Chaotification of Permanent-Magnet Synchronous Motor Drives Using Time-Delay Feedback

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Abstract –**Recent research has shown that chaos can actually be useful under certain circumstances, and there is growing interest in utilizing the very nature of chaos. Thus, a controllable chaotic motor drive, namely chaotifying a motor drive, is highly desired for practical engineering systems. This paper firstly proposes and implements a time-delay feedback method to chaotify a practical permanent-magnet synchronous motor (PMSM) drive. Based on the current-fed model and field-oriented control, the corresponding system dynamics will be approximated by first-order differential equations. Hence, the electromechanical torque will be adjusted according to the time-delay speed feedback. Consequently, chaotic motion can be achieved by tuning the feedback gain of the torque controller. Moreover, the resulted chaotic motion is easily controllable in the sense that the rotor speed boundary can be controlled precisely by the value of the speed ratio. This controllable chaotic PMSM drive potentially offers some special applications desiring chaotic motion such as fluid mixing and surface grinding. Theoretical analysis, computer simulation as well as experimental results will be given to verify the proposed method of chaotification.**

I. INTRODUCTION

Starting from the late 1980's, chaos has been identified to be a real phenomenon in power electronics. Then, many investigations into chaotic behavior of dc-dc converters were conducted in the 1990's. Recently, the investigation of chaos in motor drives has been accelerated [1]-[4]. Moreover, recent research has shown that chaos can actually be useful under certain circumstances, and there is growing interest in utilizing the very nature of chaos. For example, chaos is thought to be important in fluid mixing, in the human brain and heartbeat regulation, and in secure communication and signal processing. In electrical and mechanical engineering, chaos can be utilized for reducing electromagnetic interference (EMI) and acoustic noise of switched-mode power supplies, preventing mechanical resonance and improving the efficiency of abrasive machines. It was also proved that a chaotic vibratory roller has 12.2% higher efficiency than a traditional one. Thus, a controllable chaotic motor drive, namely chaotifying a motor drive, is highly desired for practical engineering systems.

Chaotification or unticontrol of chaos, which aims at making a nonchaotic dynamical system chaotic, has attracted increasing attention of engineers in recent years, because some very desirable features of chaos provides new ways of solving many non-traditional problems. Both non-feedback methods, or called open-loop control, and feedback control methods such as OYG method and time-delay feedback [5]-[7] have been developed in power electronics for different applications like secure communication and EMI reduction. However, to the best of authors' knowledge, the investigation of chaotification of motor drives was almost absent in literature.

This paper firstly proposes and implements a time-delay feedback method to chaotify a practical permanent-magnet synchronous motor drive. Based on the current-fed model and field-oriented control, the corresponding system dynamics will be approximated by first-order differential equations. Hence, the electromechanical torque will be adjusted according to the time-delay speed feedback. Consequently, chaotic motion can be achieved by tuning the feedback gain of the torque controller. Moreover, the resulted chaotic motion is easily controllable in the sense that the boundary of the rotor speed can be controlled precisely by the value of the speed ratio. This controllable chaotic PMSM drive potentially offers some special applications desiring chaotic motion such as fluid mixing and surface grinding. Theoretical analysis, computer simulation as well as experimental results will be given to verify the proposed method of chaotification.

II. SYSTEM MODELING AND ANALYSIS

A three-phase PMSM can be modeled in $d-q$ frame by the following equations:

$$\frac{di_d}{dt} = (v_d - R_s i_d + \omega_e L_q i_q)/L_d \quad (1)$$

$$\frac{di_q}{dt} = (-R_s i_q - \omega_e L_d i_d - \omega_e \psi_m)/L_q \quad (2)$$

$$\frac{d\omega}{dt} = \left[\frac{3}{2}\frac{P}{2}[\psi_m i_q - (L_d - L_q) i_d i_q] - B\omega - T_L\right]\Big/J \quad (3)$$

where i_d, i_q and v_d, v_q are the stator currents and voltages in the d-axis and q-axis, respectively; L_d and L_q are the stator inductance in the d-axis and q-axis, respectively; R_s

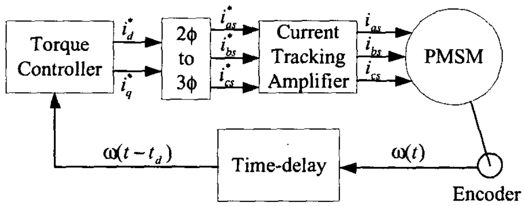


Fig. 1. Control diagram of a current-fed 3-phase PMSM.

is the stator resistance; ψ_m is the constant magnetic flux; P is the number of poles; ω_e is the electrical speed and $\omega = 2\omega_e / P$ is the mechanical rotor speed; J is the rotor inertia and B_m is the viscosity friction coefficient.

Fig. 1 shows the control diagram for chaotification of the PMSM.

In order to utilize the advantages of vector control, i_d is set to zero so that T_e can simply be expressed as:

$$T_e = \frac{3}{2}\frac{P}{2}\psi_m i_q \tag{4}$$

If the current tracking loop is sufficiently fast enough, equation (1) and (2) can be eliminated. Thus, the system dynamics can be simplified as

$$J\frac{d\omega}{dt} = T_e - B_m\omega \tag{5}$$

By choosing the feedback law as

$$T_e = \xi\mu B_m f(\omega(t-t_d)/\xi) \tag{6}$$

where $f(\cdot)$ is an integrable bounded function, t_d is the time-delay constant, ξ and μ are two adjustable positive constants, the systems dynamics can be rewritten as:

$$\frac{d\omega(t)}{dt} = -\eta\omega(t) + \xi\eta\mu f(\omega(t-t_d)/\xi) \tag{7}$$

where $\eta = B_m/J$. Dividing time t into intervals t_d, the solution of (7) can be expressed as an iterative form:

$$\begin{aligned}\omega_{n+1}(t) &= e^{-\eta t}\omega_n(t_d) \\ &+ \xi\eta\mu\int_0^t e^{-\eta(t-s)} f(\omega_n(s)/\xi)ds\end{aligned} \tag{8}$$

where $t \in [0, t_d)$. If the time-delay constant t_d is much larger than the system time constant $1/\eta$, $e^{-\eta t}$ can be well approximated by a scaled delta function $\delta(t)/\eta$. Consequently, the iterative solution (8) can be written as:

$$\omega_{n+1}(t) \approx \xi\mu f(\omega_n(t)/\xi) \tag{9}$$

Defining the normalized rotor angular speed Ω as:

$$\Omega = \frac{\omega}{\xi} \tag{10}$$

Solution (9) becomes

$$\Omega_{n+1} \approx \mu f(\Omega_n) \tag{11}$$

The criterion of chaos for this iterative map is the famous period three imply chaos theorem [8]: Let J be an interval and map $F: J \to J$ be continuous. Assume there is a point $a \in J$ for which the points $b = F(a)$, $c = F(b) = F^2(a)$ and $d = F^3(a)$ satisfy

$$d \le a < b < c (\text{or } d \ge a > b > c) \tag{12}$$

Then for every $k = 1, 2, \cdots$, there is a periodic point in J having period k.

If the function $f(\cdot)$ is defined as $f(x) = \sin(x)$, equation (11) becomes:

$$\Omega_{n+1} \approx \mu \sin(\Omega_n) \tag{13}$$

Different searching methods can be used to verify the existence of points that satisfy condition (12) for fixed sufficiently large μ. For example, $a = 0.3$ is such a point for $\mu = 3.54$. Hence the map defined by equation (13) is chaotic for sufficiently large μ.

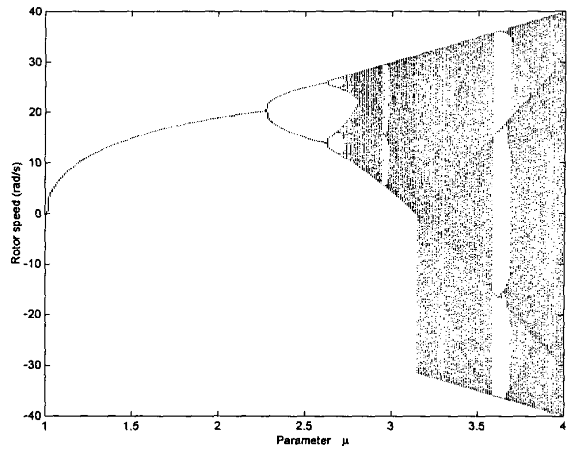


Fig. 2. Bifurcation diagram of the normalized rotor speed.

The bifurcation diagram of map (13) is plotted in Fig. 2. This figure shows that for a small value of μ ($0 < \mu \le 2.25$), map (13) will finally reach a stable fixed point. In the middle range of μ ($2.25 < \mu \le 2.69$), map (13) becomes a periodic oscillation. With the further increase of μ ($\mu > 2.69$), complicate behavior occurs since both chaos and periodic oscillations can appear with different value of μ.

Although equation (8) can be approximated by equation (9) when $\eta t_d >> 1$, it should be pointed out that the dynamics described by equation (8) and equation (9) respectively are not the same completely even though $\eta t_d >> 1$. Beyond the chaotic regime, similar bifurcations of equation (8) to those of equation (9) can be expected if $\eta t_d >> 1$. However, inside the chaotic regime, the memory effect in equation (8) makes its dynamics singular free, which differs from equation (9) [9]. In the practical PMSM drive system, this memory effect is caused by the viscosity force of the rotor. Obviously, no matter how fast the motor dynamical response is, the rotor speed can only change smoothly, which means singular free. Despite this, nevertheless, equation (9) still can be used to predict the motion tendency of equation (8) when $\eta t_d >> 1$.

III. Implementation Considerations

From the previous discussion, with the proposed time-delay controller, the close-loop PMSM drive system may either reach an equilibrium point, or oscillate periodically, or even exhibit chaotic behavior. No matter which kind of motion the system will finally demonstrate, for practical and safety consideration, the controller, first of all, should be such a one that makes the system behavior bounded.

Although there is difficulty in investigating directly from differential equation (7) or its iterative solution (8) whether the rotor speed will converge as time t goes to infinity, the rotor speed can be easily found bounded when $\eta t_d >> 1$.

From equation (9), for finite positive μ and ξ,

$$|\omega_{n+1}(t)| \approx \xi\mu|f(\omega_n(t)/\xi)| \tag{14}$$

Therefore, the rotor speed will be bounded if $f(\cdot)$ is bounded. According to the proposed controller, the stator currents will be finite as well once the rotor speed is kept finite.

On the other hand, it should be noted that the values of parameters μ and ξ are not totally chosen free. They are limited by motor torque capability. From equation (6), they should satisfy the following condition:

$$\xi\mu B_m M \leq T_{\max} \tag{15}$$

where $|f(\cdot)| \leq M$ and $T_{\max}$ is the maximum torque that the motor can generate.

For chaotic motions, although the instantaneous behavior can not be predicted, this motion is known to be bounded in certain range since chaos is a bounded random-like state, which is one attracting advantages of utilizing chaos. Thus, it is important to control the boundary of chaotic states.

To generate such chaotic motions with desired boundaries, parameters μ and ξ can be adjusted according to the normalized speed bifurcation diagram as shown in Fig. 2. The merit of the normalized speed bifurcation chart lies in the fact that it provides a base to control the boundary of the chaotic motion. Namely, in order to obtain a chaotic motion in certain required speed region, the normalized speed bifurcation diagram can be plotted firstly using the given initial condition. Then fix the bifurcation parameter μ according to the bifurcation diagram. The required chaotic motion can be achieved consequently by adjusting the speed ratio ξ. For example, with a combination of $\mu = 4$ and $\xi = 10$, a chaotic speed bounded in [−40,40]rad/s can be realized.

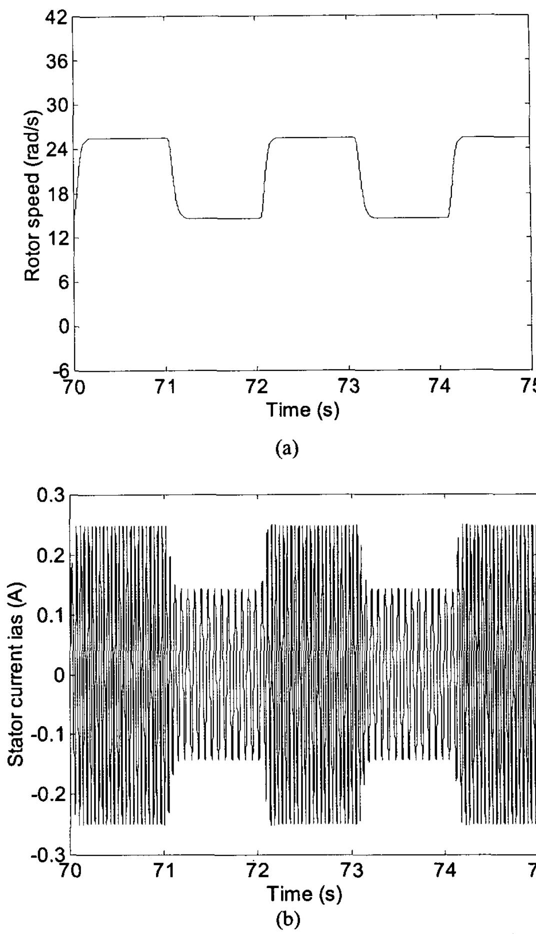
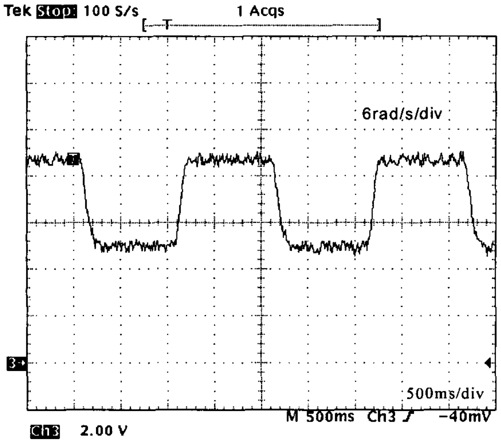


Fig. 3. Simulated periodic waveforms with $\mu = 2.55$ and $\xi = 10$: (a) ω_r; (b) i_{as}.

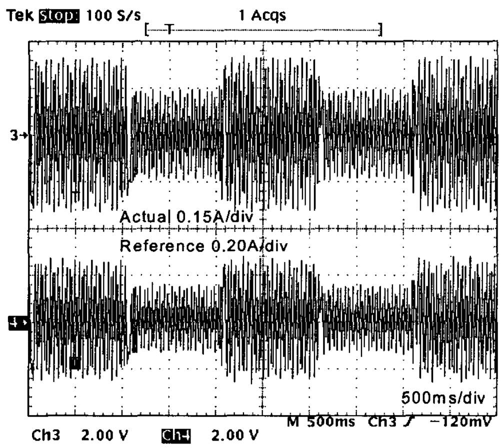
IV. Simulations and Experimental Results

A practical three-phase PMSM by Sanyo Denki with $J = 0.144\times10^{-4}\text{Kg}\cdot\text{m}^2$ and $B_m = 5.416\times10^{-4}\text{Nm/rad}\cdot\text{s}^{-1}$, $P = 8$, $L_d = 11.5\text{mH}$, $L_q = 11.5\text{mH}$ and $\psi_m = 0.0283\text{Wb}$ is used for exemplification. When the time-delay constant is chosen to be $t_d = 1\text{s}$, $\eta t_d = 37.6 >> 1$ can guarantee the validation of solution (13).

A dSPACE DS1102 DSP control board is used as the controller. In order to obtain a prototype working environment and to save code development time, MATLAB Real Time Workshop and SIMULINK are adopted as interfacing software with the DSP controller. The sampling frequency of the DSP controller is 5KHz. The time-delay is realized digitally by the DSP controller. Namely, the rotor



(a)



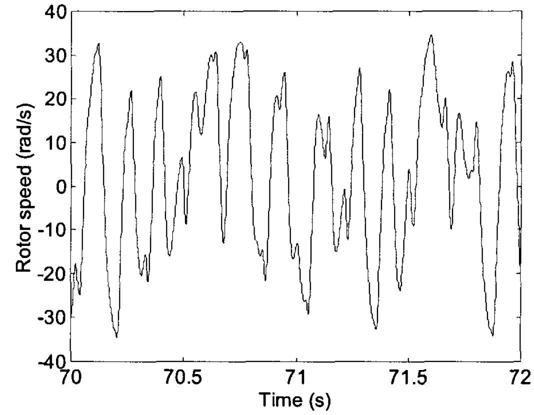
(b)

Fig. 4. Measured periodic waveforms with $\mu = 2.55$ and $\xi = 10$: (a) ω_r; (b) i_{as}.

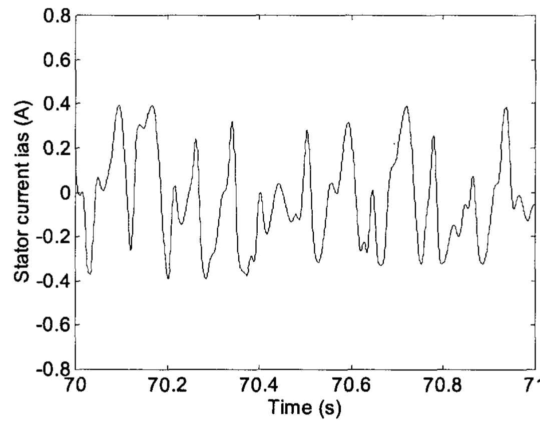
speed sampled from A/D is passed to and stored in the DSP. Then the DSP controller generates the reference current commands according to the delayed rotor speed stored in memory and passes them to the current tracking amplifier.

Based on the proposed time-delay feedback, the motor initially operates at a fixed point with a small value of μ. With an increase of μ, periodic motion occurs. With a further increase of μ, the system exhibits chaotic motion.

Fig. 3 shows the simulated speed and current waveforms under period-2 operation when $\mu = 2.55$ and $\xi = 10$, whereas Fig 4 shows the measured speed and current waveforms under the same μ and ξ. As expected, the experimental measurement closely matches with the simulation waveforms. Furthermore, this period-2 oscillation can be easily visualized in Fig. 2 with $\mu = 2.55$.



(a)



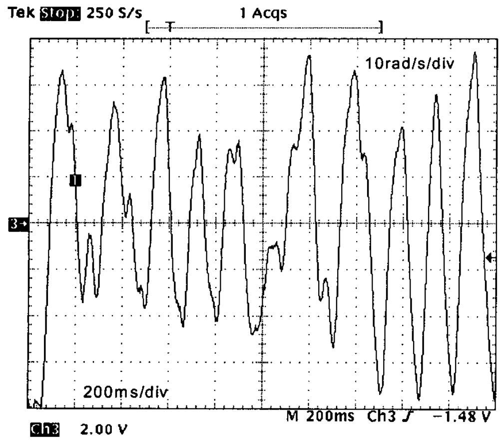
(b)

Fig. 5. Simulated chaotic waveforms with $\mu = 4$ and $\xi = 10$: (a) ω_r; (b) i_{as}.

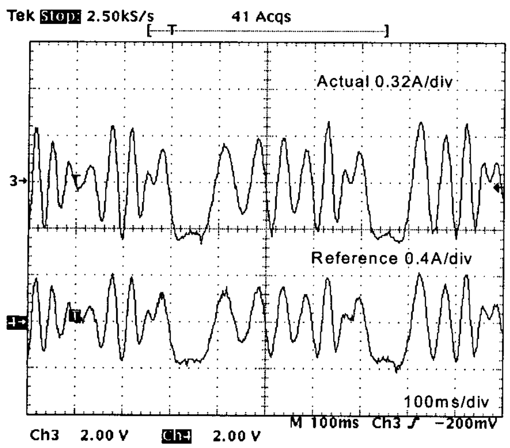
On the other hand, Fig. 5 shows the simulated chaotic speed and current waveforms when $\mu = 4$ and $\xi = 10$, while Fig 6 shows the measured chaotic speed and current waveforms with the same μ and ξ. It can be found that they are chaotic and offer similar boundaries. Notice that chaotic waveforms cannot be directly compared since they are random-like but bounded.

V. Conclusion

In this paper, a time-delay feedback method has been proposed and implemented to chaotify a practical PMSM drive. This controllable chaotic PMSM drive potentially offers some special applications desiring chaotic motion such as fluid mixing and surface grinding. Theoretical analysis, computer simulation as well as experimental measurement have been given to verify the proposed chaotification.



(a)



(b)

Fig. 6. Measured chaotic waveforms with $\mu = 4$ and $\xi = 10$: (a) ω_r; (b) i_{as}.

VI. Acknowledgment

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VII. References

[1] J. H. Chen, K. T. Chau and C. C. Chan, "Analysis of chaos in current-mode controlled dc drive systems," *IEEE Transactions on Industrial Electronics*, vol. 47, 2000, pp. 67-76.

[2] Z. Suto, I. Nagy and E. Masada, "Avoiding chaotic processes in current control of ac drive," *IEEE Power Electronics Specialists Conference*, 1998, pp. 255 -261.

[3] N. Hemati, "Strange attractors in brushless dc motors," *IEEE Transactions on Circuits and Systems - I*, vol. 41, 1994, pp. 40-45.

[4] J. H. Chen, K. T. Chau and Q. Jiang, "Analysis of chaotic behavior in switched reluctance motors using voltage PWM regulation," *Journal of Electric Power Components and Systems*, vol. 29, 2001, pp. 211-227.

[5] W. C. Y. Chan and C. K. Tse, "Studies of routes to chaos for current-programmed DC/DC converters," *Proceeding of 27th IEEE Annual Conference on Power Electronics Specialists*, 1996, vol. 1, pp. 789-795.

[6] R. S. Bueno and J. L. R. Marrero, "Application of the OGY method to the control of chaotic DC-DC converters: theory and experiments," *Proceeding of the 2000 IEEE International Symposium on Circuits and Systems*, 2000, vol. 2, pp. 369 –372.

[7] X. F. Wang, G. Q. Zhong, K. S. Tang, K. F. Man and Z. F. Liu, "Generating chaos in Chua's circuit via time-delay feedback," *IEEE Transactions on Circuits and systems I: Fundamental Theory and Applications*, vol. 48, no. 9, 2001, pp. 1151-1156.

[8] T. Y. Li and J. A. Yorke, "Period three implies chaos," The American Mathematical Monthly, vol. 82, 1975, pp. 481-485.

[9] K. Ikeda and K. Matsumoto, "High-dimensional chaotic behavior in systems with time-delayed feedback," *Physica D*, vol. 29, 1989, pp. 223-235.