MULTI-USER MIMO ANTENNA SYSTEMS USING SEMI-ORTHOGONAL SPACE DIVISION MULTIPLEXING AND SINGLE-USER QR-TRIANGULAR DETECTION

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ABSTRACT

Orthogonal space division multiplexing (OSDM) can be achieved by utilizing multiple antennas at the base station (BS) and all mobile stations (MS) (or multi-user MIMO) jointly. Recently in [1, 2], two iterative methods have been proposed to obtain the antenna weighting for realizing OSDM in the downlink. However, both suffer from high complexity if the number of BS antennas or users involved is large, as the computational complexity grows with the number of users to the fourth power. In this paper, by allowing the signal streams for a user to be non-orthogonal, we propose a semi-OSDM system for complexity reduction. In the proposed semi-OSDM system, the signals of multiple users are still orthogonal, but for each user, the multiple signaling streams are made triangular instead so that a simple backward substitution with symbol cancellation can be performed to maintain an accurate signal detection. Remarkably, the antenna weighting for semi-OSDM can be expressed in a closed form. Simulation results show that semi-OSDM has mild performance degradation, but it requires much lower computational complexity (by an order of magnitude) when compared with existing OSDM systems.

1. INTRODUCTION

Conventionally, multi-user wireless communication is achieved by multiplexing each user in an uncoupled time slot, frequency band or spreading code (e.g., GSM, PCS, and IS-95, etc.). Analogously, support of multiple users can also be accomplished in space. This arises from the fact that users are generally situated at different locations and user separation can be done by distinguishing their spatial signatures.

For uplink communication (from mobiles to base), maximum likelihood (ML) multi-user detection [3] or array signal processing using multiple receiving antennas [4] can be used for space division multiple access (SDMA). In the downlink (from base to mobiles), however, neither of them is possible since mobile station (MS) usually cannot afford a computationally intensive ML multi-user detector or have a large number of antenna elements. To accommodate co-channel users in the downlink, it is therefore the responsibility for the base station (BS) to have techniques that are able to separate the multi-user signals in space during transmission.

This idea has been investigated in the past, but mainly on the signal-to-interference plus noise ratio (SINR) balancing (e.g., [5]).

In other words, the users are not truly uncoupled, rather the objective is to maintain every user's SINR at a pre-set value for acceptable signal reception. Orthogonal space division multiplexing (OSDM) for downlink transmission has not been studied until recently in [1, 2]. In these works, multi-user signals are sent in disjoint space through joint use of multiple antennas at the BS and (optionally) all MS. The multi-user MIMO (multiple-input multiple-output) antennas are optimized iteratively for OSDM and throughput maximization. The major drawback is, however, that the proposed iterative methods require very intense computing power whose complexity grows roughly with the number of users to the fourth power, which limits the system scalability.

In this paper, we aim to reduce the complexity of an OSDM system by allowing each user to be self-interfered meaning that the multiple signaling streams of a user are no longer orthogonal. In so doing, we are able to find a closed form expression for the multi-user antenna weights, which finally reduces the overall complexity significantly. Orthogonalization between users is maintained so that they are still uncoupled to each other. Nonetheless, because of the non-orthogonality nature for every user, severe performance degradation may occur. To minimize the degradation, we propose to process individual user by QR triangularization. Thus, a simple backward substitution with symbol cancellation can be performed at the mobile side to maintain acceptable signal detection with minimal complexity. Throughout this paper, we shall refer to this system as semi-OSDM with QR triangular detection.

The paper is organized as follows. In Section II, we introduce the multi-user MIMO system model. Section III presents the transmitter and receiver structure of the proposed semi-OSDM system. Simulation setup and results are presented in Section IV. Finally, we conclude the paper in Section V.

2. SYSTEM DESCRIPTION

The system configuration of the multi-user MIMO antenna system is shown in Figure 1 (same as in [1]) where signals are transmitted from one BS to $M(> 1)$ MS, n_T antennas are located at the BS and n_{R_m} antennas are located at the mth MS. The overall system can be conveniently written as [1, 2]

$$\mathbf{y}_m = \mathbf{H}_m \left(\sum_{m'=1}^{M} \mathbf{T}_{m'} \mathbf{z}_{m'} \right) + \mathbf{n}_m \quad \forall m \qquad (1)$$

where $\mathbf{y}_m = [y_1^{(m)} \cdots y_{n_{R_m}}^{(m)}]^T \in \mathbb{C}^{n_{R_m}}$ is the received signal vector of MS m with the superscript T representing transposition, $\mathbf{z}_{m'} = [z_1^{(m')} \cdots z_{N_{m'}}^{(m')}]^T \in \mathbb{C}^{N_{m'}}$ denotes the vector sym-

bols transmitted to MS m', $\mathbf{H}_m \in \mathbb{C}^{n_{R_m} \times n_T}$ denotes the MIMO channel matrix between the BS and the mth MS, $\mathbf{n}_m \in \mathbb{C}^{n_{R_m}}$ is the complex zero-mean noise vector with power of $N_0/2$ per dimension, and $\mathbf{T}_{m'} \in \mathbb{C}^{n_T \times N_{m'}}$ is the antenna array processing for transmitting the vector symbols of user m'.

In the above formulation, it is assumed that $\mathbf{H}_m$'s have independent entries and are uncorrelated with themselves and $\mathbf{z}_m$. In addition, the values of N_m and n_{R_m} are always set equal for convenience.

3. SEMI-OSDM AND QR-TRIANGULAR DETECTION

The proposed MIMO processing orthogonalizes the signals of multiple users so that each user can be treated independently [6]. Unlike previous approaches [1, 2, 6], however, the multiple signaling streams for individual user are purposefully made triangular. In so doing, on one hand, we shall be able to express the optimal BS antenna weights in a closed form for complexity reduction. On the other hand, the performance degradation can be kept minimal using an easy-to-compute symbol cancellation.

3.1. Transmitter Processing at BS

The transmitter objective is to obtain the antenna weight matrices, $\mathbf{T}_m$, so that

$$\mathbf{T}_m|_{\text{opt}} = \arg \max_{\mathbf{T}_m} \left| \lambda_n^{(m)} \right|^2 \quad \forall m, n \quad (2)$$

where $| \cdot |$ takes the modulus of a complex number, and $\lambda_n^{(m)}$ is the resultant channel gain for the nth spatial stream of the mth MS, which is defined by

$$\mathbf{H}_m [\mathbf{T}_1 \ \mathbf{T}_2 \ \cdots \ \mathbf{T}_M]$$
$$= [\mathbf{0}_1 \ \cdots \ \mathbf{0}_{m-1} \underbrace{\mathbf{\Lambda}_m}_{\substack{m\text{th} \\ \text{sub-block} \\ \text{matrix}}} \mathbf{0}_{m+1} \ \cdots \ \mathbf{0}_M] \quad (3)$$

where

$$\mathbf{\Lambda}_m = \begin{bmatrix} \lambda_1^{(m)} & 0 & \cdots & 0 \\ \xi_{1,2}^{(m)} & \lambda_2^{(m)} & & 0 \\ \vdots & & \ddots & \vdots \\ \xi_{1,N_m}^{(m)} & \xi_{2,N_m}^{(m)} & & \lambda_{N_m}^{(m)} \end{bmatrix} \in \mathbb{C}^{N_m \times N_m}, \quad (4)$$

$\xi_{i,j}^{(m)}$ corresponds to the interference from the jth signaling stream at the ith receiving antenna of MS m, and $\mathbf{0}_{m'}$ is the m'th sub-block zero matrix of dimension $N_m \times N_{m'}$. From definition (3), each sub-block matrix corresponds to the signals transmitted from each user to mobile location m and hence by making all of them zero except for the mth user, co-channel interference (CCI) from different users are completely eliminated.

The optimization (2) is performed subject to the power constraint of $\|\mathbf{t}_n^{(m)}\| = 1$. Hence, the value of $\lambda_n^{(m)}$ will represent the resultant channel coefficient after transmitter processing.

Now, we define an equivalent multi-user channel matrix, $\mathbf{H}_e$, as

$$\mathbf{H}_e \triangleq \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_M \end{bmatrix} \in \mathbb{C}^{\left(\sum_{m=1}^{M} N_m \right) \times n_T} . \quad (5)$$

Then, the multi-user orthogonalization (3) can be written as

$$\mathbf{H}_e \mathbf{T}_m = \begin{bmatrix} \mathbf{0}_1^T \\ \vdots \\ \mathbf{0}_{m-1}^T \\ \mathbf{\Lambda}_m \\ \mathbf{0}_{m+1}^T \\ \vdots \\ \mathbf{0}_M^T \end{bmatrix} \in \mathbb{C}^{\left(\sum_{m'=1}^{M} N_{m'} \right) \times N_m} \quad \forall m. \quad (6)$$

Also, we define a LACK-m matrix, $\mathbf{H}_e^{(m)-}$, as

$$\mathbf{H}_e^{(m)-} \triangleq \begin{bmatrix} \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_{m-1} \\ \mathbf{H}_{m+1} \\ \vdots \\ \mathbf{H}_M \end{bmatrix} \in \mathbb{C}^{\left(\sum_{\substack{m'=1 \\ m' \neq m}}^{M} N_{m'} \right) \times n_T}, \quad (7)$$

which agrees with $\mathbf{H}_e$ except that the block $\mathbf{H}_m$ is missing. Therefore, the LACK-m matrix can be considered as the channels from the BS to all the MS receivers excluding MS m.

Apparently, we must have

$$\mathbf{T}_m \in \text{null} \left\{ \mathbf{H}_e^{(m)-} \right\} \quad (8)$$

to ensure (6), where $\text{null}\{\cdot\}$ denotes the nullspace of the input matrix. By letting the orthonormal basis of the $\text{null}\{\mathbf{H}_e^{(m)-}\}$ be $\mathbf{P}_m = [\mathbf{p}_1^{(m)} \ \mathbf{p}_2^{(m)} \cdots]$, we can write $\mathbf{T}_m$ in the form

$$\mathbf{T}_m = \mathbf{P}_m \mathbf{B}_m \quad (9)$$

where $\mathbf{B}_m$ denotes the coordinate transformation under the basis $\mathbf{P}_m$. Then, the remaining task is to choose $\mathbf{B}_m$ in a way that

$$\mathbf{B}_m|_{\text{opt}} = \arg \max_{\mathbf{B}_m} \left| \lambda_n^{(m)} \right|^2 \quad \forall m, n \quad (10)$$

and

$$(\mathbf{H}_m \mathbf{P}_m) \mathbf{B}_m = \mathbf{\Lambda}_m. \quad (11)$$

The problem is now reduced to a single-user MIMO channel triangularization. Similar problem of MIMO channel has been considered in recent years. The most famous example is BLAST (Bell-Labs Layered Space-Time) processing [7, 8]. Unlike these studies, however, our triangularization is done at the transmitter side, but not at the receiver end.

To solve $\mathbf{B}_m$, we use QR decomposition [9] on the matrix $(\mathbf{H}_m \mathbf{P}_m)^\dagger$ to have $(\mathbf{H}_m \mathbf{P}_m)^\dagger = \mathbf{Q}_m \mathbf{R}_m$ where $\mathbf{Q}_m$ is a unitary matrix, and $\mathbf{R}_m$ is an upper triangular matrix. It can be easily shown that (10) and (11) are performed jointly by choosing

$$\mathbf{B}_m|_{\text{opt}} = \mathbf{Q}_m|_{1 \to N_m} \quad (12)$$

where $\cdot|_{1 \to N_m}$ denotes the first N_m columns of the input matrix. Consequently,

$$\mathbf{\Lambda}_m = \mathbf{R}_m^\dagger|_{1 \to N_m} \quad (13)$$

is a lower triangular matrix in the form (4). To summarize, the optimal BS antenna weights can be found by

$$\mathbf{T}_m|_{\text{opt}} = \mathbf{P}_m \mathbf{Q}_m \quad \forall m \quad (14)$$

where $\mathbf{P}_m = \text{null}\{\mathbf{H}_e^{(m)-}\}$ and $\mathbf{Q}_m$ is the Q-factor matrix from the QR decomposition of $(\mathbf{H}_m \mathbf{P}_m)^\dagger$.

3.2. Receiver Processing at MS m

Substituting (14) into (1), we have M single-user systems (see also (16) on the next page)

$$\mathbf{y}_m = \mathbf{\Lambda}_m \mathbf{z}_m + \mathbf{n}_m \quad \text{for } m = 1, 2, \dots, M. \quad (15)$$

Let $\mathbb{D} = \{s_1, s_2, \dots, s_K\}$ be the set of K-ary modulating symbols. The first signal stream can be detected using

$$\hat{z}_1^{(m)} = \arg \min_{\hat{z}_1^{(m)} \in \mathbb{D}} \left\langle y_1^{(m)}, \lambda_1^{(m)} \hat{z}_1^{(m)} \right\rangle \quad (17)$$

where

$$\langle a, b \rangle = (a - b)(a^* - b^*). \quad (18)$$

The detected symbol, $\hat{z}_1^{(m)}$, is then used to eliminate the CCI at the other antennas for detecting other signaling streams. This simple backward substitution with symbol cancellation can be easily generalized to have the following sequential detection (SD) rule:

$$\hat{z}_n^{(m)} = \arg \min_{\hat{z}_n^{(m)} \in \mathbb{D}} \left\langle y_n^{(m)} - \sum_{i=1}^{n-1} \xi_{i,n}^{(m)} \hat{z}_i^{(m)}, \lambda_n^{(m)} \hat{z}_n^{(m)} \right\rangle. \quad (19)$$

The receiver is simple and requires only N_m steps symbol cancellation. Since the signal streams are made triangular, ideally, all of the symbols can be detected reliably in the absence of CCI. In addition, by allowing the signal streams to be non-orthogonal, more degrees of freedom can be used to maximize the resultant channel coefficients for better performance.

4. SIMULATION RESULTS

Monte Carlo simulations have been conducted to assess the system performance of the proposed semi-OSDM system for quadrature phase shift keying (QPSK). Perfect channel state information at both BS and MS is assumed. The channel is modelled as a quasi-static flat Rayleigh fading channel. For convenience, we shall use $\{n_T, [n_{R_1}(N_1), \dots, n_{R_M}(N_M)]\}$ to denote a M-user MIMO system where n_T antennas are located at the BS, and n_{R_m} antennas are located at the mth MS occupying N_m signaling streams.

4.1. Error Probability Performance

Average bit error rate (BER) results are plotted against the average E_b/N_0 per branch-to-branch. Results of the proposed semi-OSDM and OSDM using Iterative nullspace-directed singular value decomposition (Iterative NU-SVD) in [2] are provided for comparison.

In Figure 2, the system configurations we considered are: a) $\{3, [1(1), 1(1)]\}$ and b) $\{4, [1(1), 1(1)]\}$. Both consider only single-stream MS receivers (i.e., $N_m = 1 \ \forall m$). Results demonstrate that semi-OSDM and Iterative NU-SVD perform nearly the same, which agrees with the fact that when $n_{R_m} = N_m = 1 \ \forall m$, the two solutions are the same.

Figure 3 shows the average BER results when multi-stream MS receivers are considered. In this figure, performance of $\{5, [2(2), 2(2)]\}$ is investigated and only the first user's performance is shown (on average, all users perform the same [2]). As can be seen, there is a remarkable difference in BER between the proposed semi-OSDM and OSDM using Iterative NU-SVD. In particular, the first signaling stream degrades severely, but the second signaling stream improves slightly and has about 2dB energy gain

on average. To have a clearer picture on how much degradation we suffer, let's consider the required signal-to-noise ratio (SNR) at BER of 10^{-3}. Note that the SNR required for Iterative NU-SVD are 2dB and 14dB for the first and second streams, respectively whereas about 6.5dB and 11.5dB are required for semi-OSDM. Therefore, this indicates a 2dB degradation for semi-OSDM when compared to OSDM. Noticeably, however, more degradation will be suffered if lower BER is considered.

4.2. Complexity Comparison

The complexity requirements in terms of the number of floating point operations (flops) for both Iterative NU-SVD and semi-OSDM are provided in Table 1 [9] where $n_{R_m} = n_R \ \forall m$ is assumed. Some ideas can be gained by considering the complexity of $\{5, [2(2), 2(2)]\}$. For Iterative NU-SVD, at each update, the number of flops required is 4,634 and the average number of iterations for convergence is about 10.56. Hence, about 48,935 flops are required to compute the BS antenna weights for OSDM. In contrast, only 3,214 flops are needed to compute the weights for semi-OSDM. This translates to about 15 times reduction in computational complexity at the BS (with only a 2dB degradation at BER= 10^{-3}).

Apparently, the complexity of OSDM is mainly determined by the number of iterations. Results (not included in this paper) illustrate that the number of iterations has a linear dependency with the number of users and MS receiving antennas. Therefore, more saving in complexity can be obtained using semi-OSDM if the number of users or MS antennas is large.

5. CONCLUSIONS

This paper has proposed a semi-OSDM with QR triangular detection for multi-user MIMO antenna system. The proposed system provides an option to greatly reduce the system complexity while suffering an acceptable level of performance degradation, as compared to existing OSDM systems.

6. REFERENCES

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$$\begin{cases} y_1^{(m)} = \lambda_1^{(m)} z_1^{(m)} + & & & \eta_1^{(m)} \\ y_2^{(m)} = \xi_{1,2}^{(m)} z_1^{(m)} + \lambda_2^{(m)} z_2^{(m)} + & & & \eta_2^{(m)} \\ y_3^{(m)} = \xi_{1,3}^{(m)} z_1^{(m)} + \xi_{2,3}^{(m)} z_2^{(m)} + \lambda_3^{(m)} z_3^{(m)} + & & \eta_3^{(m)} \\ \quad\vdots & & & \vdots \\ y_N^{(m)} = \xi_{1,N_m}^{(m)} z_1^{(m)} + \xi_{2,N_m}^{(m)} z_2^{(m)} + \cdots + \lambda_{N_m}^{(m)} z_{N_m}^{(m)} + \eta_{N_m}^{(m)} \end{cases} \tag{16}$$

Table 1. Computational complexity requirement

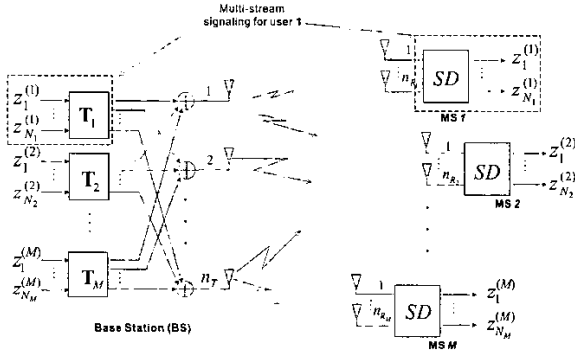|	OSDM using Iterative NU-SVD [2]			Semi-OSDM using QR triangular detection	
	Operation	Num of *flops*		Operation	Num of *flops*
	$\mathbf{H}_e$	$2Mn_T n_R^2$		$\mathbf{H}_e$	NIL
$\forall m$	$\mathbf{H}_e^{(m)-}$	NIL	$\forall m$	$\mathbf{H}_e^{(m)-}$	NIL
	$\mathbf{P}_m = \mathrm{null}\{\mathbf{H}_e^{(m)-}\}$	$2(M-1)n_R n_T^2 + 11n_T^3$		$\mathbf{P}_m = \mathrm{null}\{\mathbf{H}_e^{(m)-}\}$	Same as [2]
	$\mathbf{H}_m \mathbf{P}_m$	$2n_T n_R(n_T - (M-1)n_R)$		$\mathbf{H}_m \mathbf{P}_m$	Same as [2]
	$(\mathbf{R}^{(m)}, \mathbf{B}^{(m)}) \Leftarrow \mathrm{SVD}(\mathbf{H}_m \mathbf{P}_m)$	$4n_R^2(n_T - (M-1)n_R) + 22(n_T - (M-1)n_R)^3$		$\mathbf{B}^{(m)} \Leftarrow \mathrm{QR}((\mathbf{H}_m \mathbf{P}_m)^\dagger)$	$3n_R^2$
	$\mathbf{T}^{(m)} = \mathbf{P}_m \mathbf{B}^{(m)}$	$2n_T(n_T - (M-1)n_R)n_R$		$\mathbf{T}^{(m)} = \mathbf{P}_m \mathbf{B}^{(m)}$	Same as [2]
	Iteration	Required		Iteration	Not required

Fig. 1. System configuration of a multi-user MIMO antenna system.

Fig. 2. Average BER results versus average E_b/N_0 per branch-to-branch for systems with $N_m = 1 \; \forall m$.
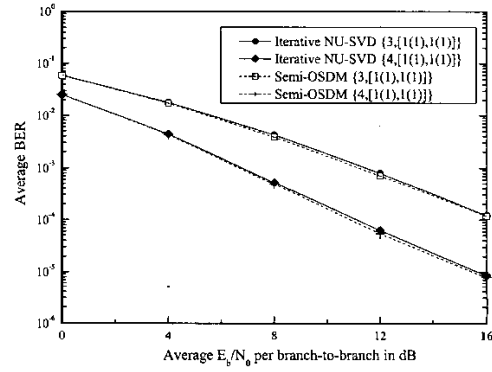

Fig. 3. Average BER results versus average E_b/N_0 per branch-to-branch for systems with $N_m = 2 \; \forall m$.

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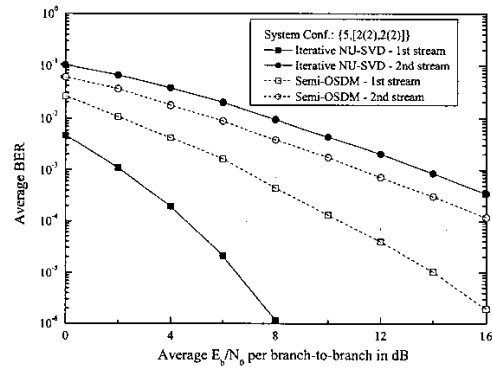[9] G. H. Golub, and Charles F. Van Loan, *"Matrix computation,"* third edition, The Johns Hopkins University Press, Baltimore and London, 1996.