THEORY AND DESIGN OF CAUSAL STABLE IIR PR COSINE-MODULATED FILTER BANKS

J. S. Mao, S. C. Chan and K. L. Ho

Department of Electrical and Electronic Engineering
The University of Hong Kong, Pokfulam Road, Hong Kong
{jsmao, scchan, klho}@eee.hku.hk

ABSTRACT

This paper proposes a novel method for designing two-channel and *M*-channel causal stable IIR PR filter banks using cosine modulation. In particular, we show that the PR condition of the two-channel IIR filter banks is very similar to the two-channel FIR case. Using this formulation, it is relatively simple to satisfy the PR condition and to ensure that the filters are causal stable. Using a similar approach, we propose a new class of *M*-channel causal stable IIR cosine modulated filter banks. Design examples are given to demonstrate the usefulness of proposed approach.

1. INTRODUCTION

Perfect reconstruction (PR) multirate filter banks have important applications in signal analysis, coding and the design of wavelet bases. Fig. 1 shows the block diagram of a *M*-channel maximally decimated filter bank. The system is called a perfect reconstruction system if the output, $\hat{x}(n)$, is identical to the input, $x(n)$, except for some constant scaling and time delay. A number of PR or nearly PR filter bank systems have been proposed and studied [1]. In the FIR filter banks, all the analysis filters and the synthesis filters are FIR filters and the PR condition is considerably simplified. In particular, the system is PR if the determinant of the polyphase matrix is equal to some delay. In case of IIR filter banks, the determinant of the polyphase matrix will in general be a minimum phase function. In addition to the more complicated PR condition, it is also very difficult to ensure that the IIR filters to be causal stable. Early attempts typically have noncausal stable filters or causal unstable filters. In [3], a new structure of two-channel IIR PR filter bank using all-pass function was proposed. Although, filters with very good stopband attenuation can be obtained, the frequency responses will exhibit a dump of about 4 dB near the transition band. Another method which is based on the transformation of a FIR prototype filter was proposed in [9]. Due to the use of the all-pass function and the transformation, both of these methods will have considerable restriction on the selection of the analysis and synthesis filters. Also, both of these approaches are difficult to generalize to the more general *M*-channel case.

The design of *M*-channel PR filter banks is inherently more difficult than the two-channel case due to the large number of design variables and PR constraints. Due to its low computational and implementation complexities and good filter quality, the cosine modulated filter bank (CMFB) has received considerably attention in recent years [2]. In particular, the design of orthogonal and birothogonal FIR CMFB has been studied in [4-6]. Attempt has also been made to employ IIR filters in the CMFB [7]. However, satisfactory design has not been obtained.

In this paper, a method for designing two-channel and *M*-channel causal stable IIR PR filter banks is proposed. In particular, we shall show that the PR condition of the two-channel IIR filter banks is very similar to the two-channel FIR case. Using this formulation, it is relatively simple to satisfy the PR condition and to ensure that the filters be causal stable. Design examples showed that causal stable PR IIR filter banks with good stopband attenuation can be obtained. Using a similar approach, we propose a new class of *M*-channel causal stable IIR filter banks using the cosine modulation. Design examples showed that causal stable PR IIR CMFB with good stopband attenuation can be obtained.

The layout of the paper is as follows: Section 2 will be devoted to the theory and design of the proposed two channel IIR PR filter banks. Their generalization to the CMFB case will be discussed in Section 3, where a number of design examples will be given. Finally, we shall summarize our results in Section 4, the conclusion.

2. TWO-CHANNEL CAUSAL STABLE IIR PR FILTER BANKS

2.1 PR Condition of IIR Filter bank

For *IIR* filter banks, the analysis and synthesis filters will be rational functions. Suppose that the analysis filters are given by,

$$H_0(z) = \frac{N_0(z)}{D_0(z)},\ H_1(z) = \frac{N_1(z)}{D_1(z)}, \tag{1}$$

where $N_i(z)$ and $D_i(z)$, $i = 0,1$, are polynomials in z. To determine its PR conditions, we need to express $H_0(z)$ and $H_1(z)$ in their polyphase representations. Observing that

$$D_i(z)D_i(-z) = \tilde{D}_i(z^2), \tag{2}$$

we can multiply the numerator and denominator of $H_i(z)$ by $D_i(-z)$ to obtain,

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$$H_i(z) = \frac{N_i(z)D_i(-z)}{D_i(z)D_i(-z)} = \frac{\tilde{N}_i(z)}{\tilde{D}_i(z^2)} . \quad (3)$$

Expressing $\tilde{N}_i(z)$ in its polyphase components:

$$\tilde{N}_i(z) = \tilde{N}_{i0}(z^2) + z^{-1}N_{i1}(z^2) , \quad (4)$$

we have

$$H_i(z) = H_{i0}(z^2) + z^{-1}H_{i1}(z^2)$$

$$= \frac{\tilde{N}_{i0}(z^2)}{\tilde{D}_i(z^2)} + z^{-1}\frac{\tilde{N}_{i1}(z^2)}{\tilde{D}_i(z^2)} . \quad (5)$$

Therefore, the polyphase matrix is given by,

$$\boldsymbol{E}(z) = \begin{bmatrix} \frac{\tilde{N}_{00}(z)}{\tilde{D}_0(z)} & \frac{\tilde{N}_{01}(z)}{\tilde{D}_0(z)} \\ \frac{\tilde{N}_{10}(z)}{\tilde{D}_1(z)} & \frac{\tilde{N}_{11}(z)}{\tilde{D}_1(z)} \end{bmatrix} . \quad (6)$$

To achieve PR, the determinant of $\boldsymbol{E}(z)$ should equal to a minimum phase function. In PR FIR filter banks, the minimum phase function will reduce to simple signal delay. In this paper, we shall limit ourselves to the latter case to simplify the overall design. In this case, we have

$$\det[\boldsymbol{E}(z)] = \frac{\tilde{N}_{00}(z)\tilde{N}_{11}(z) - \tilde{N}_{10}(z)\tilde{N}_{01}(z)}{\tilde{D}_0(z)\tilde{D}_1(z)}$$
$$= \beta \cdot z^{-d} \quad (7)$$

Equivalently, we have,

$$\tilde{N}_{00}(z)\tilde{N}_{11}(z) - \tilde{N}_{10}(z)\tilde{N}_{01}(z)$$
$$= \beta \cdot z^{-d}\tilde{D}_0(z)\tilde{D}_1(z) . \quad (8)$$

This is similar to the FIR case except that the right hand side is now a polynomial. To ensure that the filters are causal stable, the poles of $H_i(z)$ should remain inside the unit cycle. In other words, the zeros of $\tilde{D}_i(z)$ should be inside the unit cycle. For simplicity of notation, we assume that all the zeros of $\tilde{D}_i(z)$ occur in complex conjugate pairs so that

$$\tilde{D}_i(z) = K_0 \prod_{k=1}^{ND_i} (1 - p_{i,k} \cdot z^{-1})(1 - p_{i,k}^{*} \cdot z^{-1}) \quad (9)$$

Modifications to include a fixed number of real zeros are easily made. The design problem can be formulated as a constrained non-linear optimization problem which can be solved by the NCONF/DCONF subroutine in the IMSL library.

2.2 Design Procedure

Since the filter bank is biorthogonal, we have to minimize the stopband attenuation and the passband ripples of the analysis and synthesis filters. Therefore, the following object function is used :

$$\Phi = \int_0^{\omega_p} \left|1 - \left|H_0(e^{j\omega})\right|\right|^2 d\omega + \int_{\omega_s}^{\pi} \left|H_0(e^{j\omega})\right|^2 d\omega$$

$$+ \int_{\omega_s}^{\pi} \left|1 - \left|H_1(e^{j\omega})\right|\right|^2 d\omega + \int_0^{\omega_p} \left|H_1(e^{j\omega})\right|^2 d\omega .$$

Here, ω_p and ω_s are, respectively, the pass-band and stopband cutoff frequencies. Larger ω_s usually leads to larger stop-band attenuation but the overlap between adjacent filters will also be increased. The variables to be optimized are the coefficients of the polynomials $\tilde{N}_{ij}(z)$, $i, j = 0,1$, and the real and imaginary parts of $p_{i,k}$, $i = 0,1$; $k = 1,\ldots, ND_i$ ($r\_p_{i,k}$ and $i\_p_{i,k}$). Let $\boldsymbol{X}$ be the vector containing these variables, the constrained optimization can be stated as follows,

$$\min_{X} \Phi , \quad (10)$$

subjected to: $\left|p_{i,k}\right| \le 1$, $i = 0,1$, $k = 1,\ldots, ND_i$,

and the PR conditions in (8).

2.3 Initial Guess and Design Example

The convergence and computational time required for optimization are usually significantly affected by the selection of initial guess. In designing lower order two-channel IIR PR filter banks, a FIR PR filter bank with similar characteristics can first be designed and used as the numerators of the initial guess, $N_i(z)$, $i = 0,1$ in (1). The initial guess for $p_{i,k}$'s are chosen to be zero. The order of $N_i(z)$, $i = 0,1$ must be larger than that of $D_i(z)$, $i = 0,1$. Figure 2 shows a design example of a causal stable 2-channel IIR filter banks satisfying the PR conditions. The order of the numerator polynomial of the lowpass filter is 9 while that of the denominator polynomial is 4. The highpass filter has the same order as the lowpass filter. Both of them have about 40 dB of stop-band attenuation. More design examples can be found in [8].

3. *M*-CHANNEL CASUAL STABLE IIR PR CMFB

3.1 PR Condition of IIR CMFB

In CMFB, the analysis filters, $f_k(n)$, and the synthesis filters, $g_k(n)$, are obtained by modulating a prototype filter, $h(n)$.

$$f_k(n) = h(n)c_{k,n} , \; g_k(n) = h(n)\bar{c}_{k,n} ,$$
$$k = 0,1,\ldots M-1 . \quad (11)$$

M is the number of channels. The modulation we shall be using is the Extended Lapped Transform (ELT) [11]:

$$c_{k,n} = \sqrt{\frac{2}{M}} \cos\left[(2k+1)\frac{\pi}{2M}(n + \frac{M+1}{2})\right] \quad (12)$$

$\bar{c}_{k,n}$ is the time reverse of $c_{k,n}$.

Let $H_q(z)$ be the type I polyphase decomposition of the prototype filter, $h(n)$,

$$H(z) = \sum_{q=0}^{2M-1} z^{-q} H_q(z^{2M}) . \quad (13)$$

It can be shown that the frequency responses of the analysis filters can be written as [2]:

$$F_k(z) = \sum_{q=0}^{2M-1} z^{-q} c_{k,q} H_q(-z^{2M}) . \qquad (14)$$

Also, the PR condition for the general biorthogonal CMFB is given by [4-6]

$$H_k(z) H_{2M-k-1}(z) + H_{M+k}(z) H_{M-k-1}(z)$$
$$= \beta \cdot z^{-n_k} \ , \ k = 0,1,\ldots,(M/2)-1 . \qquad (15)$$

For simplicity, we assume that M is even. Similar conditions can be derived when M is an odd number. Usually n_k's are chosen to be identical for all k and it determines the delay of the filter bank. Since the analysis filters are frequency shifted versions of the prototype filter, the optimization objective function is simplified to:

$$\Phi = \int_{\omega_s}^{\pi} \left| H(e^{j\omega}) \right|^2 d\omega , \qquad (16)$$

where ω_s is the stop-band cutoff frequency whose value should be between $\frac{\pi}{2M}$ and $\frac{\pi}{M}$. For *IIR* filter banks, $H_k(z)$ will be rational functions. Using the same technique that we have developed for the two-channel case, we choose $H_k(z)$ to be

$$H_k(z) = \frac{N_k(z)}{D(z)} , \qquad k = 0,1,\ldots,2M-1 .$$

The prototype filter is then given by,

$$H(z) = \sum_{k=0}^{2M-1} z^{-k} \frac{N_k(z^{2M})}{D(z^{2M})} .$$

To ensure that the analysis and synthesis filters are causal stable, all the roots of $D(z)$ shall remain inside the unit circle. The PR condition is then given by

$$N_k(z) N_{2M-k-1}(z) + N_{M+k}(z) N_{M-k-1}(z)$$
$$= \beta \cdot z^{-n_k} D^2(z) \, , k = 0,1,\ldots,(M/2)-1 . \qquad (17)$$

Without loss of generality, we assume that $D(z)$ has the following form,

$$D(z) = K_0 \prod_{k=1}^{ND_i} (1 - p_{i,k} \cdot z^{-1})(1 - p_{i,k}^* \cdot z^{-1}) . \qquad (18)$$

3.2 Design Procedure

Let X be the vector containing the coefficients of $N_k(z)$ and the real and imaginary parts of $p_{i,k}$'s. The design problem can be formulated as the following constrained non-linear optimization problem:

$\min_X \Phi$

subjected to the PR condition in (17),

and $|p_{i,k}| \le 1 \, , i = 0,1 \, , \ k = 1,\ldots,ND_i$.

3.3 Initial Guess and Design Examples

To design the IIR CMFB, a M-Channel FIR PR CMFB with similar characteristic is first designed and used as initial guess. The initial guess for the values $p_{i,k}$'s are chosen to be zero. Fig. 3(a) shows the frequency response of the IIR prototype filter for a 4-channel PR CMFB designed using the proposed method. The order of the numerator polynomial of the prototype filter is 39 while that of the denominator is 16. It can be seen that the prototype filter has a high stopband attenuation of about 70 dB. The passband and stopband responses are flat and the roll-off is sharp. Fig. 3(b) shows the frequency response of its analysis filters. Fig. 4 shows another example where an 8-channel IIR PR CMFB was designed. Fig. 4(a) and 4(b) show the frequency responses of its IIR prototype filter and analysis filters, respectively. The order of the numerator polynomial of the prototype filter is 79 while that of the denominator is 32. The stopband attenuation of the prototype filter is about 60 dB.

4. CONCLUSION

A new method for designing two-channel and M-channel causal stable IIR PR filter banks using cosine modulation is presented. It was found that the PR condition of the two-channel IIR filter banks is very similar to the two-channel FIR case. Using this formulation, it is relatively simple to satisfy the PR condition and to ensure that the filters be causal stable. Using a similar approach, a new class of M-channel causal stable IIR cosine modulated filter banks is proposed. Design examples show that causal stable IIR filter banks with high stopband attenuation can be obtained.

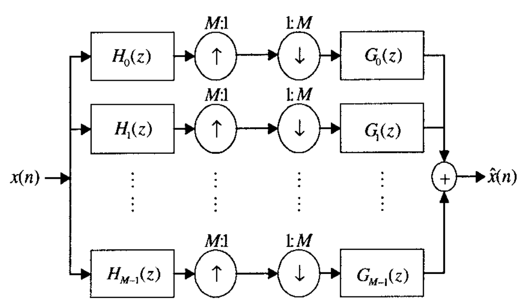

Fig. 1 M-channel maximally decimated filter banks

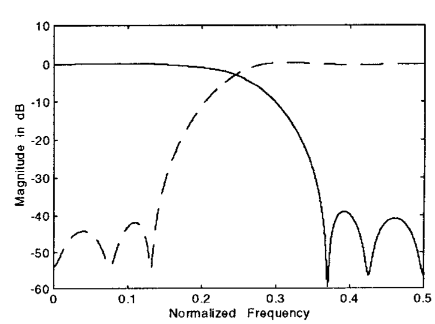

Fig. 2 Frequency response of 2-Channel IIR PR filter banks

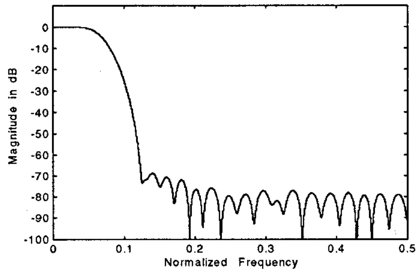

Fig. 3(a)

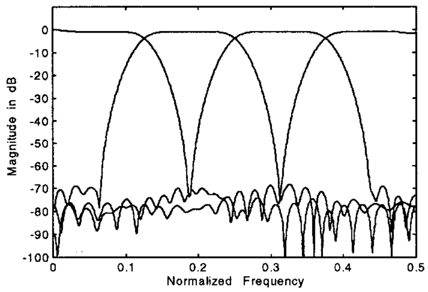

Fig. 3(b)

Fig . 3 Frequency responses of 4-channel cosine-modulated filterbanks: (a) prototype; (b)analysis filters

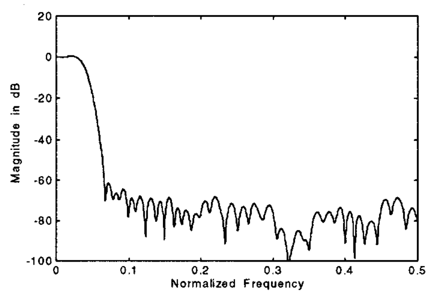

Fig. 4(a)

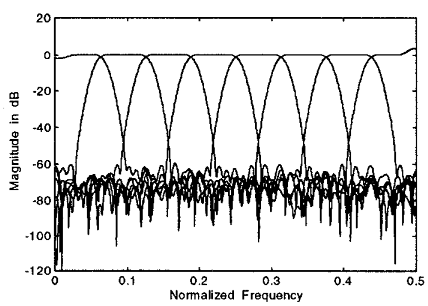

Fig. 4(b)

Fig . 4 Frequency responses of 8-channel cosine-modulated filterbanks: (a) prototype; (b) analysis filters

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