Anisotropic four-dimensional Neveu-Schwarz–Neveu-Schwarz string cosmology

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An anisotropic (Bianchi type I) cosmology is considered in the four-dimensional Neveu-Schwarz–Neveu-Schwarz (NS–NS) sector of low-energy effective string theory coupled to a dilaton and an axionlike H field within a de Sitter–Einstein frame background. The general solution of the gravitational field equations can be expressed in an exact parametric form in both Einstein and string frames. The study of the time dynamics of this universe leads to the conclusion that in the absence of a dilaton field potential or a cosmological constant the initial anisotropies do not decay and the time evolution of a pure dilaton and axion field filled Bianchi type I space-time does not lead to an isotropic phase.

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I. INTRODUCTION

Pre-big-bang cosmological models [1], based on the low-energy limit of string theory, have been intensively investigated in the recent physics literature [2–19] (for an extensive recent review of string cosmology see [20]). Generically, in these types of models the dynamics of the universe is dominated by massless bosonic fields. There are many massless fields present in the pre-big-bang scenario, such as the dilaton, graviton and moduli fields.

In the string frame, the four-dimensional Neveu-Schwarz–Neveu-Schwarz (NS–NS) effective action, which is common to both heterotic and type II string theories, is given by [21–23]

$$\hat{\mathcal{S}}=\int d^4x\sqrt{-\hat{g}}e^{-2\phi}\left\{\hat{R}+\hat{\kappa}(\nabla\phi)^2-\frac{1}{12}\hat{H}_{[3]}^2-\hat{U}\right\}, \quad (1)$$

where $H_{\mu\nu\lambda}=\partial_{[\mu}B_{\nu\lambda]}$ is an antisymmetric tensor field, $\hat{\kappa}$ is a generalized dilaton coupling constant, and $\hat{U}=\hat{U}(\phi)$ is a dilaton potential. In addition, $\hat{H}_{[3]}^2$ means the square of the H field with respect to the metric $\hat{g}_{\mu\nu}$. The low-energy string action possesses a symmetry property, called a scale factor duality, which leads us to expect that the present phase of the Universe is preceded by an inflationary pre-big-bang phase. Explicit dual solutions can be constructed for each Bianchi space-time, except the Bianchi class A type VIII and IX models [2].

By means of the conformal rescaling

$$g_{\mu\nu}=e^{-2\phi}\hat{g}_{\mu\nu}, \quad (2)$$

the action (1) can be transformed to the so-called Einstein frame as

$$\mathcal{S}=\int d^4x\sqrt{-g}\left\{R-\kappa(\nabla\phi)^2-\frac{1}{12}e^{-4\phi}H_{[3]}^2-U\right\}, \quad (3)$$

where $\kappa=6-\hat{\kappa}$, $U=e^{2\phi}\hat{U}$, and $H_{[3]}^2$ denotes the square of the antisymmetric field by $g_{\mu\nu}$.

Intensive observational study of the cosmic microwave background radiation (CMB) has shown that it seems to be almost isotropic. The significance of this isotropy for cosmology comes from the fact that, whatever the origin of this radiation, it must have propagated freely toward us from a distance of the order of Hubble radius. If there were any large-scale inhomogeneities or anisotropies in the universe, these would affect the radiation and make it appear anisotropic to us. If the radiation were exactly isotropic to all observers at all times, then the universe would have to be completely spatially isotropic and spatially homogeneous and, consequently, it would be described by a Robertson-Walker model [24]. Recent measurements based on an analysis of 42 type Ia supernovas discovered by the Supernova Cosmology Project lead to data consistent with a $\Lambda\neq 0$ isotropic flat cosmology, with the cosmological constant value comparable to the mass-energy density [25].

Therefore a physically acceptable cosmological model must lead, in a large time limit, to an isotropic geometry and also provide a mechanism for the disappearance of initial anisotropies.

The field equations derived from the string effective action, Eqs. (1) and (3), admit inflationary solutions that are driven by the kinetic energy associated with the massless fields rather than any interaction potential. A study of spatially flat and homogeneous string cosmologies, considering

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the combined effects of the dilaton, modulus, two-form potential, and central charge deficit, and using methods from the qualitative theory of differential equations (phase portrait analysis), has been presented in [5]. The effects of initial inhomogeneity and anisotropy on pre-big-bang inflation were analyzed by Veneziano [11]. Making the assumption of small initial curvature, he showed that small amounts of inhomogeneity and anisotropy do not prevent the ultimate transition to the pre-big-bang inflationary phase and concluded that pre-big-bang inflation is robust. Analytic biaxial (two scale factors only) Bianchi type I geometry has been considered in [3] for the case with nonvanishing $H_{[3]}$ but without a dilaton field potential, i.e., $U \equiv 0$. Triaxial models with the central deficit charge constrained to zero in the presence of a modulus field (representing the evolution of compact extra dimensions) have been analyzed in [4]. The general Bianchi type I space-time geometry for arbitrary dimensional dilaton gravities, with vanishing antisymmetric tensor $H_{\mu\nu\lambda}$ and in the presence of an exponential type dilaton field potential, have been obtained in both the Einstein and string frames [6].

The initial conditions in the pre-big-bang scenario have been considered in [18]. The basic postulate of the pre-big-bang cosmology has been formulated as one of ''asymptotic past trivialities,'' by which it is meant that the initial state is a generic perturbative solution of the tree-level low-energy effective action. The ''string vacuum'' is made of an arbitrary ensemble of incoming dilaton and gravitational waves. The pre-big-bang inflationary phase in the string frame is equivalent to gravitational collapse in the Einstein frame and therefore the authors of [18] conclude that initial conditions for pre-big-bang inflation are as natural as those for gravitational collapse.

These investigations show, however, that in the absence of the dilaton potential anisotropic pre-big-bang models seems to favor anisotropic Kasner type geometry [20]. The question whether the isotropy problem is naturally solved in pre-big-bang models has been raised by Kunze and Durrer [15] by analyzing the string frame behavior of the shear tensor in Bianchi class A spatially homogeneous models, with dilaton and a perfect fluid as matter sources. According to their results for these type of models initial anisotropies do not decay during pre-big-bang inflation.

It is the purpose of the present paper to consider, in the framework of a four-dimensional Bianchi type I geometry, the general effects on the dynamics, evolution, and isotropization of the early universe of a nonvanishing antisymmetric field and of a string frame exponential type dilaton field potential. An exponential potential arises in the four-dimensional effective Kaluza-Klein type theories from compactification of the higher-dimensional supergravity or superstring theories [21]. In string or Kaluza-Klein theories the moduli fields associated with the geometry of the extra dimensions may have effective exponential potentials due to the curvature of the internal spaces or to the interaction of the moduli with form fields on the internal spaces. Exponential potentials can also arise due to nonperturbative effects such as gaugino condensation [26].

In this case the general solution of the gravitational field equations can be expressed in an exact parametric form. The existence of an analytic solution simplifies the study of the properties of the basic physical parameters of the model, such as the mean anisotropy or the deceleration parameter, whose behaviors have not been explicitly investigated in the previous analysis of Bianchi type I models [3,4]. The evolution of these quantities gives a clear picture of the general physical evolution of an anisotropic space-time. As a general result we find that in the absence of a dilaton potential (or, equivalently, a cosmological constant) the mean anisotropy of the anisotropic pre-big-bang universe is a constant in the large time limit in both Einstein and string frames and hence initial anisotropies are not inflated away during the pre-big-bang evolution.

This paper is organized as follows. In Sec. II we write down the basic equations of our model. The general solution of the field equations is presented, in an exact parametric form, in Sec. III. In Sec. IV we discuss and conclude our results.

II. FIELD EQUATIONS, GEOMETRY, AND CONSEQUENCES

In the Einstein frame the field equations, which follow from variation of Eq. (3), are given by

$$R_{\mu\nu} - \kappa \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} U - \frac{1}{4} e^{-4\phi} \times \left(H_{\mu\alpha\beta} H_\nu{}^{\alpha\beta} - \frac{1}{3} g_{\mu\nu} H^2 \right) = 0, \tag{4}$$

$$\nabla_\mu (e^{-4\phi} H^{\mu\nu\lambda}) = 0, \tag{5}$$

$$\nabla^2 \phi + \frac{1}{6\kappa} e^{-4\phi} H^2 - \frac{1}{2\kappa} \frac{\partial U}{\partial \phi} = 0. \tag{6}$$

Moreover, the H field must satisfy the integrability condition (Bianchi identity) $\partial_{[\mu} H_{\nu\lambda\rho]} = 0$.

In four dimensions, every three-form field can be dualized to a pseudoscalar. Thus, an appropriate ansatz for the H field is [3]

$$H^{\mu\nu\lambda} = \frac{1}{\sqrt{-g}} e^{4\phi} \epsilon^{\mu\nu\lambda\rho} \partial_\rho h, \tag{7}$$

where $\epsilon^{\mu\nu\lambda\rho} = -\delta^\mu_{[0} \delta^\nu_1 \delta^\lambda_2 \delta^\rho_{3]}$ is the total antisymmetric tensor and $h = h(t)$ is the Kalb-Ramond axion field. Then the field equation (5) is satisfied automatically and the Bianchi identity becomes

$$\partial_\mu (\sqrt{-g} e^{4\phi} \partial^\mu h) = 0. \tag{8}$$

Moreover, we shall assume that in the string frame the dilaton field potential is of exponential type [21,26]

$$\hat{U}(\phi) = \Lambda e^{-2\phi}, \tag{9}$$

with Λ a non-negative constant (de Sitter space-time). Therefore in the Einstein frame the effect of the potential is identical to that of a cosmological constant, $U(\phi)=\Lambda$.

In the Einstein frame the line element of a Bianchi type I space-time, which is the anisotropic generalization of the flat Robertson-Walker geometry, is given by

$$ds^2=-dt^2+\sum_{i=1}^{3} a_i^2(t)(dx^i)^2. \tag{10}$$

With the ansatz (7),(9), the field equations (4),(6),(8) take the form

$$3\dot{\theta}+\sum_{i=1}^{3}\theta_i^2+\kappa\dot{\phi}^2+\frac{1}{2}e^{4\phi}\dot{h}^2-\frac{1}{2}\Lambda=0, \tag{11}$$

$$\frac{1}{V}\frac{d}{dt}(V\theta_i)-\frac{1}{2}\Lambda=0, \quad i=1,2,3, \tag{12}$$

$$\ddot{h}+3\theta\dot{h}+4\dot{\phi}\dot{h}=0, \tag{13}$$

$$\frac{1}{V}\frac{d}{dt}(V\dot{\phi})-\frac{1}{\kappa}e^{4\phi}\dot{h}^2=0, \tag{14}$$

where we have introduced the volume scale factor $V:=\Pi_{i=1}^{3}a_i$, directional Hubble factors $\theta_i:=\dot{a}_i/a_i$, $i=1,2,3$, and the mean Hubble factor $\theta:=\Sigma_{i=1}^{3}\theta_i/3=\dot{V}/3V$.

We also introduce two basic physical observational quantities in cosmology: the mean anisotropy parameter $A:=\Sigma_{i=1}^{3}(\theta_i-\theta)^2/3\theta^2$ and the deceleration parameter $q=d\theta^{-1}/dt-1$. For an isotropic expansion $A\equiv 0$. The sign of the deceleration parameter indicates whether the universe inflates. The positive sign corresponds to ''standard'' decelerating models whereas negative sign indicates inflation.

By summing Eqs. (12) we obtain

$$\frac{1}{V}\frac{d}{dt}(V\theta)=\frac{1}{2}\Lambda, \tag{15}$$

which, together with Eqs. (12), leads to

$$\theta_i=\theta+K_iV^{-1}, \quad i=1,2,3, \tag{16}$$

with K_i, $i=1,2,3$, being constants of integration satisfying the condition $\Sigma_{i=1}^{3}K_i=0$.

It is worth noticing that, in this framework, the geometry of the considered universe, which is described by $a_i(t)$, $i=1,2,3$, is determined only by the existence of the cosmological constant Λ and is ''decoupled'' from the matter fields ϕ and H. (The effect of matter fields is presented in the magnitude of the parameters, i.e., constants of integration.)

From Eq. (15) we obtain the time evolution of the mean Hubble factor,

$$\theta(\tau)=\sqrt{\frac{\Lambda}{6}}\coth\tau, \tag{17}$$

leading to

$$V(\tau)=V_0\sinh\tau, \tag{18}$$

$$a_i(\tau)=a_{i0}\sinh^{\alpha_i^+}\frac{\tau}{2}\cosh^{\alpha_i^-}\frac{\tau}{2}, \quad i=1,2,3, \tag{19}$$

where $\tau:=\sqrt{3\Lambda/2}(t-t_0)$ and $\alpha_i^{\pm}:=1/3\pm\sqrt{2/3\Lambda}K_i/V_0$. The mean anisotropy and the deceleration parameter are given by

$$A(\tau)=\frac{2K^2}{\Lambda V_0^2}\operatorname{sech}^2\tau, \tag{20}$$

$$q(\tau)=3\operatorname{sech}^2\tau-1, \tag{21}$$

where $K^2=\Sigma_{i=1}^{3}K_i^2$.

III. GENERAL SOLUTION OF THE FIELD EQUATIONS

Equation (13) can be integrated to give

$$\dot{h}=Ce^{-4\phi}V^{-1}, \tag{22}$$

with $C\geqslant 0$, a constant of integration. Thus the dynamics of the dilaton field in the Einstein frame is described by the following differential equation:

$$\sinh\tau\frac{d}{dt}(\sinh\tau\dot{\phi})=\frac{C^2}{\kappa V_0^2}e^{-4\phi}, \tag{23}$$

with the general solution

$$e^{2\phi(\tau)}=\varphi_0^2\left(\tanh^{\omega}\frac{\tau}{2}+\tanh^{-\omega}\frac{\tau}{2}\right), \tag{24}$$

where we denote $\omega:=\sqrt{8\phi_0/3\Lambda}/V_0$, $\varphi_0^2:=\sqrt{C^2/8\kappa\phi_0}$, and $\phi_0>0$ is a constant of integration. The antisymmetric tensor field is given by

$$h(\tau)=h_0+\frac{\kappa\sqrt{\phi_0}}{C}\frac{\tanh^{2\omega}\frac{\tau}{2}-1}{\tanh^{2\omega}\frac{\tau}{2}+1}, \tag{25}$$

with h_0 an arbitrary constant.

The integration constants must satisfy the consistency condition

$$K^2=\Lambda V_0^2-\kappa\phi_0, \tag{26}$$

which follows from Eq. (11).

In the case of a vanishing cosmological constant, $\Lambda\equiv 0$, the general solution in the Einstein frame of the gravitational field equations for a Bianchi type I geometry with dilaton and Kalb-Ramond axion fields is given by

$$\theta(t)=\frac{1}{3t}, \quad V(t)=V_0t, \tag{27}$$

$$a_i(t)=a_{i0}t^{1/3+K_i/V_0},\quad i=1,2,3, \tag{28}$$

$$A=3K^2V_0^{-2}=\text{const},\quad q=2=\text{const}, \tag{29}$$

$$e^{2\phi(t)}=\varphi_0^2(t^\alpha+t^{-\alpha}), \tag{30}$$

$$h(t)=h_0-\frac{2\kappa\sqrt{\phi_0}}{C}(t^{2\alpha}+1)^{-1}, \tag{31}$$

together with the consistency condition

$$K^2=\frac{2}{3}V_0^2-\kappa\phi_0, \tag{32}$$

where $\alpha=2\sqrt{\phi_0}/V_0$.

In order to find the general solution of the gravitational field equations in the string frame with the line element

$$d\hat{s}^2=-d\hat{t}^2+\sum_{i=1}^{3}\hat{a}_i^2(\hat{t})(dx^i)^2, \tag{33}$$

we must perform the conformal transformation (2). To obtain a simpler mathematical form of the equations we shall introduce a new variable $\eta=\tanh\pi/2$, $\eta\in[0,1]$ and denote $\hat{\varphi}_0:=\sqrt{8/3\Lambda}\varphi_0$. Then the string frame time evolution of the Bianchi type I space-time with dilaton and Kalb-Ramond axion fields and an exponential type dilaton potential can be expressed in the following exact parametric form:

$$\hat{t}(\eta)=\hat{t}_0+\hat{\varphi}_0\int\frac{\sqrt{\eta^\omega+\eta^{-\omega}}}{1-\eta^2}d\eta, \tag{34}$$

$$\hat{V}(\eta)=\hat{V}_0(\eta^\omega+\eta^{-\omega})^{3/2}\frac{\eta}{1-\eta^2}, \tag{35}$$

$$\hat{\theta}(\eta)=\frac{1-\eta^2}{3\hat{\varphi}_0\eta\sqrt{\eta^\omega+\eta^{-\omega}}}\left(\frac{3\omega}{2}\frac{\eta^\omega-\eta^{-\omega}}{\eta^\omega+\eta^{-\omega}}+\frac{1+\eta^2}{1-\eta^2}\right), \tag{36}$$

$$\hat{a}_i(\eta)=\hat{a}_{i0}\frac{\eta^{\alpha_i^+}\sqrt{\eta^\omega+\eta^{-\omega}}}{(1-\eta^2)^{1/3}}\quad i=1,2,3, \tag{37}$$

$$\hat{A}(\eta)=\frac{2K^2}{\Lambda V_0^2}\left(\frac{3\omega}{2}\frac{\eta^\omega-\eta^{-\omega}}{\eta^\omega+\eta^{-\omega}}+\frac{1+\eta^2}{1-\eta^2}\right)^{-2}, \tag{38}$$

$$\hat{q}(\eta):=\frac{d}{d\hat{t}}\hat{\theta}^{-1}-1=\frac{1-\eta^2}{\hat{\varphi}_0\sqrt{\eta^\omega+\eta^{-\omega}}}\frac{d}{d\eta}\hat{\theta}^{-1}-1, \tag{39}$$

$$\hat{h}(\eta)=h_0+\frac{\kappa\sqrt{\phi_0}}{C}\frac{\eta^{2\omega}-1}{\eta^{2\omega}+1}, \tag{40}$$

$$e^{2\phi(\eta)}=\varphi_0^2(\eta^\omega+\eta^{-\omega}). \tag{41}$$

In the case of a vanishing cosmological constant the string frame solution of the gravitational field equations with dilaton and axion fields is given again in a parametric form by

$$\hat{t}(t)=\hat{t}_0+\varphi_0\int\sqrt{t^\alpha+t^{-\alpha}}dt, \tag{42}$$

$$\hat{V}(t)=\hat{V}_0t(t^\alpha+t^{-\alpha})^{3/2}, \tag{43}$$

$$\hat{\theta}(t)=\varphi_0^{-1}\left(\frac{\alpha}{2}\frac{t^\alpha-t^{-\alpha}}{t^\alpha+t^{-\alpha}}+\frac{1}{3}\right)\frac{1}{t(t^\alpha+t^{-\alpha})^{1/2}}, \tag{44}$$

$$\hat{a}_i(t)=\hat{a}_{i0}t^{1/3+K_i/V_0}\sqrt{t^\alpha+t^{-\alpha}},\quad i=1,2,3, \tag{45}$$

$$\hat{A}(t)=\frac{K^2}{3V_0^2}\left(\frac{\alpha}{2}\frac{t^\alpha-t^{-\alpha}}{t^\alpha+t^{-\alpha}}+\frac{1}{3}\right)^{-2}, \tag{46}$$

$$\hat{q}(t)=2-\frac{\alpha\left(\frac{1}{3}\frac{t^\alpha-t^{-\alpha}}{t^\alpha+t^{-\alpha}}+\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\frac{t^\alpha-t^{-\alpha}}{t^\alpha+t^{-\alpha}}+\frac{1}{3}\right)^2}, \tag{47}$$

$$\hat{h}(t)=h_0-\frac{2\kappa\sqrt{\phi_0}}{C}(t^{2\alpha}+1)^{-1}, \tag{48}$$

$$e^{2\phi(t)}=\varphi_0^2(t^\alpha+t^{-\alpha}). \tag{49}$$

IV. DISCUSSIONS AND FINAL REMARKS

In the present paper we have presented the exact solution of the gravitational field equations for a Bianchi type I space-time with dilaton and axion fields in both the Einstein and string frames.

In the presence of a cosmological constant the evolution of the Bianchi type I universe starts in the Einstein frame from a singular state, but with finite values of the mean anisotropy and deceleration parameter. In the large time limit the mean anisotropy tends to zero, $A\to0$, and the universe ends in an isotropic inflationary de Sitter phase with a negative deceleration parameter, $q<0$. In the large time limit the dilaton and axion fields become constants, $\lim_{t\to\infty}h(t)=h_0=\text{const}$, $\lim_{t\to\infty}e^{2\phi(t)}=\varphi_0^2=\text{const}$. Moreover, in the Einstein frame, the dynamics and evolution of the universe is determined only by the presence of a cosmological constant (or a dilaton field potential) and there is no direct coupling between the metric and the dilaton and axion fields.

In the string frame the dilaton and axion fields are coupled to the metric, the character of the cosmological evolution being strongly dependent on both fields. The string frame time variation of the volume scale factor of the Bianchi type I space-time for different values of the parameter ω is presented in Fig. 1. Depending on the values of ω there are two

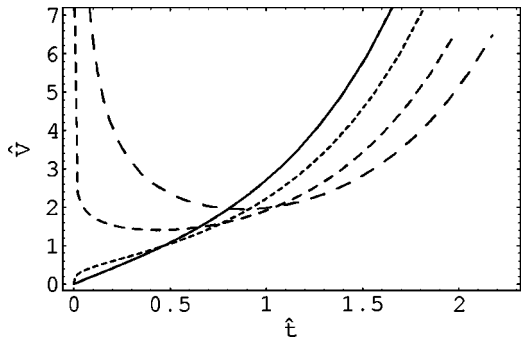

FIG. 1. String frame evolution of the volume scale factor $\hat{V}$ for different values of the parameter ω: $\omega=0.1$ (solid curve), $\omega=0.5$ (dotted curve), $\omega=0.75$ (short dashed curve), and $\omega=0.9$ (long dashed curve). We have used the normalization $\hat{V}_0=1$ and $\hat{t}_0=0$

distinct types of behavior. In the first type of evolution, corresponding to $\omega<2/3$, the universe starts from a singular state with zero values of the scale factors, $\hat{a}_i(0)=0$, $i=1,2,3$, and expands indefinitely. In the second case, when $\omega>2/3$, the Bianchi type I universe starts its evolution with infinite values of the scale factors and collapses to a bounce state, corresponding to minimum finite nonzero values of the scale factors. From this nonsingular state the universe starts to expand, ending in an isotropic inflationary era. The values of the physical quantities at the bounce correspond to the values of η satisfying the equation $d\hat{V}/d\eta=0$ or

$$\frac{(\eta^{\omega}+\eta^{-\omega})^{3/2}}{1-\eta^2}\left(\frac{3\omega}{2}\frac{\eta^{\omega}-\eta^{-\omega}}{\eta^{\omega}+\eta^{-\omega}}+\frac{1+\eta^2}{1-\eta^2}\right)=0. \qquad (50)$$

The string frame evolution of the mean anisotropy parameter $\hat{A}$ is represented in Fig. 2. Independent of which type of evolution is classified by the value of ω, in the presence of an exponential type dilaton potential and of an axion field, the Bianchi type I universe always isotropizes in the large time limit, $\hat{A}\to 0$ for $\hat{t}\to\infty$. But the dynamics of the mean anisotropy factor is very different for the two types of evolution. For $\omega>2/3$, the mean anisotropy increases to an infinite value during the collapse to the bounce and then, during

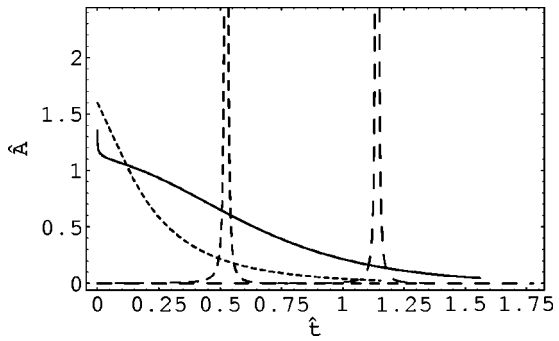

FIG. 2. String frame time variation of the mean anisotropy parameter $\hat{A}$ for different values of ω: $\omega=0.1$ (solid curve), $\omega=0.5$ (dotted curve), $\omega=0.75$ (short dashed curve), and $\omega=0.9$ (long dashed curve). We have used the normalization $2K^2/V_0^2\Lambda=1$.

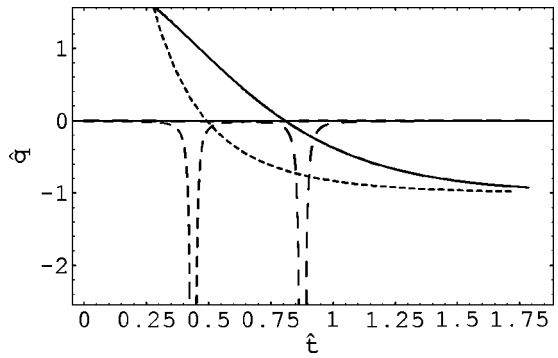

FIG. 3. Dynamics of the deceleration parameter $\hat{q}$ in the string frame for different values of ω: $\omega=0.1$ (solid curve), $\omega=0.5$ (dotted curve), $\omega=0.75$ (short dashed curve), and $\omega=0.9$ (long dashed curve).

the expansionary period, tends rapidly to zero. Hence in this case the post-big-bang type expansionary evolution of the Bianchi type I universe starts with nonsingular scale factors and with maximum (infinite) anisotropy. For $\omega<2/3$ the mean anisotropy of the Bianchi type I space-time monotonically decreases from a maximum finite value to zero.

The variation of the deceleration parameter $\hat{q}$ is represented in Fig. 3. In the string frame and in the presence of a dilaton potential the large time evolution is generally inflationary for all times and for all ω. For $\omega<2/3$, the absence of the bounce state, the dilaton and axion field filled universe starts its evolution from a noninflationary state and accelerated expansion occurs only in the large time limit. For evolutions characterized by a minimum of the scale factors the deceleration parameter is zero at the initial stage of evolution and decreases rapidly during the contracting phase, tending to infinity in the moment when the universe reaches the bounce state. During the expansionary phase the deceleration parameter increases rapidly and in the large time limits tends to zero, $\hat{q}\to 0$ for $\hat{t}\to\infty$. Thus in this case the universe ends at the exact limit separating inflationary and noninflationary evolutions.

In Figs. 4 and 5 we have represented the string frame time

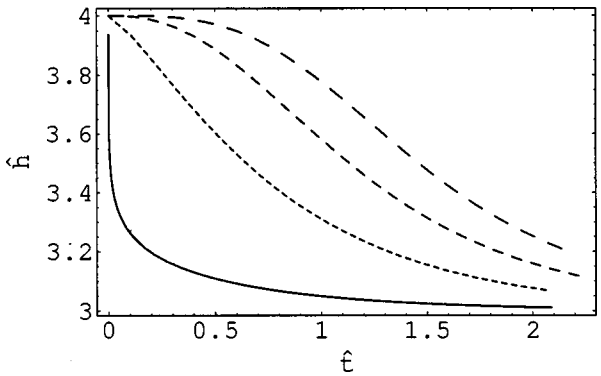

FIG. 4. String frame evolution of the Kalb-Ramond axion field $\hat{h}$ for different values of ω: $\omega=0.1$ (solid curve), $\omega=0.5$ (dotted curve), $\omega=0.75$ (short dashed curve), and $\omega=0.9$ (long dashed curve). We have used the normalizations $\hat{\varphi}_0=1$, $\kappa^2\phi_0/C^2=1$, and $h_0=3$.

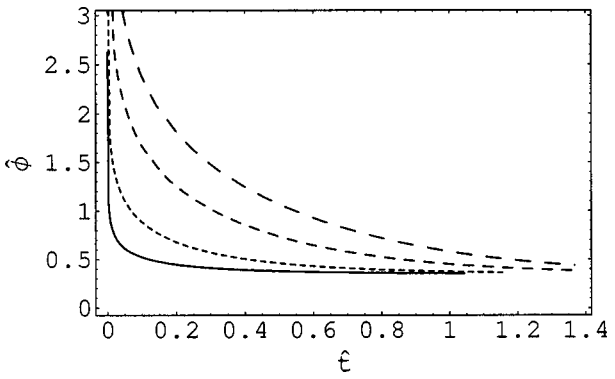

FIG. 5. Variation of the dilaton field as a function of string frame cosmological time for different values of ω: $\omega=0.3$ (solid curve), $\omega=0.5$ (dotted curve), $\omega=0.75$ (short dashed curve), and $\omega=0.9$ (long dashed curve). We have used the normalizations $\hat{\varphi}_0=1$ and $\varphi_0^2=3\Lambda/8$.

evolution of the dilaton and axion fields, respectively. Qualitatively the behavior of these fields is similar to the Einstein frame evolution; in both frames in the large time limit the axion and the dilaton become constants. In the string frame the general character of the evolution is independent of the value of ω and of the presence or absence of the bounce state. As a result of the coupling between the dilaton and axion fields, the string frame evolution of this model is strongly influenced by the presence of the axion. For a vanishing axion field, $h\equiv 0$, the dilaton field equation (14) gives $e^{2\phi(\tau)}=\varphi_0^2\tanh^{\omega_1}(\tau/2)$, with ω_1 a non-negative constant. The presence of the axion field fundamentally modifies the small time behavior of the dilaton field and, consequently, the string frame evolution of the geometry. In the presence of the axion for $\tau\to 0$ we have $e^{2\phi(\tau)}\to\infty$ while for $h=0$, $e^{2\phi(\tau)}\to 0$. In the large time limit in both cases the dilaton tends to a constant value. In the string frame the scale factor is given by $\hat{a}_i=e^{\phi}a_i$, $i=1,2,3$. For $h=0$ we obtain $\hat{a}_i\sim\sinh^{\alpha_i^+ +\omega_1/2}(\tau/2)\cosh^{\alpha_i^- -\omega_1/2}(\tau/2)$, $i=1,2,3$, and in the limit $\tau\to 0$ we always have $\hat{a}_i\to 0$, $i=1,2,3$, since all α_i and ω_1 are non-negative constants. For $h\neq 0$ the string frame scale factors behave like $\hat{a}_i\sim\sqrt{\tanh^{2\omega}(\tau/2)+1}\sinh^{\alpha_i^+ -\omega/2}(\tau/2)\cosh^{\alpha_i^- +\omega/2}(\tau/2)$, $i=1,2,3$. In the small time limit $\hat{a}_i\sim\sinh^{\alpha_i^+ -\omega/2}(\tau/2)$, $i=1,2,3$, and depending on the sign of $\alpha_i^+-\omega/2$ we obtain the two distinct types of evolution already mentioned. Therefore the string frame small time evolution and the character of the initial singularity in this frame essentially depend on the axion field.

In the absence of a cosmological constant or a dilaton field potential the universe does not isotropize. In this case the Einstein frame mean anisotropy is constant for all times and the evolution is anisotropic, of the well-known Kasner type, with an initial singularity at $t=0$. In the string frame the evolution of the scale factors and of the volume element of the universe is qualitatively similar to the $\Lambda\neq 0$ case, with the presence of a bounce state, corresponding to values of the parameter t so that $d\hat{V}/dt=0$ and $t_{\min}=[(3\alpha/2-1)/(3\alpha/2+1)]^{1/2\alpha}$. For $\alpha>2/3$ the scale factors and the volume element collapse from an initial state with infinite values to a nonzero minimum value from which the expanding universe emerges. For $\alpha<2/3$ the Bianchi type I space-time indefinitely expands from an initial singular state. In the large time limit the mean anisotropy tends to a constant nonzero value,

$$\hat{A}=\frac{K^2}{3V_0^2}\left(\frac{\alpha}{2}+\frac{1}{3}\right)^{-2},$$

for $\hat{t}\to\infty$, hence showing that also in the string frame the anisotropic Bianchi type I universe will never experience a smooth transition to an isotropic flat Robertson-Walker type phase. The deceleration parameter in both frames is positive for all times and an inflationary evolution is also impossible. In the string frame for $\hat{t}\to\infty$ we have $\hat{q}=4/(3\alpha+2)$. In the same limit the axion field tends to a constant value while the dilaton field has a logarithmic dependence upon the string frame cosmological time: $\phi(\hat{t})\sim\ln(\hat{t})$.

Recently, several open problems of the pre-big-bang cosmological scenario have been pointed out. One measure of the naturalness of a cosmological scenario is its sensitivity to initial conditions. The effect of spatial curvature in pre-big-bang inflation has been analyzed by [27]. They have shown that in this model the end of inflation is fixed, while its beginning is delayed by the curvature. Too much curvature, of either sign, shortens the duration of the inflationary era to the point that the flatness and horizon problems are not solved. Thus, pre-big-bang inflation requires fine-tuning of initial conditions to solve this basic cosmological problems. In the generic case of the pre-big-bang scenario, inflation will solve cosmological problems only if the universe at the onset of inflation is extremely large and homogeneous from the very beginning, with the size of the homogeneous part greater than $10^{19}l_s$ and with the total mass of the inflationary domain greater than $10^{72}M_s$, where l_s is the stringy length and $M_s\sim l_s^{-1}$ [28]. Also a regime of eternal inflation does not occur in this model. Therefore, as the authors of [28] conclude, ''the current version of pre-big-bang scenario cannot replace usual inflation even if one solves the graceful exit problems.''

The results of the present paper strongly support these conclusions, obtained by using other methods and considering the effects of the dilaton field only. If the pre-big-bang scenario cannot solve the isotropization problem, even with the inclusion of the H field in the model, then it requires a very special initial state in which the universe is already homogeneous and isotropic. The dilaton and the axion field decouple from the geometry in the case of a Bianchi type I geometry and the dynamics of the universe is determined by the dilaton field potential only. These difficulties can be traced back to the deficiencies of the basic physical model used, based on the zeroth order in α' (the inverse string tension parameter) truncation of the string energy effective action. The inclusion of the first order terms in α' in the field equations can lead to a quite different model of the evolution of the very early universe.

Therefore string cosmological models involving only pure dilaton and axion fields resulting from the zeroth order of the

low-energy string effective action do not have, at least in the case of Bianchi type I anisotropic geometries, the ability of providing realistic cosmological models. To obtain a transition from an anisotropic state to an isotropic inflationary one the ''good old'' cosmological constant is still the key ingredient.

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