A FACTORIZATION-BASED PROJECTIVE RECONSTRUCTION ALGORITHM WITH CIRCULAR MOTION CONSTRAINT

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ABSTRACT

In this paper, we propose a projective reconstruction algorithm for a circular motion image sequence. We first formulate the circular motion constraint in the Euclidean frame, and then deduce its expression in a projective frame. The circular motion constraint is gradually enforced during the iterations of a projective reconstruction. This approach can be used to deal with both constant and varying intrinsic parameters. Experimental results for synthetic and real data are presented to illustrate the performance and improvements of our approach over methods based on general motion.

1. INTRODUCTION

Circular motion image sequence is obtained when images are captured by a camera on a rotating arm or a fixed camera pointing towards an rotating object. Constructing 3D models from these special configurations has attracted a lot of attentions [5, 2, 3].To make use of the huge 2D point correspondences, Niem and Buschmann [5] provided a system for constructing 3D models from well-controlled circular motion (i.e., cameras are calibrated and the exact camera positions are measured). Fitzgibbon et al. [2] proposed to reduce the constraints to unknown constant intrinsic parameters over all the views. In [2], the reconstruction ambiguity for circular motion image sequence is discussed. However, the problem of varying intrinsic parameters for auto-focusing and zooming cameras is not addressed. Tang and Hung [6] proposed a factorization-based method by solving a sequence of weighted least-squares (WLS) problems to achieve minimization of 2D reprojection error.

In this paper, we propose a factorization-based method for projective reconstruction for a circular motion image sequence. The condition of constant intrinsic parameters [2] is relaxed to varying intrinsic parameters. We will give a formulation of the circular motion constraint in both the Euclidean and projective frames. During the iterative process in projective reconstruction, the constraint is enforced gradually.

The paper is organized as follows. The projective reconstruction problem is formulated in Section 2. Some background on the factorization-based method for projective reconstruction is given in Section 3. The theory of circular motion constraint is given in

Section 4. In Section 5, we formulate the problem of projective reconstruction with circular motion constraint. In Section 6, the case of constant intrinsic parameters is shown to be a special case of the varying parameter problem. The generalized algorithm is given in Section 7. Experimental results for synthetic and real data are presented in Section 8 to illustrate the performance and improvements of our approach over methods based on general motion. Concluding remarks are given in Section 9.

Notation: The Hadamard product of two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ of the same size is denoted

$$A \quad B = [a_{ij}b_{ij}].$$

2. PROBLEM FORMULATION

Consider multiple cameras with projection matrices P_i viewing a set of 3D points X_j captured as image points x_{ij} on the image planes of the cameras as

$$\lambda_{ij}x_{ij} = P_iX_j \tag{1}$$

where λ_{ij} is the projective depth for x_{ij}. For a known circular motion image sequence, the ith view is rotated around the Z-axis with a known angle θ_i w.r.t. the first view. Assuming that the projection matrix of the first view in Euclidean frame is $K_1[R_1|t_1]$ with $\theta_1 = 0$, the problem of projective reconstruction with known circular motion constraint is formulated as

$$\min_{P_i, X_j} \sum_{i,j} \left| x_{ij} - \frac{1}{P_i^3 X_j} P_i X_j \right|_2^2 \tag{2}$$

subject to $P_i H = \beta_i K_i [R_1|t_1] R_z(\theta_i)$

where $H \in \Re^{4 \times 4}$ is the projective transformation to upgrade the reconstructed projective frame to the Euclidean frame, β_i is the scaling factor for the ith camera, and P_i^3 is the 3rd row of P_i.

3. BACKGROUND

3.1. Factorization-based method for projective reconstruction

In the factorization approach to projective reconstruction, a basic problem is to estimate the depths λ_{ij} for the image points $x_{ij} = [u_{ij} \ v_{ij} \ 1]^T$ so that the scaled measurement matrix $[\lambda_{ij}x_{ij}]$ can be factorized as a rank-4 product PX, where P is the joint projection

Algorithm 1 Factorization-based method for Projective Reconstruction in [6]

1. Initialization: if P^o and X^o are not provided as initial guesses, they are initialized by the rank-4 approximation of $\left\{\frac{1}{\beta^o_{ij}} x_{ij}\right\}$ where β^o_{ij} are set to 1 and missing entries are filled by the centroids of the visible 2D points for that view. $k = 1$ to start the algorithm.

2. Put $k = k + 1$.
   Fix X^{k-1} and P^{k-1}, evaluate β^k by solving
   $$\min_{\beta^k} F_P(P^{k-1}, X^{k-1}, \beta^k).$$

3. Fix β^k and P^{k-1}, evaluate X^k by solving
   $$\min_{X^k} F_P(P^{k-1}, X^k, \beta^k).$$

4. Fix β^k and X^k, evaluate P^k by solving
   $$\varepsilon^1_k = \min_{P^k} F_P(P^k, X^k, \beta^k).$$

5. Repeat steps 2, 3 and 4 until ε^1_k converges.

6. Output k, P^k, β^k and X^k.

matrix and X is a shape matrix containing the 3D points. Due to occlusion some points cannot be seen in all the views. Let us define the set $\mathcal{A}$ as

$$\mathcal{A} = \left\{ (i, j) \in \Re^2 \mid X_j \text{ is visible in the } i^{th} \text{ view} \right\}.$$

In [6], the cost function F_P of the factorization-based method is proposed as

$$F_P(P, X, \beta) = \sum_{(i,j) \in \mathcal{A}} \left\| \begin{bmatrix} 1 \\ 1 \\ \gamma_{ij} \end{bmatrix} \left\{ \begin{bmatrix} u_{ij} \\ v_{ij} \\ 1 \end{bmatrix} - \beta_{ij} P_i X_j \right\} \right\|^2 \tag{3}$$

where $F_P(P, X, \beta)$ is the cost function of the factorization-based method, $\beta_{ij} = \frac{1}{\lambda_{ij}}$ and γ_{ij} is the weighting factor for the 2D point x_{ij} and usually chosen as $\max(|u_{ij}|, |v_{ij}|)$. A sequence of weighted linear least-squares (WLS) problems is formulated iteratively on $F_P(P, X, \beta)$ for solving one of P, X and λ as free parameters while fixing the other two parameters, as given in Algorithm 1.

3.2. Reconstruction Ambiguity

It has been shown in [2] that for a camera with fixed intrinsic parameters undergoing known circular motion, if the first camera centre is at position $T = [t \; 0 \; 0]^T$ and the camera is rotated around the Z-axis of the world coordinate system, the projection matrices P_i in the Euclidean frame for the ith view can be written as

$$P_i = M \; [I \mid T] \, R_z(\theta_i) \tag{4}$$

where M is a homography including the fixed intrinsic parameters K and a rotation R_0 of the camera about its centre: $M = K R_0$, I is the 3×3 identity matrix, and $R_z(\theta_i)$ is the rotation matrix of

rotating by angle θ_i about the Z-axis. It is proved in [2] that if a reconstructed projection matrix $P'_i = P_i H$ is determined with a set of cameras of the form (4), then H can be parameterized as

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & \beta & 1 \end{bmatrix}.$$

Hence, P'_i can be expanded as

$$P'_i = M \; [I \mid T] \, R_z(\theta_i) \, H. \tag{5}$$

The result of [2] is also valid for cameras with varying intrinsic parameters K_i. That is for a camera with varying intrinsic parameters undergoing known circular motion, M is replaced by $M_i = K_i R_0$ for all cameras in (4). Due to the special form of H, (5) can be rewritten as

$$P'_i = M_i \; [I \mid T] \, H R_z(\theta_i). \tag{6}$$

4. CIRCULAR MOTION CONSTRAINT FOR PROJECTIVE RECONSTRUCTION

It will be helpful in the following sections to consider that the object is fixed and that the camera rotates about a fixed axis around the object. The world coordinate system is chosen as in Section 3.2.

For controlled circular motion, we may assume:

1. The camera centres lie on a circle in the motion plane.

2. The rotation angles θ_i between the ith view and the first view are known.

Under the above assumptions, we can deduce that in the Euclidean frame the camera centre O_i for the ith view satisfies the following equation

$$O_i = R_z^T(\theta_i) O_1 \tag{7}$$

where O_1 is the camera centre of the first view. (7) will be referred to as the circular motion constraint. Our purpose is to find a projective reconstruction method which ensures that the camera centres satisfy (7) in some Euclidean frame.

It has been shown in Section 3.2 that the projection matrix can be expressed as (6). For any 3×3 non-singular matrix D, we can insert $D D^{-1}$ between K_i and $R_0[I|T]$ without changing the value of P_i. Denote $R_0[I|T]$ as P_0, the projection matrix can be written as

$$P'_i = K_i D D^{-1} P_0 H R_z(\theta_i) = A_i B R_z(\theta_i) \tag{8}$$

where $A_i = K_i D$ is a 3×3 non-singular matrix, $B = D^{-1} P_0 H$ is a 3×4 matrix which remains unchanged for each camera. If we constrain D to be an upper triangular matrix, A_i is also an upper triangular matrix. The projective ambiguity now exists in the part of $A_i B$ instead of the whole projection matrix. The projection matrix P'_i can be upgraded to P_i in the Euclidean frame as:

$$P_i = P'_i H^{-1} = A_i B H^{-1} R_z(\theta_i). \tag{9}$$

Since the camera centre can be calculated as the right null vector of its projection matrix [1] and A_i is a 3×3 non-singular matrix, C_i can be calculated as $C_i = null(B H^{-1} R_z(\theta_i))$. Hence, we have

$$C_i = R_z^T(\theta_i) C_1. \tag{10}$$

(10) shows that if we can find a projective reconstruction with the form (8), we can ensure that the camera centres satisfy the circular motion constraint in the Euclidean frame.

5. A FACTORIZATION-BASED PROJECTIVE RECONSTRUCTION ALGORITHM WITH CIRCULAR MOTION CONSTRAINT

To enforce the projection matrices to have the form (8), let us introduce the cost function,

$$F_{CM}(A_i, B, X, \beta) =$$

$$\sum_{(i,j)\in\mathcal{A}} \left\| \begin{bmatrix} 1 & \cdot \\ & 1 \\ & & \gamma_{ij} \end{bmatrix} \left\{ \begin{bmatrix} u_{ij} \\ v_{ij} \\ 1 \end{bmatrix} - \beta_{ij} A_i B R_z(\theta_i) X_j \right\} \right\|^2 \tag{11}$$

where A_i is an upper triangular matrix.

5.1. From general motion to circular motion

Given a reconstructed projective frame from Section 3.1, the camera centres can be expressed as: $O_i = null(P_i)$, $i = 1, \ldots, n$. Obviously, in the projective frame the camera centres do not satisfy (7) and P_i does not have the form of (8). But there exists a projective transformation H_1 which will transform O_i into a frame where (7) is satisfied. The next theorem shows that under such a transformation, the projection matrices will have the form (8).

Theorem 1 *If we can find a 4×4 non-singular matrix H_c to transform the projective frame so that the camera centre O'_i and the principle plane p'^3_i of the ith camera in the new projective frame should satisfy the following equations*

$$O'_i = R_z^T(\theta_i)O'_1$$
$$p'^{3T}_i = p'^{3T}_1 R_z(\theta_i)$$

where p'^{3T}_i is the third row of the ith camera matrix P'_i. Both O'_i and p'^{3T}_i are scaled so their last entries are equal to one. Then the projection matrices P'_i in the new projective frame can be expressed as

$$P'_i = P_i H_c^{-1} = K_i P_0 H R_z(\theta_i),$$

i.e. the projection matrices are in form of (8).

Due to the limit of space, we omit the proof for Theorem 1.

5.2. Decomposition of Projection matrices

According to Theorem 1, after transformation with the matrix H_1 the new projection matrices P'_i can be written as

$$P'_i = A_i B R_z(\theta_i). \tag{12}$$

From (12), we have $P'_i R_z^T(\theta_i) = A_i B$. Since all the row vectors of the matrix $A_i B$ are linear combinations of the three independent row vectors of B, if we stack all $P'_i R_z^T(\theta_i)$ $(i = 1, \ldots, n)$ into one $3n \times 4$ matrix P, the rank of P should still be 3. By means of Singular Value Decomposition, P can be decomposed as $P = USV^T$. B can be chosen as $B = V_3^T$, where V_3 contains the first three rows of V.

6. ALGORITHM FOR CONSTANT INTRINSIC PARAMETERS

When the camera intrinsic parameters K_i are kept constant over all the views, it can be considered as a special case of our circular motion constraint. The above algorithm can be modified slightly to handle this scenario. The constant intrinsic parameters can be treated as $A_i = I, \forall i$. In this case there is no need to estimate A_i.

Algorithm 2 Generalized Algorithm for Circular Motion

1. Use the results of Algorithm 1 as k, P^k, β^k and X^k.

2. Evaluate B^k as in Sec.5.2.

3. Put $k = k + 1$.

4. Fix β^{k-1}, B^{k-1} and X^{k-1}, evaluate A_i^k by solving

$$\min_{A_i^k} F_{CM}(A_i^k, B^{k-1}, X^{k-1}, \beta^{k-1}).$$

5. Fix A_i^k, β^{k-1} and X^{k-1}, evaluate B^k by solving

$$\min_{B^k} F_{CM}(A_i^k, B^k, X^{k-1}, \beta^{k-1}).$$

6. Fix A_i^k, B^k and X^{k-1}, evaluate β^k by solving

$$\min_{\beta^k} F_{CM}(A_i^k, B^k, X^{k-1}, \beta^k).$$

7. Fix A_i^k, B^k and β^k, evaluate X^k by solving

$$\varepsilon_k = \min_{X^k} F_{CM}(A_i^k, B^k, X^k, \beta^k).$$

8. Repeat steps 3-7 until ε_k converges. After ε_k has converged, enforce A_i^k to be an upper triangular matrix in Step 4 and repeat steps 3-7 until ε_k converges again.

9. Output A_i^k, B^k, β^k, X^k and $P_i^k = A_i^k B^k R_z(\theta_i)$.

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7. THE GENERALIZED ALGORITHM

The complete generalized algorithm for circular motion constraint with varying and constant intrinsic parameters is given in Algorithm 2. Since (11) is a quadrilinear (trilinear for the cases of constant intrinsic parameters) minimization problem in A_i, B, X and β, (11) is expressed as four different WLS problems where A_i, B, X and β are evaluated one by one iteratively while keeping the others unchanged. The superscript k indicates that the variables are updated in the kth iteration. The constraint of A_i being upper triangular matrices is enforced after the algorithm converges with A_i being general 3×3 non-singular matrices. For the case of constant intrinsic parameters, Step 4 for estimating A_i in the algorithm can be skipped.

8. EXPERIMENTAL RESULTS

8.1. Synthetic Data

In this simulation, we compare the results of our algorithm with the method given in [6]. The test scene consists of 30 points uniformly distributed within the unit sphere centered at the origin of the world coordinate system. The synthetic camera moving on the x-y plane along a circular path of 7 meter radius captures 8 images. Gaussian noise with standard deviation ranging from 0.5 pixels to 4.5 pixels is added to the image points. The image size is about 1000×1200 pixels.

To measure the quality of the reconstruction, we calculate the 2D reprojection error and 3D residual of the Euclidean distances between the reconstructed scene points and their ground truth positions. The graphs show the median value over 8 trials.
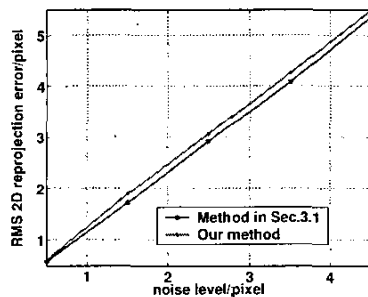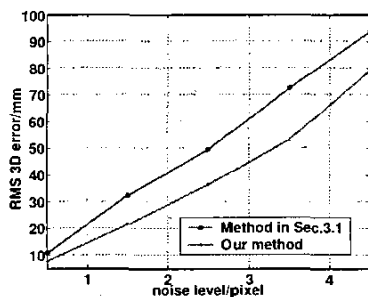
Fig. 1. 2D Reprojection Error

Fig. 2. 3D Error

8.1.1. Performance on 2D Reprojection Error

Fig. 1 shows the 2D reprojection error of the two algorithms against different noise levels. It is reasonable that the 2D reprojection error of our algorithm is slightly larger than the method in [6], because the degree of freedom is less.

8.1.2. Performance on 3D Error

A transformation H is computed by an optimization method to upgrade the reconstructed shape X to the ground truth M so that $M_i = HX_j$ has minimal 3D error. In Fig. 2, it can be seen that the reconstructed scene is closer to the ground truth under the circular motion constraint.

8.2. Real Data

The projective reconstruction algorithm is tested with a sequence of images taken by a static camera looking at a model house on a turntable. The intrinsic parameters of the camera are fixed while capturing the images so there is no need to estimate A_i. The
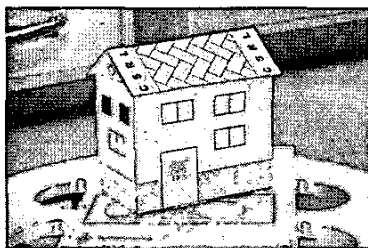

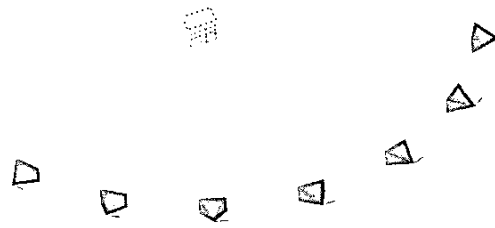**Fig. 3.** An image for the circular motion image sequence

Fig. 4. A Reconstructed Scene

turntable is rotated by an angle 10° incrementally for each image. A total of 7 images are taken and one of the images is shown in Fig. 3. The size of the images is 2000 × 3000 pixels. There are 33 corresponding points which are matched manually and visible to all the views. Our implementation of Algorithm 2 took 726 iterations to converge which required 273 seconds on a 3.4GHz PC Pentium-4 with 1GB RAM. The RMS 2D reprojection error is 1.107 pixels. Fig. 4 shows the recovered the Euclidean structure and motion. The 3D points are shown and the cameras are represented by pyramids.

9. CONCLUSION

In this paper, we propose a new projective reconstruction method for minimizing 2D reprojection error for a circular motion image sequence. This generalized algorithm can be applied to image sequences with varying or constant intrinsic parameters. With the circular motion constraint, the accuracy of reconstruction can be improved in terms of 3D error and the reconstructed projective frame can provide accurate results for the camera calibration methods relying on epipolar geometry (e.g., [4]).

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