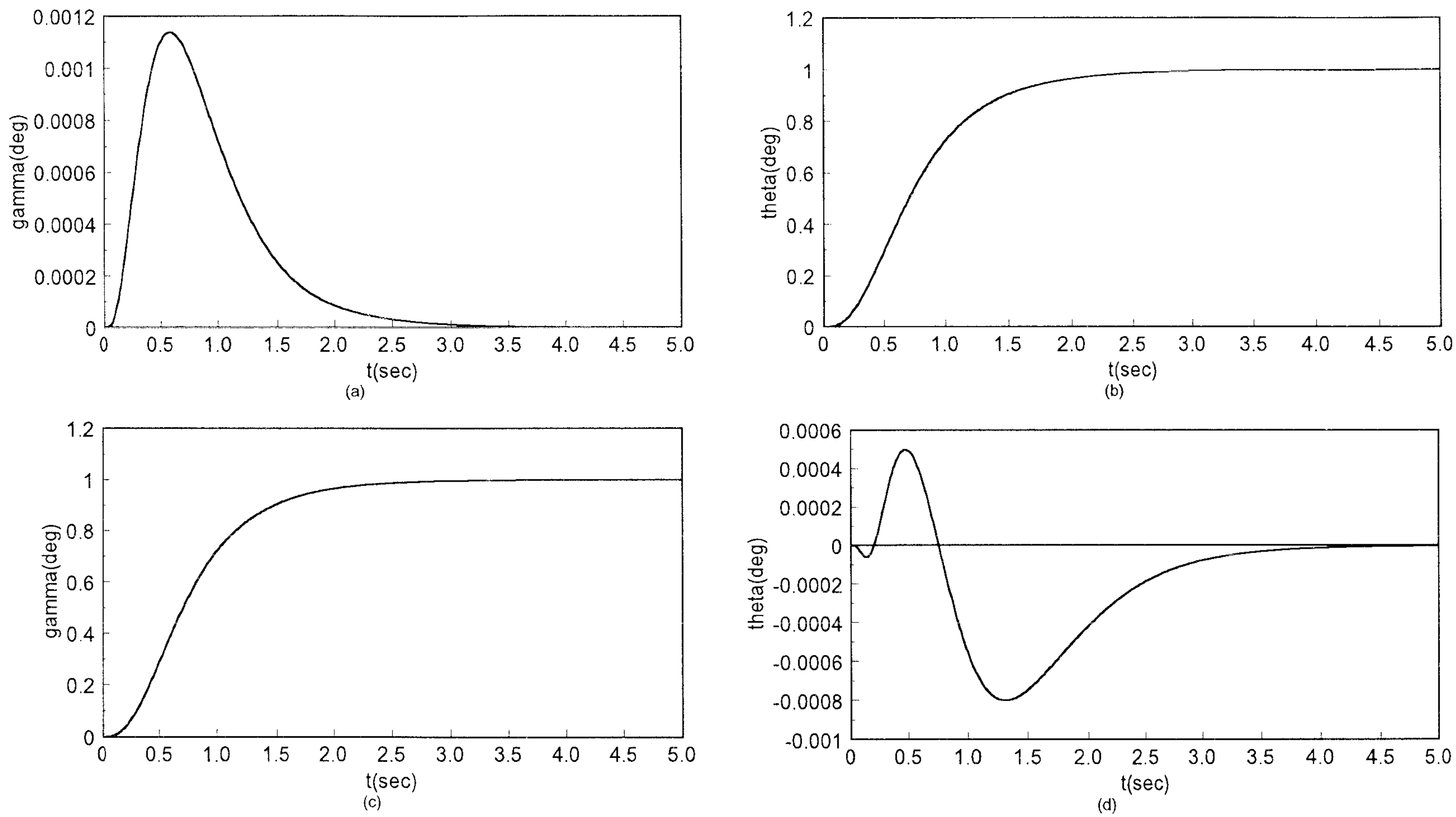

Figure 6: Model-following responses of a genetically tuned unconstrained PID controller for the F-16 aircraft