MULTIPLIER-LESS DISCRETE SINUSOIDAL AND LAPPED TRANSFORMS USING SUM-OF-POWERS-OF-TWO (SOPOT) COEFFICIENTS

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Abstract

This paper proposes a new family of multiplier-less discrete cosine and sine transforms called the SOPOT DCTs and DSTs. The fast algorithm of Wang [10] is used to parameterize all the DCTs and DSTs in terms of certain (2x2) matrices, which are then converted to SOPOT representation using a method previously proposed by the authors [7]. The forward and inverse transforms can be implemented with the same set of SOPOT coefficients. A random search algorithm is also proposed to search for these SOPOT coefficients. Experimental results show that the (2x2) basic matrix can be implemented, on the average, in 6 to 12 additions. The proposed algorithms therefore require only $O(N\log_2 N)$ additions, which is very attractive for VLSI implementation. Using these SOPOT DCTs/DSTs, a family of SOPOT Lapped Transforms (LT) is also developed. They have similar coding gains but much lower complexity than their real-valued counterparts.

I. INTRODUCTION

Discrete Cosine and Sine transforms (DCTs and DSTs) are frequently used in signal and image processing applications. For example, the DCT-II, which is usually referred to as the DCT, is commonly used in image, video and audio coding. The four types of DCTs (DSTs) are also useful in implementing the lapped transforms [4, 5] and the cosine modulated filter banks (CMFB) [1-3]. Recently, there is a considerable interest in designing filter banks with low arithmetic complexity. Filter banks using integer coefficients [1], and the sum-of-powers-of-two (SOPOT) representation [6, 7] were proposed. One important problem with integer CMFB and integer lapped transform [8] is that it is very difficult to design orthogonal integer matrices which resemble the various sinusoidal transforms when the size of that transform increases. Up to now, only integer DCT-IV up to order 8 has been reported. In order to overcome this difficulty, the authors have recently proposed a multiplier-less DCT-IV for the implementation of the conventional CMFB [7], which is based on the SOPOT representation. Multiplier-less DCT-IV up to 1024 and higher can be designed. In this paper, we further generalize this technique to cover the four types of DSTs and DCTs. The fast decomposition algorithm of Wang [10] is used to parameterize all the DCTs and DSTs in terms of a set of basic (2x2) matrices. These matrices, which are closely related to the (2x2) rotation matrix, are then converted into SOPOT representation using the method we have introduced in [7]. This allows us to implement both the forward and inverse transforms with the same set of SOPOT coefficients. Moreover, as the proposed SOPOT DCTs and DSTs are derived from the fast algorithms of Wang [10], it only requires $O(N\log_2 N)$ additions and the implementation of $O(N\log_2 N)$ (2x2) basic matrices as mentioned earlier, where N is the length of the transforms. As each (2x2) basic matrix can be implemented, on the average, in 6 to 12 additions, the proposed multiplier-less transforms require only $O(N\log_2 N)$ additions, which is very attractive for VLSI implementation. Using these SOPOT DCTs/DSTs, a family of SOPOT Lapped Transforms is also developed. They have similar coding gains but lower implementation complexity than their real-valued counterparts. It should be noted that another (8×8) multiplier-less DCT-II, called binDCT, can also be obtained from the Gauss-Jordan factorization and the lifting structure [9]. Our approach differs from [9] in that it is based on the fast DCT/DST algorithms of Wang which can be generalized to different types of sinusoidal transforms with different transform lengths. Moreover, the parameterization is based on a rotation-like (2x2) matrix, which is expected to have better numerical properties, especially when the transform size is large. The approach described here can also be generalized to design SOPOT approximation to discrete Fourier transform (DFT) and discrete Hartley transform (DHT) [15].

This paper is organized as follows: Sections II and III are devoted to the definition of the four types of DCTs and DSTs, and their fast algorithms. The proposed SOPOT DCTs and DSTs are discussed in Section IV followed by several design examples in Section V. Finally, conclusions are drawn in section VI.

II. THE SINUSOIDAL TRANSFORMS: DCT AND DST

According to Wang [10], there are four different types of discrete cosine transforms (DCT) and discrete sine transforms (DST). These transforms have been developed at various times by Ahmed et al [11], Kitajima [12], Jain [13] and Kekre and Solanki [14]. The definitions of the DCTs and DSTs are given as follows,

A. Four types of DCT Matrices

$$\left[C^{I}_{N+1}\right]_{k,n} = \sqrt{2/N}\{\varepsilon_k\varepsilon_n\cos(kn\pi/N)\},\quad k,n=0,1,\ldots,N. \tag{2.1a}$$

$$\left[C^{II}_{N}\right]_{k,n} = \sqrt{2/N}\{\varepsilon_k\cos(k(n+1/2)\pi/N)\},\quad k,n=0,1,\ldots,N-1. \tag{2.1b}$$

$$\left[C^{III}_{N}\right]_{k,n} = \sqrt{2/N}\{\varepsilon_n\cos(n(k+1/2)\pi/N)\},\quad k,n=0,1,\ldots,N-1. \tag{2.1c}$$

$$\left[C^{IV}_{N}\right]_{k,n} = \sqrt{2/N}\{\cos((k+1/2)(n+1/2)\pi/N)\},\quad k,n=0,1,\ldots,N-1. \tag{2.1d}$$

B. Four types of DST Matrices

$$\left[S^{I}_{N-1}\right]_{k,n} = \sqrt{2/N}\{\sin(kn\pi/N)\},\quad k,n=1,2,\ldots,N-1. \tag{2.2a}$$

$$\left[S^{II}_{N}\right]_{k,n} = \sqrt{2/N}\{\varepsilon_k\sin(k(n-1/2)\pi/N)\},\quad k,n=1,2,\ldots,N. \tag{2.2b}$$

$$\left[S^{III}_{N}\right]_{k,n} = \sqrt{2/N}\{\varepsilon_n\sin(n(k-1/2)\pi/N)\},\quad k,n=1,2,\ldots,N. \tag{2.2c}$$

$$\left[S^{IV}_{N}\right]_{k,n} = \sqrt{2/N}\{\sin((k+1/2)(n+1/2)\pi/N)\},\quad k,n=0,1,\ldots,N-1. \tag{2.2d}$$

ε_i is equal to $1/\sqrt{2}$ for $i=0,N$ and 1 otherwise. The superscripts and subscripts represent respectively the type and the size of the transforms. Also, let $X_C^K(k)$, $K=I$ to IV, and $x(n)$ be the type K cosine transformed and the input sequences, respectively. For simplicity, it is assumed that the scaling factors are absorbed into $X_C^K(k)$ or $x(n)$ so that ε_k and ε_n are taken as unity in the subsequent development. We now summarize the decompositions for the various DCTs and DSTs to construct their multiplier-less transmations.

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III. DECOMPOSITION OF DCTs and DSTs

DCT-I

From the definition of DCT-I [10], it can be shown that, for N an even number, its even and odd parts can be decomposed as

$$X_N^{C-I}(2k) = \sum_{n=0}^{(N/2)-1}[x(n)+x(N-n)]\cos\frac{\pi nk}{(N/2)} + (-1)^k x(N/2),$$

$$X_N^{C-I}(2k+1) = \sum_{n=0}^{(N/2)-1}[x(n)-x(N-n)]\cos\frac{\pi n(2k+1)}{(N/2)},$$

$$\text{where } k = 0,1,\ldots,N/2-1. \qquad (3\text{-}1)$$

Therefore, an DCT-I can be computed by a DCT-I and a DCT-III of lower order. (3-1) can be written more compactly as the following matrix representation

$$C_{N+1}^{I} = P_{N+1}(C_{N/2+1}^{I} \oplus C_{N/2}^{III})A_{N+1}, \qquad (3\text{-}2)$$

where P_{N+1} is a permutation matrix that accomplishes the permutation of the even and odd indexed parts, $A_{N+1} = \frac{1}{\sqrt{2}}\begin{bmatrix} I_{N/2} & & J_{N/2} \\ & \sqrt{2} & \\ I_{N/2} & & -J_{N/2} \end{bmatrix}$, and I_N and J_N are order N identity and exchange matrices, respectively.

DCT-II and DCT-III

Similarly, a DCT-II with an even length N can be decomposed into an (N/2)-point DCT-II and DCT-IV.

$$X_N^{C-II}(2k) = \sum_{n=0}^{(N/2)-1}[x(n)+x(N-n-1)]\cos\frac{\pi(2n+1)k}{2(N/2)}, \qquad (3\text{-}3a)$$

$$X_N^{C-II}(2k+1) = \sum_{n=0}^{(N/2)-1}[x(n)-x(N-n-1)]\cos\frac{\pi(2n+1)(k+1)}{2(N/2)},$$

$$\text{where } k = 0,1,\ldots,(N/2)-1. \qquad (3\text{-}3b)$$

In matrix notation, (3-3) reads

$$C_N^{II} = P_N(C_{N/2}^{II} \oplus C_{N/2}^{IV})B_N, \qquad (3\text{-}4)$$

where P_N is a permutation matrix that accomplishes the permutation of the even and odd indexed parts and $B_N = \begin{bmatrix} I_{N/2} & J_{N/2} \\ I_{N/2} & -J_{N/2} \end{bmatrix}$. The decomposition for DCT-III can be obtained by observing that DCT-III and DCT-II are inverses of each other. It then follows from (3-4) that

$$C_N^{III} = [C_N^{II}]^T = B_N^T(C_{N/2}^{III} \oplus C_{N/2}^{IV})P_N^T. \qquad (3\text{-}5)$$

DCT-IV

To obtain the decomposition for DCT-IV

$$X_N^{C-IV}(k) = \sum_{n=0}^{N-1}x(n)\cos\frac{\pi(2n+1)(2k+1)}{4N},\quad k = 0,1,\ldots,N-1. \qquad (3\text{-}6)$$

Let's define the following sequences

$$Y_N(k) = X_N^{C-IV}(k)\cos\phi_k + X_N^{C-IV}(N-k-1)\sin\phi_k, \qquad (3\text{-}7a)$$

$$Y_N(N-k-1) = X_N^{C-IV}(k)\sin\phi_k - X_N^{C-IV}(N-k-1)\cos\phi_k, \qquad (3\text{-}7b)$$

where $k = 0,1,\ldots,N-1$ and $\phi_k = \pi(2k+1)/4N$. It can be shown [10] that $X_N^{C-IV}(k)$ can be expressed in terms of $Y_N(k)$ as follows

$$X_N^{C-IV}(k) = Y_N(k)\cos\phi_k + Y_N(N-k-1)\sin\phi_k, \qquad (3\text{-}8a)$$

$$X_N^{C-IV}(N-k-1) = Y_N(k)\sin\phi_k - Y_N(N-k-1)\cos\phi_k, \qquad (3\text{-}8b)$$

where $k = 0,1,\ldots,N-1$. $Y_N(k)$ can be computed via DCT-III and DST-III of lower order as follows,

$$Y_N(k) = x(0) + \sum_{n=1}^{N/2+1}[x(2n-1)+x(2n)]\cos(k+\frac{1}{2})(2n)\frac{\pi}{N}, \qquad (3\text{-}11a)$$

$$Y_N(N-k-1) = (-1)^k x(N-1) + \sum_{n=1}^{N/2-1}[x(2n-1)-x(2n)]\sin(k+\frac{1}{2})(2n)\frac{\pi}{N}, \qquad (3\text{-}11b)$$

The order-N/2 DCT-III and DST-III can furthur be decomposed into DST-IV and DST-IV of lower order. In matrix notation, we have

$$C_N^{IV} = T_N(C_{N/2}^{III} \oplus S_{N/2}^{III})B_N', \qquad (3\text{-}12)$$

where $T_N = \begin{bmatrix} R_{1/4N} & & & \\ & R_{3/4N} & & \\ & & \ddots & \\ & & & R_{(N-1)/4N} \end{bmatrix}$,

$R_r = \begin{bmatrix} \cos r\pi & \sin r\pi \\ \sin r\pi & -\cos r\pi \end{bmatrix}$, and $B_N' = \begin{bmatrix} I_{N/2} & I_{N/2} \\ I_{N/2} & -I_{N/2} \end{bmatrix}$.

Similar decompositions can be derived for the DSTs, and are omitted here due to page limitation.

IV. MULTIPLIER-LESS DISCRETE SINUSOIDAL TRANSFORMS

The main difficulty in constructing a multiplier-less transformation is that the coefficients of the matrix transformation and its inverse cannot in general be expressed in terms of SOPOT coefficients. Let's consider the following simple matrix in (3-12)

$$R_\theta = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$$

If $\cos\theta$ and $\sin\theta$ are expressed directly in terms of SOPOT coefficients, say α and β. The inverse of $\tilde{R}_\theta = \begin{bmatrix} \alpha & \beta \\ \beta & -\alpha \end{bmatrix}$ is $\tilde{R}_\theta^{-1} = \frac{1}{\sqrt{\alpha^2+\beta^2}}\begin{bmatrix} \alpha & \beta \\ \beta & -\alpha \end{bmatrix}$. As α and β are SOPOT coefficients, the term $\frac{1}{\sqrt{\alpha^2+\beta^2}}$ cannot in general be expressed as SOPOT coefficient. The basic idea of the proposed multiplier-less sinusoidal transforms is based on the following factorization of the matrix R_θ and its inverse.

$$R_\theta = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix} = \begin{bmatrix} 1 & -\tan(\theta/2) \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \sin\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan(\theta/2) \\ 0 & -1 \end{bmatrix}, \qquad (4\text{-}1a)$$

$$R_\theta^{-1} = \begin{bmatrix} 1 & \tan(\theta/2) \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -\sin\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan(\theta/2) \\ 0 & 1 \end{bmatrix}. \qquad (4\text{-}1b)$$

Since these factorizations of R_θ and R_θ^{-1} involve the same set of coefficients, i.e. $\sin\theta$ and $\tan(\theta/2)$, they can be directly quantized to SOPOT coefficients as follows

$$R_\theta \approx S_\theta = \begin{bmatrix} 1 & -\beta_\theta \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \alpha_\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \beta_\theta \\ 0 & -1 \end{bmatrix}, \qquad (4\text{-}2)$$

where α_θ and β_θ are respectively SOPOT approximations to $\sin\theta$ and $\tan(\theta/2)$. These coefficients are represented as

$$\alpha_\theta = \sum_{k=1}^{t} a_k 2^{b_k} \text{ where } a_k \in \{-1,1\} \text{ and } b_k \in \{-r,\ldots,-1,0,1,\ldots r\}, \qquad (4.3)$$

r is the range of the coefficients and t is the number of terms being used in each coefficient. Replacing R_θ in the fast sinusoidal transform algorithms in Section III, the desired multiplier-less sinusoidal transforms can be obtained. Length $N = 2^m$ DCT-II, -III, -IV, and length $N = 2^m + 1$ DCT-I can be generated where m is a positive integer. Similar SOPOT DSTs can be generated. The remaining problem is to search for the SOPOT coefficients such that certain criteria are minimized subject to a given implementation complexity. For example, in signal coding applications, the coding gain of the transform can be used as the criterion for minimization. For multiplier-less digital filters employing SOPOT coefficients, the total number of terms in the SOPOT coefficients is usually used as a measure of its implementation complexity. It can be seen that this is a combinatorial optimization problem. In this paper, a random search algorithm is used to perform this discrete optimization. More precisely, a random vector with all its elements bounded by ±1 is first multiplied by a scaling factor s, and is added to the parameter vector containing the real-value of α_θ and β_θ. It is then quantized to the nearest SOPOT coefficients. The objective function is then evaluated for this SOPOT candidate. The one with the best performance at a given number of additions is recorded. The search continues until the maximum allowed number of trials is exceeded. The scale factor controls the size of the neighborhood to be searched. A number of solutions with different tradeoffs between implementation complexity and performance are then obtained. For the DCTs and DSTs, the mean squared error between the impulse responses of the candidate vector and the real-valued transform is used as the performance measurement. Also, when the number of channels, in the power of two, being more than about 64, all the transformation can be calculated using the early mentioned decomposition. The previous generated lower order DCTs/ DSTs can be used to reduce the searching time.

V. DESIGN EXAMPLES

We now present some design examples for the DCTs and DSTs.

SOPOT DCT-I, II, III and IV

Table 1 shows the parameters of the proposed SOPOT DCT-I, -II, -III and –IV with $m = 4$. It can be seen that the multiplication with the SOPOT coefficients α_θ and β_θ can be implemented, on the average, as approximately 2 additions per coefficient. The frequency responses of an 8-channel SOPOT DCT-II and an 64-channel SOPOT DCT-IV are plotted in figure 2 and 3, respectively. The coding gain performance and the implementation complexity of the proposed SOPOT DCTs and their real-valued counterparts are further compared in table 2. It can be seen that the SOPOT DCTs have similar coding gains as their real-valued counterparts.

$\mathbf{R}_\theta$	α_θ	β_θ
$\mathbf{R}_{1/4}$	$2^{0}-2^{-2}-2^{-5}$	$2^{-1}-2^{-3}+2^{-5}$
$\mathbf{R}_{1/4}$	$2^{0}-2^{-2}-2^{-5}$	$2^{-1}-2^{4}-2^{-5}+2^{-7}$
$\mathbf{R}_{1/4}$	$2^{0}-2^{-2}-2^{-5}$	$2^{-1}-2^{4}-2^{-5}+2^{-7}$
$\mathbf{R}_{1/8}$	$2^{-1}-2^{-3}$	$2^{-2}-2^{-4}$
$\mathbf{R}_{1/4}$	$2^{0}-2^{-2}-2^{-5}$	$2^{-1}-2^{-3}+2^{-5}$
$\mathbf{R}_{1/8}$	$2^{-1}-2^{-3}$	$2^{-2}-2^{-4}$
$\mathbf{R}_{1/16}$	$2^{-2}-2^{-4}+2^{-7}$	$2^{-3}-2^{-5}+2^{-8}$
$\mathbf{R}_{5/16}$	$2^{0}-2^{-3}-2^{-5}$	$2^{-1}+2^{-5}$
$\mathbf{R}_{1/4}$	$2^{-1}+2^{-2}-2^{-4}+2^{-6}$	$2^{-1}-2^{-4}-2^{-5}+2^{-7}$

Table 1a. Coefficients of the 17-channel SOPOT DCT-I

$\mathbf{R}_\theta$	α_θ	β_θ
$\mathbf{R}_{1/4}$	$2^{0}-2^{-2}-2^{-5}$	$2^{-1}-2^{-3}+2^{-5}$
$\mathbf{R}_{1/8}$	$2^{-1}-2^{-3}+2^{-7}$	$2^{-2}-2^{-4}+2^{-7}$
$\mathbf{R}_{1/16}$	$2^{-2}-2^{-4}+2^{-7}$	$2^{-3}-2^{-5}$
$\mathbf{R}_{5/16}$	$2^{0}-2^{-3}-2^{-5}$	$2^{-1}+2^{-5}$
$\mathbf{R}_{1/4}$	$2^{0}-2^{-2}-2^{-5}$	$2^{-1}-2^{-3}+2^{-5}$
$\mathbf{R}_{1/32}$	$2^{-3}-2^{-5}+2^{-8}$	$2^{-4}-2^{-6}$
$\mathbf{R}_{5/32}$	$2^{-1}-2^{-5}$	2^{-2}
$\mathbf{R}_{9/32}$	$2^{0}-2^{-2}+2^{-5}-2^{-7}$	$2^{-1}-2^{-5}+2^{-8}$
$\mathbf{R}_{13/32}$	$2^{0}-2^{-5}-2^{-6}+2^{-8}$	$2^{0}-2^{-2}-2^{-6}+2^{-8}$
$\mathbf{R}_{1/8}$	$2^{-1}-2^{-3}+2^{-7}$	$2^{-2}-2^{-4}+2^{-7}$
$\mathbf{R}_{1/4}$	$2^{-1}+2^{-2}-2^{-4}+2^{-6}$	$2^{-1}-2^{-4}-2^{-5}+2^{-7}$

Table 1b. Coefficients of the 16-channel SOPOT DCT-II and DCT-III

$\mathbf{R}_\theta$	α_θ	β_θ
$\mathbf{R}_{1/64}$	$2^{-4}-2^{-6}$	$2^{-5}-2^{-7}$
$\mathbf{R}_{5/64}$	2^{-2}	2^{-3}
$\mathbf{R}_{9/64}$	$2^{-1}-2^{-4}-2^{-7}$	$2^{-2}-2^{-5}+2^{-7}$
$\mathbf{R}_{13/64}$	$2^{-1}+2^{-3}-2^{-5}$	$2^{-2}+2^{-4}+2^{-6}+2^{-8}$
$\mathbf{R}_{17/64}$	$2^{0}-2^{-2}-2^{-7}$	$2^{-1}-2^{-4}+2^{-8}$
$\mathbf{R}_{21/64}$	$2^{0}-2^{-3}-2^{-6}$	$2^{-1}+2^{-4}-2^{-8}$
$\mathbf{R}_{25/64}$	$2^{0}-2^{-4}+2^{-7}$	$2^{-1}+2^{-2}-2^{-4}+2^{-6}$
$\mathbf{R}_{29/64}$	$2^{0}-2^{-6}+2^{-8}$	$2^{0}-2^{-3}-2^{-6}$
$\mathbf{R}_{1/16}$	$2^{-2}-2^{-4}+2^{-8}$	$2^{-3}-2^{-5}+2^{-8}$
$\mathbf{R}_{5/16}$	$2^{0}-2^{-3}-2^{-6}$	$2^{-1}+2^{-5}+2^{-8}$
$\mathbf{R}_{1/8}$	$2^{-1}-2^{-3}+2^{-7}$	$2^{-2}-2^{-4}+2^{-6}-2^{-8}$
$\mathbf{R}_{1/4}$	$2^{-1}+2^{-2}-2^{-4}+2^{-6}$	$2^{-1}-2^{-4}-2^{-5}+2^{-7}$

Table 1c. Coefficients of the 16-channel SOPOT DCT-IV

SOPOT LAPPED TRANSFORMS

As mentioned earlier, one of the applications of the proposed SOPOT DCTs/DSTs is to implement multiplier-less lapped transform and CMFB. In this section, we shall present several design examples on SOPOT Lapped Transforms. Further results on the design of different types of SOPOT CMFB will be reported elsewhere. Figure 1 shows the general structure of an *M*-channel Lapped Transform (LT) with $M = 8$. The *M*-channel LT is an *M*-channel perfect reconstruction filter bank with filter length 2*M*. The proposed SOPOT Lapped Transform (LT) is obtained by replacing the C_N^{II}, $C_{N/2}^{II}$ and $S_{N/2}^{IV}$ with their SOPOT counterparts obtained in the previous sub-section.

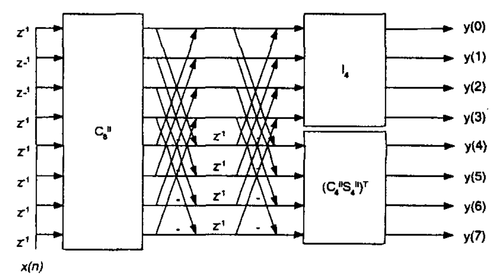

Figure 1. Flow graph of 8-channel LT.

In designing the SOPOT LT, we have used the coding gain as the performance measure.

$$G = \frac{1}{M(\prod_{i=0}^{M-1} A_k B_k)^{1/M}}, \tag{5-1}$$

where $A_k = \sum_{j=0}^{N-1}\sum_{i=0}^{N-1} h_k(j)h_k(i)\rho^{|i-j|}$ and $B_k = \frac{1}{M}\sum_{j=0}^{N-1} g_k^2(j)$.

$h_k(n)$ and $g_k(n)$ are the impulse responses of the *k*-th analysis and synthesis filters of length *N*. Here the input is assumed to be a first-order auto-regressive process with correlation coefficient $\rho = 0.95$.

The coding gains of Lapped Transform (LT) and the proposed SOPOT LT are given in table 3. It can be seen that the coding gains of the SOPOT LTs are very close to their real-valued counterparts. However, only about two additions are required to implement each multiplication in the SOPOT LT. As another illustration, the frequency response of an 8-channel SOPOT LT is given in Figure 4.

VI. CONCLUSIONS

A new family of multiplier-less discrete cosine and sine transforms called the SOPOT DCTs and DSTs is presented. They are derived from the fast algorithms of Wang [10] by parameterizing all the DCTs and DSTs in terms of certain (2x2) matrices. Using a method previously proposed by the authors [7], these (2x2) matrices are then converted to SOPOT representation. The forward and inverse transforms can then be implemented with the same set of SOPOT coefficients. A random search algorithm is also proposed to search for these SOPOT coefficients. Experimental results show that the (2x2) matrix can be implemented, on the average, in 6 to 12 additions. The proposed algorithms therefore require only $O(N\log_2 N)$ additions, which is very attractive for VLSI implementation. Using these SOPOT DCTs/DSTs, a family of SOPOT Lapped Transforms (LT) is also developed. They have similar coding gains but much lower complexity than their real-valued counterparts.

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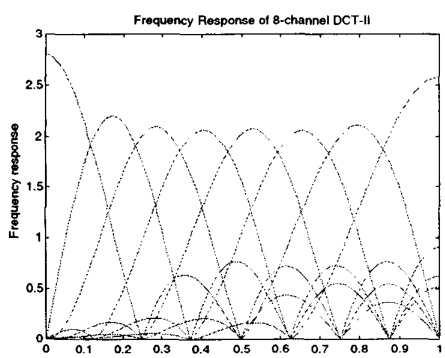

Figure 2. Frequency Response of 8-channel SOPOT DCT-II.

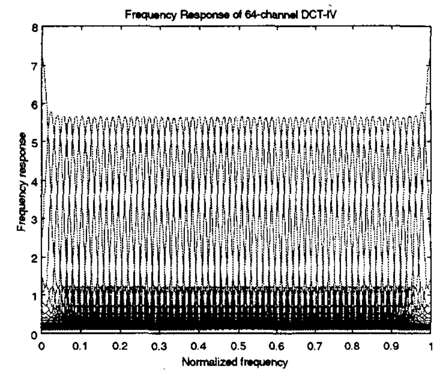

Figure 3. Frequency Response of 64-channel SOPOT DCT-IV.

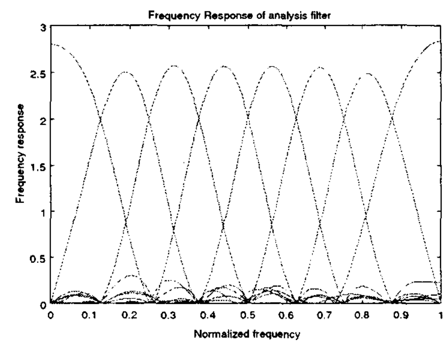

Figure 4. Frequency Response of 8-channel SOPOT LT.

	DCT-I			DCT-II & III		DCT-IV	
5-channel	3.00	(4)	4-channel	3.00	(4)	2.67	(6)
9-channel	2.88	(8)	8-channel	2.80	(10)	2.75	(12)
17-channel	2.94	(18)	16-channel	2.95	(22)	3.00	(24)
33-channel	3.50	(40)	32-channel	3.07	(46)	2.85	(48)
65-channel	3.27	(86)	64-channel	2.95	(94)	2.84	(96)
129-channel	3.10	(180)	128-channel	2.89	(190)	2.95	(94)

Table 2. The average number of SOPOT terms and number of parameters represented in SOPOT inside the brackets of the discrete cosine transform (DCT) type-I, II, III and IV.

	Gain of LOT	Gain of Multiplier-less LT	Average no. of SOPOT terms	
8-channel	9.2189	9.2135	2.81	(20)
16-channel	9.7593	9.7556	2.86	(44)
32-channel	9.9729	9.9408	3.02	(92)
64-channel	10.0541	10.0501	2.95	(188)
128-channel	10.0847	10.0819	2.91	(474)

Table 3. Coding gains of LOT and multiplier-less LT and its corresponding average number of SOPOT terms per coefficient. The number of parameters (α_n and β_n) is shown in the brackets.