Development of Bidding Strategies in Electricity Markets Using Possibility Theory

Li Yang, Fushuan Wen, F.F. Wu, Yixin Ni and Jiaju Qiu

Abstract-- **In the electricity market environment, bidding strategies employed by generation companies may have significant impacts on their own benefits, and on the operating behaviors of an electricity market as well. Hence, how to develop optimal bidding strategies for generation companies or how to analyze strategic behaviors of them and hence to figure out the potential market power abuse is now a very active research area. A possibility theory based approach is proposed in this work for building optimal bidding strategies for generation companies. Based on historical bidding data, the available (production cost) data before the power industry restructuring and experts' heuristic knowledge, the well-known fuzzy set theory is employed to represent the estimated bidding behaviors of rival generation companies, and a fuzzy programming model is next developed and a solving method followed. The approach is especially suitable for those electricity markets recently launched, since sufficient historical bidding data is not available and hence probability methods cannot be employed. Finally, a sample example with six generation companies participating in an electricity market is served for demonstrating the essential features of the presented approach.**

Index Terms-- **Bidding strategy, fuzzy programming, electricity market, possibility theory.**

I. INTRODUCTION

THE power industry restructuring is undergoing in many countries around the world, and as the result, competition has been introduced in the generation sector through bid-based operation. In the new and competitive environment, profit-maximization is a primary objective of generation companies or power suppliers. Theoretically, in a perfectly competitive electricity market, all power suppliers are price takers, and the optimal bidding strategy for them is simply to bid their marginal costs. However, it is well known that the emergent electricity market structure is more akin to oligopoly than perfect market competition, due to special features of the electricity supply industry such as, a limited number of producers, large investment size (barrier to entry), transmission constraints and transmission losses. In an oligopoly market, the profit of a supplier depends, to a great extent, on his own strategies and his rivals' strategies as well. Without exception, in an electricity market bidding strategies employed by generation companies may have significant impacts on their own benefits, and on the operating behaviors of the market as well. Hence, how to develop optimal bidding strategies for generation companies or how to analyze strategic behaviors of them and hence to figure out the potential market power abuse is now a very active research area.

A considerable amount of literature has been produced on this subject in recent years, and in [1], a comprehensive survey was made. Broadly speaking, there are basically three ways for developing optimal bidding strategies. The first one relays on estimations of the market clearing price (MCP) in the next trading period. This method may be applicable for small generation companies whose bidding strategies have little effects on the MCP, since an implicit assumption is made in this method that the bidding strategy used by any power supplier will not have any impact on the MCP. This method may also be applicable to the power suppliers in those electricity markets in which historical bidding data is not publicly broadcasted. The second approach is game theory based. However, based on the available methods and techniques from the game theory at this moment, it is not realistic to develop practical bidding strategies. The third approach utilizes estimations of bidding behaviors of the rival participants. Most publications available on this topic are based on this method; however, research results achieved so far are still far from practical applications. The research work carried out in this paper follows the third line of approaches.

Specifically, most of approaches developed following the third line of research are based on probability analysis[2][3], which is mathematically rigorous and well accepted. However, probabilistic methods require large amounts of observations (i.e., available historical data for our problem) with good statistical characteristics. Furthermore, this kind of methods is unable to deal with human experts' linguistic information usually with fuzzy rather than crisp implications. Obviously, for recently launched electricity markets or markets in which rules were modified recently, the available data may not be sufficient for probabilistic modeling of rivals' bidding behaviors. By recognizing this problem, it is proposed in some papers such as [4] that fuzzy sets based methods are more appropriate for this purpose. However, the research work in this area is still very preliminary and a systematic method has not been developed up to now.

Given this background, a possibility theory based approach

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is proposed in this work for building optimal bidding strategies for generation companies. Based on available information, such as historical bidding data, the data available before the power industry restructuring (such as rivals' generator types and corresponding operating efficiencies), current fuel price, and experts' heuristic expert knowledge, the well-known fuzzy set theory is employed to represent the estimated bidding behaviors of competitors, and a fuzzy programming model is next developed and a solving method followed. Finally, a sample example with six generation companies participating in an electricity market is served for demonstrating the essential features of the presented approach.

II. FUNDAMENTALS OF POSSIBILITY THEORY

The possibility theory [5] was developed as a branch of the fuzzy set theory[6] by the need for a systematic way of dealing with uncertainty that is non-statistical in nature. Intuitively, possibility relates to our perception of the degree of feasibility or ease of attainment, whereas probability is associated with the degree of belief, likelihood, frequency, or proportion. Unlike probability, the concept of possibility in no way involves the notion of repeated experimentation. Hence, it is natural to use the concept of possibility when the imprecision or uncertainty in the phenomena under study cannot be well dealt with by probability analysis [7].

Although the possibility theory has been applied, in the context of power systems, to fault section estimation and state identification [8], and calculation of LOLE [9], it is still not well known for people in the circle of power engineering. Hence, a brief introduction on some fundamental knowledge such as notions and theorems concerning the possibility theory is given here with the presumption that readers have some basic knowledge of the fuzzy set theory.

Definition 1: Let U be a nonempty set, and F be the power set of U. A nonnegative real function Π is called *possibility measure*, such that

1. $\Pi(U)=1, \Pi(\emptyset)=0$

2. $\Pi(A)\geq 0, \forall A \in F$ (1)

3. $\Pi(A\cup B)=\Pi(A)\vee\Pi(B), \forall A\in F, B\in F$.

The triplet (U,F,Π) is called a *possibility space*.

Definition 2: Let (U,F,Π) be a possibility space, then a *necessity measure* N is defined by

$$N(A)=1-P(\neg A), \forall A\in F. \quad (2)$$

One of the major differences between possibility and probability could be found from the above definitions. In the possibility theory, the uncertainty of an event is described by both the measure of possibility of the event itself and the measure of the contrary event, and these two measures are weakly related. However, in the probability theory, the probability of an event completely determines the counterpart of the contrary event. The probability measure P of an event A satisfies the following relationship

$$P(A)\in[N(A),\pi(A)].$$

In [10], the credibility of a fuzzy event is defined as the average of its possibility and necessity, as detailed in the following definition.

Definition 3: Let (U,F,Π) be a possibility space, then a *credibility measure* Cr is defined by

$$Cr(A)=\frac{1}{2}(\Pi(A)+N(A)), \forall A\in F. \quad (3)$$

A fuzzy event may not happen even though its possibility is 1, and may occur even though its necessity is 0. However, the fuzzy event will be sure to happen if its credibility is 1, and will surely not happen if its credibility is 0.

There are many ways for defining the expected value of a fuzzy variable. In the following work, the definition given in [10] as detailed below will be employed.

Definition 4: Let ξ be a fuzzy variable on the possibility space (U,F,Π), then the expected value of ξ is defined by

$$E(\xi)=\int_0^{+\infty}Cr\{\xi\geq r\}dr-\int_{-\infty}^{0}Cr\{\xi\leq r\}dr. \quad (4)$$

III. PROBLEM FORMULATION

Suppose that there are n independent power suppliers participating in a pool-based electricity market, the sealed auction with uniform market clearing price is employed in the market, and the forecasted load Q is publicly broadcasted. Next, assume that each power supplier is required to submit a linear bidding function to the pool, say the jth supplier's bid is $B_j(P_j)=\alpha_j+\beta_j P_j$, subject to $P_{j\min}\leq P_j\leq P_{j\max}$, where P_j is the generation output, α_j and β_j biding coefficients, and $P_{j\max}$ and $P_{j\min}$ the generation output upper and lower limits, respectively. Upon receiving bids from suppliers, the pool determines a set of generation outputs that meet the system economic, security and reliability constraints using transparent dispatch procedures. In this work, a simplified market clearing mechanism as formulated below is utilized [2].

$$\alpha_j+\beta_j P_j=R \quad j=1,2,\cdots n \quad (5)$$

$$\sum_{j=1}^{n}P_j=Q(R) \quad (6)$$

$$Q(R)=Q_0-KR \quad (7)$$

$$P_{j\min}\leq P_j\leq P_{j\max} \quad (8)$$

Here R is the MCP, $Q(R)$ the pool load forecasted, and K the elastic coefficient of the load demand. When $P_j<P_{j\min}$, set $P_j=0$, and when $P_j>P_{j\max}$, set $P_j=P_{j\max}$ [2]. When the inequality constraints in (8) are ignored, the solutions to (5)-(7) are

$$R = \left(Q_0 + \sum_{j=1}^{n} \alpha_j / \beta_j \right) \Bigg/ \left(\sum_{j=1}^{n} 1/\beta_j + K \right) \quad (9)$$

$$P_j = (R - \alpha_j) / \beta_j \quad (10)$$

Hence for the ith supplier, the decision-making problem can be described as

$$\max_{\alpha_i, \beta_i} f(\alpha_i, \beta_i) = RP_i - C(P_i) \quad (11)$$

Subject to: (9) and (10).

Here $C(P_i)$ is the production cost function of the ith supplier.

In the sealed bid auction based electricity market, competitors' bidding data for the next period is confidential. In this case, estimating of competitors' behaviors is necessary. As discussed above, the available data may not be sufficient for rigorous probabilistic analysis in electricity markets recently established. As an alternative, the notion of the possibility theory appears more appropriate, which provides a reasonable compromise between quantitative and qualitative, objective and subjective, crisp and fuzzy.

Although it is not possible for a generation company to know the rivals' bidding strategies or bidding coefficients before the next trading-period auction clears, however, estimating the bidding coefficients is possible. The estimation of bidding coefficients could be represented as a fuzzy set based on qualitative knowledge of rivals from available data (historical bidding data and data about rivals before power industry restructuring) and heuristic knowledge. In [9], in accordance with the degree of uncertainty in the given data, a method was proposed to transform a probability distribution into a possibilistic representation using the probability/possibility consistency principle. Moreover, in [11] many methods are available on determining membership functions.

Suppose that, from the ith supplier's point of view, the bidding coefficients of the jth ($j \neq i$) supplier, α_j and β_j, can be represented as fuzzy sets $\underset{\sim}{\alpha}_j, \underset{\sim}{\beta}_j$. Thus eq. (11) can be rewritten as follows,

$$\max_{\alpha_i, \beta_i} f(\alpha_i, \beta_i) = RP_i - C(P_i) \quad (12)$$

Subject to:

$$R = \left(Q_0 + \sum_{\substack{j=1 \\ j \neq i}}^{n} \underset{\sim}{\alpha}_j / \underset{\sim}{\beta}_j + \alpha_i / \beta_i \right) \Bigg/ \left(\sum_{\substack{j=1 \\ j \neq i}}^{n} 1/\underset{\sim}{\beta}_j + 1/\beta_i + K \right)$$

$$P_i = (R - \alpha_i) / \beta_i .$$

Although $\underset{\sim}{\alpha}_j, \underset{\sim}{\beta}_j (j = 1, 2, \cdots, n; j \neq i)$ are not explicitly included in the profit function of the subject power supplier i, i.e. $f(\alpha_i, \beta_i)$, they are implicitly involved in the procedure of determining R and P_i. For the simplicity of presentation, estimations for rivals' bidding coefficients are grouped as $\xi = (\underset{\sim}{\alpha}_1, \underset{\sim}{\beta}_1, \cdots, \underset{\sim}{\alpha}_{i-1}, \underset{\sim}{\beta}_{i-1}, \underset{\sim}{\alpha}_{i+1}, \underset{\sim}{\beta}_{i+1}, \cdots, \underset{\sim}{\alpha}_n, \underset{\sim}{\beta}_n)$, and the ith supplier's profit function is then expressed as $f(\alpha_i, \beta_i, \xi)$.

Many methods are available for comparing two fuzzy variables. Here the fuzzy expected value is used, and this is consistent with the method used in probabilistic analysis. Hence, the above decision-making problem can be transformed to an expected value form,

$$\max_{\alpha_i, \beta_i} E(f(\alpha_i, \beta_i, \xi)) \quad (13)$$

Subject to:

$$f(\alpha_i, \beta_i, \xi) = RP_i - C(P_i)$$

$$R = \left(Q_0 + \sum_{\substack{j=1 \\ j \neq i}}^{n} \underset{\sim}{\alpha}_j / \underset{\sim}{\beta}_j + \alpha_i / \beta_i \right) \Bigg/ \left(\sum_{\substack{j=1 \\ j \neq i}}^{n} 1/\underset{\sim}{\beta}_j + 1/\beta_i + \right.$$

$$P_i = (R - \alpha_i) / \beta_i$$

The ith supplier's profit, $f(\alpha_i, \beta_i, \xi)$, depends on his estimations of rivals' bidding behaviors ξ and his own bidding coefficients α_i, β_i. $f(\alpha_i, \beta_i, \xi)$ is a fuzzy variable since ξ is a fuzzy one. $E(.)$ represents the expected value.

Up to now, the ith supplier's decision-making problem is to search for the best bidding coefficients α_i, β_i with the objective of maximizing expected profit, given that estimations of the rivals' bidding behaviors are known.

From the mathematical point of view, one of the two bidding coefficients of the ith supplier, i.e., α_i, β_i, is redundant. In other words, the ith supplier can fix one of these two coefficients and then determine the other by using an optimization procedure. Thus building the optimal bidding strategy is reduced to a one-parameter search problem. In this work, it is assumed that α_i is fixed by the ith supplier and β_i is determined by an optimization method.

The ith supplier's estimations of the rivals' bidding coefficients $\underset{\sim}{\alpha}_j, \underset{\sim}{\beta}_j (j = 1, 2, \cdots, n; j \neq i)$ represent his fuzzy and qualitative (or roughly quantitative) knowledge of the rivals' behaviors. The membership functions of $\underset{\sim}{\alpha}_j, \underset{\sim}{\beta}_j$ can be obtained through structure and parameter identifications. In general, $\tilde{\alpha}_j, \tilde{\beta}_j$ are not independent. However, due to the insufficiency of historical data and limited knowledge, it is difficult to obtain the joint membership functions directly. In this work, the membership functions of $\tilde{\alpha}_j$ and $\tilde{\beta}_j$ are represented by two one-dimensional Gaussian functions respectively, however, other forms of membership functions can be accommodated in the proposed method as well.

$$\mu_{\alpha_j}(x) = e^{-\frac{1}{2}(\frac{x-C_{\alpha_j}}{\sigma_{\alpha_j}})^2} \overset{\Delta}{=} N(C_{\alpha_j}, \sigma_{\alpha_j}) \quad (14)$$

$$\mu_{\beta_j}(x) = e^{-\frac{1}{2}(\frac{x-C_{\beta_j}}{\sigma_{\beta_j}})^2} \overset{\Delta}{=} N(C_{\beta_j}, \sigma_{\beta_j}). \quad (15)$$

Here C_{α_j}, C_{β_j} denote the centers, σ_{α_j}, σ_{β_j} the spreads or widths. The possibility that the jth supplier's bidding coefficients (α_j, β_j) are (x_j, y_j) can be expressed as follows,

$$\mu_j(x_j, y_j) = \mu_{\alpha_j}(x_j) \wedge \mu_{\beta_j}(y_j) . \quad (16)$$

With regard to the correlation between $\tilde{\alpha}_j$ and $\tilde{\beta}_j$, it is more likely that the jth supplier chooses $(C_{\alpha_j} \pm \varepsilon, C_{\beta_j} \mp \varepsilon)$ $(\varepsilon > 0)$ rather than $(C_{\alpha_j} \pm \varepsilon, C_{\beta_j} \pm \varepsilon)$. However, this kind of heuristic knowledge cannot be described by eqs. (14)-(16). For this purpose, a fuzzy correlation term as defined in [12] is employed here to represent the correlation between $\tilde{\alpha}_j$ and $\tilde{\beta}_j$ as follows

$$\mu_{\beta_j|\alpha_j}(y, x) = e^{-\frac{1}{2}(\frac{x-C_{\alpha_j}}{C_{\alpha_j}} + \frac{y-C_{\beta_j}}{C_{\beta_j}})^2} \quad (17)$$

$\mu_{\beta_j|\alpha_j}(y, x)$ is the possibility that β_j is y given that α_j is x. In fact, the fuzzy correlation is defined as a kind of conditional possibility distributions[12]. Based on this, the possibility that the jth supplier will choose $(\alpha_j, \beta_j) = (x_j, y_j)$ is modified as:

$$\mu_j(x_j, y_j) = \mu_{\alpha_j}(x_j) \wedge \mu_{\beta_j}(y_j) \wedge \mu_{\beta_j|\alpha_j}(y_j, x_j) \quad (18)$$

From the ith supplier's point of view, the possibility that the rivals bid the parameters included in $\xi = (\alpha_1, \beta_1, \cdots, \alpha_{i-1}, \beta_{i-1}, \alpha_{i+1}, \beta_{i+1}, \cdots, \alpha_n, \beta_n)$ is

$$\mu(\xi) = \mu_1(\alpha_1, \beta_1) \wedge \cdots \wedge \mu_{i-1}(\alpha_{i-1}, \beta_{i-1}) \wedge \mu_{i+1}(\alpha_{i+1}, \beta_{i+1}) \wedge \cdots \wedge \mu_n(\alpha_n, \beta_n) \quad (19)$$

IV. FUZZY SIMULATION ALGORITHM OF EXPECTED VALUE

In accordance with Definition 4, we have

$$E(f(\alpha_i, \beta_i, \xi)) = \int_0^{+\infty} Cr\{f(\alpha_i, \beta_i, \xi) \ge r) dr - \int_{-\infty}^0 Cr\{f(\alpha_i, \beta_i, \xi) \le r) dr \quad (20)$$

A fuzzy simulation algorithm introduced in [10] can be used to estimate $E(f(\alpha_i, \beta_i, \xi))$. Randomly generate $\alpha_{1l}, \beta_{1l}, \alpha_{2l}, \beta_{2l}, \cdots, \alpha_{nl}, \beta_{nl}$ and set $\xi_l = (\alpha_{1l}, \beta_{1l}, \alpha_{2l}, \beta_{2l}, \cdots, \alpha_{nl}, \beta_{nl})$, $\mu(\xi_l) = \mu_1(\alpha_{1l}, \beta_{1l}) \wedge \cdots \wedge \mu_n(\alpha_{nl}, \beta_{nl})$, $l = 1, 2, \cdots m$, respectively, from the ε-level sets of $\tilde{\alpha}_j, \tilde{\beta}_j (j = 1, 2, \cdots, n; j \ne i)$. Here ε is a sufficiently small number, and m is a sufficiently large positive integer representing sampling times. The integration terms in eqn. (20) can be obtained by discrete integration for H times, and H is a sufficiently large number. Then, for any $r \ge 0$, the credibility $Cr\{f(\alpha_i, \beta_i, \xi) \ge r\}$ can be estimated by

$$Cr\{f(\alpha_i, \beta_i, \xi) \ge r\} = \frac{1}{2}(\max_{l=1,2\cdots,m} \{\mu(\xi_l) \mid f(\alpha_i, \beta_i, \xi_l) \ge r\} + 1 - \max_{l=1,2\cdots,m} \{\mu(\xi_l) \mid f(\alpha_i, \beta_i, \xi_l) < r\}) \quad (21)$$

and for any $r < 0$, the credibility $Cr\{f(\alpha_i, \beta_i, \xi) \le r\}$ can be estimated by

$$Cr\{f(\alpha_i, \beta_i, \xi) \ge r\} = \frac{1}{2}(\max_{l=1,2\cdots,m} \{\mu(\xi_l) \mid f(\alpha_i, \beta_i, \xi_l) \le r\} + 1 - \max_{l=1,2\cdots,m} \{\mu(\xi_l) \mid f(\alpha_i, \beta_i, \xi_l) > r\}) \quad (22)$$

Given a pair of (α_i, β_i), the fuzzy simulation algorithm proceeds as follows,

1) Initialize the sampling size, m, and set the sampling counter l=0;
2) Set l=l+1;
3) Randomly generate ξ_l by sampling from ξ. Given the bidding parameter set $(\alpha_i, \beta_i, \xi_l)$, MCP, R_l and the ith supplier's dispatched output P_{il} can be obtained by running the market clearing procedure denoted by eqs. (5)-(8). Hence the ith supplier's profit

$f(\alpha_i, \beta_i, \xi_l)$ can then be obtained.

4) If $l<m$, go back to 2), otherwise go to 5);
5) Set
$$w_{\min} = \min(f(\alpha_i,\beta_i,\xi_1), f(\alpha_i,\beta_i,\xi_2), \cdots, f(\alpha_i,\beta_i,\xi_m)),$$
$$w_{\max} = \max(f(\alpha_i,\beta_i,\xi_1), f(\alpha_i,\beta_i,\xi_2), \cdots, f(\alpha_i,\beta_i,\xi_m));$$
6) Set the expected value E=0;
7) Randomly generate r from $[w_{\min}, w_{\max}]$;
8) If $r \geq 0$, then set $E = E + Cr\{f(\alpha_i,\beta_i,\xi) \geq r\}$.
9) If $r < 0$, then set $E = E - Cr\{f(\alpha_i,\beta_i,\xi) \geq r\}$;
10) Repeat 7)-9) for H times.
11) $E(f(\alpha_i,\beta_i,\xi)) = w_{\min} \vee 0 + w_{\max} \wedge 0 + E\cdot(w_{\max} - w_{\min})/H.$

V. Algorithmic Procedures of Building Optimal Bidding Strategy

The procedures for the ith supplier to build the optimal bidding strategy are as follows,

1) Fix the bidding coefficient α_i;
2) Based on the one-dimension search algorithm, determine β_i from $[\beta_{i*}, \beta_i^*]$ which is the possible range of β_i and could be defined large if experience is not much;
3) Use the fuzzy simulation algorithm as introduced before to get the expected value $E(f(\alpha_i,\beta_i,\xi))$;
4) Repeat 2)-3), and choose the optimal value of β_i which maximizes $E(f(\alpha_i,\beta_i,\xi))$ as the optimal bidding strategy.

VI. Numerical Example

An example with six generators is used for demonstrating the essential features of the method. As a preliminary research, suppose that each supplier owns only one generator. The generation production cost data is listed in Table I. The forecasted system load Q_0 is 350 MW.

Many simulations are carried out, but only test results for three cases are detailed below due to space limitation. In the simulations, m is specified to be15000 and H 5000.

A. Case

Suppose that the second power supplier (generator 2) is our subject of research. Assume that the second supplier fixes $\alpha_2 = 2.1$ and his estimations of the rivals' bidding parameters are listed in Table II.

TABLE I
GENERATION DATA

Gen No.	a_i	b_i	c_i	$P_{i\min}$ (MW)	$P_{i\max}$ (MW)
1	0.0	2.0	0.00375	50	200
2	0.0	1.75	0.0175	20	80
3	0.0	1.0	0.0625	15	50
4	0.0	3.25	0.00834	10	35
5	0.0	3.0	0.025	10	30
6	0.0	3.0	0.025	12	40

*Generation cost function: $C_i(P_i) = a_i + b_i P_i + c_i P_i^2$

TABLE II
ESTIMATED RIVALS' BIDDING PARAMETERS BY THE SECOND SUPPLIER

Gen No.	c_{α_j}	σ_{α_j}	c_{β_j}	σ_{β_j}
1	2.4	$0.1C_{\alpha_1}$	0.0113	$0.1C_{\beta_1}$
3	1.3	$0.1C_{\alpha_3}$	0.0688	$0.1C_{\beta_3}$
4	3.575	$01.C_{\alpha_4}$	0.0117	$0.1C_{\beta_4}$
5	3.3	$0.1C_{\alpha_5}$	0.0325	$0.1C_{\beta_5}$
6	3.3	$0.1C_{\alpha_6}$	0.0275	$0.1C_{\beta_6}$

In the case with inelastic load, i.e. K=0, the optimal bidding strategy obtained is $\beta_2 = 0.0357$ and the corresponding expected profit is 82.09.

B. Case

Suppose that this case is the same as Case 1, except that the load is elastic, and K=20. The optimal bidding strategy obtained is then $\beta_2 = 0.0315$ and the corresponding expected profit reduces to 61.93. Compared with Case 1, the optimal bidding price is decreased and the expected profit then reduced. This implies that the market power of the second supplier is limited by the demand price elasticity. This result is consistent with that obtained by a probability-based method in [2].

C. Case

Suppose that K=20, and our subject of research changes to the fifth supplier. Assume that the fifth supplier's estimations of the second supplier's bidding parameters are c_{α_2}=2.1, $\sigma_{\alpha_2} = 0.21$, c_{β_2}=0.035, $\sigma_{\beta_2} = 0.0035$, and those of the other suppliers are the same as Case 1 (see Table 2). Moreover, suppose the fifth supplier fixes $\alpha_5 = 3.3$, and the optimal bidding strategy obtained is $\beta_5 = 0.0288$ and the corresponding expected profit is 7.13. If the fifth supplier could have better estimations of the sixth supplier's bidding parameters, such as $\sigma_{\alpha_6} = 0.105$, $\sigma_{\beta_2} = 0.00135$, then in this case the optimal bidding strategy obtained is $\beta_5 = 0.0324$, and the expected profit increases to 7.57. It is clear that the accuracy of estimations of the rivals' bidding behaviors has a definite impact on the expected profit, and hence, it is very important for the suppliers to make full use of available information so as to get better estimations, and as a result, more profit.

VII. CONCLUDING REMARKS

In the electricity market environment, the production cost information of generators is commercially confident. Hence, it is not practical to build optimal bidding strategies under complete and perfect information assumption. Intuitively, it is more realistic to develop optimal bidding strategies by estimating competitors' bidding behaviors. However, for those electricity markets recently established or whose structure and rules recently modified, historical data may not be sufficient for using probabilistic methods to model the bidding behaviors of rivals. As an alternative, a possibility theory based method is proposed in this paper, which could accommodate uncertainties and incomplete and insufficient information. It should be stressed that the emphasis of this work is to establish a framework for building optimal bidding strategies for generation companies under imperfect and insufficient information, rather than to develop a method on estimating rivals' bidding behaviors. Given any estimated rivals' bidding behaviors represented by fuzzy sets, the method could be used to develop a bidding strategy for the subject generation company. However, as a preliminary research, only the problem of developing bidding strategies in a single-period auction is addressed in this paper, and inter-temporal operating constraints for start-up and shutdown of a generator and power system network constraints are not taken into account, and these will be done in later studies.

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X. BIOGRAPHIES

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