IIR Notch Filtering - Comparisons of Four Adaptive Algorithms for Frequency Estimation

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Abstract: This paper compares the parameter estimation accuracy of four adaptive algorithms for frequency estimation when applied to an IIR digital notch filter. All four algorithms were subjected to the same experimental conditions and the variance of parameter estimates are compared to the Cramer Rao Lower Bound. Results show that the RML yielded the most accurate parameter estimates although its computational burden is quite high. The AML produced good parameter estimates and it has the advantages of proven convergence properties as well as lower computational burden over the RML. For applications where the signal to noise ratio is moderate it is shown that the AGB algorithm may be suitable, particularly where minimal computational burden is desired.

1. Introduction

Adaptive Infinite Impulse Response (IIR) notch filtering has received considerable attention in recent times by various researchers [1-10]. An issue that requires consideration in all adaptive notch filtering applications is to evaluate and select an appropriate adaptive algorithm. One of the problems which arises when examining previously published work is that different authors have used different algorithms with notch filtering under different conditions and so it is difficult to ascertain which of these algorithms yields the best estimation accuracy.

In an attempt to resolve this issue four adaptive algorithms, including: the Stochastic Gauss Newton (SGN) [4,8,9,10], the Recursive Maximum Likelihood (RML) [4,9,10], the Approximate Maximum Likelihood (AML) [4], and the Approximate Gradient-Based (AGB) [1] algorithm were simulated under the same conditions. These four algorithms are applied to a notch filter parametization [1]. The parameter estimates obtained from the various algorithms are compared with the Cramer Rao Lower Bound (CRLB). Note that the SGN algorithm and the RML algorithm are quite similar. In fact, they are both of recursive maximum likelihood generic form except that the SGN algorithm uses a prediction error in its parameter update, while the RML uses a residual error sequence. Various researchers [8,9] in the area of IIR notch filtering have used the above nomenclature to differentiate between the two methods. The simulation results are summarized in this paper and it is concluded that the RML algorithm yields the best results. It was also found that although the AML is not statistically efficient, its performance is very good. This is particularly interesting in view of the fact that the AML is the least computationally expensive and is the only one that does not require monitoring for convergence among the first three algorithms considered. On the question of the estimate bias [5,10], it was found from extensive Monte Carlo simulations that it is minimal especially as the notch filter approached ideal characteristics.

2. The Modified Notch Filter.

The modified notch filter parametization [1] is expressed as follows:

$$H(z^{-1}) = \prod_{i=1}^{m} \frac{1 + \beta a_i z^{-1} + \beta^2 z^{-2}}{1 + \alpha a_i z^{-1} + \alpha^2 z^{-2}} \quad (2.1)$$

where $a_i = -2\text{Cos}(\omega_i)$ for $-\pi \le \omega_i \le \pi$, while $0 \le \beta \le 1$, $0 \le \alpha < \beta \le 1$. Equation (2.1) can be expressed in polynomial form and is given by:

$$H(z^{-1}) = \frac{\sum_{i=0}^{2m} d_i \beta^i z^{-i}}{\sum_{i=0}^{2m} d_i \alpha^i z^{-i}} \quad (2.2)$$

where $d_0 = d_{2m} = 1$ and $d_{2m-i} = d_i$ for $i = 1,\cdots, m-1$. We consider the case where the input y(t) to the notch filter consists of m sinusoids embedded in noise, i.e., $y(t) = u(t)+n(t)$, the output can be expressed as:

$$v(t) = H(z^{-1})y(t) \quad (2.3)$$

where $u(t) = \sum_{i=1}^{m} C_i \sin(\omega_i t + \phi_i)$ describes the sum of m sinusoids with magnitude C_i, frequency ω_i and phase ϕ_i. The term n(t) represents a zero mean white Gaussian noise signal with variance σ^2. When an input sequence {y(t)} is applied to $H(z^{-1})$ then the output sequence {v(t)} is measured through direct computation using Equation (2.3).

3. Filter Parameter Estimation

The algorithms considered are:

(a) the Stochastic Gauss-Newton algorithm [4,8,9,10],
(b) the Recursive Maximum Likelihood algorithm [4,9,10],
(c) the Approximate Maximum Likelihood algorithm [4],
(d) the Approximate Gradient-Based Algorithms [1].

Although the above algorithms have been proposed and used by various authors [1,4,8,9], it is difficult to properly evaluate their results since the algorithms were implemented under different conditions. The aim, therefore, is to apply the SGN, the RML, the AML and the AGB algorithms to the modified notch filter parametization as defined by Equation (2.2) under identical signal and noise conditions. The derivation of these adaptive algorithms are based on a polynomial notch filter model while the Cramer-Rao Lower Bound derived in [2] is for a cascaded notch filter model application. This means that for a 2nd order notch filter model, both the CRLB and simulation results are consistent and can therefore be compared. Due to page limits, details of the four algorithms will not be given here and readers are referred to [1,4,8,9].

4. Simulation Results

In this section, results are presented which compare the estimation accuracy obtained from experimental Monte Carlo simulations and the approximate CRLB for finite N derived in [2]. All four algorithms, the SGN, the RML, the AML and the AGB were implemented and executed under the same conditions. It is expected that the frequency estimation accuracy for a single sinusoid in additive white Gaussian noise will approach the CRLB, provided the adaptive algorithm is statistically efficient and the number of data samples is reasonably large.

Example 1. The first case considered is for a single sinusoid in white Gaussian noise where the signal-to-noise ratio is 12dB. The results are shown in Table 1 for various data lengths; that is, N = 128, 500, 1000, 2000. Each result is based on one hundred independent experiments.

Experimental variance was obtained by considering the last estimate of each of the one hundred different experiments. It is clear that the RML algorithm is at least one order of magnitude better, in terms of parameter estimation accuracy, than the other two algorithms. The AML algorithm, despite being statistically inefficient, performed extremely well, and on the whole somewhat better than the SGN algorithm. The AGB algorithm clearly provided the lowest parameter estimation accuracy, although it is still quite acceptable.

In general, the parameter estimation accuracy obtained from Monte Carlo simulations does indeed approach the CRLB for the statistical efficient RML algorithm. The validity of the CRLB results were also verified by considering the case where N tends to infinity. This was achieved by considering the variance of the last one thousand parameter estimates in one hundred experiments of two thousand samples each. With the same signal conditions as given in Table 1, the experimental variance using the RML algorithm was found to be $3.220*10^{-9}$. This result is smaller than for the case indicated in Table 1 where N = 2000 (i.e., var $(\hat{\theta})$ = $8.470 *10^{-9}$) and is clearly approaching the derived CRLB which is calculated to be equal to $6.329* 10^{-10}$.

Regarding the bias associated with adaptive notch filter parameter estimates, it is noted from the analysis of [10], that for a single sinusoid case the order of the bias is proportional to$(\beta-\alpha)^5$, while for the more general case, it is shown [5] that the bias is between $(\beta-\alpha)^2$ to $(\beta-\alpha)^3$.

In this example the expected order of the bias is 10^{-5} which, in fact, agrees very well with the results obtained and shown in Table 1. The actual parameter θ is zero (i.e., the sinusoidal frequency is 0.25 Hz), and therefore, $\hat{\theta}_{ave}$ represents the estimate bias.

The next case considered is where the signal-to-noise ratio is reduced to 0dB. The corresponding results are shown in Table 2. Once again all algorithms performed quite well, however, their performance was significantly inferior compared with the expected CRLB. In particular, their bias tend to increase.

Table 3 shows the computational burden associated with each of the four algorithms for single iteration of an adaptive second order notch filter. Clearly the AGB algorithm is by far the least computationally expensive while the RML has the highest computational burden. Note that any additional computational overhead associated with the possible need for convergence monitoring has not been included in the results shown in Table 3.

In conclusion it can be stated that, the three algorithms converged in all simulation trials and under different signal-to-noise ratio conditions for the single sinusoid in noise case.

Example 2: In this example the input y(t) is given by:

$$y(t) = 3\sin[2\pi(0.125)t]+\sin[2\pi(0.35)t]+ 2\sin[2\pi(0.2)t] +n(t) \qquad (4.1)$$

where again n(t) is a white noise sequence with zero mean and unity variance. The following initial conditions were used: P(0) = 100, initial parameter estimates were $f_1 = 0$, $f_2 = 0.15$, $f_3 = 0.28$, and for the time-varying forgetting factor, $\lambda_0 = 0.99$ and $\lambda(0) = 0.95$. In addition, $\beta = 1$ while α was made time-varying according to the following expression:

$$\alpha(t) = \alpha_0\, \alpha(t-1) + (1-\alpha_0- \xi) \qquad (4.2)$$

where $\alpha_0 = 0.99$, $\alpha(0) = 0.8$ and ξ is a very small number. The time-varying pole position [9] improved the accuracy of the estimated frequencies. This is due to the fact that, initially, the notch filter has a large bandwidth and as the adaptive algorithm converges to the desired frequency, the bandwidth reduces resulting in a very sharp notch filter. It also assumes that the number of sinusoids are known a priori.

All four algorithms were implemented and the parameter estimation accuracy performance was consistent with the previous example when convergence occurred. The simulation results for the AML algorithm (with an input signal as per Equation (4.1) are shown in Figure 1 and are very similar to parameter trajectories obtained using the SGN, the RML and the AGB. Clearly, all three frequencies have been identified and eliminated by the notch filter. In this case the choice of initial estimates was not found to be critical since each of the cascaded modules converged to one of three local minima associated with one of the sinusoids present in the input signal.

It was found that for situations involving multiple sinusoids in noise where low signal-to-noise ratios around 0dB were used, both the SGN, the RML and the AGB became unstable or did not converge. This is consistent with observations reported by previous researchers [7,4]. The AML, on the other hand, converged in all of the simulations and proved to be the most robust of all four algorithms. Further, although the AML is not asymptotically efficient in the statistical sense, its performance was found to be very good. The fact that it is the simplest of the three non-gradient based algorithms, as far as implementation is concerned, is an added bonus.

5. Conclusion

In this paper some performance aspects of the adaptive notch filter were considered. Four adaptive estimation algorithms (SGN, RML, AML and AGB) which had been previously proposed and implemented with notch filtering were simulated under the same conditions. Their performance was evaluated for the modified digital notch filter parametization. It can be concluded that the RML provided better results when convergence occurred, however, this algorithm is also more computationally expensive and requires stability monitoring. The SGN algorithm was found to be slightly worse than the RML and it too suffered from the non-convergence problem in low SNR multiple sinusoid cases. The AML is computationally simpler and converged in all simulations, and it also requires no stability monitoring. In brief, the AML performed very well and its estimates did not appear to be significantly biased. For situations where the computational burden is of primary importance and the signal to noise ratios are moderate then the AGB algorithm provides a very worthwhile solution.

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7 References

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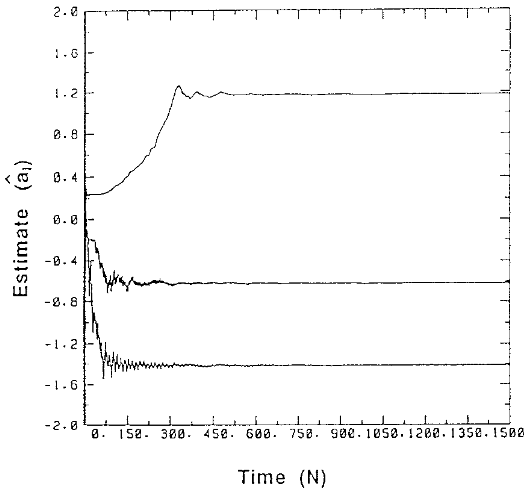

Figure 1 Parameter Estimate $(\hat{a}_i)$ Trajectory Versus Time (N). Actual Parameter Values are: a_1 = -1.414, a_2 = -0.618, a_3 = 1.176.

Alg	N	$\hat{\theta}$ave	Variance $(\hat{\theta})$	CRLB
SGN	128	$-2.356*10^{-3}$	$8.318*10^{-4}$	$1.545*10^{-7}$
RML	128	$-1.447*10^{-3}$	$6.263*10^{-6}$	$1.545*10^{-7}$
AML	128	$9.093*10^{-3}$	$2.099*10^{-4}$	$1.545*10^{-7}$
AGB	128	$1.241*10^{-3}$	$2.282*10^{-4}$	$1.545*10^{-7}$
SGN	500	$-8.722*10^{-4}$	$2.516*10^{-5}$	$1.013*10^{-8}$
RML	500	$-1.005*10^{-3}$	$1.810*10^{-7}$	$1.013*10^{-8}$
AML	500	$5.604*10^{-4}$	$4.918*10^{-6}$	$1.013*10^{-8}$
AGB	500	$2.959*10^{-4}$	$1.425*10^{-4}$	$1.013*10^{-8}$
SGN	1000	$-5.112*10^{-4}$	$8.131*10^{-6}$	$2.532*10^{-9}$
RML	1000	$-7.866*10^{-4}$	$3.647*10^{-8}$	$2.532*10^{-9}$
AML	1000	$1.061*10^{-4}$	$9.616*10^{-7}$	$2.532*10^{-9}$
AGB	1000	$-9.445*10^{-5}$	$8.229*10^{-6}$	$2.532*10^{-9}$
SGN	2000	$1.260*10^{-4}$	$4.396*10^{-7}$	$6.329*10^{-10}$
RML	2000	$-7.551*10^{-4}$	$8.470*10^{-9}$	$6.329*10^{-10}$
AML	2000	$2.871*10^{-5}$	$1.924*10^{-7}$	$6.329*10^{-10}$
AGB	2000	$-6.556*10^{-5}$	$1.757(10^{-6}$	$6.329(10^{-10}$

Table 1 Statistical results for a single sinusoid (f=0.25Hz and A=4) with n(t)~G(0,1) and α =0.9, β = 1.0, λ_0 =0.99, $\lambda(0)$ = 0.95 and P(0) = 100 and μ=0.001. Each result is based on one hundred independent experiments with initial estimate being 0.1 Hz. Actual Value is θ = 0.0.

Algorithm	Mult/Div	Add/Sub
RML	37	32
SGN	31	23
AML	30	22
AGB	15	13

Table 3 Computational burden associated with the RML, SGN, AML and AGB algorithms for a single iteration of a second order adaptive notch filter

Alg	N	$\hat{\theta}$ave	Variance $(\hat{\theta})$	CRLB
SGN	128	$-1.013*10^{-2}$	$5.905*10^{-3}$	$2.343*10^{-6}$
RML	128	$-7.326*10^{-2}$	$2.298*10^{-2}$	$2.343*10^{-6}$
AML	128	$1.003*10^{-2}$	$3.245*10^{-3}$	$2.343*10^{-6}$
AGB	128	$6.994*10^{-2}$	$1.247*10^{-2}$	$2.343*10^{-6}$
SGN	500	$-4.128*10^{-3}$	$1.780*10^{-3}$	$1.536*10^{-7}$
RML	500	$-1.295*10^{-2}$	$7.688*10^{-6}$	$1.536*10^{-7}$
AML	500	$3.105*10^{-3}$	$1.682*10^{-4}$	$1.536*10^{-7}$
AGB	500	$6.335*10^{-2}$	$5.373*10^{-3}$	$1.536*10^{-7}$
SGN	1000	$-1.113*10^{-3}$	$1.566*10^{-4}$	$3.838*10^{-8}$
RML	1000	$-1.249*10^{-2}$	$2.472*10^{-6}$	$3.838*10^{-8}$
AML	1000	$-7.539*10^{-4}$	$2.799*10^{-5}$	$3.838*10^{-8}$
AGB	1000	$6.004*10^{-2}$	$1.007*10^{-3}$	$3.383*10^{-8}$
SGN	2000	$-2.667*10^{-4}$	$4.077*10^{-6}$	$9.597*10^{-9}$
RML	2000	$-1.251*10^{-2}$	$1.067*10^{-6}$	$9.597*10^{-9}$
AML	2000	$-5.853*10^{-4}$	$1.068*10^{-5}$	$9.597*10^{-9}$
AGB	2000	$5.199*10^{-2}$	$1.352*10^{-4}$	$9.597*10^{-9}$

Table 2 Statistical results for a single sinusoid (f=0.25Hz and A=1) with n(t)~G(0,1) and α =0.9, β = 1.0, λ_0 =0.99, $\lambda(0)$ = 0.95 and P(0) = 100 and μ=0.001. Each result is based on one hundred independent experiments with initial estimate being 0.1 Hz. Actual Value is θ = 0.0.