DECOMPOSITION OF NETWORK OF QUEUES WITH SELF-SIMILAR TRAFFIC

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Abstract

Jackson's network of queues model greatly simplifies the performance analysis of telecommunication networks with Poisson traffic arrivals and exponential service times. It reduces the analysis of a network into the analysis of individual communication links, each of which may be modeled as an M/M/m queue. Motivated by the growing significance of self-similar traffic in modeling broadband network traffic, we propose a new network of queues model for telecommunication networks. Our model resembles Jackson's model except that the arrival is self-similar and the service time is deterministic. It captures the characteristics of modern high speed cell-based networks. We hypothesize a result analogous to Jackson's Theorem, that each node of this network model behaves as a G/D/1 queue with self-similar arrival. Based on this hypothesis, many network-wide performance measures, such as the end-to-end delay, can be evaluated in a simple fashion. Our hypothesis is strongly supported by three facts, namely, the sum of independent self-similar processes, the random splitting of self-similar processes, and the output process of a deterministic service time queue with self-similar input are all self-similar.

Keywords: Self-similar traffic, Jackson's Theorem, network of queues.

1 Introduction

Traditional telecommunication traffic engineering has been based on the assumptions of exponential distributions and Markovian models. The memoryless properties such models possess lead to high model tractability. Performance models based on these assumptions have proved to be surprisingly successful. The main reason is that these assumptions are actually plausible in many practical situations. For instance, the use of Poisson processes to describe call arrivals in circuit-switched networks has been validated by empirical studies. In the past twenty years, packet-switched networks such as X.25, Frame Relay and Asynchronous Transfer Mode (ATM) have become more and more important. Traditional queueing analysis is continually being applied on such packet-switched networks. However, the validity of these Markovian assumptions on traffic processes has not been verified against real traffic data.

This situation has changed significantly over the past few years as many traffic measurements show that actual traffic in various packet-switched networks can be clearly distinguished from traffic generated from traditional traffic models. In particular, it has been demonstrated that these traffic exhibit so-called self-similarity. These studies include investigations of high-resolution Ethernet LAN traces [1, 2], Wide Area Network (WAN) traffic[3], ISDN traffic (16kps)[4], VBR video traffic[5, 6, 7] and many others. It has also been shown in many of the above studies that Markovian based models fail to predict the queueing performance of systems with self-similar traffic. Traditional traffic models tend to overestimate the system performance.

In [8], Taqqu et al. studied the underlying processes which lead to self-similar traffic. They have shown that the superposition of many on/off sources with strictly alternating on- and off-periods and whose on- and off-periods exhibit the *Noah Effect* (i.e., have high variability or infinite variance) can produce aggregate network traffic that exhibits the *Joseph Effect* (i.e., is self-similar or long-range dependent). This explains why many actual traffic patterns in high-speed networks show self-similarity. In fact, it is reasonable to expect that self-similar traffic will constitute a major component of the traffic in future broadband networks. This has led many researchers to study a brand new area in queueing theory, fractal queueing.

Since fractal queueing is still in its infant stage, there is not much analytical result available in the literature. Moreover, most studies have focused on the performance of a single queue with self-similar arrival. For example, Norros and his colleagues[9, 10, 11] have developed some analytical results for queues driven by Fractional Brownian Motion (FBM) arrival processes (A simple mathematical model for self-similar random processes). In this paper, we take one step further and study networks of queues with self-similar traffic.

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We propose a network model that resembles the Jackson's network in classical queueing theory. Jackson's network of queues model greatly simplifies the performance analysis of telecommunication networks with Poisson traffic arrivals and exponential service times. It reduces the analysis of the network into the analysis of individual communication links, each of which may be modeled as an M/M/m queue. In our proposed model, instead of Poisson arrivals, we have self-similar arrivals. Instead of exponential service times at each node, we have deterministic service times since the transmission time of a fixed size packet (called a cell in ATM networks) is deterministic. We hypothesize a result similar to Jackson's Theorem. Our hypothesis is based on three supporting facts regarding self-similar processes. These three facts are: the sum of independent self-similar processes is self-similar, the random splitting of self-similar process results in self-similar processes, and the output process of a G/D/1 queue with self-similar arrival is self-similar. Together with the queueing performance of a single fractal queue, our hypothesis provides a simple way of solving fractal queueing networks. We believe this will be extremely useful in the performance analyses of both current and future high speed packet-switched networks.

This paper is organized as follows. Section 2 describes the basic definitions of self-similar random processes. In Section 3, we present the legacy model for telecommunication networks and then propose a modern version of this model. In Section 4, we describe our hypothesis and its implication, together with the three supporting facts. We conclude our paper in Section 5.

2 Self-similar random processes

We adopt the definition of self-similar random processes in [12]. Let $X = \{X_t, t = 1, 2, ...\}$ be a semi-infinite segment of a covariance-stationary stochastic process of discrete argument t, $t \in I_1 = \{1, 2, ...\}$. Let $\mu = EX_t$ and $x_t = X_t - \mu$. The process X and $x = \{x_t\}$ share the same autocorrelation function:

$$r(k) = \frac{E\{x_t x_{t+k}\}}{E x_t^2} \quad (1)$$

They also have the same variance, denoted by σ^2. Now define

$$X_t^{(m)} = \frac{1}{m} \sum_{i=1}^{m} X_{(t-1)m+i} \quad (2)$$

The process $X^{(m)} = \{X_t^{(m)}, t = 1, 2, ...\}$ is called the aggregated process of X. Let $r^{(m)}(k)$ and $\sigma^{(m)2}$ denote the autocorrelation function and variance of $X^{(m)}$.

According to [12], a second-order self-similar process can be defined as follows.

Definition: Process X is called *exactly second-order self-similar* (e.s.o.s.s.) with parameter $H = 1 - \beta/2$, $0 < \beta < 1$, iff its autocorrelation $r(k)$ is:

$$r(k) = g(k) \triangleq \frac{1}{2}[|k-1|^{2-\beta} - 2|k|^{2-\beta} + |k+1|^{2-\beta}] \quad (3)$$

There are several other equivalent definitions. One of them relates the variances of the process and its aggregated process:

$$\sigma^{(m)2} = \sigma^2 m^{-\beta} \quad (4)$$

The discrete time Fourier Transform (DTFT) of $g(k)$ is given by

$$h(\lambda) \triangleq \frac{\sin^2 \pi\lambda \sum_{\ell=-\infty}^{\infty} |\lambda + \ell|^{\beta-3}}{\int_{-\frac{1}{2}}^{\frac{1}{2}} \sin^2 \pi\mu \sum_{\ell=-\infty}^{\infty} |\mu + \ell|^{\beta-3} d\mu} \quad (5)$$

Hence, in the frequency domain, e.s.o.s.s. process has a power spectrum $f_x(\lambda)$, defined as the DTFT of $\sigma^2 r(k)$, of the following form:

$$f_x(\lambda) = \sigma^2 h(\lambda) \quad (6)$$

There is another concept called asymptotically second-order self-similarity. In [12], it is shown that

$$\lim_{k\to\infty} \frac{r(k)}{k^{-\beta}} = \text{constant} \quad (7)$$

implies

$$\lim_{m\to\infty} \frac{\sigma^{(m)2}}{m^{-\beta}} = \text{constant} \quad (8)$$

which further implies

$$\lim_{m\to\infty} r^{(m)}(k) = g(k) \quad (9)$$

In addition, (9) is used as the definition of asymptotically second-order self-similarity. In this paper, we choose to define it using (8), which can be thought of as a generalization of the definition of e.s.o.s.s. process given in (4).

Definition: Process X is called *asymptotically second-order self-similar* (a.s.o.s.s.) with parameter $H = 1 - \beta/2, 0 < \beta < 1$, iff (8) holds.

Self-similarity manifests itself in a number of ways. The three most common properties are (1) slowly decaying variance, (2) long range dependency, and (3) $1/f$-noise. Interested readers are referred to [13] and the references there of.

There are many formal mathematical models for self-similar random processes. Two representative models are the fractional Gaussian noise[14] and the fractional autoregressive integrated moving-average (ARIMA) processes[15, 16]. Our results in this paper do not depend on any specific mathematical models. It applies to any second-order self-similar process.

3 A new model for telecommunication networks

3.1 Legacy model for telecommunication networks

The legacy model for telecommunication networks is best represented by the Jackson's network of queues. Jackson [17] studied an arbitrary network of queues, in which there are N nodes, where the ith node consists of m_i exponential servers of service rates $1/\mu_i$. Moreover, the external arrival to the ith node is a Poisson process at rate γ_i. Upon finishing service at node i, a customer will be routed to node j with probability r_{ij}, or departs the network with probability $1-\sum_{j=1}^{N} r_{ij}$. The overall arrival rates to a given node i, denoted by λ_i, can be determined by solving the following set of equations:

$$\lambda_i = \gamma_i + \sum_{j=1}^{N} \lambda_i r_{ji} \qquad i = 1, 2, ..., N \qquad (10)$$

The amazing result that Jackson showed is that each node in the network behaves as if it were an independent M/M/m system with arrival rate λ_i. Let the states of the system be $(k_1, k_2, ..., k_N)$ where k_i denotes the number of customers at node i and $p(k_1, k_2, ..., k_N)$ be the equilibrium probability of this state. Let $p_i(k_i)$ be the marginal probability distribution of having k_i customers at node i. Jackson's Theorem states that the joint distribution for all nodes in the network is equal to the product of each of the marginal distribution. In other words,

$$p(k_1, k_2, ..., k_N) = p_1(k_1) p_2(k_2) \cdots p_N(k_N) \qquad (11)$$

where $p_i(k_i), i = 1, 2, ..., N$ has the same distribution as the number of customers in an M/M/m system with arrival rate λ_i.

Jackson's Theorem has been found very useful in conventional queueing analysis and telecommunication traffic engineering.

3.2 The self-similar network

Motivated by the fact that real network traffic exhibits self-similarity, we propose a new model for telecommunication networks, called the *self-similar network*. Figure 1 shows a telecommunication network consisting of N nodes. The model is basically the same as Jackson's network. There are external arrivals of packets to each node. Upon completion of service at node i, a packet will be routed to another node j with probability r_{ij} so that the probability for it to depart from the network is equal to $1-\sum_{i=1}^{N} r_{ij}$. However, there are several differences to capture the characteristics of modern telecommunication systems.

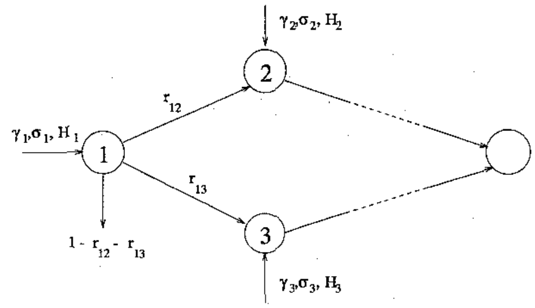

Figure 1: Proposed model for telecommunication networks.

Firstly, instead of external Poisson arrivals, we have second-order self-similar arrival processes, which can be characterized by the means γ, variances σ^2 and Hurst parameters H. Secondly, instead of exponential service time, we have deterministic service time since the time needed to transmit a fixed-size packet (or cell in an ATM network) is constant. Thus if $N = 1$, we have a $G/D/1$ system where the arrival process is general but self-similar. We further assume that packets are served in a FCFS basis.

4 The Decomposition Hypothesis

Based on three facts (to be discussed in the following) regarding self-similar traffic, we propose the following hypothesis.

The Decomposition Hypothesis: Each node of a self-similar network behaves as a G/D/1 queue with second order self-similar arrival.

This hypothesis provides an efficient way to evaluate many network-wide performance measures in a modern Jackson network. Without the hypothesis, there is currently no simple way of obtaining network-wide performance for such a network. To illustrate the utility of this hypothesis, consider the problem of finding the end-to-end average delay T_{ij} between node i and node j in an N-node network. Let t_{ij} denote the average delay on the link between node i and node j, assuming there is such a direct link. T_{ij} can be obtained by solving the following set of equations.

$$T_{ij} = \sum_{(i,k)\in\mathcal{A}} r_{ik}(t_{ik} + T_{kj}) \qquad (12)$$

where $\mathcal{A}$ denotes the set of all direct links in the network.

The difficulty of the problem lies in the evaluation of t_{ij}'s. But with our hypothesis, t_{ij} can be easily obtained by assuming the particular link (i,j) to be a G/D/1 queue with self-similar arrival, whose mean, variance and Hurst parameter are given. A number of researchers have attempted to characterize such individual link performance. See, for example, [9, 11, 18, 19].

To see why we believe in the Decomposition Hypothesis, we look into the underlying reasons for Jackson's Theorem to hold. Jackson's Theorem is basically supported by three facts:

1. The sum of two independent Poisson processes is Poisson.
2. The random splitting of a Poisson process is Poisson.
3. Burke's Theorem[20], which says that the steady state output process of a stable M/M/m queue with arrival rate λ and service-time parameter μ for each of the m servers is in fact a Poisson process with the same rate λ. Burke also showed that the output process is independent of any other processes in the system.

We strongly believe that our hypothesis holds since we also have the corresponding supporting facts for the modern Jackson's network.

Fact 1 *The sum of self-similar processes is self-similar.*

Fact 2 *A self-similar process subject to random splitting is self-similar.*

Fact 3 *The output process of a G/D/1 queue with self-similar arrival is self-similar.*

We will discuss the above three facts in the following subsections.

4.1 Fact 1

The following theorem regarding the merging of self-similar processes have been proved.

Theorem 1 *Let X and Y be two uncorrelated e.s.o.s.s. processes with Hurst parameters H_x and H_y respectively. The process $Z = X + Y$ is e.s.o.s.s. if and only if $H_x = H_y$, in which case the Hurst parameter of Z, $H_z = H_x$. If $H_x \neq H_y$, Z is a.s.o.s.s. with $H_z = \max(H_x, H_y)$. If X and Y are not e.s.o.s.s, but a.s.o.s.s. with Hurst parameters H_x and H_y respectively. Z is a.s.o.s.s. with Hurst parameter $H_z = max(H_x, H_y)$[21]. If X and Y are e.s.o.s.s. or a.s.o.s.s. processes which are correlated in such a way that $cov(X_t, Y_{t+k}) = cg(k)$ with parameter β_{xy}, where c is a positive constant and $g(k)$ is defined as in (3), then Z is a.s.o.s.s. with Hurst parameter $H_z = \max(H_x, H_y, H_{xy})$, where $H_{xy} = 1 - \beta_{xy}/2$[22].*

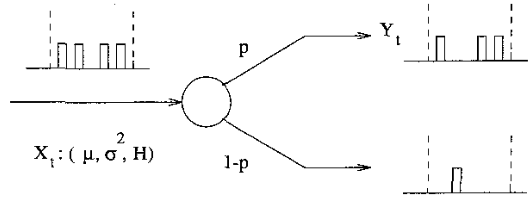

Figure 2: Cell-by-cell random splitting policy.

4.2 Fact 2

In [21], the effect of splitting a self-similar process using the *independent splitting operation* is studied. With independent splitting, a process X_t is split into two subprocesses Y_t with probability $p (0 < p < 1)$, X_t becomes Z_t with probability $1 - p$. It is shown in this work that:

Theorem 2 *If X_t is e.s.o.s.s. with parameter H, then the splitted processes Y_t and Z_t are not e.s.o.s.s., but a.s.o.s.s. with the same parameters H. If X_t is a.s.o.s.s. with parameter H, then the splitted processes Y_t and Z_t are also a.s.o.s.s. with the same parameters H.*

We consider another splitting policy, called the *cell-by-cell random splitting policy*. Suppose $X = (X_t, t = 0, 1, 2...)$ represents an ATM traffic stream, with X_t being the number of cells at time t. In this splitting operation, each cell of X_t has a probability p of going to a splitted stream Y, independent of any other cells. Figure 2 illustrates the case of two-way splitting. Our result applies to the general situation of multiple-way splitting.

We are able to show that Theorem 2 is true under cell-by-cell random splitting[22].

The splitted processes under both splitting policies, however, are not uncorrelated. The cross-covariances between Y and Z in both cases are given by[22]:

$$cov(Y_t, Z_{t+k}) = p(1-p)\sigma^2 r(k) \qquad (13)$$

where σ^2 and $r(k)$ are the variance and autocorrelation function of process X respectively. This and the fact (see Theorem 1) that self-similar processes correlated in the form $cg(k)$ merge to self-similar process strongly suggest the possibility that any two processes in our network model are either uncorrelated, or they are correlated in the special form of $cg(k)$.

4.3 Fact 3

Fact 3 characterizes the output process of a deterministic service time queue with self-similar input, as illustrated in Figure 3.

We found it convenient to work in the frequency domain when dealing with the characteristics of the

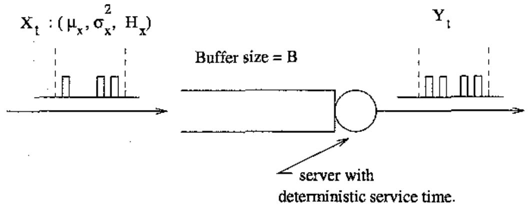

Figure 3: A deterministic service time queue with self-similar input.

output process. The definition for e.s.o.s.s. processes is given in (6).

It is well known that a queueing system is generally nonlinear. However, the study in [23] revealed that the input/output relationship in a certain low frequency band behaves linearly or very nearly so. This is identified by a coherence spectrum of approximately one in a low frequency band. In other words, the low frequency traffic stays intact as it passes through a queueing system. Although their work is based on MMPP/M/1/K queues, our numerical study[22] indicates the same result for single server queues with self-similar arrivals and deterministic service time. Therefore, we assume that the power spectral density of the output process, $f_y(\lambda)$, to be of the following form:

$$f_y(\lambda) = \begin{cases} f_x(\lambda) & |\lambda| < \lambda_c \\ q(\lambda) & \lambda_c < |\lambda| < 1/2 \end{cases} \tag{14}$$

where $f_x(\lambda)$ is the power spectrum of the input process, λ_c is some constant in the interval $(0, \frac{1}{2})$, and $q(\lambda)$ is a bounded function characterizing the high frequency component of the output power spectrum. We are now ready to prove an important result.

Theorem 3 *If the input process X of a deterministic service time queue is e.s.o.s.s. with parameter $H = 1 - \beta/2$, and the power spectrums of the input and output processes are related as described in (14), then the output process is a.s.o.s.s. with the same Hurst parameter.*

Proof: The power spectrum of the aggregated output process $Y^{(m)}$ is given by:

$$f_y^{(m)}(\lambda) = \sum_{k=-\infty}^{\infty} E\{y_t^{(m)} y_{t+k}^{(m)}\} e^{-j2\pi\lambda k} \tag{15}$$

where $y_t = Y_t - \mu_y$ is the centralized version of Y_t.

It can be shown[22] that this is related to $f_y(\lambda)$ as follows:

$$f_y^{(m)}(\lambda) = \frac{\sin^2 \pi\lambda}{m^3} \sum_{n=-\lceil \frac{m}{2} \rceil}^{\lfloor \frac{m}{2} \rfloor} \frac{f_y(\frac{\lambda+n}{m})}{\sin^2 \pi(\frac{\lambda+n}{m})} \tag{16}$$

Since X is e.s.o.s.s., $f_x(\lambda)$ is equal to $\sigma^2 h(\lambda)$ given in (6), so,

$$f_y(\frac{\lambda+n}{m}) = f_x(\frac{\lambda+n}{m}) = c \sin^2 \pi \frac{\lambda+n}{m} \sum_{\ell=\infty}^{\infty} |\frac{\lambda+n}{m} + \ell|^{\beta-3} \tag{17}$$

for $|\frac{\lambda+n}{m}| < \lambda_c$, or equivalently, for $n \in (\lceil -m\lambda_c - \lambda \rceil, \lfloor m\lambda_c - \lambda \rfloor)$, where c is a positive constant.

Hence,

$$\begin{aligned} \frac{f_y^{(m)}(\lambda)}{m^{-\beta}} &= \frac{\sin^2 \pi\lambda}{m^{3-\beta}} (\sum_{n=\lceil -m\lambda_c - \lambda \rceil}^{\lfloor m\lambda_c - \lambda \rfloor} c \sum_{\ell=-\infty}^{\infty} |\frac{\lambda+n}{m}|^{\beta-3} + \\ &\quad 2 \sum_{n=\lfloor m\lambda_c - \lambda \rfloor + 1}^{\lfloor \frac{m}{2} \rfloor} \frac{q(\frac{\lambda+n}{m})}{\sin^2 \pi(\frac{\lambda+n}{m})}) \\ &= c \sin^2 \pi\lambda \sum_{n=\lceil -m\lambda_c - \lambda \rceil}^{\lfloor m\lambda_c - \lambda \rfloor} \sum_{\ell=-\infty}^{\infty} |\lambda + n + m\ell|^{\beta-3} + \\ &\quad 2 \frac{\sin^2 \pi\lambda}{m^{3-\beta}} (\sum_{n=\lfloor m\lambda_c - \lambda \rfloor + 1}^{\lfloor \frac{m}{2} \rfloor} \frac{q(\frac{\lambda+n}{m})}{\sin^2 \pi(\frac{\lambda+n}{m})}) \end{aligned}$$

Letting m go to infinity, the second term goes to zero since both $q(\lambda)$ and $1/\sin^2 \pi(\frac{\lambda+n}{m})$ are bounded, and there are at most $m/2$ terms in the sum. For the first term, all terms in the second summation go to zero except those for which $\ell = 0$. Besides, the limits of the first summation go to $\pm\infty$. Thus,

$$\lim_{m \to \infty} \frac{f_y^{(m)}(\lambda)}{m^{-\beta}} = c \sin^2 \pi\lambda \sum_{n=-\infty}^{\infty} |\lambda + n|^{\beta-3} = c' h(\lambda) \tag{18}$$

where c' is another constant. Taking inverse DTFT on both sides and setting $k = 1$, we arrive at[1],

$$\lim_{m \to \infty} \frac{\sigma_y^{(m)2}}{m^{-\beta}} = c' \tag{19}$$

That is, Y is a.s.o.s.s. with Hurst parameter $H = 1 - \beta/2$.

Q.E.D.

We are currently working on the corresponding result when the input process is a.s.o.s.s. instead of e.s.o.s.s. Preliminary study shows strong evidence that the statement also holds for a.s.o.s.s. arrivals.

[1] Assuming the uniform convergence of the function $f_y^{(m)}(\lambda) e^{j2\pi\lambda k}$ such that we can exchange the order of limit and integration.

5 Conclusion

Motivated by the fact that future network traffic is self-similar and its queueing performance is significantly different from what conventional Markovian type models predict, we proposed a model for telecommunication networks with self-similar traffic and deterministic service times. This model is similar to Jackson's network of queues model in classical queueing analysis. We have hypothesized a result analogous to the Jackson's Theorem. Our hypothesis is strongly backed by three facts, namely, the sum of independent self-similar processes is self-similar, the random splitting of self-similar processes are self-similar, and the output process of a G/D/1 queue with self-similar input is self-similar. With our hypothesis, many network-wide performance measures such as the end-to-end delay can be obtained in a much simpler fashion. Each node can be considered separately as a G/D/1 queue with self-similar arrival, whose mean, variance and Hurst parameter can be determined. We believe that our work greatly simplifies the performance analysis of future packet-based networks.

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