The Effect of Branch Correlation in Dual MRC, SC and SWC Diversity Systems for Noncoherent MFSK over Nakagami-*m* Fading Channels

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Abstract

This paper presents closed form expressions for the average symbol error rate (SER) of noncoherent M-ary frequency shift keying (MFSK) with dual branch maximal ratio combining (MRC), selection combining (SC) and switched combining (SWC) space diversity receptions over correlated Nakagami-m fading channels. Numerical results demonstrate the error performance degradation due to the effects of correlated branches in the three diversity combining systems. They also show that there is similar performance degradation in different diversity combining systems when correlation coefficient ρ *is increased from 0 to 0.5. Furthermore, the error performance of employing MRC and SC diversity receptions are very close to each other when both the branch correlation and channel fading are severe.*

1. Introduction

Over the years, compensation techniques for multipath channel fading in wireless communications have attracted much attention (see, e.g., [1, 2] and references therein). Diversity combining, which skillfully combines multiple replicas of the received signals, has long been recognized as one of the effective compensation techniques for combating detrimental effects of channel fading. Three of the well-known methods to combine these multipath components are MRC, SC and SWC [1]. It is well known that the improvement on error performance of any space diversity combining system will be increased when the number of diversity branches is increased. However, a high number of diversity branches is not feasible for most practical systems, because of the limited space available. In addition, the marginal improvement of an additional branch in a diversity system is inversely proportional to the number of existing branches in the diversity system. Therefore, dual branch diversity combining systems are the most popular and important among all. When dual branches are employed, correlation between them is important because the two antennae are closely spaced in practical systems. Thus, it is of interest to study the effects of branch correlation on error performance of employing these three diversity combining systems.

Previous work [3] has studied the error performance of binary phase shift keying (PSK) and frequency shift keying (FSK) with dual MRC diversity system in a correlated Nakagami-*m* fading environment. In [4], extension to *L*-fold MRC diversity system was considered. In addition, [5] has also investigated the error performance of binary PSK and FSK with SC diversity systems in correlated Nakagami-*m* fading channels. Furthermore, [6] has evaluated the average BER of noncoheret FSK with dual switched diversity system in correlated Nakagami-*m* fading channels and [7] has extended the work to consider the case of *M*-ary Differential PSK. This paper encompasses previous works to consider the case of noncoherent MFSK with the three dual diversity combining systems over Nakagami-*m* fading channels, and compares the differences on performance degradations due to the effects of branch correlation.

This paper is organized as follows. Section II will describe some background of the diversity combining systems. Sections III-V will present the performance analyses of employing MRC, SC and SWC, respectively. Section VI provides numerical results and discussion. Finally, Section VII gives some concluding remarks.

2. Background

In consideration of a communication system where the mobile communication fading channel is modeled as

Nakagami-m distribution, the probability density function (pdf) of the instantaneous received signal-to-noise ratio (SNR) per symbol λ_i on the ith branch in a receiver with dual diversity combining is given by [8]

$$p(\lambda_i) = \left(\frac{m}{\Omega}\right)^m \frac{\lambda_i^{m-1}}{\Gamma(m)} \exp\left(-\frac{m}{\Omega}\lambda_i\right) \tag{1}$$

where m is the fading severity parameter with values from 0.5 to ∞, Ω is the average SNR per symbol per branch and $\Gamma(.)$ denotes the gamma function [9]. Note that the two branches are assumed to have the same values of m and Ω. With the assumption of equal noise power in both branches, the instantaneous SNR per symbol γ at the output of the receiver can be obtained and will be discussed in the following sections. With these channel statistics, the average SER at the output of a receiver can be calculated by averaging the conditional probability of error in the presence of additive white gaussian noise (AWGN) over the pdf of γ, i.e.,

$$\overline{P} = \int_0^\infty P_e(\gamma) p(\gamma) d\gamma \tag{2}$$

where $P_e(\gamma)$ is the conditional probability of symbol error for a particular communications system on the assumption that γ is known. With the increasing use of M-ary orthogonal signaling with noncoherent reception in current commercial and military systems, the evaluation of its error performance is thus important. The well-known conditional SER for noncoherent MFSK is given by [10]

$$P_{e,MFSK} = \frac{1}{M}\sum_{i=2}^{M}(-1)^i \binom{M}{i} \exp\left\{-\left(1-\frac{1}{i}\right)\gamma\right\} . \tag{3}$$

3. Error Performance of MRC

MRC is known as the optimal combining technique by maximizing the SNR of the combined signal at the output of a receiver. With the use of dual MRC diversity reception for communication systems in correlated Nakagami-m fading channel, the pdf of the instantaneous SNR per symbol γ of the combined signal is given by [4]

$$p_{MRC}(\gamma) = \frac{\left(\frac{m\gamma}{\Omega}\right)^{2m-1} \exp\left(-\frac{m\gamma}{\Omega(1-\rho)}\right) {}_1F_1\left(m; 2m; \frac{2m\rho\gamma}{\Omega(1-\rho)(1+\rho)}\right)}{\left(\frac{\Omega}{m}\right)(1-\rho^2)^m \Gamma(2m)} \tag{4}$$

where ρ is the correlation coefficient and ${}_1F_1(a;b;x)$ is the confluent hypergeometric function [9]. Note that constant correlation model is assumed. After substitutions of (4) and (3) into (2), the average SER of noncoherent MFSK with dual branch MRC diversity system in correlated Nakagami-m fading channel can be written as

$$\overline{P}_{e,MRC} = \frac{1}{M}\sum_{i=2}^{M}(-1)^i \binom{M}{i} \left(\frac{m}{\Omega}\right)^{2m} \frac{(1-\rho^2)^{-m}}{\Gamma(2m)} \times \int_0^\infty \gamma^{2m-1} \exp\left\{-\left(1-\frac{1}{i}+\frac{m}{\Omega(1-\rho)}\right)\gamma\right\} {}_1F_1\left(m; 2m; \frac{2m\rho\gamma}{\Omega(1-\rho^2)}\right) d\gamma \tag{5}$$

$$\overline{P}_{e,MRC} = \frac{1}{M}\sum_{i=2}^{M}(-1)^i \binom{M}{i} \left(\frac{m}{\Omega}\right)^{2m} (1-\rho^2)^{-m} \times \left(1-\frac{1}{i}+\frac{m}{\Omega(1-\rho)}\right)^{-2m} \left(1-\frac{2m\rho}{(1+\rho)[m+\Omega(1-\rho)(1-1/i)]}\right) \tag{6}$$

and finally, it can be simplified to

$$\overline{P}_{e,MRC} = \sum_{i=2}^{M} \frac{\frac{(-1)^i}{M}\binom{M}{i}}{[(1+(\frac{\Omega}{m})(1+\rho)(1-\frac{1}{i}))(1+(\frac{\Omega}{m})(1-\rho)(1-\frac{1}{i}))]^m} \tag{7}$$

4. Error Performance of SC

SC method of combining multipath components is to select the diversity branch with the largest SNR. The main advantages of SC over MRC are relative simplicity and lower cost. Consider a receiver implementing dual SC diversity reception in correlated Nakagami-m fading channel, the pdf of the instantaneous SNR per symbol γ of the combined signal at the output of the receiver is given by [5]

$$p_{SC}(\gamma) = \sum_{k=0}^{\infty}\sum_{r_1=0}^{k}\sum_{r_2=0}^{k}\sum_{n=1}^{2} \frac{\Delta(k, r_1, r_2)\gamma^{m+r_n-1}}{(\Omega/m)^{m+r_n}} \times \exp\left(-\frac{\gamma}{\Omega/m}\right) \prod_{j=1, j\neq n}^{2} \gamma\left(m+r_j, \frac{\gamma}{\Omega/m}\right) \tag{8}$$

where $\gamma(a, b)$ is the first incomplete gamma function [9] and

$$\Delta(k,r_1,r_2)=\binom{k}{r_1}\binom{k}{r_2}\frac{(-1)^{r_1+r_2}(m)_k\rho^{2k}}{k!\Gamma(m+r_1)\Gamma(m+r_2)} \quad (9)$$

where $(a)_b$ is the Pochhammer's symbol [9]. With substitutions of (8) and (3) into (2), the average SER of noncoherent MFSK with dual branch SC diversity system in correlated Nakagami-m fading channel can be calculated by

$$\overline{P}_{e,SC}=\frac{1}{M}\sum_{i=2}^{M}(-1)^i\binom{M}{i}\sum_{k=0}^{\infty}\sum_{r_1=0}^{k}\sum_{r_2=0}^{k}\Delta(k,r_1,r_2)\left\{\int_0^\infty\frac{\gamma^{m+r_1-1}}{(\Omega/m)^{m+r_1}}\times\exp\left[-\left(1-\frac{1}{i}+\frac{m}{\Omega}\right)\gamma\right]\gamma\left(m+r_2,\frac{m\gamma}{\Omega}\right)d\gamma+\int_0^\infty\frac{\gamma^{m+r_2-1}}{(\Omega/m)^{m+r_2}}\exp\left[-\left(1-\frac{1}{i}+\frac{m}{\Omega}\right)\gamma\right]\gamma\left(m+r_1,\frac{m\gamma}{\Omega}\right)d\gamma\right\} \quad (10)$$

Using a tabulated relation in [11], (10) becomes

$$\overline{P}_{e,SC}=\frac{1}{M}\sum_{i=2}^{M}\sum_{k=0}^{\infty}\sum_{r_1=0}^{k}\sum_{r_2=0}^{k}(-1)^i\binom{M}{i}\frac{\Delta(k,r_1,r_2)\Gamma(2m+r_1+r_2)}{(2+(\Omega/m)(1-1/i))^{2m+r_1+r_2}}\times\left\{\frac{1}{m+r_2}{}_2F_1\left(\begin{matrix}1,2m+r_1+r_2\\ m+r_2+1\end{matrix};\frac{1}{2+(\Omega/m)(1-1/i)}\right)+\frac{1}{m+r_1}{}_2F_1\left(\begin{matrix}1,2m+r_1+r_2\\ m+r_1+1\end{matrix};\frac{1}{2+(\Omega/m)(1-1/i)}\right)\right\} \quad (11)$$

where ${}_2F_1(a,b;c;x)$ is the Gauss hypergeometric function [9].

5. Error Performance of SWC

In the above two combining methods, different amounts of knowledge of all the received branch signals in one form or another are required. A simpler and cheaper though less efficient combining technique is SWC, which can be viewed as a suboptimum implementation of SC. One of the switching strategies is used here [6]. Note that one switches the branch only at discrete instant of time. If branch 1 is being used at t_{i-1}, one switches to branch 2 if a local power measurement for branch 1 at t_i is below a threshold value ξ, regardless of the local power measurement for branch 2 at t_i. Switching from branch 2 to branch 1 is performed similarly. Note that the rate of branch switching is reduced with respect to that of SC and thus a reduction of transient effects due to switching. When a receiver implements dual SWC diversity reception in correlated Nakagami-m fading channel, the pdf of the instantaneous SNR per symbol γ of the combined signal at the output of the receiver is given by [6]

$$p_{SWC}(\gamma)=\begin{cases}A(\gamma), & \gamma\le\xi\\ \dfrac{m}{\Omega\,\Gamma(m)}\left(\dfrac{m\gamma}{\Omega}\right)^{m-1}\exp\left(-\dfrac{m\gamma}{\Omega}\right)+A(\gamma), & \gamma>\xi\end{cases} \quad (12)$$

$$A(\gamma)=\int_0^\xi\frac{m^{m+1}\gamma^{(m-1)/2}y^{(m-1)/2}\exp[-m(\gamma+y)/\Omega(1-\rho^2)]}{\Gamma(m)\,\Omega^{m+1}\rho^{m-1}(1-\rho^2)}\times I_{m-1}\left(\frac{2m\rho}{\Omega(1-\rho^2)}\sqrt{y\gamma}\right)dy \quad (13)$$

where $I_v(a)$ is the modified Bessel function of the first kind of order v [9]. With substitutions of (12) and (3) into (2), the average SER of noncoherent MFSK with dual branch SWC diversity system in correlated Nakagami-m fading channel can be evaluated by

$$\overline{P}_{e,SWC}=\frac{1}{M}\sum_{i=2}^{M}(-1)^i\binom{M}{i}\left\{\int_0^\infty\exp\left\{-\left(1-\frac{1}{i}\right)\gamma\right\}A(\gamma)d\gamma+\left(\frac{m}{\Omega}\right)^m\int_\xi^\infty\frac{\exp[-(1-1/i+m/\Omega)\gamma]\gamma^{m-1}}{\Gamma(m)}d\gamma\right\} \quad (14)$$

Let the first integral in (14) be I_i. After further simplifications, I_i can be written as

$$I_i=\frac{1}{\Gamma(m)(1+(1-1/i)(\Omega/m))^m}\gamma\left[m,\frac{m\xi}{\Omega}\left(\frac{1+(1-1/i)(\Omega/m)}{1+(1-1/i)(\Omega/m)(1-\rho^2)}\right)\right] \quad (15)$$

and thus the average SER can finally be expressed as

$$\overline{P}_{e,SWC}=\frac{1}{M}\sum_{i=2}^{M}(-1)^i\binom{M}{i}\frac{1}{\Gamma(m)(1+(1-1/i)(\Omega/m))^m}\times\left\{\gamma\left[m,\frac{m\xi}{\Omega}\left(\frac{1+(1-1/i)(\Omega/m)}{1+(1-1/i)(\Omega/m)(1-\rho^2)}\right)\right]+\Gamma\left[m,\frac{m\xi}{\Omega}\left(1+(1-1/i)(\Omega/m)\right)\right]\right\} \quad (16)$$

where $\Gamma(a,b)$ is the second incomplete gamma function [9]. The optimum threshold level, say ξ_o, in a minimum probability of error sense is usually determined by employing standard differentiation technique on (16).

There exists an optimum threshold ξ_o for every average SNR level and thus, at any average SNR, the minimum average SER could be achieved only by adapting ξ_o to the actual value of the average SNR on each branch. However, it would be difficult for any practical system, thus fixed switching threshold is employed. One of the fixed threshold strategies [7] is used in this paper. It operates the switching with the fixed threshold $\xi_R=\xi_o(\Omega_R)$, that is, the optimum one in correspondence of the average SNR $\Omega_R=(\Omega_a+\Omega_b)/2$, where (Ω_a, Ω_b) is the average SNR range of interest.

6. Numerical Results and Discussion

By using the closed form expressions in (7), (11) and (16), the numerical values for the average SER of noncoherent MFSK with the three dual branch diversity combining systems are plotted in figures 1 to 6. Note that the average SER of noncoherent MFSK with no diversity (only one branch) is also plotted in all the figures. Figures 1 to 3 show the average SER of noncoherent MFSK with dual branch MRC, SC and SWC diversity receptions in correlated Nakagami-*m* fading channel of ρ=0,0.5,0.9 and m=1 for M=4,8,16, respectively. The corresponding numerical values with m=5 are shown in figures 4 to 6.

As for the case of M=4 and m=1 in figure 1, the required SNR per symbol for an average SER of 10^{-3} are 18dB, 18.8dB and 21.9dB for MRC with branch correlation ρ of 0, 0.5 and 0.9, respectively. The corresponding values for SC and SWC are 19.5dB, 20.2dB, 22dB and 21.95dB, 22.3dB, 25.8dB, respectively. The required average SNR per symbol is 33dB when no diversity is employed. As expected, more performance improvement can be achieved when MRC is employed, while SWC would render less performance improvement for the same number of branches and same correlation coefficients. There is only slight performance degradation when ρ is increased from 0 to 0.5, but the degradation is more severe when ρ is increased from 0.5 to 0.9. This phenomenon can be observed in all the three diversity combining systems. In addition, the amount of performance degradation is similar for the three diversity combining systems (except the divergence part of SWC curves) at any particular average SER below 10^{-2} when ρ is increased from 0 to 0.5 and from 0.5 to 0.9. Note that the divergence of the performance curves for the SWC systems are probably due to the use of fixed threshold level instead of the optimum threshold level. Although, diversity systems employing MRC perform better than those of SC under the same channel condition, the error performance of employing both combining methods are similar for ρ=0.9. Therefore, dual diversity systems using MRC can be replaced by using SC when the branch correlation is high. Furthermore, the error performance improvement of employing SWC with ρ=0 is close to that of employing MRC and SC with ρ=0.9 (except the divergence part of the error performance curves for SWC). Although there is performance degradation when the correlation is high, significant performance improvement can still be obtained for all diversity combining systems. Similar conclusions can be drawn from figures 2 and 3.

Figures 4 to 6 show the case of m=5 and some more observations. By comparing figures 1 and 4, less performance improvement is observed for all the three diversity combining systems when m=5. The reason is that in a less severe fading channel with no diversity reception, the system can already perform quite well on its own. Hence, not much improvement can be done with the use of diversity combining systems. Contrary to the case of m=1, the error performance improvement of using SC is significantly worse than that of using MRC even when ρ is high. Thus, MRC has an absolute advantage over SC when the channel experiences less severe fading. Also, the error performance of diversity systems using SWC are so close to the case of no diversity, it surrenders the usefulness of SWC diversity systems even when ρ is low. However, considerable performance improvement can still be achieved by employing MRC and SC even when ρ is high.

5. Conclusions

This paper derived some closed form expressions for the average SER of noncoherent MFSK with dual branch MRC, SC and SWC diversity systems in correlated Nakagami-*m* fading channels. Comparisons of the effects of branch correlation on the three diversity combining systems have been studied. It shows that SC can be used in place of MRC in severe fading channel when the branch correlation is high. Results also show that significant performance improvement can be achieved for most of the considered cases even when the branch correlation is high. Furthermore, diversity combining systems are more valuable for communications systems in severe fading channels than in good channels.

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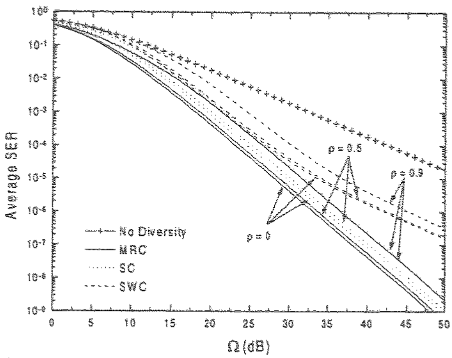

Figure 1. Average SER (versus Ω) of noncoherent *M*=4 FSK with dual MRC, SC and SWC diversity receptions in correlated Nakagami-*m* fading channels with *m*=1 and *ρ*=0,0.5,0.9.

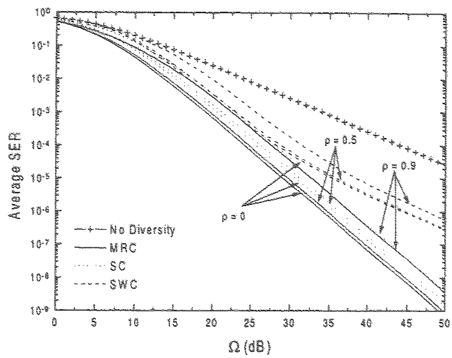

Figure 2. Average SER (versus Ω) of noncoherent *M*=8 FSK with dual MRC, SC and SWC diversity receptions in correlated Nakagami-*m* fading channels with *m*=1 and *ρ*=0,0.5,0.9.

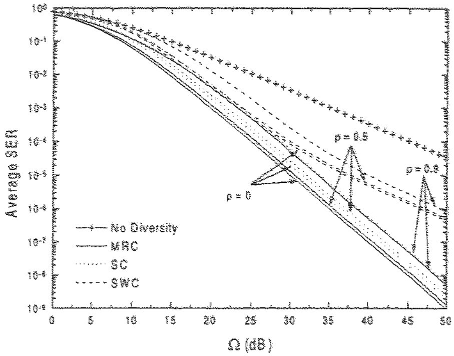

Figure 3. Average SER (versus Ω) of noncoherent *M*=16 FSK with dual MRC, SC and SWC diversity receptions in correlated Nakagami-*m* fading channels with *m*=1 and *ρ*=0,0.5,0.9.

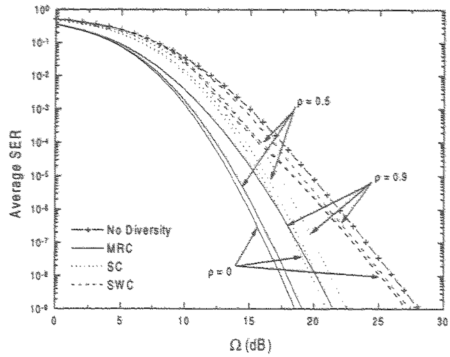

Figure 4. Average SER (versus Ω) of noncoherent *M*=4 FSK with dual MRC, SC and SWC diversity receptions in correlated Nakagami-*m* fading channels with *m*=5 and *ρ*=0,0.5,0.9.

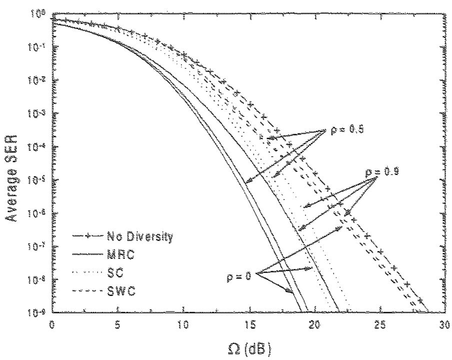

Figure 5. Average SER (versus Ω) of noncoherent *M*=8 FSK with dual MRC, SC and SWC diversity receptions in correlated Nakagami-*m* fading channels with *m*=5 and *ρ*=0,0.5,0.9.

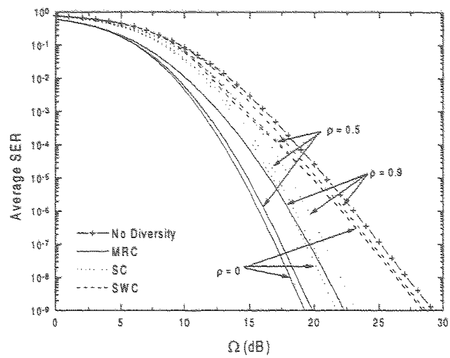

Figure 6. Average SER (versus Ω) of noncoherent *M*=16 FSK with dual MRC, SC and SWC diversity receptions in correlated Nakagami-*m* fading channels with *m*=5 and *ρ*=0,0.5,0.9.