THE THEORY AND DESIGN OF RECOMBINATION NONUNIFORM FILTER-BANKS WITH LINEAR-PHASE ANALYSIS/SYNTHESIS FILTERS

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Abstract

This paper studies the theory and design of a class of linear-phase (LP) nonuniform filter-banks (FBs) called recombination nonuniform FBs (RNFBs). It is based on a recombination structure, where certain channels of an M-channel uniform FB are merged by synthesis filters of transmultiplexer (TMUX). It is assumed that the numbers of channels of the FB and TMUX are coprime to each other so that it is possible to obtain linear-time invariant (LTI) analysis/synthesis filters, instead of linear periodic time varying (LPTV) filters. The spectral supports of the analysis filters are analyzed, and the existence and matching conditions to obtain LP RNFBs with good frequency characteristics are then derived. The LTI representation of the analysis filters and the use of cosine-roll-off characteristics allow us to design the analysis filters by the REMEZ exchange algorithm. Design examples of LP nearly perfect reconstruction (NPR) RNFBs are given to demonstrate the effectiveness of the proposed method.

I. Introduction

The theory and design of uniform perfect recombination (PR) filter-banks (FBs) has been widely studied [1]. In certain applications such as image, audio and speech analysis and coding, PR FBs with nonuniform frequency spacing usually offer considerable flexibility in frequency partitioning. This has attracted considerable interests in designing nonuniform FBs [3-6]. One popular method is the direct structure proposed in [4]. However, the design usually involves nonlinear optimization with considerable number of parameters. Another useful approach is the indirect method proposed in [5], where certain channels of a uniform FB are merged using the synthesis filters of a recombination FB or transmultiplexer (TMUX). It was shown recently in [6] that it is possible to achieve PR in recombination nonuniform FB (RNFB). Moreover, if the number of channels of the uniform FB and recombination TMUX are coprime, then the analysis filters of the resulting RNFB admit an equivalent linear time invariant (LTI) representation. In other words, the frequency responses of the analysis filters can be optimized directly, which considerably simplifies the design procedure. A class of RNFBs based on the cosine modulation FBs (CMFBs) was also proposed. By imposing a simple matching condition on the filter length, RN-CMFB with low design and implementation complexities and good frequency characteristics can be obtained. One advantage of the RNFB is that the PR property is structurally imposed as long as the original uniform FB and recombination TMUXs are PR. Further, dynamic recombination of consecutive channels in the original uniform FB by pre-designed TMUXs is possible [9]. The method in [2] is also based on the merging approach. However, the recombination FB is a simple transformation. It is useful in coding application where it is unnecessary to create a single subband with good frequency characteristic. In array processing and other applications involving partial spectrum reconstruction, then a general recombination TMUX has to be used.

In this paper, the theory and design of a class of RNFBs with linear-phase (LP) filters are proposed. The LP property is attractive because it does not introduce phase distortion in the subband signals. For simplicity, we also assume that the numbers of channels of the uniform FB and recombination TMUX are coprime to each other so that it is possible to obtain LTI analysis/synthesis filters, instead of linear periodic time varying (LPTV) filters. This simplifies the design procedure while offering reasonable flexibility in choosing the sampling factors. For example, the number of channels in the uniform FB can be chosen as an even number, while those for the recombination TMUXs can be chosen as odd numbers. It should be noted that PR RNFB without the coprime condition is also feasible. However their analysis is rather involved and it will not be considered in this paper. Rather than the LP paraunitary FB in [7] and the LP FBs by cosine and sine modulation in [8], the LP uniform FBs we introduced are obtained by using REMEZ algorithm and transition band being cosine roll-off. This allows a simple design and ideal-bandwidth of the LP filters. Finally, design examples are given to show that nearly perfect reconstruction LP RNFBs with good filter quality can be achieved by the proposed method.

The organization of the paper is as follows: the spectral supports of the analysis filters are first analyzed in section III after an overview of the principle of RNFBs in section II. Based on this analysis, the conditions for obtaining LP RNFBs with good frequency characteristics are then derived in Section IV. The LTI representation of the analysis filters and the use of cosine-roll-off characteristic allow us to design the analysis filters by the REMEZ exchange algorithm. Design examples of LP nearly perfect reconstruction (NPR) RNFBs are given in Section V. Finally, conclusions are drawn in section VI.

II. Principle of RNFBs

The recombination structure for an L-band nonuniform FB is shown in Fig. 1. In this structure, m_l sub-channels of an M-channel uniform FB are merged by m_l-channel TMUXs, where $l = 0,\ldots,L-1$, producing nonuniform subbands with a sampling rate of m_l/M. At the synthesis section, the analysis filters of the TMUX are used to produce the m_l subbands for the M-channel uniform FB. Similar merging can be performed for other consecutive channels. It was observed in [6] that if the uniform FB and the TMUXs are PR, then the nonuniform FB would also be PR. This is because if the input signal of the TMUX can be approximately reconstructed at the synthesis side (enclosed by the dotted line in Fig. 1), the merging operation is equivalent to the introduction of a certain delay in the m_l channels. If these delays are compensated in other branches, the whole structure can be treated as an M-channel uniform PR FB and the PR condition is ensured. This structural PR property simplifies the design of the nonuniform FB, which was already demonstrated by the RN-CMFBs in [6,9]. Because the analysis filters of the RNFB are in general LPTV, their design can be complicated. Fortunately, if M and m_l are coprime, it is possible to come up with an LTI representation of the analysis/synthesis filters by interchanging the order of the decimator and interpolator [6]. This will be given in the next section where the spectral support of the analysis filters will be studied.

III. Spectral Support of the Analysis Filters

To begin with, let us study the equivalent LTI representation of the RNFB. Consider the merging of the

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subbands in Fig. 1. For the time being, let us ignore the multiplication with the constant $c_0 \sim c_{M-1}$, which does not affect the PR condition. It is known that if m_l and M are coprime, then the decimator (M) and the interpolator (m_l) can be interchanged. Moreover, using the noble identity, $H_{r_l+i}(z)$ and $G_{l,i}(z)$ can be moved across the interpolator and decimator, where $i = 0,\dots,m_l - 1$ and r_l are the starting index of the merged channels of the M-channel uniform FB. It gives the following equivalent LTI representation of the analysis filter:

$$\hat{H}_l(z) = \sum_{i=0}^{m_l-1} H_{r_l+i}(z^{m_l}) G_{l,i}(z^M) . \quad (1)$$

For notational convenience, we shall use $H(\omega)$ to denote $H(e^{j\omega})$. Hence

$$\hat{H}_l(\omega) = \sum_{i=0}^{m_l-1} H_{r_l+i}(m_l\omega) G_{l,i}(M\omega)$$
$$= e^{-j\omega} \sum_{i=0}^{m_l-1} H_{r_l+i}(m_l\omega) G_{l,i\_FB}(M\omega) = e^{-j\omega} D(\omega) , \quad (2)$$

where $D(\omega) = \sum_{i=0}^{m_l-1} H_{r_l+i}(m_l\omega) G_{l,i\_FB}(M\omega)$. (3)

Here, the TMUX with synthesis filters $G_{l,i}(\omega)$ is obtained from an 1-skewed FB with synthesis filters $G_{l,i\_FB}(\omega)$, where $G_{l,i}(\omega) = e^{-j\omega} G_{l,i\_FB}(\omega)$. Let

$$H_{r_l+i}(m_l\omega) = A_{r_l+i}(\omega) = A^+_{r_l+i}(\omega) + A^-_{r_l+i}(\omega) ,$$
$$G_{l,i\_FB}(M\omega) = B_i(\omega) = B^+_i(\omega) + B^-_i(\omega) . \quad (4)$$

Since the periods of $H_{r_l+i}(\omega)$ and $G_{l,i\_FB}(\omega)$ are 2π, the period of $A_{r_l+i}(\omega)$ is $2\pi/m_l$ and that of $B_i(\omega)$ is $2\pi/M$. $A^+_{r_l+i}(\omega)$ and $A^-_{r_l+i}(\omega)$ correspond to the responses of $A_{r_l+i}(\omega)$ for positive and negative values of ω in $[-\frac{\pi}{m_l}, \frac{\pi}{m_l}]$, respectively. Similarly, $B^+_i(\omega)$ and $B^-_i(\omega)$ correspond to the responses of $B_i(\omega)$ in $[-\frac{\pi}{M}, \frac{\pi}{M}]$.

Our objective is to study the conditions on $H_{r_l+i}(\omega)$ and $G_l(\omega)$ such that $\hat{H}_l(\omega)$ will possess the desirable frequency characteristics. To simplify the analysis, we assume that (i) the stopband cutoff frequencies of $H_{r_l+i}(\omega)$ lie within $[((r_l+i)\frac{\pi}{M} - \frac{\pi}{2M}), ((r_l+i+1)\frac{\pi}{M} + \frac{\pi}{2M})]$, (ii) the stopband attenuation is sufficiently high that it can be treated as zero outside the region. Thence, only adjacent channels overlap in their magnitude responses. This also applies to $G_{l,i\_FB}(\omega)$ with the stopband cutoff frequencies inside the range $[(i\frac{\pi}{m_l} - \frac{\pi}{2m_l}), ((i+1)\frac{\pi}{m_l} + \frac{\pi}{2m_l})]$. Denote the frequency support of a function $Q(\omega)$ as $Supp(Q)$. In other words, if $\omega \notin Supp(Q)$, then $Q(\omega) = 0$. It can be shown that

$$Supp(H_{r_l+i}(\omega)) = [2p\pi + ((r_l+i)\frac{\pi}{M} - \frac{\pi}{2M}), 2p\pi + ((r_l+i+1)\frac{\pi}{M} + \frac{\pi}{2M})]$$
$$\cup [2p\pi - ((r_l+i+1)\frac{\pi}{M} + \frac{\pi}{2M}), 2p\pi - ((r_l+i)\frac{\pi}{M} - \frac{\pi}{2M})],$$
$$Supp(G_{l,i\_FB}(\omega)) = [2q\pi + (i\frac{\pi}{m_l} - \frac{\pi}{2m_l}), 2q\pi + ((i+1)\frac{\pi}{m_l} + \frac{\pi}{2m_l})]$$
$$\cup [2q\pi - ((i+1)\frac{\pi}{m_l} + \frac{\pi}{2m_l}), 2q\pi - (i\frac{\pi}{m_l} - \frac{\pi}{2m_l})],$$

where p and q are integers. Since the period of $H_{r_l+i}(m_l\omega) G_{l,i\_FB}(M\omega)$ is 2π, we need only focus on the range $-\pi \le \omega \le \pi$. Inside which, p and q satisfy

$$|p| \le \lfloor m_l/2 \rfloor \text{ and } |q| \le \lfloor M/2 \rfloor . \quad (5)$$

Here, $\lfloor x \rfloor$ denotes the nearest integer less than or equal to x. Similarly, we have

$$Supp(A_{r_l+i}(\omega)) = Supp(A^+_{r_l+i}(\omega)) \cup Supp(A^-_{r_l+i}(\omega))$$
$$= \left\{ [\frac{2p\pi}{m_l} + (r_l+i-\frac{1}{2})\frac{\pi}{m_l M}, \frac{2p\pi}{m_l} + (r_l+i+\frac{3}{2})\frac{\pi}{m_l M}], p = \pm 0, \pm 1, \pm 2, \cdots \right\}$$
$$\cup \left\{ [\frac{2p\pi}{m_l} - (r_l+i+\frac{3}{2})\frac{\pi}{m_l M}, \frac{2p\pi}{m_l} - (r_l+i-\frac{1}{2})\frac{\pi}{m_l M}], p = \pm 0, \pm 1, \pm 2, \cdots \right\},$$
$$Supp(B_i(\omega)) = Supp(B^+_i(\omega)) \cup Supp(B^-_i(\omega))$$
$$= \left\{ [\frac{2q\pi}{M} + (i-\frac{1}{2})\frac{\pi}{m_l M}, \frac{2q\pi}{M} + (i+\frac{3}{2})\frac{\pi}{m_l M}], q = \pm 0, \pm 1, \pm 2, \cdots \right\}$$
$$\cup \left\{ [\frac{2q\pi}{M} - (i+\frac{3}{2})\frac{\pi}{m_l M}, \frac{2q\pi}{M} - (i-\frac{1}{2})\frac{\pi}{m_l M}], q = \pm 0, \pm 1, \pm 2, \cdots \right\}.$$

We now turn to $D(\omega)$ in (3). Substituting (4) into (3), we get $D(\omega) = \sum_{j=1}^{4} D_j(\omega)$ where

$$D_1(\omega) = \sum_{i=0}^{m_l-1} A^+_{r_l+i}(\omega) B^+_i(\omega) , \quad D_2(\omega) = \sum_{i=0}^{m_l-1} A^+_{r_l+i}(\omega) B^-_i(\omega) ;$$
$$D_3(\omega) = \sum_{i=0}^{m_l-1} A^-_{r_l+i}(\omega) B^+_i(\omega) , \quad D_4(\omega) = \sum_{i=0}^{m_l-1} A^-_{r_l+i}(\omega) B^-_i(\omega) . \quad (6)$$

Only the terms $D_1(\omega)$ and $D_2(\omega)$ need to be studied as $D_3(\omega)$ and $D_4(\omega)$ are their conjugates.

$D_1(\omega)$: Careful examination show that $D_1(\omega) = \sum_{i=0}^{m_l-1} A^+_{r_l+i}(\omega) B^+_i(\omega)$ constitutes the desired passband of $\hat{H}_l(\omega)$ and $A^+_{r_l+i}(\omega) B^+_i(\omega)$ has a length of $I = 2\pi/(m_l M)$. Detailed derivation can be found in [9].

$D_2(\omega)$: On the other hand, $D_2(\omega) = \sum_{i=0}^{m_l-1} A^+_{r_l+i}(\omega) B^-_i(\omega)$ in (6) give rise to undesirable spurious response from the overlapping of the transition bands of $A^+_{r_l+i}$ and B^-_i . There are two such supports in $D_2(\omega)$, denoted by F^+_i and F^-_i, each having a length of $I/2$. Further, it can be shown that $F^+_{i+1} = F^-_i$. By introducing the indicator function of F as E_F, i.e. $E_F(\omega) = \begin{cases} 1, & \text{if } \omega \in F \\ 0, & \text{if } \omega \notin F \end{cases}$, we have $E_{F^+_{i+1}} = E_{F^-_i}$ and

$$D_2(\omega) = \sum_{k=0}^{m_l-1} A^+_{r_l+i} B^-_i = \sum_{i=0}^{m_l-1} (A^+_{r_l+i} B^-_i E_{F^+_i} + A^+_{r_l+i} B^-_i E_{F^-_i})$$
$$= A^+_{r_l} B^-_0 E_{F^+_0} + \sum_{i=0}^{m_l-2} (A^+_{r_l+i+1} B^-_{i+1} E_{F^+_{i+1}} + A^+_{r_l+i} B^-_i E_{F^-_i})$$
$$+ A^+_{r_l+m_l-1} B^-_{m_l-1} E_{F^-_{m_l-1}} . \quad (7)$$

The term, $\sum_{i=0}^{m_l-2} (A^+_{r_l+i} B^-_i E_{F^-_i} + A^+_{r_l+i+1} B^-_{i+1} E_{F^+_{i+1}})$, which we call it the "cross-term", is the source of the spurious response. It causes bumpings in the stopband of $\hat{H}_l(\omega)$, which was first observed in [6] for RN-CMFB. Next, we will establish the condition such that the spurious response in (7) can be forced to zero, by imposing appropriate conditions on the constant c_{r_l+i} and a matching condition of the two filters $A_{r_l+i}(\omega)$ and $B_i(\omega)$ in LP RNFB. The analysis above is similar to that of RN-CMFB and more details of the derivation can be found in [9]. Apart from the assumptions in (i) and (ii), the analysis is valid for any filters, $H(\cdot)$ and $G(\cdot)$, i.e. LP or nonlinear, FIR or IIR.

IV. Linear-Phase RNFBs

Before proceeding to the selection of c_{r_l+i} and the matching conditions, let us briefly review the basic property of LP filters: for a LP filter $H(e^{j\omega})$ with a symmetric (anti-symmetric) impulse response $h(n)$, then we say $H(e^{j\omega})$ is symmetric (anti-symmetric). In addition, we have for symmetric $H(e^{j\omega})$,

$$H(e^{j\omega}) = e^{-j\omega N/2}|H_R(\omega)|\,; \tag{8a}$$

and for anti-symmetric $H(e^{j\omega})$,

$$H(e^{j\omega}) = \begin{cases} je^{-j\omega N/2}|H_R(\omega)|, & H_R(\omega) \geq 0 \\ -je^{-j\omega N/2}|H_R(\omega)|, & H_R(\omega) < 0, \end{cases} \tag{8b}$$

where $H_R(\omega)$ is a real-valued function called the *amplitude response* of $H(e^{j\omega})$. N is the filter length of $h(n)$. In addition, $H_R(\omega) = H_R(-\omega)$ for symmetric and $H_R(\omega) = -H_R(-\omega)$ for anti-symmetric filters.

If $A_{r_l+i}(\omega)$ and $B_i(\omega)$ are symmetric, we have from (8a):

$$\begin{aligned} A^+_{r_l+i}(\omega) &= e^{-j\alpha\omega}\left|A^R_{r_l+i}(\omega)\right|, \; B^+_i(\omega) = e^{-j\beta\omega}\left|B^R_i(\omega)\right|, \\ A^-_{r_l+i}(\omega) &= e^{-j\alpha\omega}\left|A^R_{r_l+i}(\omega)\right|, \; B^-_i(\omega) = e^{-j\beta\omega}\left|B^R_i(\omega)\right|. \end{aligned} \tag{9a}$$

Similarly, if $A_{r_l+i}(\omega)$ and $B_i(\omega)$ are anti-symmetric, we have from (8b):

$$\begin{aligned} A^+_{r_l+i}(\omega) &= je^{-j\alpha\omega}\left|A^R_{r_l+i}(\omega)\right|, \; B^+_i(\omega) = je^{-j\beta\omega}\left|B^R_i(\omega)\right|, \\ A^-_{r_l+i}(\omega) &= -je^{-j\alpha\omega}\left|A^R_{r_l+i}(\omega)\right|, \; B^-_i(\omega) = -je^{-j\beta\omega}\left|B^R_i(\omega)\right|. \end{aligned} \tag{9b}$$

Here, $A^R_{r_l+i}(\omega)$ and $B^R_i(\omega)$ are the *amplitude responses*, $\alpha = m_l N_M/2$ and $\beta = MN_{m_l}/2$.

Here, we consider two types of LP FBs: (i) analysis filters having *same symmetry* (i.e. all symmetric) and (ii) analysis filter having *alternate symmetry* (i.e. alternative symmetric and anti-symmetric). In the case of *alternate symmetry*, for simplicity, we assume that $A_{r_l+i}(\omega)$ and $B_i(\omega)$ are symmetric when i is even, and anti-symmetric when i is odd. In the following, we will establish the condition on c_{r_l+i} in the above FBs for the existence of LP LTI FBs with good frequency characteristics. Although the constant c_{r_l+i} do not affect the PR condition, they are very important for achieving a good frequency response as we shall show in the following section. Our investigation starts from the analysis of $D_1(\omega)$.

Passband Flatness:

$$D_1(\omega) = \sum_{i=0}^{m_l-1} c_{r_l+i} A^+_{r_l+i}(\omega) B^+_i(\omega)$$

$$= c_{r_l} A^+_{r_l}(\omega)B^+_0(\omega) + c_{r_l+1}A^+_{r_l+1}(\omega)B^+_1(\omega) + c_{r_l+2}A^+_{r_l+2}(\omega)B^+_2(\omega)\cdots$$

$$= \begin{cases} e^{-j(\alpha+\beta)\omega}\left(c_{r_l}\left|A^R_{r_l}(\omega)\right|\left|B^R_0(\omega)\right| + c_{r_l+1}\left|A^R_{r_l+1}(\omega)\right|\left|B^R_1(\omega)\right| + c_{r_l+2}\left|A^R_{r_l+2}(\omega)\right|\left|B^R_2(\omega)\right| + \cdots\right) \\ \text{for same symmetry} \\ e^{-j(\alpha+\beta)\omega}\left(c_{r_l}\left|A^R_{r_l}(\omega)\right|\left|B^R_0(\omega)\right| - c_{r_l+1}\left|A^R_{r_l+1}(\omega)\right|\left|B^R_1(\omega)\right| + c_{r_l+2}\left|A^R_{r_l+2}(\omega)\right|\left|B^R_2(\omega)\right| - \cdots\right) \\ \text{for oppositesymmetry.} \end{cases} \tag{10}$$

As mentioned earlier, $D_1(\omega)$ constitutes the passband of $\hat{H}_l(\omega)$. Therefore, at the transition band of the magnitude responses, the term inside the bracket of (10) should add up to a constant. To this end, c_{r_l+i} and c_{r_l+i+1} should satisfy

$$\begin{cases} c_{r_l+i} = c_{r_l+i+1}, & \text{for same symmetry} \\ c_{r_l+i} = -c_{r_l+i+1}, & \text{for alternativ e symmetry.} \end{cases} \tag{11}$$

Spurious Response Suppression:

As mentioned earlier, $D_2(\omega)$, other than F^+_0 and F^-_{m-1}, should correspond to the stopband of $\hat{H}_l(\omega)$, and the cross-term might be appeared. Let us rewrite $D_2(\omega)$, after including the constant c_{r_l+i}, as

$$\begin{aligned} D_2(\omega) = \sum_{k=0}^{m_l-1} c_{r_l+i}A^+_{r_l+i}B^-_i &= c_{r_l}A^+_{r_l}B^-_0 E_{F^+_0} \\ &+ \sum_{i=0}^{m_l-2}(c_{r_l+i+1}A^+_{r_l+i+1}B^-_{i+1}E_{F^+_{i+1}} + c_{r_l+i}A^+_{r_l+i}B^-_i E_{F^-_i}) \\ &+ c_{r_l+m_l-1}A^+_{r_l+m_l-1}B^-_{m_l-1}E_{F^-_{m_l-1}}. \end{aligned} \tag{12}$$

The term inside the bracket (.) can be written as

$$\begin{aligned} &c_{r_l+i}A^+_{r_l+i}B^-_i E_{F^-_i} + c_{r_l+i+1}A^+_{r_l+i+1}B^-_{i+1}E_{F^+_{i+1}} \\ &= e^{-j(\alpha+\beta)\omega}\left(c_{r_l+i}\left|A^R_{r_l+i}(\omega)\right|\left|B^R_i(\omega)\right| + c_{r_l+i+1}\left|A^R_{r_l+i+1}(\omega)\right|\left|B^R_{i+1}(\omega)\right|\right), \end{aligned} \tag{13}$$

for both same and alternate symmetries. In order to eliminate the cross-term, we need to choose

$$c_{r_l+i} = -c_{r_l+i+1}. \tag{14}$$

(11) and (14) together suggest that filters in the LP RNFB must have alternate symmetry and c_{r_l+i} should satisfy (14). This is a necessary condition for LP LTI RNFBs to have good frequency characteristics. Hence, we call it **existence condition**.

Having satisfied the condition in (14), (13) becomes

$$\begin{aligned} &c_{r_l+i}A^+_{r_l+i}B^-_i E_{F^-_i} + c_{r_l+i+1}A^+_{r_l+i+1}B^-_{i+1}E_{F^+_{i+1}} \\ &= e^{-j(\alpha+\beta)\omega}c_{r_l+i}\left(\left|A^R_{r_l+i}(\omega)\right|\left|B^R_i(\omega)\right| - \left|A^R_{r_l+i+1}(\omega)\right|\left|B^R_{i+1}(\omega)\right|\right). \end{aligned}$$

Obviously, if $\left|A^R_{r_l+i}(\omega)\right|\left|B^R_i(\omega)\right| = \left|A^R_{r_l+i+1}(\omega)\right|\left|B^R_{i+1}(\omega)\right|$, then the cross-term will be zero. This additional condition on $A_{r_l+i}(\omega)$ and $B_i(\omega)$ can be realized as the following **matching condition**:

$$A_{r_l+i}(\omega) = B_i(\omega),$$

or equivalently $H_{r_l+i}(m_l\omega) = G_i(M\omega)$. (15)

Due to page limitation, the proof is omitted, and interested readers can refer to [9] for a similar analysis using the RN-CMFB.

With the cross-term eliminated, $D_2(\omega)$ in (12) reduces to

$$D_2(\omega) = c_{r_l}A^+_{r_l}B^-_0 E_{F^+_0} + c_{r_l+m_l-1}A^+_{r_l+m_l-1}B^-_{m_l-1}E_{F^-_{m_l-1}}. \tag{16}$$

It can be proved that $\omega \in F^+_0$ and $\omega \in F^-_{m-1}$ belongs to the transition band of $\hat{H}_l(\omega)$.

Finally, for simplicity, we choose $c_{r_l+i} = \begin{cases} 1, & \text{for even } i \\ -1, & \text{for odd } i \end{cases}$ in the design examples to be followed. Hence, the equivalent LTI filters can be written simply as follows:

$$\hat{H}_l(z) = \sum_{i=0}^{m_l-1}(-1)^i H_{r_l+i}(z^{m_l})G_{l,i}(z^M). \tag{17}$$

V. Design Method

In the recombination structure, the original and the recombination FBs can be designed separately (first for the uniform FB and then the recombination TMUXs) as long as they satisfy the matching condition above. For the current examples, the uniform LP FBs have *equal* filter lengths. The filters have alternate symmetry. The transition band of the analysis filters follow a cosine roll-off characteristic so that the FB is approximately power complementary and is NPR. Because of this simplification, the desired responses of the analysis filters are completely determined, the REMEZ algorithm in MATLAB can be employed to minimize the maximum approximation error. Therefore, the design of the proposed NPR LP RNFBs is very simple and good filter quality can be achieved, as we shall demonstrate below.

Consider an LP RNFB with sampling factors (3/4, 1/4). It is constructed by merging the first three channels of an 4-channel uniform LP FB with an 3-channel recombination LP TMUX. In the original FB, $H_0(z)$ and $H_2(z)$ are chosen to be symmetric and $H_1(z)$ and $H_3(z)$ are anti-symmetric. In the recombination TMUX, $G_0(z)$ and $G_2(z)$ are chosen to be symmetric, while $G_1(z)$ is anti-symmetric. The lengths of the 3- and 4-channel FBs are 63 and 84, respectively. As mentioned earlier, the use of a cosine-roll-off transition band allows us to solve the problem as a filter design problem, which in turn can be solved using the REMEZ algorithm. Also, the matching condition in (15) is much easier to be satisfied by choosing $N_M/M = N_{m_l}/m_l$. In this example, the passband and stopband cutoff frequencies of the analysis filters are: $H_0(z)$: $\omega_{p_0} = 0.1\times(2\pi)$, $\omega_{s_0} = 0.15\times(2\pi)$; $H_1(z)$: $\omega_{s_1} = 0.1\times(2\pi)$, $\omega_{p_1} = 0.15\times(2\pi)$, $\omega_{p_2} = 0.225\times(2\pi)$ and $\omega_{s_3} = 0.275\times(2\pi)$. The cutoff frequencies of the other analysis filters can are similarly defined. Here, ω_{p_i} and ω_{s_i} are respectively the passband and stopband cutoff frequencies of the filters. The relative weighting of the passband and stopband errors are chosen to be identical. The equivalent structure of the FB with LTI analysis filters is shown in Fig. 2(a). The magnitude responses of the equivalent LTI analysis filters are shown in Fig. 2(b). This system is approximately PR and the reconstruction and aliasing errors are 10^{-3} .

The second example is an LP RNFB with sampling factors (2/5, 3/5). By using (15) and (17), we get the equivalent LTI analysis filters shown in Fig. 3. The lengths of the 2-, 3- and 5-channel FBs are 50, 75 and 125 respectively. The reconstruction and aliasing errors of the system are 10^{-3} .

VI. Conclusion

This paper presents the theory and design of a class of LP nonuniform FBs called RNFBs. The spectral supports of the analysis filters are analyzed, and the existence and matching conditions to obtain LP RNFBs with good frequency characteristics are derived. Design examples of the LP NPR RNFBs are given to demonstrate the effectiveness of the proposed method.

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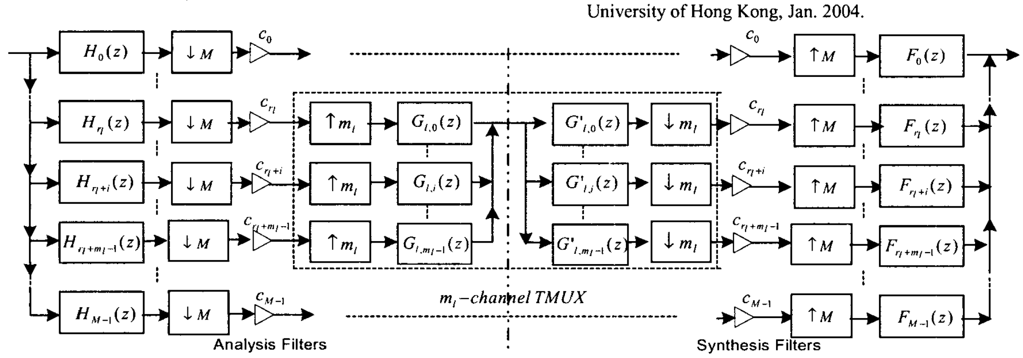

Fig. 1. Structure of recombination nonuniform filter-bank

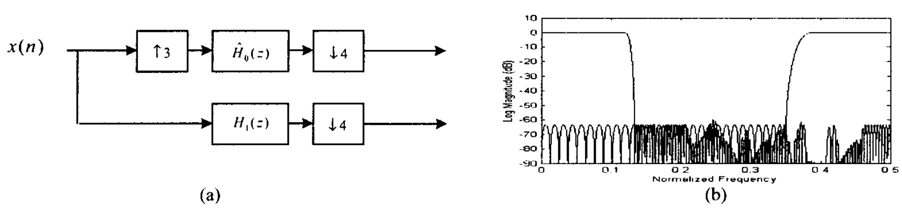

Fig. 2. LP RNFB with sampling factors (3/4,1/4). (a) Equivalent structure of the analysis banks with LTI analysis filter. (b) Magnitude responses.

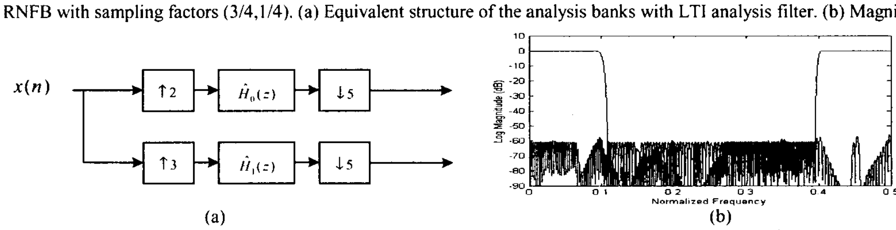

Fig. 3. LP RNFB with sampling factors (2/5,3/5). (a) Equivalent structure of the analysis banks with LTI analysis filters. (b) Magnitude responses.