ISCAS 2000 - IEEE International Symposium on Circuits and Systems, May 28-31, 2000, Geneva, Switzerland

# DESIGN OF TWO-CHANNEL PR FIR FILTER BANKS WITH LOW SYSTEM DELAY

*J. S. Mao, S. C. Chan and K. L. Ho*

Department of Electrical and Electronic Engineering,

The University of Hong Kong, Pokfulam Road, Hong Kong

## ABSTRACT

In this paper, a new approach for designing two-channel PR FIR filter banks with low system delay is proposed. It is based on the generalization of the structure previously proposed by Phoong et al [6]. Such structurally PR filter banks are parameterized by two functions $\alpha(z)$ and $\beta(z)$ which can be chosen as linear-phase FIR or allpass functions to construct FIR/IIR filter banks with good frequency characteristics. In this paper, the more general case of using different nonlinear-phase FIR functions for $\beta(z)$ and $\alpha(z)$ is studied. As the linear-phase requirement is relaxed, higher stopband attenuation can still be achieved at low system delay. The design of the proposed low-delay filter banks is formulated as a complex polynomial approximation problem, which can be solved by the Remez exchange algorithm or analytic formula with very low complexity. The usefulness of the proposed algorithm is demonstrated by several design examples.

## 1 INTRODUCTION

Perfect reconstruction (PR) multirate filter banks have important applications in signal analysis, coding and the design of wavelet bases. Figure 1 shows the block diagram of a two-channel maximally decimated filter bank. The theory of PR filter banks has been extensively studied in the literature [1]-[6]. Because of the low delay requirement in subband coding [2], subband adaptive filtering [3] and other applications, there is a growing interest in designing PR filter banks with low system delay. Nayebi, Smith and Barnwell [7] were the first to consider the design of low-delay Perfect Reconstruction filter banks. The problem was solved using the conjugate gradient algorithm. One problem with the optimization approach is that the filter banks so obtained are in general not PR (pseudo PR). This is also a major problem of other related works based on optimization techniques [7,11]. One solution to this problem is to employ filter banks that are inherently or structurally PR. The designs of such structurally PR low-delay filter banks were recently reported in [8-10], which require unconstrained optimization of highly nonlinear objective functions.

In this paper, a new method for designing two-channel PR FIR filter banks with low system delay is proposed. It is based on the generalization of the structure previously proposed by Phoong et al [6]. Such structurally PR filter banks are parameterized by two functions $\alpha(z)$ and $\beta(z)$ which can be chosen as linear-phase FIR or allpass functions to construct FIR/IIR filter banks with good frequency characteristics. In this paper, we consider the more general case where $\alpha(z)$ and $\beta(z)$ are nonlinear-phase FIR functions. As the linear-phase requirement is relaxed, the length of $\alpha(z)$ and $\beta(z)$ are no longer restricted by the delay parameters of the filter banks. Hence, higher stopband attenuation can still be achieved at low system delay. The design of the proposed low-delay filter banks is formulated as a complex polynomial approximation problem, which can be solved by the Remez exchange algorithm or analytic formula with very low complexity. Another advantage of the proposed low-delay filter bank over the optimization approach [7,11] is that it is PR even under coefficient quantization. The proposed technique is also applicable to the case where $\alpha(z)$ and $\beta(z)$ are all-pass and type-II linear-phase functions, respectively [4]. These filter banks are very attractive because of their low design and implementation complexities and capable of suppressing the bumping problem in the transition band [4,6].

The paper is organized as follows: in section 2, a brief summary of the two-channel structurally PR filter bank proposed in [6] is given together with the basic idea behind the proposed low-delay FIR filter banks. Details of the proposed design method and several design examples are given in section 3 Finally, the conclusions are drawn in section 4.

## 2 TWO-CHANNEL STRUCTURALLY PR FILTER BANKS

Consider a two-channel critically decimated multirate filter bank as shown in Fig. 1. The aliasing is eliminated if the synthesis filters are chosen as $G_0(z) = -H_1(-z)$ and $G_1(z) = H_0(-z)$, where $H_0(z)$ and $H_1(z)$ are respectively the low-pass and high-pass analysis filters. In [4,6], the analysis filters are chosen as

$$H_0(z) = \frac{(z^{-2N} + z^{-1}\beta(z^2))}{2}, \tag{2-1}$$

$$H_1(z) = -\alpha(z^2)H_0(z) + z^{-2M-1}. \tag{2-2}$$

It can be seen from (2-1) and (2-2) that the PR condition is satisfied for any choices of $\alpha(z)$ and $\beta(z)$. In [4,6], FIR/IIR filter banks are obtained by choosing $\beta(z)$ and $\alpha(z)$ as causal stable all-pass functions and type-II linear-phase functions, respectively. The special case of using identical $\alpha(z)$ and $\beta(z)$ is studied and the delay parameter $M$ is chosen as $2N-1$. The design of $\beta(z)$ (and $\alpha(z)$) can be accomplished by noting that $H_0(z)$ (and $H_1(z)$) will become ideal lowpass filters if $\beta(z)$ (and $\alpha(z)$) has the following magnitude and phase responses

$$\beta(e^{j2\omega}) = \begin{cases} e^{-j(2N-1)\omega} & for \quad \omega \in [0, \pi/2] \\ e^{-j(2N-1)\omega \pm \pi} & for \quad \omega \in (\pi/2, \pi]. \end{cases} \tag{2-3}$$

To design a linear-phase FIR filter bank, $\beta(z)$ has to be a type-II linear-phase function with length $N_\beta = 2N$, an even number. Furthermore, if $\alpha(z) = \beta(z)$, the delay parameter $M$ must be equal

0-7803-5482-6/99/$10.00 '2000 IEEE

to $(2N-1)$. It can be seen that linear-phase FIR filter bank with $\alpha(z) \neq \beta(z)$ can also be designed by properly choosing the delay parameter $M$ and the length of $\alpha(z)$, $N_\alpha$. This has previously been reported and studied by a number of authors [4,5]. More precisely, the delay parameter $M$ is equal to $N + N_\alpha/2 - 1$, and the total system delay $n_0$ is therefore equal to $2N + 2M + 1$. In the proposed low-delay filter bank, $\beta(z)$ and $\alpha(z)$ are nonlinear-phase FIR functions. The system delay is still given by $n_0 = 2N + 2M + 1$, but the length of $\beta(z)$ and $\alpha(z)$, $N_\beta$ and $N_\alpha$ can be greater than $2N$ and $2(M - N + 1)$, unlike their linear-phase constraints. Therefore, higher stopband attenuation can still be achieved at low system delay.

## 3 THE PROPOSED METHOD

### 3.1 Design of Lowpass Filter

Let $\omega_{p_0}$ and $\omega_{s_0}$ be respectively the passband and stopband cutoff frequencies of $H_0(z)$. Similarly, let $\omega_{p_1} = \omega_{s_0} = \omega_s$ and $\omega_{s_1} = \omega_{p_0} = \omega_p$ be the passband and stopband cutoff frequencies of $H_1(z)$. The desired frequency response of $H_0(z)$ is:

$$H_d(e^{j\omega}) = \begin{cases} e^{-j2N\omega} & 0 \le \omega \le \omega_p \\ 0 & \omega_s < \omega \le \pi \end{cases}. \quad (3\text{-}1)$$

Where $N$ is a positive integer. In a low-delay filter bank, the delay parameter $2N$ will in general be smaller than $N_\beta$, the length of $\beta(z)$, i.e. $N < N_\beta/2$. To design $\beta(z)$ with nonlinear-phase, let's decompose it into its even and odd parts:

$$\beta(z) = \beta_e(z) + \beta_o(z). \quad (3\text{-}2)$$

As the length of $\beta(z)$ is even, $\beta_e(z)$ and $\beta_o(z)$ are type-II and type-IV linear-phase FIR functions, respectively. Similarly, when the length of $\beta(z)$ is odd, it can be decomposed into type-I and type-III FIR functions. For simplicity, only the even length case will be considered here. Detail of the odd length case can be found in [14]. For the even length case, $\beta_e(z)$ and $\beta_o(z)$ can be expressed in terms of the following polynomials,

$$\beta_e(e^{j\omega}) = e^{-j\omega(N_\beta - 1)/2}\cos(\omega/2)P_e^{(\beta)}(\cos\omega),$$

$$\beta_o(e^{j\omega}) = je^{-j\omega(N_\beta - 1)/2}\sin(\omega/2)P_o^{(\beta)}(\cos\omega). \quad (3\text{-}3)$$

Where

$$P_e^{(\beta)}(\cos\omega) = \sum_{k=0}^{L_\beta} a_k(\cos\omega)^k, \; P_o^{(\beta)}(\cos\omega) = \sum_{k=0}^{L_\beta} b_k(\cos\omega)^k.$$

$L_\beta$ is the order of the polynomials $P_e^{(\beta)}(x)$ and $P_o^{(\beta)}(x)$, and is equal to $(N_\beta/2) - 1$. The error function $E(\omega) = H_0(e^{j\omega}) - H_d(e^{j\omega})$ is then given by

$$E(\omega) = \frac{1}{2}e^{-j2\omega N}\left\{e^{-j2\omega N_\Delta}P^{(\beta)}(e^{j\omega}) - 1\right\}, \; \omega \in [0, \pi/2], \quad (3\text{-}4)$$

Where $N_\Delta = N_\beta/2 - N$, and

$$P^{(\beta)}(e^{j\omega}) = \cos\omega \cdot P_e^{(\beta)}(\cos(2\omega)) + j \cdot \sin\omega \cdot P_o^{(\beta)}(\cos(2\omega)).$$

The desired response of $P^{(\beta)}(e^{j\omega})$ is seen to be $P_d^{(\beta)}(e^{j\omega}) = e^{j2\omega N_\Delta}$, which is recognized as a complex polynomial approximation problem. This can be solved by a variety of methods [12,15]. For simplicity, the complex polynomial is decomposed into its real and imaginary parts and is solved by two independent real Chebyshev approximations [13]. The desired functions of $P_e^{(\beta)}(\cos(2\omega))$ and $P_o^{(\beta)}(\cos(2\omega))$ are then given by

$$\hat{P}_e^{(\beta)}(\cos(2\omega)) = \cos(2\omega N_\Delta)/\cos\omega,$$

$$\hat{P}_o^{(\beta)}(\cos(2\omega)) = \sin(2\omega N_\Delta)/\sin\omega, \qquad 0 < \omega < \pi/2. \quad (3\text{-}5)$$

Writing $x = \cos(2\omega)$, the two Chebyshev approximation problems can be written as,

$$a_{k,opt} = \arg\min_{a_k}\max\left|W_e(x)(P_e^{(\beta)}(x) - \hat{P}_e^{(\beta)}(x))\right|,$$

$$b_{k,opt} = \arg\min_{b_k}\max\left|W_o(x)(P_o^{(\beta)}(x) - \hat{P}_o^{(\beta)}(x))\right|, \quad (3\text{-}6)$$

Where $\tilde{x}_s < x_s = \cos(2\omega_p)$, $I_x = (-1, \tilde{x}_s] \cup [x_s, 1]$, and

$$W_e(x) = \cos(0.5 \cdot \arccos(x)), \quad W_o(x) = \sin(0.5 \cdot \arccos(x)). \quad (3\text{-}7)$$

The interval $(0, \tilde{x}_s]$ is an optional disjoint interval to control the values of $P_e^{(\beta)}(\cos(2\omega))$ and $P_o^{(\beta)}(\cos(2\omega))$ in the transition band of $H_0(e^{j\omega})$. (3-6) can readily be solved using the function REMEZ in the signal processing Toolbox of MATLAB.

### 3.2 Design of Highpass Filter

It can be seen from (2-2) that the frequency response of $H_1(z)$ depends on both the lowpass filter $H_0(z)$ and the function $\alpha(z)$. The ideal frequency response of $H_1(e^{j\omega})$ is :

$$H_d(e^{j\omega}) = \begin{cases} 0 & 0 \le \omega \le \omega_p \\ e^{-j\omega(2M+1)} & \omega_s \le \omega \le \pi \end{cases}. \quad (3\text{-}8)$$

The error function $E(\omega)$ of the highpass filter $H_1(z)$ is defined as

$$E(\omega) = e^{-j\omega(2M+1)} - \alpha(e^{2j\omega})H_0(e^{j\omega}) - H_d(e^{j\omega}). \quad (3\text{-}9)$$

(3-9) implies that the magnitude of $\alpha(e^{2j\omega})$ is almost equal to one, except around $\omega = \pi/2$ where it is even smaller. It then follows from (2-2) that the passband ripple of $H_1(e^{j\omega})$ is approximately equal to the stopband error of $H_0(e^{j\omega})$. This allows us to minimize only the stopband attenuation of $H_1(e^{j\omega})$ using $\alpha(e^{2j\omega})$, instead of minimizing (3-9) over the pass- and stop-bands, and relies on the high stopband attenuation of $H_0(e^{j\omega})$ to achieve a small passband ripple. First of all, let's consider the case where $\alpha(z)$ is an even-length FIR filter with length $N_\alpha$. The odd case can be derived similarly and will be described in [14]. Using again the even and odd parts decomposition, $\alpha(z)$ can be expressed as $\alpha_e(z)+\alpha_o(z)$, where

$$\alpha_e(e^{j\omega}) = e^{-j\omega(N_\alpha-1)/2}\cos(\omega/2)P_e^{(\alpha)}(\cos\omega),$$
$$\alpha_o(e^{j\omega}) = je^{-j\omega(N_\alpha-1)/2}\sin(\omega/2)P_o^{(\alpha)}(\cos\omega). \quad (3\text{-}10)$$

and $P_e^{(\alpha)}(\cos\omega) = \sum_{k=0}^{L_\alpha} \tilde{a}_k(\cos\omega)^k$, $P_o^{(\alpha)}(\cos\omega) = \sum_{k=0}^{L_\alpha} \tilde{b}_k(\cos\omega)^k$.

$L_\alpha$ is the order of the polynomials and is equal to $(N_\alpha/2)-1$.

Let the lowpass filter $H_0(e^{j\omega})$ be written as

$$H_0(e^{j\omega}) = A(e^{j\omega})e^{-j2\omega N}, \quad (3\text{-}11)$$

Where $A(e^{j\omega})$ is a complex function and is approximately equal to one for $\beta(\omega)$ with sufficient high order. Substituting (3-8), (3-10) and (3-11) into (3-9), one obtains

$$E(\omega) = e^{-j\omega(2M+1)}\left\{1 - e^{-j2\omega M_\Delta}P^{(\alpha)}(e^{j\omega})\cdot A(e^{j\omega})\right\}, \quad \omega\in[0,\pi/2], \quad (3\text{-}12)$$

Where $M_\Delta = (N_\alpha/2)+(N-M)-1$, and

$$P^{(\alpha)}(e^{j\omega}) = \left(\cos\omega\cdot P_e^{(\alpha)}(\cos(2\omega)) + j\cdot\sin\omega\cdot P_o^{(\alpha)}(\cos(2\omega))\right). \quad (3\text{-}13)$$

The ideal response of $P^{(\alpha)}(e^{j\omega})$ is

$$P_d^{(\alpha)}(e^{j\omega}) = e^{j2\omega M_\Delta}A^{-1}(e^{j\omega}), \quad (3\text{-}14)$$

$$\hat{P}_e^{(\alpha)}(\cos(2\omega)) = \mathrm{Re}\left(A^*(e^{j\omega})e^{j2\omega M_\Delta}\right)/\left(\left|A(e^{j\omega})\right|^2\cos\omega\right), \quad (3\text{-}15)$$

$$\hat{P}_o^{(\alpha)}(\cos(2\omega)) = \mathrm{Im}\left(A^*(e^{j\omega})e^{j2\omega M_\Delta}\right)/\left(\left|A(e^{j\omega})\right|^2\sin\omega\right). \quad (3\text{-}16)$$

Writing $x=\cos(2\omega)$, the problem can be solved using the Chebyshev approximation as given in (3-6), with weighting functions

$$\tilde{W}_e(x) = (\cos(0.5\cdot\arccos(x))\left|A(e^{j0.5\arccos(x)})\right|^2),$$

$$\tilde{W}_o(x) = (\sin(0.5\cdot\arccos(x))\left|A(e^{j0.5\arccos(x)})\right|^2). \quad (3\text{-}17)$$

They again can be solved by the Remez exchange algorithm.

### 3.3 Design Examples

*Example 1:* In this design example, $\beta(z)$ and $\alpha(z)$ are non linear-phase FIR functions with lengths $N_\beta=8$ and $N_\alpha=10$. The orders of the polynomials $P_e^{(\beta)}(x)$ and $P_o^{(\beta)}(x)$ are $L_\beta=3$, and the orders of $P_e^{(\alpha)}(x)$ and $P_o^{(\alpha)}(x)$ are $L_\alpha=4$. The delay parameters $N$ and $M$ are respectively 2 and 5. The overall system delay is $n_0=15$ samples. Figure 2(a) plots the impulse responses of $\beta(z)$ and $\alpha(z)$ designed by the proposed method using the Remez exchange algorithm. Figure 2(b) displays the magnitude responses of $\beta(z)$ (solid line) and $\alpha(z)$ (dashed line). Figures 2(c) and 2(d) display, respectively, the magnitude and the group delay responses of the analysis filters $H_0(z)$ and $H_1(z)$. It can be seen from figure 2(c) that the stopband attenuation of $H_0(z)$ and $H_1(z)$ are about 43 dB and 40 dB, respectively, and the passband and stopband cutoff frequencies are $\omega_p=0.34\pi$ and $\omega_s=0.66\pi$. Both $H_0(z)$ and $H_1(z)$ are approximately linear phase in their passbands. In order to compare the proposed method with the conventional methods in [7,8], the cutoff frequencies and system delay are chosen to be the same as the design examples in [Fig.5, 7] and [Fig.3, 8]. The stopband attenuations of analysis filters in [7] are about 39 dB. The stopband attenuations of $H_0(z)$ and $H_1(z)$ in [8] are respectively 45 dB and 40 dB. Therefore the proposed analysis filters have comparable passband and stopband performance as that in [7] and [8]. By adjusting the cutoff frequencies to $\omega_p=0.24\pi$ and $\omega_s=0.76\pi$, and $\omega_p=0.4\pi$ and $\omega_s=0.6\pi$, the proposed method can still provide comparable stopband attenuation as that in [Fig.1, 9] (50 dB vs. 51 dB), and higher stopband attenuation than that in [Fig.2, 11] (30 dB vs. 25 dB). Moreover, the design complexity of the proposed method is very low and there is no reconstruction error, which is always present in other methods based on constrained nonlinear optimization [7][11]. Furthermore, as mentioned earlier, the proposed filter bank structure is more robust to coefficient quantization than that in [8]. Compared with the unconstrained nonlinear optimization method in [9,10], the proposed method is very simple to apply and filter banks with very good frequency characteristics can be obtained. Figure 3 displays the magnitude responses of the analysis filters $H_0(z)$ and $H_1(z)$ designed by the proposed least squares method. The stopband attenuation of $H_0(z)$ and $H_1(z)$ are approximately the same as the minimax design in Figure 3, but the passband ripples of $H_0(z)$ and $H_1(z)$ around $\omega=0$ or $\omega=\pi$ are lower in the least squares case.

*Example 2:* In this example, a two-channel low-delay FIR filter bank with higher order is designed. The lengths of $\beta(z)$ and $\alpha(z)$ are chosen to be odd numbers, $N_\beta=13$ and $N_\alpha=15$. The orders of the corresponding Chebyshev polynomials $P_e^{(\beta)}(x)$ and $P_o^{(\beta)}(x)$ are $L_{\beta_e}=6$, and $L_{\beta_o}=5$, respectively, while those of $P_e^{(\alpha)}(x)$ and $P_o^{(\alpha)}(x)$ are $L_{\alpha_e}=7$ and $L_{\alpha_o}=6$. The overall system delay of the filter bank is $n_0=23$ samples with $N=3$ and $M=8$. Figure 4 plots the magnitude responses of the analysis filters $H_0(z)$ and $H_1(z)$. It can be seen that the stopband attenuation of $H_0(z)$ and $H_1(z)$ is about 39 dB and their passband and stopband cutoff frequencies are respectively $\omega_p=0.41\pi$ and $\omega_s=0.59\pi$. It is also observed that due to the higher system delay, the transition band is sharper than that of example 1.

## 4 CONCLUSION

In this paper, a new approach for designing two-channel PR FIR filter banks with low system delay is presented. It is based on the use of nonlinear-phase FIR functions in the PR structure previously proposed by Phoong et al [6]. Because the linear-phase requirement is relaxed, the lengths of $\beta(z)$ and $\alpha(z)$ are no longer restricted by the delay parameters of the filter banks. Hence, higher stopband attenuation can be achieved at low system delay. The design of the proposed low-delay filter banks is formulated as a complex polynomial approximation problem, which is solved by the Remez exchange algorithm or analytic formula with very low complexity. Several design examples are given to demonstrate the usefulness of the proposed method.

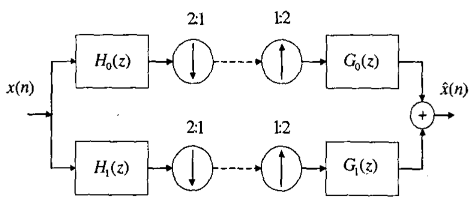
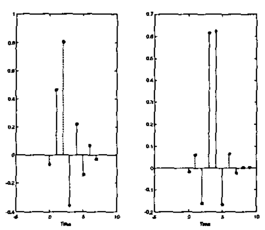



Fig. 1 Two-channel maximally decimated multirate filter bank.



$\beta(z)$ $\alpha(z)$

Fig. 2(a)

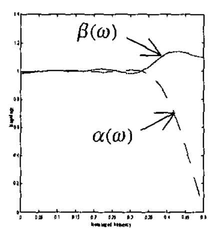



Fig. 2(b)

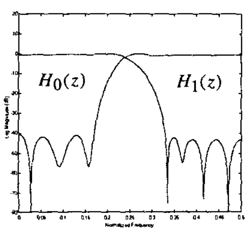



Fig. 2(c)

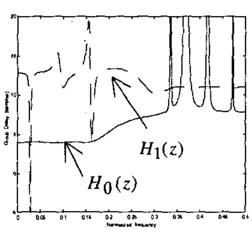



Fig. 2(d)

Fig. 2 Analysis filters in example 1 designed by the Remez exchange algorithm (a) impulse responses of $\beta(z)$ and $\alpha(z)$; (b) magnitude responses of $\beta(z)$ and $\alpha(z)$; (c) magnitude responses of analysis filters $H_0(z)$ and $H_1(z)$; (d) their group delay responses.

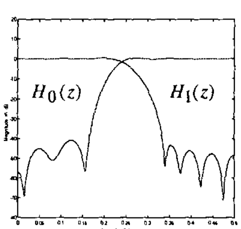



Fig. 3

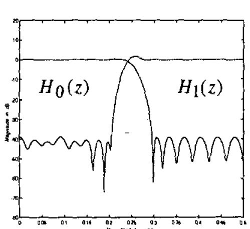



Fig. 4

Fig. 3 Magitude responses of $H_0(z)$ and $H_1(z)$ in example 1 designed by least squares criteria.

Fig. 4 Magitude responses of $H_0(z)$ and $H_1(z)$ in examlpe 2 designed by Remez Exchange algorithm.

# REFERENCES


[1] P. P. Vaidyanathan, *Multirate systems and filter banks.* Englewood Cliffs, NJ: Prentice Hall, 1992.

[2] M. Vetterli and J. Kovacevic, *Wavelets and subband coding.* Englewood Cliffs, NJ: Prentice Hall, 1995.

[3] A. N. Akansu and M. J. T. Smith, *Subband and Wavelet Transforms: Design and Applications.* Kluwer Academic Publishers, Boston: 1996.

[4] C. W. Kim and R. Ansari, "FIR/IIR exact reconstruction filter bank with applications to subband coding of images," *Proc. 34th Midwest CAS Symp.*, Vol. 1, pp.227-230, Monterey, CA, May 1991.

[5] H. Kiya, M. Yae, and M. Iwahashi, "A linear-phase two-channel filter bank allowing perfect reconstruction," *Proc. IEEE ISCAS'1992*, Vol. 2, pp.951-954, May 1992.

[6] S. M. Phoong, C. W. Kim, P. P. Vaidyanathan and Rashid Ansari, "A new class of two-channel biorthogonal filter banks and wavelet bases," *IEEE Trans. Signal Processing*, Vol.43, No. 3, pp.649-664, March 1995.

[7] K. Nayebi, T. P. Barnwell III, Mark J. T. Smith, "Low delay FIR filter banks: design and evaluation," *IEEE Trans. Signal Processing*, Vol.42, No. 1, pp.24-31, Jan. 1994.

[8] H. Ochi, M. Ohta, S. Kinjo, "Linear Programming Design of Two-Channel Perfect-Reconstruction Biorthogonal Filter Banks -Linear-phase and Low-delay -," *Proc. IEEE 40th Midwest Symposium on Circuits and Systems*, Vol. 2, pp.969 – 972, Aug. 1997.

[9] G. Schuller, M. J. T. Smith, "A new algorithm for efficient low delay filter bank design," *Proc. IEEE ICASSP'95*, Vol. 2, pp.1472 – 1475, May 1995.

[10] R. Gandhi, S. K. Mitra, "Design of Two-Channel Low Delay Perfect Reconstruction Filter Banks," *Proc. 32nd International Asilomar Conference on Signals, Systems & Computers*, Vol. 2, pp1655 – 1659, Nov. 1998.

[11] S. J. Yang, J. H. Lee, B. C. Chieu, "Minimax design of two-channel low-delay perfect-reconstruction FIR filter banks," *IEE Proc. –Vis. Image Signal Process.*, Vol. 146, No. 1, pp.15-24, Feb 1999.

[12] L. J. Karam, J. H. McClellan, "Complex Chebyshev approximation for FIR filter design," *IEEE Trans. Circuits and Systems II: Analog and Digital Signal Processing*, Vol. 42, No. 3, pp207-216, Mar. 1995.

[13] S. C. Pei, J. J. Shyu, "Design of complex FIR filters with arbitrary complex frequency responses by two real Chebyshev approximations," *IEEE Trans. Circuits and Systems I: Fundamental Theory and Applications*, Vol. 44, No. 2, pp.170-174, Feb. 1997.

[14] J. S. Mao, S. C. Chan and K. L. Ho, "Design and Multiplierless Implementation of a Class of two-Channel PR FIR Filter Banks and wavelets with Low System Delay," submitted for publication, Oct. 1999.

[15] T.W. Parks and C. S. Burrus, *Digital filter design.* New York: Wiley, 1987.