A CLASS OF BIORTHOGONAL NONUNIFORM COSINE-MODULATED FILTER BANKS WITH LOWER SYSTEM DELAY

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ABSTRACT

In this paper, the theory and design of a class of perfect-reconstruction (PR) biorthogonal nonuniform cosine-modulated filter banks (CMFB) is proposed. It is based on a recombination or merging structure previously proposed by the authors [1]. By relaxing the original CMFB and the recombination transmultiplexer (TMUX) to be biorthogonal, nonuniform CMFBs with lower system delay can be obtained. This increases the possible choices of the prototype filters and greatly reduces the overall system delay of the nonuniform filter bank. A new method is also introduced to suppress the spurious response resulting from the mismatch in the transition bands of the two biorthogonal CMFBs. Design example shows that nonuniform CMFBs with good stopband attenuation, lower system delay and implementation complexity can be realized using the proposed method.

I. INTRODUCTION

Nonuniform filter banks with perfect-reconstruction (PR) property are desirable in applications such as audio coding. The PR condition of nonuniform filter bank was first studied in [2], where a structure for *P*-band nonuniform QMF bank was proposed. As the analysis and synthesis filters in this filter bank are linear time-invariant (LTI), certain compatibility condition has to be satisfied to construct a PR system. In [3], Cox has proposed a two-stage structure for pseudo PR nonuniform filter bank by merging certain outputs from a uniform filter bank using the synthesis filters of another filter bank with smaller number of channels. More recently, the authors have shown that this structure is capable of achieving PR if the original filter bank and the recombination transmultiplexer (TMUX) are PR [1]. A family of orthogonal nonuniform cosine-modulated filter banks (CMFB) was also proposed, where a coprime condition on the numbers of channels in the original filter bank and the recombination TMUX is imposed so that the analysis and synthesis filters become LTI. This considerably simplifies the design of such nonuniform CMFBs. In this paper, we further generalize this class of nonuniform filter banks to the biorthogonal case. In addition to the increased freedom offered by the more general prototype filters, they can be used to realize nonuniform filter banks with much lower system delay than their orthogonal counterparts, which is very important in delay sensitive applications. Since the original CMFB and the recombination TMUX all come from biorthogonal CMFBs, the design procedure is different from that in the orthogonal case. A new method for designing these biorthogonal nonuniform CMFBs is therefore developed in this paper. Design example shows that nonuniform CMFBs with high stopband attenuation, lower system delay and implementation complexity can be obtained by the proposed method. The rest of the paper is organized as follows: In Section II, the principle of the proposed biorthogonal nonuniform CMFBs is introduced. The theory of CMFB and some design problems related to the proposed biorthogonal nonuniform CMFBs such as spectrum inversion and transition band matching are described in Section III. The design procedure, design example, and comparison between biorhogonal and orthogonal nonuniform CMFBs are given in Section IV. Finally, conclusions are drawn in Section V.

II. COSINE-MODULATED NONUNIFORM FILTER BANKS

The two-stage structure of Cox [3], called indirect or recombination (merging) method here, is shown in Fig. 1. The first m_0 channels of an *M*-channel uniform filter bank are merged using the synthesis filters of an m_0-channel uniform filter bank, $G_{o,i}(z)$. At the synthesis section, the analysis filters, $G'_{o,i}(z)$, is used to produce the m_0 subbands for the *M*-channel uniform filter bank. Similar merging can be performed for other consecutive channels. For example, an *L*-band nonuniform filter bank has *L* merged outputs indexed using *l* from 0 to *L*-1. The *l*-th output is obtained by merging successively the m_l outputs from $H_{r_l+i}(z)$, $i = 0,...m_l - 1$, where for the *l*-th band of the nonuniform filter bank, r_l is the starting index of the merged channels in the *M*-channel uniform filter bank and it is given by

$$r_l = \begin{cases} 0, & l = 0 \\ \sum_{k=0}^{l-1} m_k, & l = 1,\dots,L-1 \end{cases}$$

In Cox's original work, the analysis and synthesis filter banks were derived from the pseudo-quadrature mirror filters similar to the CMFB, hence it is not PR. It can be seen from Fig. 1 that if $G_{o,i}(z)$ and $G'_{o,i}(z)$ form a PR TMUX, then it is equivalent to introducing a certain delay in the m_0 channels. If this delay is compensated in other branches of the nonuniform filter bank, the entire system will be PR. The PR condition for a TMUX is closely related to its filter bank counterpart [4], and their design procedures are quite similar. More precisely, the PR condition of a TMUX is identical to that of a 1-skewed PR filter bank. This means that we can obtain a PR TMUX from a 1-skewed PR filter bank, which in turn can be obtained from a standard PR filter bank by choosing its filters as $H'_k(z) = H_k(z)$ and $F'_k(z) = z^{-1}F_k(z)$, $k = 0,\dots,M-1$, where $\{H_k(z), F_k(z)\}$ and $\{H'_k(z), F'_k(z)\}$ are the analysis and synthesis filters for a standard PR filter bank and a 1-skewed PR filter bank, respectively. In Fig. 1, if $G_{o,i}(z)$ and $G'_{o,i}(z)$ is a 1-skewed PR

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filter bank, then the merging and decomposition operations enclosed in dotted line constitute a PR TMUX. In what follows, we shall call this filter bank the recombination or merging filter bank.

One possible problem with this structure is the possibility of spectrum inversion, where the spectrum in the merged subbands are inverted if the starting index of the merged channels in an M-channel uniform filter bank is an odd number [3, 5]. This can easily be corrected by multiplying the merged output with the sequence $(-1)^n$ before the upsamplers [3]. A more serious problem, however, is that the analysis (synthesis) filters are in general linear periodically time-varying (LPTV). Because of this reason, there is no equivalent LTI filter representation like the direct structure in [5], making the optimization quite complicated. Fortunately, under the condition that M and m_0 are coprime, the decimators and the interpolators can be interchanged [5], and the analysis filters become LTI, giving rise to the equivalent filter representation in Fig. 2. For simplicity, the first subscript of $G_{o,i}(z)$ is dropped. Using the Noble identity [6], one gets,

$$\tilde{H}_0(z) = \sum_{i=0}^{m_0-1} H_i(z^{m_0}) G_i(z^M). \quad (1)$$

The design of this nonuniform filter bank with $H_i(z)$ and $G_i(z)$ derived from the orthogonal CMFB was discussed in [1]. A limitation of this nonuniform orthogonal CMFB is its rather long system delay. In this work, biorthogonal PR CMFB [7, 8] is used not only as the original filter bank but also as the recombination or merging filter bank to reduce the system delay. This also increases the freedom in choosing the prototype filters to meet other possible requirements. The design of this new class of nonuniform CMFBs is discussed in the following section.

III. BIORTHOGONAL NONUNIFORM CMFBS AND PROTRUSION CANCELLATION

In CMFB, the analysis and synthesis filters $h_k(n)$ and $f_k(n)$ are obtained respectively by modulating the prototype filter $h(n)$ with a cosine modulation as follows,

$$h_k(n) = 2h(n)\cos\left[\frac{(2k+1)\pi}{2M}(n - \frac{N-1}{2}) + (-1)^k \frac{\pi}{4}\right],$$

$$f_k(n) = 2h(n)\cos\left[\frac{(2k+1)\pi}{2M}(n - \frac{N-1}{2}) - (-1)^k \frac{\pi}{4}\right],$$

$$k = 0,1,\ldots, M-1, \quad n = 0,1,\ldots, N-1. \quad (2)$$

For simplicity, the filter length N is assumed to be $2mM$, where m is a positive integer. CMFB with other lengths can also be generated from this special case using the technique proposed in [9]. Let $H(z) = \sum_{q=0}^{2M-1} z^{-q} P_q(z^{2M})$ be the type I polyphase decomposition of the prototype filter $h(n)$. For perfect reconstruction, $H(z)$ has to satisfy the following PR condition [7]:

$$P_k(z)P_{2M-k-1}(z) + P_{M+k}(z)P_{M-k-1}(z) = \beta \cdot z^{-s},$$

$$k = 0,1,\ldots, M-1. \quad (3)$$

for some nonzero constant β and positive integer s. If the CMFB is orthogonal, s is equal to $m-1$, due to the linear-phase property of the prototype filter. The overall system delay is then fixed at $2mM-1$ samples. In the biorthogonal case, the linear-phase property is relaxed and a value of s smaller than $m-1$ can be used. The system delay of the biorthogonal CMFB is then reduced to $2(s+1)M-1$ samples. As we mentioned earlier, PR TMUXs can be generated from 1-skewed PR filter banks. And a 1-skewed PR filter bank $\{H_k'(z), F_k'(z)\}$ can be obtained from a standard PR filter bank $\{H_k(z), F_k(z)\}$ [4] by choosing the filters as $H_k'(z) = H_k(z)$ and $F_k'(z) = z^{-1}F_k(z)$. If a standard PR filter bank has a system delay of $2(s+1)M-1$ samples, then the system delay of a 1-skewed PR filter bank is $2(s+1)M$ samples. Thus, PR TMUX, which starts from 1-skewed PR CMFB, has a delay of $2(s+1)$ samples. If the PR TMUX is derived from an orthogonal CMFB, then the delay of the TMUX will be $2m$ samples. While in the biorthogonal case, since the value of s can be smaller than $m-1$, the delay of the PR TMUX can then be reduced.

We now calculate the total system delay. If s and s_l, $l = 0,\ldots, L-1$, are respectively the delay parameters of the M-channel and m_l-channel CMFBs, the total system delay of the nonuniform CMFB is $2(s+1)M - 1 + M \cdot max\{2(s_l+1)\}$ samples, where $2(s+1)M-1$ and $2(s_l+1)$ are the system delay of the original M-channel CMFB and the recombination m_l-TMUX. By using biorthogonal CMFBs as the original and recombination filter banks, the system delay of the nonuniform CMFB can therefore be greatly reduced.

Next, we consider the design of the proposed biorthogonal nonuniform CMFBs. From (1), it can be seen that the quality of $\tilde{H}_l(z)$ depends on the frequency responses of $H_i(z)$ and $G_i(z)$. If two independent CMFBs are merged together, protrusions or spurious response will appear in the stopband of $\tilde{H}_l(z)$. This is illustrated in Fig. 3(a) for the frequency response of $\tilde{H}_l(z)$ in a two-band nonuniform filter bank with decimation factors $(\frac{4}{7}, \frac{3}{7})$. In this example, $L = 2$, $m_0 = 4$ and $m_1 = 3$. The first four channels and the remaining three channels of a 7-channel biorthogonal CMFB are separately merged by a 4-channel and a 3-channel biorthogonal CMFBs. Due to the mismatch in the transition bands of the filters, $H_i(z)$ and $G_i(z)$, $\tilde{H}_l(z)$ will exhibit protrusion in the stopband of the interpolated filters. Fortunately, we found that if the transition band of the filters, $H_i(z)$ and $G_i(z)$, are matched to each other, protrusions or spurious response will be suppressed, as shown in Fig. 3(b). More precisely, the transition band of the m_l-channel CMFB, after interpolation by a factor of M, should be equal to that of the M-channel CMFB, after interpolation by m_l. Therefore, it is possible to design the M-channel biorthogonal CMFB first by minimizing the following objective function

$$\min_{h} \Phi = \int_{\omega_s}^{\pi} \left|H(e^{j\omega})\right|^2 d\omega, \quad (4)$$

subject to the PR constraint in (3),

where $\boldsymbol{h}$ is the vector containing the impulse response of the prototype filter, $h(n)$. The stopband attenuation of $H(z)$ is minimized, instead of $H_k(z)$, because $H_k(z)$'s are frequency shifted versions of the prototype filter. The cut-off frequency ω_s depends on the desired transition bandwidth and is chosen normally between $\frac{\pi}{2M}$ and $\frac{\pi}{M}$.

Duo to the nonlinear-phase nature of the prototype filter in the biorthogonal CMFB, the design procedure is quite different from the orthogonal case. In fact, after designing the original CMFB using (4), the recombination CMFB $G_i(z)$ is obtained by solving the following constrained optimization,

$$\min_{g} \Phi = \alpha\Phi_1 + (1-\alpha)\Phi_2 , \qquad (5)$$

subject to the PR constraint in (3),

where

$$\Phi_1 = \int_{\frac{M}{m_l}\omega_s}^{\pi} \left|G(e^{j\omega})\right|^2 d\omega ,$$

$$\Phi_2 = \int_{\frac{M}{m_l}\omega_p}^{\frac{M}{m_l}\omega_s} \left|G(e^{j\omega}) - H(e^{j\frac{m_l}{M}\omega})\right|^2 d\omega ,$$

g is the vector containing the impulse response of the prototype filter, $g(n)$, α is a weighting factor between 0 and 1, ω_p is the passband cutoff frequency of $H(e^{j\omega})$, and $\frac{M}{m_l}\omega_p$ and $\frac{M}{m_l}\omega_s$ are the passband and the stopband cutoff frequencies of $G(e^{j\omega})$, respectively. The second term in (5) is used to suppress the spurious response of the nonuniform CMFB by matching the transition bands of the two CMFBs.

IV. DESIGN PROCEDURE AND EXAMPLE

The design procedure can be summarized as follows:

Given the decimation ratios $\left\{ {m_l}/{M} \right\}$, $l = 0,1,\ldots, L-1$, with m_l coprime to M and $\sum_{l=0}^{L-1} m_l = M$.

(1) Design an M-channel CMFB with a desired cutoff frequency ω_s using (4).

(2) According to the frequency response of the prototype filter of the M-channel CMFB, design an m_l-channel CMFB using (5).

(3) If all starting indices r_l are even, the CMFB can be merged directly by the synthesis filters.

(4) If any r_l is odd, the sequence $(-1)^n$ should be multiplied to the corresponding channels before the subband merging.

We give an example to illustrate the effectiveness of the proposed method. Fig. 4 shows the frequency response of a biorthogonal nonuniform CMFB with decimation factors $(\frac{4}{7}, \frac{3}{7})$. In this example, the lengths of the 3-channel, 4-channel and 7-channel biorthogonal CMFBs are 30, 40 and 70, respectively. The delay parameters s and s_l in (3) for the M-channel and m_l-channel CMFBs are chosen to be 3, which is smaller than $m-1=4$ in the orthogonal case. The original and recombination CMFBs can in principle have different values of delay parameters. Table 1 compares the system delay and stopband attenuation between the orthogonal and biorthogonal PR nonuniform CMFBs. It is clear that the proposed nonuniform CMFB has a comparable stopband attenuation but lower system delay than its orthogonal counterpart. The optimizations in (4) and (5) are performed using the DCONF program in the IMSL library. DCONF converges in about 100 iterations, and the violation of PR constraints is of the order of 10^{-15}. It can be concluded from this and many other examples not shown here due to space limitation that nonuniform biorthogonal CMFBs with high stopband attenuation, low system delay, and low implementation complexity can be designed by the proposed method.

V. CONCLUSION

The theory and design of a class of biorthogonal PR nonuniform CMFBs, based on a recombination or merging structure in [1], is presented. By imposing a coprime condition on the numbers of channels in the original CMFB and the recombination TMUX, the analysis and synthesis filters become LTI, which considerably simplifies the design of such nonuniform filter banks. Using biorthogonal CMFBs, instead of their orthogonal counterparts, as the original and recombination or merging filter banks, nonuniform CMFBs with comparable stopband attenuation but lower system delay can be obtained by the proposed method.

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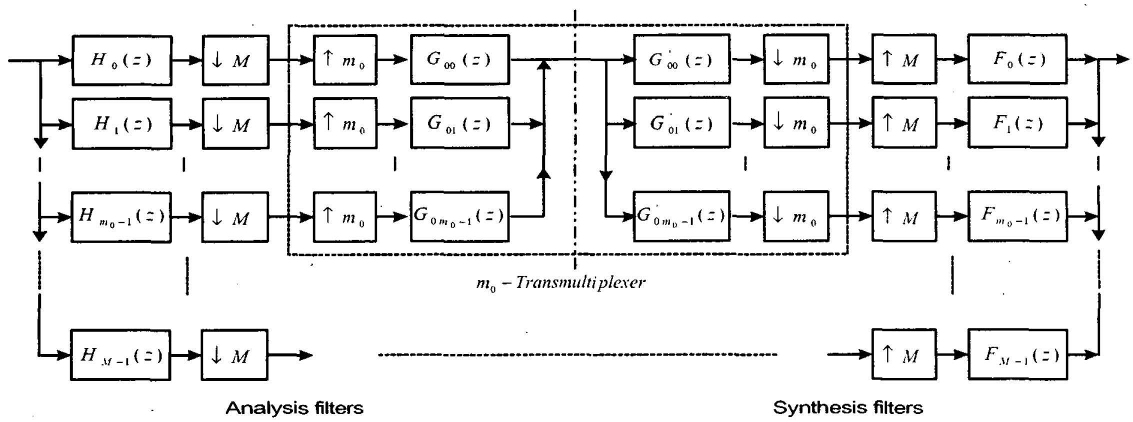


Fig. 1. Structure of nonuniform filter bank (only the first m_0 branches are plotted)

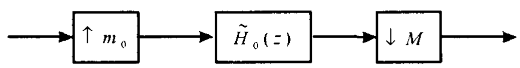
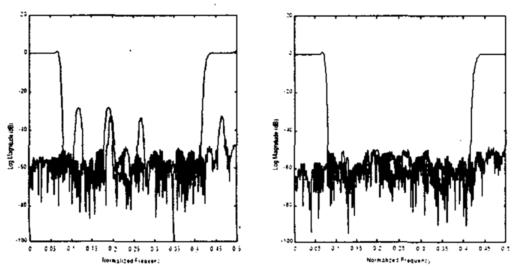


Fig. 2. Equivalent structure when M and m_0 are coprime (analysis filters of the nonuniform filter bank).



(a) (b)

Fig. 3. The protrusion cancellation in nonuniform filter bank with sampling factors $(\frac{4}{7},\frac{3}{7})$.

Frequency responses of $\tilde{H}_l(z)$ when the transition bands of $H_i(z)$ and $G_i(z)$ are (a) mismatched, (b) matched.

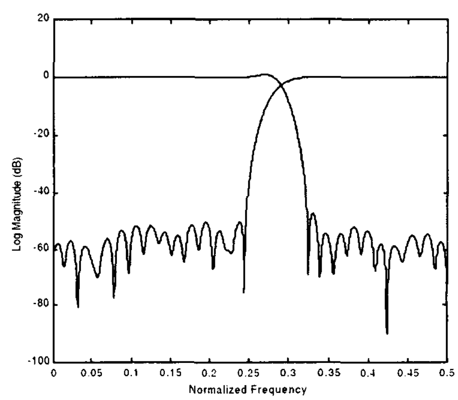


Fig. 4. Frequency responses of the proposed nonuniform CMFB with sampling factors $(\frac{4}{7},\frac{3}{7})$.

Nonuniform filter bank	m_1 m_0 M	Filter length	s_1 s_0 s	Stopband attenuation (dB)	Overall system delay (samples)
$(\frac{4}{7},\frac{3}{7})$ orthogonal	3 4 7	30 40 70	4 4 4	52	139
$(\frac{4}{7},\frac{3}{7})$ biorhtogonal	3 4 7	30 40 70	3 3 3	48	111

Table 1. Comparison between orthogonal and biorthogonal nonuniform CMFBs