A Low-Complexity Multitone-CDMA Communication Technique

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***Abstract* — Digital implementation of multitone-(MT-) CDMA systems often involves considerable complexity. Assuming the spreading-sequence length, N, is a multiple of the number of subcarriers, M, this paper proposes a new digital MT-CDMA technique that achieves low-implementation complexity. In particular, for the case of rectangular chip waveform, it is shown that the MT-CDMA technique proposed earlier and the technique proposed in this paper require $\alpha + \alpha \log_2 \alpha N$ and $1 + (M/N)\log_2 M$ multiplications per chip, respectively, where $\alpha \geq 1$ is the number of samples per chip. The reduction in complexity is particularly considerable when N is large.**

I. INTRODUCTION

Vandendorpe's multitone- (MT-) CDMA technique [1] combines orthogonal frequency division multiplexing (OFDM) and direct-sequence spread-spectrum (DSSS) technique in such a way that, by using a larger number of subcarriers for parallel symbol transmission, the processing gain of a MT-CDMA system can be made substantially greater than that of a single-carrier DSSS system under the same-bandwidth condition. This greater processing gain can be utilized in, for instance, a more effective suppression of the narrowband interference [2]. Other advantages due to a greater processing gain can be found in [1] and [3]. Despite a number of advantages, little attention has been given to the implementation complexity issue of MT-CDMA systems. It is shown in Section II that digital implementation of MT-CDMA systems based on the signal model of [1][1] often involves considerable complexity. In Section III, we propose a new digital MT-CDMA technique that can greatly reduce the implementation complexity. The proposed technique can be applied if the spreading-sequence length is a multiple of the number of subcarriers. Implementation efficiency of the proposed technique is also demonstrated. Finally, conclusions are given in Section IV.

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[1] In this paper, the technique proposed by Vandendorpe in [1] is referred to as the conventional MT-CDMA technique.

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II. IMPLEMENTATION COMPLEXITY OF CONVENTIONAL MT-CDMA SYSTEMS

A. *Transmitter complexity*

Let M be the number of orthogonal subcarriers that are used for parallel transmission of symbols in a MT-CDMA system. At the transmitter, the incoming symbol stream having a rate of M/T_s symbols per second is first serial-to-parallel converted into M lower-rate substreams each of which has a symbol duration of T_s seconds. These M substreams are respectively modulated onto M orthogonal subcarriers that are uniformly spaced by f_Δ Hz. The M modulated subcarriers are multiplexed to give a narrowband OFDM signal, which is then spread by a spreading sequence of length N to form a MT-CDMA signal. The complex envelope of a MT-CDMA signal, $s(t)$, is given by

$$s(t) = \sum_{i=-\infty}^{\infty} s_i(t) \tag{1}$$

where

$$s_i(t) = \sum_{m=0}^{M-1} I_{i,m} e^{j2\pi m f_\Delta t} \sum_{n=0}^{N-1} a_n \psi(t - iT_s - nT_c). \tag{2}$$

In (2), $I_{i,m}$ is the ith symbol transmitted on the mth subcarrier, $T_c = T_s/N$ is the chip duration, $\{a_0, a_1, \dots, a_{N-1}\}$ is the spreading sequence satisfying $|a_n| = 1$, $n = 0, 1, \dots, N-1$, and (t) is the chip waveform. If the chip waveform is rectangular, orthogonality among subcarriers before spreading is maintained by setting $f_\Delta = /T_s$ [1]. If bandlimited pulse shaping is used, the selection of f_Δ is not trivial. It is apparent that $s(t)$ can be realized by individually generating $s_i(t)$, $i = 0, \pm 1, \pm 2, \dots$, according to (2). To generate $s_i(t)$ from $I_{i,m}$, $m = 0, 1, \dots, M-1$, one can first realize a narrowband OFDM signal

$$x_i(t) = \sum_{m=0}^{M-1} I_{i,m} e^{j2\pi m f_\Delta t} \tag{3}$$

followed by spreading $x_i(t)$ with a spectral spreading function

$$y_i(t) = \sum_{n=0}^{N-1} a_n \psi(t - iT_s - nT_c). \tag{4}$$

The digital approach for realizing $s_i(t)$ is based on the IDFT technique to generate the narrowband OFDM signal.

Table 1. Number of per-chip multiplications for implementing a transmitter and a receiver's symbol detector in a conventional MT-CDMA system.

	Number of multiplications per chip		
N	$\alpha = 1$	$\alpha = 2$	$\alpha = 4$
16	5	12	28
32	6	14	32
64	7	16	36
128	8	18	40
256	9	20	44
512	10	22	48
1024	11	24	52

Suppose rectangular chip waveform is used, so that $f_\Delta = \ /T_s$, and $\ (t) = 1$ for $t \in [0, T_c)$ and $\ (t) = 0$ otherwise. It follows that we only need to generate $x_i(t)$ and $y_i(t)$ for the interval $t \in [iT_s, (i+1)T_s)$. It is known that $x_i(t)$ can be realized by taking IDFT on $I_{i,m}$'s [4]. A Q-point IDFT yields Q samples that represent $x_i(t)$ at Q time instants, and $x_i(t)$ can be reconstructed from these Q samples by interpolation. The minimum possible value of Q is $Q = M$, corresponding to the case of Nyquist sampling on $x_i(t)$. Since the narrowband signal $x_i(t)$ is subsequently spread by a wideband signal $y_i(t)$, an M-point discrete-time representation of $x_i(t)$ does not provide sufficient number of points for subsequent spreading and hence interpolation is required. As sampling is done at the Nyquist rate, reconstruction of $x_i(t)$ can be accomplished only by perfect interpolation based on the sinc function, a task which requires considerable computation. Furthermore, perfect interpolation requires a knowledge of $x_i(t)$ values at time instants t that are outside the range $t \in [iT_s, (i+1)T_s)$. To eliminate the need of interpolation, one can use oversampling (i.e., $Q > M$). A discussion in [5] also indicates that oversampling is always desirable for reducing the complexity required by interpolation and filtering in practical implementation of OFDM systems. Since $y_i(t)$ is a spectral spreading waveform consisting of N chips, and since $x_i(t)$ is varying within a chip duration, αN samples of $x_i(t)$ are required to provide sufficient points for spreading with $y_i(t)$, where $\alpha \geq 1$ (an integer) is the number of points modeling the variation of $x_i(t)$ within a chip duration. The αN samples of $x_i(t)$ are generated by an IDFT of $I_{i,m}$, $m = 0, 1, \ldots, M-1$, appended with $\alpha N - M$ zeroes. Finally, $s_i(t)$ can be generated from its αN samples by D/A conversion followed by filtering or more often by a sample-and-hold circuit.

The number of multiplications required to generate $s_i(t)$ can be estimated as follows. An αN-point IDFT is used to generate $x_i(t)$, requiring $\alpha N \log_2 \alpha N$ multiplications [6]. The subsequent spreading of (the αN-point representation of) $x_i(t)$ with $y_i(t)$ requires another αN multiplications. A sample-and-hold circuit is used to generate $s_i(t)$ from its αN samples so that no multiplication is needed. The total number of multiplications is therefore $\alpha N + \alpha N \log_2 \alpha N$. It follows that the number of multiplications per chip for the MT-CDMA technique of [1] is $\alpha + \alpha \log_2 \alpha N$. Table 1 lists the numbers of multiplications per chip for different combinations of N and α. The very high numbers of per-chip multiplications required for all cases indicate that implementation of MT-CDMA transmitters is quite complex.

When a bandlimited chip pulse is used instead of a rectangular one, $x_i(t)$ and $y_i(t)$ are required to be generated beyond the interval $t \in [iT_s, (i+1)T_s)$, so that an IDFT involving more than αN points is required. More computation than the case of rectangular chip waveform is needed.

B. Receiver complexity

The receiver is required to provide an estimate of $I_{i,m}$ based on the received signal in the presence of noise and interference. For illustration purposes, we consider a simple case that the received signal, $r(t)$, is modeled by

$$r(t) = s(t) + J(t) \tag{5}$$

where $J(t)$ is the undesired disturbance due to noise and interference. We furthermore assume that (a) the receiver is able to synchronize at the received signal, and (b) intersymbol-interference-free pulse shaping is used for $s(t)$. An analog receiver usually provides an estimate of $I_{i,m}$, denoted by $\hat{I}^{(A)}_{i,m}$, by means of matched filtering:

$$\hat{I}^{(A)}_{i,m} = C^{-1}\int_{-\infty}^{\infty} r(t) y_i^*(t) e^{-j2\pi m f_\Delta t} dt \tag{6}$$

where $C = N\int_{-\infty}^{\infty} |\psi(t)|^2 dt$ is a scale constant.

The following describes a method for a digital receiver to estimate $I_{i,m}$ when rectangular chip waveform is used. Note that we only need to process $r(t)$ for the interval $t \in [iT_s, (i+1)T_s)$. At the receiver, the received signal is sampled and A/D-converted. Suppose that Q equally-spaced samples of $r(t)$ are obtained, namely, $r(t_{k,Q})$, $k = 0, 1, \ldots, Q-1$, where $t_{k,Q} = (i + k/Q)T_s$. Eqn. (6) suggests that an estimate of $I_{i,m}$, denoted by $\hat{I}_{i,m}$, can be obtained by

$$\hat{I}_{i,m} = Q^{-1}\sum_{k=0}^{Q-1} r(t_{k,Q}) y_i^*(t_{k,Q}) e^{-j2\pi mk/Q} \tag{7}$$

for $m = 0, 1, \ldots, M-1$. That is, $\hat{I}_{i,m}$ is the mth output of a Q-point DFT of $r(t_{k,Q}) y_i^*(t_{k,Q})$, $k = 0, 1, \ldots, Q-1$. The validity of using (7) to obtain an estimate of $I_{i,m}$ becomes apparent by substituting (5) into (7). It yields

$$\hat{I}_{i,m} = I_{i,m} + Q^{-1}\sum_{k=0}^{Q-1} J(t_{k,Q}) a^*_{\lfloor kN/Q \rfloor} e^{-j2\pi mk/Q} \quad \textbf{(8)}$$

where $\lfloor x \rfloor$ is the largest integer less than or equal to x. Note that the minimum value of Q is M because it is required to compute M estimates, i.e., $\hat{I}_{i,m}$, $m = 0, 1, \ldots, M-1$. Also note that $\hat{I}_{i,m}$ closely approximates $\hat{I}^{(A)}_{i,m}$ if Q is sufficiently large.

To minimize the amount of computation in obtaining $\hat{I}_{i,m}$, one might use the minimum possible value of Q, that is, $Q = M$. However, (8) indicates that the *effective* processing gain for suppressing the undesired disturbance $J(t)$ is only M. Therefore, using M points in the processing of $r(t)$ is insufficient to fully utilize the available processing gain inherent in the MT-CDMA signal. Oversampling is thus required. To fully utilize the available processing gain, at least N points are required. Furthermore, it is usually desirable to approximate matched filtering as closely as possible in order to maximize the signal-to-noise ratio. To do this, we need to take into consideration the variation of the desired signal $s(t)$ over a chip duration. It follows that αN samples of $r(t)$ are required for processing, where $\alpha \geq 1$ is the same parameter used at the transmitter side for modeling the variation of the desired signal within a chip duration.

The number of multiplications required to compute $\hat{I}_{i,m}$ is estimated by setting $Q = \alpha N$. Multiplication of $r(t_{k,Q})$ and $y_i^*(t_{k,Q})$ involves αN multiplications. The Q-point DFT requires $\alpha N \log_2 \alpha N$ multiplications. It follows that the number of multiplications per chip is given by $\alpha + \alpha \log_2 \alpha N$. Consequently, implementation complexity of a receiver for symbol detection[2] is the same as that of a transmitter. Numerical figures of the number of per-chip multiplications required for the symbol detector are listed in Table 1.

If bandlimited pulse shaping is used, $y_i(t)$ is not limited to the interval $t \in [iT_s, (i+1)T_s)$. More points are required to approximate the integration carried out in (6). More computation than the case of rectangular chip waveform is involved.

III. THE PROPOSED LOW-COMPLEXITY TECHNIQUE

A. Signal model

One of the factors that leads to the requirement of oversampling in generating a MT-CDMA signal and in detecting transmitted symbols is that $x_i(t)$ and $s_i(t)$ vary during a chip duration. If they can be made piecewise constant over a number of chips, some demand on oversampling can be reduced. This idea forms the basis of the proposed technique.

The proposed technique uses an M-point IDFT. Since an M-point IDFT yields M outputs, it is required to increase the size of these M outputs to N samples in order to form an N-point discrete-time signal for multiplication with the spreading code. Piecewise-constant interpolation, which does not require computation, can be used to accomplish this purpose. Let

$$w_{i,k} = \sum_{m=0}^{M-1} I_{i,m} e^{j2\pi mk/M}, \quad k = 0, 1, \cdots, M-1, \quad \textbf{(9)}$$

be the M-point-IDFT outputs of $I_{i,0}, I_{i,1}, \ldots, I_{i,M-1}$. We first partition the N chips of the spreading sequence into M blocks, each of which consists of N/M chips. Notice that N/M must be an integer. This constraint is required to be satisfied for the MT-CDMA system that uses the proposed technique. For the sake of convenience, let

$$N_{ba} = \frac{N}{M}. \quad \textbf{(10)}$$

The kth IDFT output, $w_{i,k}$, is used to modulate consecutive N_{ba} chips in the kth block. The complex envelope of the MT-CDMA signal generated by the proposed technique, $\tilde{s}(t)$, is given by

$$\tilde{s}(t) = \sum_{i=-\infty}^{\infty} \tilde{s}_i(t) \quad \textbf{(11)}$$

where

$$\tilde{s}_i(t) = \sum_{n=0}^{N-1} a_n \psi(t - iT_s - nT_c) \cdot w_{i,\lfloor n/N_{ba} \rfloor}. \quad \textbf{(12)}$$

Substituting (9) into (12) gives

$$\tilde{s}_i(t) = \sum_{m=0}^{M-1} I_{i,m} \sum_{n=0}^{N-1} a_n e^{j2\pi m \lfloor n/N_{ba} \rfloor / M} \psi(t - iT_s - nT_c). \quad \textbf{(13)}$$

Eqn. (13) reveals that the proposed MT-CDMA technique can be viewed as a multicode DSSS transmission scheme using a specific choice of M spreading sequences. The mth spreading sequence is given by $\{a_n e^{j2\pi m \lfloor n/N_{ba} \rfloor / M}\}_{n=0,1,\cdots,N-1}$. It is easy to show that the mth and m'th spreading sequences, $m \neq m'$, are orthogonal. Note that if intersymbol-interference-free pulse shaping is used, orthogonality among the spreading sequences implies that $I_{i,m}$ can be recovered from $\tilde{s}(t)$ by

$$I_{i,m} = C^{-1} \int_{-\infty}^{\infty} \tilde{s}(t) \sum_{n=0}^{N-1} a_n^* e^{-j2\pi m \lfloor n/N_{ba} \rfloor / M} \times \psi^*(t - iT_s - nT_c) dt \quad \textbf{(14)}$$

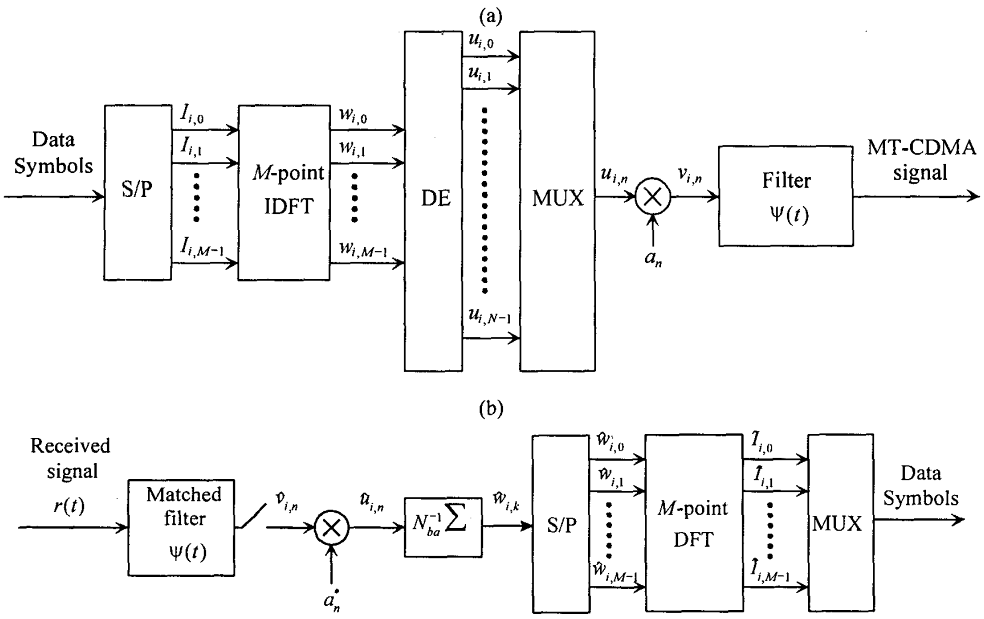
where C is a scale constant the same as that of (6).

B. Digital implementation of transmitter and receiver

The transmitter for the proposed technique is shown in Fig. 1a. The procedure for realizing $\tilde{s}_i(t)$ follows from (12) and is listed as follows.

[2] Note that we do not take into account the signal processing requirement for other parts of the receiver such as the synchronization circuit.

Fig. 1. (a) A transmitter and (b) a symbol detector of a receiver for the proposed technique. (S/P: serial-to-parallel converter; MUX: multiplexer; DE: data expansion unit)



1) Generate $w_{i,k}$, $k = 0, 1, \ldots, M-1$, by an M-point IDFT according to (9).
2) Construct $u_{i,n} = w_{i,\lfloor n/N_{ba}\rfloor}$, $n = 0, 1, \ldots, N-1$, by a simple data expansion unit.
3) Generate $v_{i,n} = a_n u_{i,n}$, $n = 0, 1, \ldots, N-1$.
4) Generate $\tilde{s}_i(t)$ by feeding $v_{i,0}, v_{i,1}, \ldots, v_{i,N-1}$ into a transmitter filter having an impulse response (t).

Fully-digital implementation of the transmitter is possible by using a digital transmitter filter. The generated $\tilde{s}_i(t)$ is D/A-converted and filtered to obtain the transmitted signal.

To illustrate the construction of a receiver, we consider a simple case that the received signal, $\tilde{r}(t)$, is corrupted by disturbance $J(t)$ due to noise and interference and is modeled by

$$\tilde{r}(t) = \tilde{s}(t) + J(t). \tag{15}$$

Assume that the received signal can be synchronized at the receiver. An estimate of the transmitted symbol $I_{i,m}$ based on $\tilde{r}(t)$, denoted by $\hat{I}_{i,m}$, is obtained by matched filtering similar to (14). That is,

$$\hat{I}_{i,m} = C^{-1}\int_{-\infty}^{\infty} \tilde{r}(t) \sum_{n=0}^{N-1} a_n^* e^{-j2\pi m\lfloor n/N_{ba}\rfloor/M} \times \psi^*(t - iT_s - nT_c)dt \tag{16}$$

To gain more insight into the estimation procedure, we express (16) as

$$\hat{I}_{i,m} = M^{-1}\sum_{k=0}^{M-1} e^{-j2\pi mk/M}\left\{\frac{1}{N_{ba}} \times \sum_{n=kN_{ba}}^{(k+1)N_{ba}-1} a_n^* \frac{\int_{-\infty}^{\infty}\tilde{r}(t)\psi^*(t - iT_s - nT_c)dt}{\int_{-\infty}^{\infty}|\psi(t)|^2 dt}\right\}. \tag{17}$$

A block diagram for the receiver is plotted in Fig. 1b. The procedure for obtaining $\hat{I}_{i,m}$ is as follows.

1) Process $\tilde{r}(t)$ by a matched filter matched to (t), i.e., a chip matched filter. Sample the matched filter output at a rate of $/T_c$ per second, and obtain $\hat{v}_{i,0}, \hat{v}_{i,1}, \ldots, \hat{v}_{i,N-1}$ at appropriate sampling instants, where

$$\hat{v}_{i,n} = \frac{\int_{-\infty}^{\infty}\tilde{r}(t)\psi^*(t - iT_s - nT_c)dt}{\int_{-\infty}^{\infty}|\psi(t)|^2 dt}. \tag{18}$$

2) Compute $\hat{u}_{i,n} = \hat{v}_{i,n} a_n^*$, $n = 0, 1, \ldots, N-1$.
3) Compute

$$\hat{w}_{i,k} = N_{ba}^{-1}\sum_{n=kN_{ba}}^{(k+1)N_{ba}-1}\hat{u}_{i,n}, \quad k = 0, 1, \cdots, M-1. \tag{19}$$

4) Obtain $\hat{I}_{i,m}$'s by an M-point DFT:

$$\hat{I}_{i,m} = M^{-1}\sum_{k=0}^{M-1}\hat{w}_{i,k} e^{-j2\pi mk/M}, \quad m = 0, 1, \cdots, M-1. \tag{20}$$

Notice that the averaging process and the DFT given by (19) and (20), respectively, are most suitable to be implemented in the digital domain. When a digital chip matched filter is used to process the A/D-converted received signal $\tilde{r}(t)$, the receiver can be made fully digital.

Table 2. Number of per-chip multiplications for implementing a transmitter and a receiver's symbol detector in a system using the proposed technique.

N/M	Number of multiplications per chip
16/1	1.0000
32/2	1.0625
64/4	1.1250
128/8	1.1875
256/16	1.2500
512/32	1.3125
1024/64	1.3750
64/1	1.0000
128/2	1.0156
256/4	1.0312
512/8	1.0469
1024/16	1.0625

C. *Implementation efficiency and discussion*

At the transmitter, the generation of $w_{i,k}$'s from an M-point IDFT and the computation of $v_{i,n}$'s from a_n's and $u_{i,n}$'s require $M\log_2 M$ and N multiplications, respectively. Since rectangular chip waveform is used, $\tilde{s}_i(t)$ can be directly realized by D/A conversion of computed $v_{i,n}$'s followed by using a sample-and-hold circuit to maintain each of $v_{i,n}$'s for a duration of T_c seconds. No additional multiplication is involved. The number of per-chip multiplications for the proposed technique is therefore $1 + N_{ba}^{-1}\log_2 M$. At the receiver, the received signal is sampled and A/D-converted. The chip matched filter can be implemented by an accumulator, so that multiplication is not needed. The computation of $\hat{u}_{i,n}$'s by multiplying a_n^*'s with $\hat{v}_{i,n}$'s requires N multiplications. The generation of $\hat{w}_{i,k}$'s from $\hat{u}_{i,n}$'s by (19) does not involve multiplication. The final step of taking an M-point DFT requires $M\log_2 M$ multiplications. The number of per-chip multiplications required for symbol detection at the receiver is therefore $1 + N_{ba}^{-1}\log_2 M$, which is the same as that for signal generation at the transmitter.

Table 2 lists the numbers of per-chip multiplications for different combinations of N and M. It is apparent that the numbers of per-chip multiplications are close to 1 for all cases, indicating that the implementation complexity required by the proposed technique is very low. Comparing the results of Table 2 with those for the conventional MT-CDMA technique listed in Table 1 reveals that significant reduction of implementation complexity is achieved by using the proposed MT-CDMA technique. The reduction is especially substantial when N is large. Since the objective of MT-CDMA communications is to operate a system at a large processing gain N [1], [3], the significance of the proposed technique to practical implementation of systems is apparent.

If bandlimited pulse shaping is used, the conventional MT-CDMA technique requires a higher-dimensional IDFT/DFT than it does for the case of rectangular chip waveform (see Section II). More computation is thus required. However, an M-point IDFT/DFT can still be used for the proposed technique. The only additional requirements are: (a) to replace the sample-and-hold circuit with a pulse shaping filter for the transmitter; and (b) to replace the accumulator with a chip matched filter matched to a bandlimited pulse for the receiver. Although these additional requirements involve some computation, the proposed technique still provides a reduction in computation requirements over the conventional MT-CDMA technique because the saving in carrying out IDFT/DFT is significant.

IV. CONCLUSIONS

We have proposed a technique that enables low-complexity implementation for MT-CDMA communications. In digital implementation of MT-CDMA systems using rectangular chip waveform, analysis has shown that the numbers of multiplications per chip required by the conventional MT-CDMA technique and the proposed one are $\alpha + \alpha\log_2 \alpha N$ and $1 + N_{ba}^{-1}\log_2 M$, respectively. The proposed technique thus achieves a reduction in implementation complexity. The reduction in complexity has been found to be particularly considerable when the processing gain N is large. It has also been shown that a reduction is also obtained when bandlimited pulse shaping is used.

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