A Delay-Dependent Approach to Robust H_∞ Filtering for Uncertain Distributed Delay Systems

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***Abstract*—This paper is concerned with the problem of robust H_∞ filtering for linear systems with both discrete and distributed delays, which are subject to norm-bounded time-varying parameter uncertainties. Both the state and measurement equations are assumed to have discrete and distributed delays. A delay-dependent condition for the existence of H_∞ filters is proposed, which is less conservative than existing ones in the literature. Via solutions to certain linear matrix inequalities, general full-order filters are designed that ensure asymptotic stability and a prescribed H_∞ performance level, irrespective of the parameter uncertainties. An illustrative example is provided to demonstrate the effectiveness and the reduced conservatism of the proposed method.**

***Index Terms*—H_∞ filtering, distributed delay, linear matrix inequality, robust filtering, time-delay systems, uncertain systems.**

I. Introduction

OVER the past few years, a great deal of interest has been devoted to the study of H_∞ filtering problem, which is concerned with the design of estimators such that the $\mathcal{L}_2$-induced norm (for continuous systems) or l_2-induced norm (for discrete systems) from the noise signal to the estimation error is less than a prescribed level [1], [11], [15]. In the H_∞ setting, the noises are assumed to be arbitrary deterministic signals with bounded energy (or average power). Compared with traditional Kalman filtering, the H_∞ filtering approach does not require knowledge of the statistical properties of the external noises. In addition, H_∞ filtering is insensitive to uncertainty in the exogenous signal statistics as well as to uncertainty in dynamic models [19]. These features make the H_∞ filtering technique useful in many applications [3], [18].

Time delays are frequently encountered in many practical engineering systems, such as communication, electronics, hydraulic, and chemical systems. It is now well known that time delay is one of the main causes of instability and poor performance of a control system [5], [13], [14]. Therefore, there has been an increasing interest in the control and estimation for time-delay systems, and a great number of results on these topics have been reported in the literature; see, e.g., [10], [12], [13], and the references therein. Recently, the H_∞ filtering problem for time-delay systems was studied in [17], where a Riccati equation approach was developed to solve the problem. When time delays appear in both the state and measurements, a sufficient condition for the existence of H_∞ filters was proposed in [6], in which a design method based on Riccati equations was presented. In the case when time delays and parameter uncertainties appear simultaneously, the robust H_∞ filtering problem was solved in [21] and [25] via a Riccati equation approach and a linear matrix inequality (LMI) approach, respectively. The corresponding results for the discrete case can be found in [16], [20], and [23]. It is noted that all the above H_∞ filtering results are derived for systems with discrete delays. When the number of summands in a system equation is increased and the differences between neighboring argument values are decreased, systems with distributed delays will arise. One application of distributed delay systems can be found in the modeling of feeding systems and combustion chambers in a liquid monopropellant rocket motor with pressure feeding [4], [7]. Very recently, the robust H_∞ filtering problem for such systems has been dealt with in [24], where sufficient conditions for the existence of H_∞ filters have been obtained in terms of LMIs. However, it should be pointed out that the aforementioned results for both the discrete delay case and distributed delay case are delay-independent, that is, they do not include any information on the size of delays. It is known that delay-dependent conditions are generally less conservative than delay-independent ones, especially when the size of the delay is small. Although delay-dependent results on the robust H_∞ filtering problem for systems with discrete delays were presented in [8] and [9], respectively, no delay-dependent H_∞ filtering results on distributed delay systems are available in the literature, which motivates the present study.

This paper deals with the problem of robust H_∞ filtering for linear uncertain systems with both discrete and distributed delays. The time delays are assumed to appear in both the state and measurement equations, and the parameter uncertainties are assumed to be time-varying but norm-bounded which appear in all the matrices in both the state and measurement equations. A delay-dependent condition for the existence of H_∞ filters is proposed and an LMI approach is developed, which is less conservative than existing ones in the literature. A general full order filter is sought to guarantee that the resulting error system is asymptotically stable and satisfies a prescribed H_∞ performance level for all admissible uncertainties. Desired H_∞ filters can be obtained by the solution to certain LMIs, which can be solved numerically and efficiently by resorting to standard numerical algorithms [2]. Finally, an illustrative example is pro-

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vided to demonstrate the less conservatism and effectiveness of the proposed method.

Notation. Throughout this paper, the notation $X \geq Y$ (respectively, $X > Y$) for real symmetric matrices X and Y means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). The superscript "T" represents the transpose. I is an identity matrix with appropriate dimension. $\mathcal{L}_2[0,\ \infty)$ is the space of square-integrable vector functions over $[0,\ \infty)$. The notation $|\cdot|$ refers to the Euclidean vector norm, whereas $\|\cdot\|_2$ stands for the usual $\mathcal{L}_2[0,\infty)$ norm. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

II. Problem Formulation

Consider the following uncertain distributed delay system:

$$(\Sigma): \quad \begin{aligned} \dot{x}(t) &= A(t)x(t) + A_{d1}(t)x(t-\tau_1) \\ &\quad + A_{d2}(t)\int_{t-\tau_2}^{t} x(s)ds + B_1\omega(t) \quad (1)\\ y(t) &= C(t)x(t) + C_{d1}(t)x(t-\tau_1) \\ &\quad + C_{d2}(t)\int_{t-\tau_2}^{t} x(s)ds + B_2\omega(t) \quad (2)\\ z(t) &= Lx(t) + L_{d1}x(t-\tau_1) \quad (3)\\ x(t) &= \varphi(t), \quad \forall\, t \in [-\tau, 0] \quad (4)\end{aligned}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $y(t) \in \mathbb{R}^r$ is the measurement; $z(t) \in \mathbb{R}^q$ is the signal to be estimated, and $\omega(t) \in \mathbb{R}^p$ is the noise input which belongs to $\mathcal{L}_2[0,\infty)$. The scalars $\tau_1 > 0$, $\tau_2 > 0$ represent the time delays of the system; $\tau = \max(\tau_1,\tau_2)$, $\varphi(t)$ is a real-valued continuous initial function on $[-\tau, 0]$. B_1, B_2, L, and L_{d1} are known real constant matrices. The matrices $A(t)$, $A_{d1}(t)$, $A_{d2}(t)$, $C(t)$, $C_{d1}(t)$, and $C_{d2}(t)$ are of the form

$$\begin{aligned} A(t) &= A + \Delta A(t), \quad A_{d1}(t) = A_{d1} + \Delta A_{d1}(t)\\ A_{d2}(t) &= A_{d2} + \Delta A_{d2}(t), \quad C(t) = C + \Delta C(t)\\ C_{d1}(t) &= C_{d1} + \Delta C_{d1}(t), \quad C_{d2}(t) = C_{d2} + \Delta C_{d2}(t)\end{aligned}$$

where A, A_{d1}, A_{d2}, C, C_{d1}, and C_{d2} are known real constant matrices, and $\Delta A(t)$, $\Delta A_{d1}(t)$, $\Delta A_{d2}(t)$, $\Delta C(t)$, $\Delta C_{d1}(t)$, and $\Delta C_{d2}(t)$ are unknown matrices representing time-varying parameter uncertainties, which are assumed to be of the form

$$\begin{bmatrix} \Delta A(t) & \Delta A_{d1}(t) & \Delta A_{d2}(t) \\ \Delta C(t) & \Delta C_{d1}(t) & \Delta C_{d2}(t)\end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} F(t) \begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix} \quad (5)$$

where M_1, M_2, N_1, N_2, and N_3 are known real constant matrices, and $F(\cdot): \mathbb{R} \rightarrow \mathbb{R}^{k\times l}$ is an unknown time-varying matrix function satisfying

$$F(t)^T F(t) \leq I, \quad \forall\, t. \quad (6)$$

The uncertain matrices $\Delta A(t)$, $\Delta A_{d1}(t)$, $\Delta A_{d2}(t)$, $\Delta C(t)$, $\Delta C_{d1}(t)$, and $\Delta C_{d2}(t)$ are said to be admissible if both (5) and (6) hold.

Remark 1: The distributed delay model in (1)–(4) can be used to describe some real systems, such as feeding systems and combustion chambers in a liquid monopropellant rocket motor with pressure feeding; see, e.g., [4], [7], and the references therein.

For system (Σ), we now consider the following general full-order filter for the estimate of $z(t)$:

$$(\Sigma_f): \quad \begin{aligned} \dot{\hat{x}}(t) &= A_f\hat{x}(t) + B_f y(t) \quad (7)\\ \hat{z}(t) &= C_f\hat{x}(t) \quad (8)\end{aligned}$$

where $\hat{x}(t) \in \mathbb{R}^n$ and $\hat{z}(t) \in \mathbb{R}^q$. A_f, B_f, and C_f are matrices to be determined. Denote

$$e(t) = \begin{bmatrix} x(t)^T & \hat{x}(t)^T \end{bmatrix}^T, \quad \tilde{z}(t) = z(t) - \hat{z}(t). \quad (9)$$

Then, the filtering error dynamics from the systems (Σ) and (Σ_f) can be obtained as

$$(\tilde{\Sigma}): \quad \begin{aligned} \dot{e}(t) &= A_c(t)e(t) + A_{cd1}(t)He(t-\tau_1) \\ &\quad + A_{cd2}(t)H\int_{t-\tau_2}^{t} e(s)ds + B_c\omega(t) \quad (10)\\ \tilde{z}(t) &= L_c e(t) + L_{cd1}He(t-\tau_1) \quad (11)\end{aligned}$$

where

$$A_c(t) = A_c + \Delta A_c(t), \quad A_{cd1}(t) = A_{cd1} + \Delta A_{cd1}(t) \quad (12)$$

$$A_{cd2}(t) = A_{cd2} + \Delta A_{cd2}(t), \quad A_c = \begin{bmatrix} A & 0 \\ B_f C & A_f \end{bmatrix} \quad (13)$$

$$A_{cd1} = \begin{bmatrix} A_{d1} \\ B_f C_{d1} \end{bmatrix}, \quad A_{cd2} = \begin{bmatrix} A_{d2} \\ B_f C_{d2} \end{bmatrix} \quad (14)$$

$$\Delta A_c(t) = \begin{bmatrix} \Delta A(t) & 0 \\ B_f \Delta C(t) & 0 \end{bmatrix} \quad (15)$$

$$\Delta A_{cd1}(t) = \begin{bmatrix} \Delta A_{d1}(t) \\ B_f \Delta C_{d1}(t) \end{bmatrix} \quad (16)$$

$$\Delta A_{cd2}(t) = \begin{bmatrix} \Delta A_{d2}(t) \\ B_f \Delta C_{d2}(t) \end{bmatrix}, \quad B_c = \begin{bmatrix} B_1 \\ B_f B_2 \end{bmatrix} \quad (17)$$

$$H = \begin{bmatrix} I & 0 \end{bmatrix}, \quad L_c = \begin{bmatrix} L & -C_f \end{bmatrix}, \quad L_{cd1} = L_{d1}. \quad (18)$$

The purpose of this paper is to develop delay-dependent conditions for the existence of robust H_∞ filters for the uncertain distributed delay system (Σ). Specifically, for given scalars $\overline{\tau}_1 > 0$ and $\overline{\tau}_2 > 0$, we are concerned with finding an asymptotically stable filter (Σ_f) in the form of (10) and (11) such that for any constant time delays τ_1 and τ_2 satisfying $0 < \tau_1 \leq \overline{\tau}_1$, $0 < \tau_2 \leq \overline{\tau}_2$ the filtering error system $(\tilde{\Sigma})$ is asymptotically stable, and

$$\|\tilde{z}\|_2 < \gamma\, \|\omega\|_2 \quad (19)$$

under zero-initial conditions for any nonzero $\omega(t) \in \mathcal{L}_2[0,\ \infty)$ and all admissible uncertainties, where $\gamma > 0$ is a given scalar.

Before concluding this section, we present the following lemma, which will be used in the proof of our main results in Section III.

Lemma: [25] Let $\mathcal{D}$, $\mathcal{S}$, and F be real matrices of appropriate dimensions with F satisfying $F^T F \leq I$. Then, for any scalar $\epsilon > 0$

$$\mathcal{D}F\mathcal{S} + (\mathcal{D}F\mathcal{S})^T \leq \epsilon^{-1}\mathcal{D}\mathcal{D}^T + \epsilon\mathcal{S}^T\mathcal{S}.$$

III. Main Results

The following theorem is essential for solving the robust H_∞ filtering problem formulated in the previous section.

Theorem 1: For any delays τ_1 and τ_2 satisfying $0 < \tau_1 \leq \bar{\tau}_1$ and $0 < \tau_2 \leq \bar{\tau}_2$, the filtering error system $(\tilde{\Sigma})$ in (10) and (11) is robustly asymptotically stable and (19) is satisfied under zero-initial conditions for any nonzero $\omega(t) \in L_2[0,\infty)$ and all admissible uncertainties if there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Z > 0$, W_1, W_2 and a scalar $\epsilon > 0$ such that the LMI in (20), shown at the bottom of the page, holds, where

$$\Upsilon_1 = PA_c + A_c^T P + H^T\left(Q_1 - W_1 - W_1^T\right)H \tag{21}$$

$$\Upsilon_2 = PA_{cd1} + H^T\left(W_1 - W_2^T\right) \tag{22}$$

$$\Upsilon_3 = W_2 + W_2^T - Q_1 \tag{23}$$

$$\tilde{N}_1 = [\,N_1 \quad 0\,], \quad \tilde{N}_2 = N_2, \quad \tilde{N}_3 = N_3 \tag{24}$$

$$\tilde{M}_1 = \begin{bmatrix} M_1 \\ B_f M_2 \end{bmatrix}. \tag{25}$$

Proof: From (20), it can be seen that there exists a scalar $\delta > 0$ such that we have (26), shown at the bottom of the page. By Schur complement equivalence, it follows from (26) that we have (27), shown at the bottom of the page. Note

$$[\,\Delta A_c(t) \quad \Delta A_{cd1}(t) \quad \Delta A_{cd2}(t)\,] = \tilde{M}_1 F(t)\,[\,\tilde{N}_1 \quad \tilde{N}_2 \quad \tilde{N}_3\,]. \tag{28}$$

Then, using Lemma 1, we have the first equation at the bottom of the next page. This, together with (27), gives that, for any

$$\begin{bmatrix}
\Upsilon_1+\bar{\tau}_2^2 H^T Q_2 H+\epsilon\tilde{N}_1^T\tilde{N}_1 & \Upsilon_2+\epsilon\tilde{N}_1^T\tilde{N}_2 & PA_{cd2}+\epsilon\tilde{N}_1^T\tilde{N}_3 & PB_c & \bar{\tau}_1 H^T W_1 & L_c^T & \bar{\tau}_1 A_c^T H^T Z & P\tilde{M}_1 \\
\Upsilon_2^T+\epsilon\tilde{N}_2^T\tilde{N}_1 & \Upsilon_3+\epsilon\tilde{N}_2^T\tilde{N}_2 & \epsilon\tilde{N}_2^T\tilde{N}_3 & 0 & \bar{\tau}_1 W_2 & L_{cd1}^T & \bar{\tau}_1 A_{cd1}^T H^T Z & 0 \\
A_{cd2}^T P+\epsilon\tilde{N}_3^T\tilde{N}_1 & \epsilon\tilde{N}_3^T\tilde{N}_2 & \epsilon\tilde{N}_3^T\tilde{N}_3-Q_2 & 0 & 0 & 0 & \bar{\tau}_1 A_{cd2}^T H^T Z & 0 \\
B_c^T P & 0 & 0 & -\gamma^2 I & 0 & 0 & \bar{\tau}_1 B_c^T H^T Z & 0 \\
\bar{\tau}_1 W_1^T H & \bar{\tau}_1 W_2^T & 0 & 0 & -\bar{\tau}_1 Z & 0 & 0 & 0 \\
L_c & L_{cd1} & 0 & 0 & 0 & -I & 0 & 0 \\
\bar{\tau}_1 ZHA_c & \bar{\tau}_1 ZHA_{cd1} & \bar{\tau}_1 ZHA_{cd2} & \bar{\tau}_1 ZHB_c & 0 & 0 & -\bar{\tau}_1 Z & \bar{\tau}_1 ZH\tilde{M}_1 \\
\tilde{M}_1^T P & 0 & 0 & 0 & 0 & 0 & \bar{\tau}_1\tilde{M}_1^T H^T Z & -\epsilon I
\end{bmatrix} < 0 \tag{20}$$

$$\begin{bmatrix}
\Upsilon_1+\bar{\tau}_2^2 H^T Q_2 H+\epsilon\tilde{N}_1^T\tilde{N}_1+\delta I & \Upsilon_2+\epsilon\tilde{N}_1^T\tilde{N}_2 & PA_{cd2}+\epsilon\tilde{N}_1^T\tilde{N}_3 & PB_c & \bar{\tau}_1 H^T W_1 & L_c^T & \bar{\tau}_1 A_c^T H^T Z & P\tilde{M}_1 \\
\Upsilon_2^T+\epsilon\tilde{N}_2^T\tilde{N}_1 & \Upsilon_3+\epsilon\tilde{N}_2^T\tilde{N}_2 & \epsilon\tilde{N}_2^T\tilde{N}_3 & 0 & \bar{\tau}_1 W_2 & L_{cd1}^T & \bar{\tau}_1 A_{cd1}^T H^T Z & 0 \\
A_{cd2}^T P+\epsilon\tilde{N}_3^T\tilde{N}_1 & \epsilon\tilde{N}_3^T\tilde{N}_2 & \epsilon\tilde{N}_3^T\tilde{N}_3-Q_2 & 0 & 0 & 0 & \bar{\tau}_1 A_{cd2}^T H^T Z & 0 \\
B_c^T P & 0 & 0 & -\gamma^2 I & 0 & 0 & \bar{\tau}_1 B_c^T H^T Z & 0 \\
\bar{\tau}_1 W_1^T H & \bar{\tau}_1 W_2^T & 0 & 0 & -\bar{\tau}_1 Z & 0 & 0 & 0 \\
L_c & L_{cd1} & 0 & 0 & 0 & -I & 0 & 0 \\
\bar{\tau}_1 ZHA_c & \bar{\tau}_1 ZHA_{cd1} & \bar{\tau}_1 ZHA_{cd2} & \bar{\tau}_1 ZHB_c & 0 & 0 & -\bar{\tau}_1 Z & \bar{\tau}_1 ZH\tilde{M}_1 \\
\tilde{M}_1^T P & 0 & 0 & 0 & 0 & 0 & \bar{\tau}_1\tilde{M}_1^T H^T Z & -\epsilon I
\end{bmatrix} < 0. \tag{26}$$

$$\begin{bmatrix}
\Upsilon_1+\bar{\tau}_2^2 H^T Q_2 H+\delta I & \Upsilon_2 & PA_{cd2} & PB_c & \bar{\tau}_1 H^T W_1 & L_c^T & \bar{\tau}_1 A_c^T H^T Z \\
\Upsilon_2^T & \Upsilon_3 & 0 & 0 & \bar{\tau}_1 W_2 & L_{cd1}^T & \bar{\tau}_1 A_{cd1}^T H^T Z \\
A_{cd2}^T P & 0 & -Q_2 & 0 & 0 & 0 & \bar{\tau}_1 A_{cd2}^T H^T Z \\
B_c^T P & 0 & 0 & -\gamma^2 I & 0 & 0 & \bar{\tau}_1 B_c^T H^T Z \\
\bar{\tau}_1 W_1^T H & \bar{\tau}_1 W_2^T & 0 & 0 & -\bar{\tau}_1 Z & 0 & 0 \\
L_c & L_{cd1} & 0 & 0 & 0 & -I & 0 \\
\bar{\tau}_1 ZHA_c & \bar{\tau}_1 ZHA_{cd1} & \bar{\tau}_1 ZHA_{cd2} & \bar{\tau}_1 ZHB_c & 0 & 0 & -\bar{\tau}_1 Z
\end{bmatrix}
+\epsilon^{-1}\begin{bmatrix} P\tilde{M}_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \bar{\tau}_1 ZH\tilde{M}_1 \end{bmatrix}\begin{bmatrix} P\tilde{M}_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \bar{\tau}_1 ZH\tilde{M}_1 \end{bmatrix}^T
+\epsilon\begin{bmatrix} \tilde{N}_1^T \\ \tilde{N}_2^T \\ \tilde{N}_3^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\begin{bmatrix} \tilde{N}_1^T \\ \tilde{N}_2^T \\ \tilde{N}_3^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T < 0. \tag{27}$$

delays τ_1 and τ_2 satisfying $0<\tau_1\le\overline{\tau}_1$ and $0<\tau_2\le\overline{\tau}_2$, we have (29), shown at the bottom of the page, where

$$\tilde{\Upsilon}_1(t)=PA_c(t)+A_c(t)^TP+H^T\left(Q_1-W_1-W_1^T\right)H \quad (30)$$

$$\tilde{\Upsilon}_2(t)=PA_{cd1}(t)+H^T\left(W_1-W_2^T\right). \quad (31)$$

Now, we show the robust asymptotic stability of the filtering error system $(\tilde{\Sigma})$. To this end, we consider (10) with $\omega(t)=0$, that is

$$\dot{e}(t)=A_c(t)e(t)+A_{cd1}(t)He(t-\tau_1)+A_{cd2}(t)H\int_{t-\tau_2}^{t}e(s)ds. \quad (32)$$

Define the following Lyapunov function candidate for system (32):

$$V(t)=V_1(t)+V_2(t)+V_3(t)+V_4(t)+V_5(t) \quad (33)$$

where

$$\begin{aligned}
V_1(t)&=e(t)^TPe(t)\\
V_2(t)&=\int_{t-\tau_1}^{t}e(\alpha)^TH^TQ_1He(\alpha)d\alpha\\
V_3(t)&=\int_{t-\tau_2}^{t}\left[\int_s^t e(\theta)^TH^Td\theta\right]Q_2\left[\int_s^t He(\theta)d\theta\right]ds\\
V_4(t)&=\int_0^{\tau_2}\int_{t-\beta}^{t}(\alpha-t+\beta)\,e(\alpha)^TH^TQ_2He(\alpha)d\alpha d\beta\\
V_5(t)&=\int_{-\tau_1}^{0}\int_{t+\beta}^{t}\dot{e}(\alpha)^TH^TZH\dot{e}(\alpha)d\alpha d\beta.
\end{aligned}$$

Then, by Lemma 1, the time derivative of $V(t)$ along the trajectory of the system (32) is given by

$$\dot{V}_1(t)=2e(t)^TP\left[A_c(t)e(t)+A_{cd1}(t)He(t-\tau_1)+A_{cd2}(t)H\int_{t-\tau_2}^{t}e(s)ds\right] \quad (34)$$

$$\dot{V}_2(t)=e(t)^TH^TQ_1He(t)-e(t-\tau_1)^T\times H^TQ_1He(t-\tau_1) \quad (35)$$

$$\begin{bmatrix}
P\Delta A_c(t)+\Delta A_c(t)^TP & P\Delta A_{cd1}(t) & P\Delta A_{cd2}(t) & 0 & 0 & 0 & \overline{\tau}_1\Delta A_c(t)^TH^TZ\\
\Delta A_{cd1}(t)^TP & 0 & 0 & 0 & 0 & 0 & \overline{\tau}_1\Delta A_{cd1}(t)^TH^TZ\\
\Delta A_{cd2}(t)^TP & 0 & 0 & 0 & 0 & 0 & \overline{\tau}_1\Delta A_{cd2}(t)^TH^TZ\\
0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0\\
\overline{\tau}_1ZH\Delta A_c(t) & \overline{\tau}_1ZH\Delta A_{cd1}(t) & \overline{\tau}_1ZH\Delta A_{cd2}(t) & 0 & 0 & 0 & 0
\end{bmatrix}
\le\epsilon\begin{bmatrix}\tilde{N}_1^T\\ \tilde{N}_2^T\\ \tilde{N}_3^T\\ 0\\ 0\\ 0\\ 0\end{bmatrix}\begin{bmatrix}\tilde{N}_1^T\\ \tilde{N}_2^T\\ \tilde{N}_3^T\\ 0\\ 0\\ 0\\ 0\end{bmatrix}^T+\epsilon^{-1}\begin{bmatrix}P\tilde{M}_1\\ 0\\ 0\\ 0\\ 0\\ 0\\ \overline{\tau}_1ZH\tilde{M}_1\end{bmatrix}\begin{bmatrix}P\tilde{M}_1\\ 0\\ 0\\ 0\\ 0\\ 0\\ \overline{\tau}_1ZH\tilde{M}_1\end{bmatrix}^T.$$

$$\begin{bmatrix}
\tilde{\Upsilon}_1(t)+\tau_2^2H^TQ_2H+\delta I & \tilde{\Upsilon}_2(t) & PA_{cd2}(t) & PB_c & \tau_1H^TW_1 & L_c^T & \tau_1A_c(t)^TH^TZ\\
\tilde{\Upsilon}_2(t)^T & \Upsilon_3 & 0 & 0 & \tau_1W_2 & L_{cd1}^T & \tau_1A_{cd1}(t)^TH^TZ\\
A_{cd2}(t)^TP & 0 & -Q_2 & 0 & 0 & 0 & \tau_1A_{cd2}(t)^TH^TZ\\
B_c^TP & 0 & 0 & -\gamma^2I & 0 & 0 & \tau_1B_c^TH^TZ\\
\tau_1W_1^TH & \tau_1W_2^T & 0 & 0 & -\tau_1Z & 0 & 0\\
L_c & L_{cd1} & 0 & 0 & 0 & -I & 0\\
\tau_1ZHA_c(t) & \tau_1ZHA_{cd1}(t) & \tau_1ZHA_{cd2}(t) & \tau_1ZHB_c & 0 & 0 & -\tau_1Z
\end{bmatrix}<0 \quad (29)$$

$$\dot{V}_3(t) \le \frac{\tau_2^2}{2} e(t)^T H^T Q_2 H e(t) - \eta(t)^T Q_2 \eta(t) + \int_{t-\tau_2}^{t} (\theta - t + \tau_2)\, e(\theta)^T H^T Q_2 H e(\theta) d\theta \quad (36)$$

$$\dot{V}_4(t) = \frac{\tau_2^2}{2} e(t)^T H^T Q_2 H e(t) - \int_{t-\tau_2}^{t} (\alpha - t + \tau_2)\, e(\alpha)^T H^T Q_2 H e(\alpha) d\alpha \quad (37)$$

$$\dot{V}_5(t) = \tau_1 \dot{e}(t)^T H^T Z H \dot{e}(t) - \int_{t-\tau_1}^{t} \dot{e}(\alpha)^T H^T Z H \dot{e}(\alpha) d\alpha \quad (38)$$

where

$$\eta(t) = \int_{t-\tau_2}^{t} He(\theta) d\theta.$$

Therefore, noting

$$e(t-\tau_1) = e(t) - \int_{t-\tau_1}^{t} \dot{e}(\alpha) d\alpha,$$

and using (34)–(38), we have

$$\begin{aligned}\dot{V}(t) \le\; & 2e(t)^T P \left[A_c(t)e(t) + A_{cd1}(t)He(t-\tau_1) + A_{cd2}(t)\eta(t)\right] \\ & + e(t)^T H^T \left(Q_1 + \tau_2^2 Q_2\right) He(t) \\ & - e(t-\tau_1)^T H^T Q_1 He(t-\tau_1) - \eta(t)^T Q_2 \eta(t) \\ & - \int_{t-\tau_1}^{t} \dot{e}(\alpha)^T H^T Z H \dot{e}(\alpha) d\alpha \\ & + \tau_1 \left[A_c(t)e(t) + A_{cd1}(t) \times He(t-\tau_1) + A_{cd2}(t)\eta(t)\right]^T \\ & \times H^T Z H \left[A_c(t)e(t) + A_{cd1}(t)He(t-\tau_1) + A_{cd2}(t)\eta(t)\right] \\ & + 2e(t)^T H^T W_1 H \int_{t-\tau_1}^{t} \dot{e}(\alpha) d\alpha - 2e(t)^T H^T \\ & \times W_1 H \left[e(t) - e(t-\tau_1)\right] \\ & + 2e(t-\tau_1)^T H^T W_2 H \int_{t-\tau_1}^{t} \dot{e}(\alpha) d\alpha - 2e(t-\tau_1)^T \\ & \times H^T W_2 H \left[e(t) - e(t-\tau_1)\right] \\ =\; & \frac{1}{\tau_1} \int_{t-\tau_1}^{t} \xi(t,\alpha)^T \Omega(t) \xi(t,\alpha) d\alpha \end{aligned} \quad (39)$$

where we have the two equations at the bottom of the page. Note that (29) implies (40), shown at the bottom of the page. Applying the Schur complement formula to (40), we obtain

$$\Omega(t) + \begin{bmatrix} \delta I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} < 0. \quad (41)$$

This, together with (39), gives

$$\dot{V}(t) \le -\delta \left|x(t)\right|^2. \quad (42)$$

Therefore, along a similar line as in the proof of [26, Th. 1], it follows from (42) that the filtering error system $(\tilde{\Sigma})$ is robustly asymptotically stable for any delays τ_1 and τ_2 satisfying $0 < \tau_1 \le \overline{\tau}_1$, $0 < \tau_2 \le \overline{\tau}_2$.

Next, we will establish the H_∞ performance of the filtering error system $(\tilde{\Sigma})$ under zero initial condition. To this end, we introduce

$$J(t) = \int_0^t \left[\tilde{z}(s)^T \tilde{z}(s) - \gamma^2 \omega(s)^T \omega(s)\right] ds \quad (43)$$

where $t > 0$. Noting zero initial condition, it can be shown that for any nonzero $\omega(t) \in \mathcal{L}_2[0,\infty)$ and $t > 0$

$$J(t) \le \int_0^t \left[\tilde{z}(s)^T \tilde{z}(s) - \gamma^2 \omega(s)^T \omega(s) + \dot{V}(s)\right] ds \quad (44)$$

where $V(s)$ is defined in (33). Then, following a similar argument as in the derivation of (39), we can obtain

$$\tilde{z}(s)^T \tilde{z}(s) - \gamma^2 \omega(s)^T \omega(s) + \dot{V}(s) \le \frac{1}{\tau_1} \int_{s-\tau_1}^{s} \zeta(s,\alpha)^T \hat{\Omega}(s) \zeta(s,\alpha) d\alpha \quad (45)$$

$$\xi(t,\alpha) = \begin{bmatrix} e(t)^T & (He(t-\tau_1))^T & \eta(t)^T & (H\dot{e}(\alpha))^T \end{bmatrix}^T$$

$$\Omega(t) = \begin{bmatrix} \tilde{\Upsilon}_1(t) + \tau_2^2 H^T Q_2 H & \tilde{\Upsilon}_2(t) & PA_{cd2}(t) & \tau_1 H^T W_1 \\ \tilde{\Upsilon}_2(t)^T & \Upsilon_3 & 0 & \tau_1 W_2 \\ A_{cd2}(t)^T P & 0 & -Q_2 & 0 \\ \tau_1 W_1^T H & \tau_1 W_2^T & 0 & -\tau_1 Z \end{bmatrix} + \tau_1 \begin{bmatrix} A_c(t)^T H^T \\ A_{cd1}(t)^T H^T \\ A_{cd2}(t)^T H^T \\ 0 \end{bmatrix} Z \begin{bmatrix} A_c(t)^T H^T \\ A_{cd1}(t)^T H^T \\ A_{cd2}(t)^T H^T \\ 0 \end{bmatrix}^T.$$

$$\begin{bmatrix} \tilde{\Upsilon}_1(t) + \tau_2^2 H^T Q_2 H + \delta I & \tilde{\Upsilon}_2(t) & PA_{cd2}(t) & \tau_1 H^T W_1 & \tau_1 A_c(t)^T H^T Z \\ \tilde{\Upsilon}_2(t)^T & \Upsilon_3 & 0 & \tau_1 W_2 & \tau_1 A_{cd1}(t)^T H^T Z \\ A_{cd2}(t)^T P & 0 & -Q_2 & 0 & \tau_1 A_{cd2}(t)^T H^T Z \\ \tau_1 W_1^T H & \tau_1 W_2^T & 0 & -\tau_1 Z & 0 \\ \tau_1 Z H A_c(t) & \tau_1 Z H A_{cd1}(t) & \tau_1 Z H A_{cd2}(t) & 0 & -\tau_1 Z \end{bmatrix} < 0. \quad (40)$$

where we have the first two equations at the bottom of the page. On the other hand, by the Schur complement formula, it follows from (29) that

$$\hat{\Omega}(t) + \begin{bmatrix} \delta I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0.$$

By this and (45), we have

$$\tilde{z}(s)^T \tilde{z}(s) - \gamma^2 \omega(s)^T \omega(s) + \dot{V}(s) < 0$$

which, together with (44), implies that $J(t) < 0$ for any nonzero $\omega(t) \in \mathcal{L}_2[0, \infty)$. Thus, for any delays τ_1 and τ_2 satisfying $0 < \tau_1 \leq \overline{\tau}_1$ and $0 < \tau_2 \leq \overline{\tau}_2$, the inequality in (19) holds. This completes the proof. □

Remark 2: The stability condition for distributed delay system in (1) with $\omega(t) = 0$ can be easily inferred from Theorem 1; such a stability condition will be less conservative than that in [22] since in [22], some bounding techniques for some cross terms are used, whereas in Theorem 1, such techniques are not used.

Now, we are in a position to present a solution to the robust H_∞ filtering problem.

Theorem 2: Consider the uncertain distributed delay system (Σ), and let $\gamma > 0$ be a prescribed constant scalar. Then, for any delays τ_1 and τ_2 satisfying $0 < \tau_1 \leq \overline{\tau}_1$, $0 < \tau_2 \leq \overline{\tau}_2$, there exists a filter in the form of (7) and (8) such that the filtering error system $(\tilde{\Sigma})$ is asymptotically stable, and (19) is satisfied if there exist matrices $X > 0$, $Y > 0$, $Q_1 > 0$, $Q_2 > 0$, $Z > 0$, W_1, W_2, Λ, Φ, Ψ, and a scalar $\epsilon > 0$ such that LMIs in (46), shown at the bottom of the page, hold, and

$$X - Y > 0 \tag{47}$$

where

$$J_1 = YA + A^T Y + Q_1 + \overline{\tau}_2^2 Q_2 - W_1 - W_1^T + \epsilon N_1^T N_1 \tag{48}$$

$$J_2 = A^T X + XA + \Psi C + C^T \Psi^T + Q_1 + \overline{\tau}_2^2 Q_2 - W_1 - W_1^T + \epsilon N_1^T N_1 \tag{49}$$

$$J_3 = W_2 + W_2^T - Q_1 + \epsilon N_2^T N_2 \tag{50}$$

$$J_4 = \epsilon N_3^T N_3 - Q_2 \tag{51}$$

$$G_1 = A^T X + YA + C^T \Psi^T + \Phi^T + Q_1 + \overline{\tau}_2^2 Q_2 - W_1 - W_1^T + \epsilon N_1^T N_1 \tag{52}$$

$$G_2 = YA_{d1} + W_1 - W_2^T + \epsilon N_1^T N_2 \tag{53}$$

$$G_3 = YA_{d2} + \epsilon N_1^T N_3 \tag{54}$$

$$G_4 = XA_{d1} + \Psi C_{d1} + W_1 - W_2^T + \epsilon N_1^T N_2 \tag{55}$$

$$G_5 = XA_{d2} + \Psi C_{d2} + \epsilon N_1^T N_3 \tag{56}$$

and the matrix function $U(K_1, K_2)$ is defined as

$$U(K_1, K_2) = XK_1 + \Psi K_2.$$

In this case, a desired robust H_∞ filter is given in the form of (7) and (8) with parameters as follows:

$$\begin{aligned} A_f &= S_1^{-1} \Phi Y^{-1} S_2^{-T}, \quad B_f = S_1^{-1} \Psi \\ C_f &= \Lambda Y^{-1} S_2^{-T} \end{aligned} \tag{57}$$

where S_1 and S_2 are any nonsingular matrices satisfying

$$S_1 S_2^T + XY^{-1} = I. \tag{58}$$

$$\zeta(s, \alpha) = \begin{bmatrix} e(s)^T & (He(s - \tau_1))^T & \eta(s)^T & \omega(s)^T & (H\dot{e}(\alpha))^T \end{bmatrix}^T$$

$$\hat{\Omega}(s) = \begin{bmatrix} \tilde{\Upsilon}_1(s) + \tau_2^2 H^T Q_2 H + L_c^T L_c & \tilde{\Upsilon}_2(s) + L_c^T L_{cd1} & PA_{cd2}(s) & PB_c & \tau_1 H^T W_1 \\ \tilde{\Upsilon}_2(s)^T + L_{cd1}^T L_c & \Upsilon_3 + L_{cd1}^T L_{cd1} & 0 & 0 & \tau_1 W_2 \\ A_{cd2}(s)^T P & 0 & -Q_2 & 0 & 0 \\ B_c^T P & 0 & 0 & -\gamma^2 I & 0 \\ \tau_1 W_1^T H & \tau_1 W_2^T & 0 & 0 & -\tau_1 Z \end{bmatrix} + \tau_1 \begin{bmatrix} A_c(s)^T H^T \\ A_{cd1}(s)^T H^T \\ A_{cd2}(s)^T H^T \\ B_c^T H^T \\ 0 \end{bmatrix} Z \begin{bmatrix} A_c(s)^T H^T \\ A_{cd1}(s)^T H^T \\ A_{cd2}(s)^T H^T \\ B_c^T H^T \\ 0 \end{bmatrix}^T.$$

$$\begin{bmatrix} J_1 & G_1 & G_2 & G_3 & YB_1 & \overline{\tau}_1 W_1 & L^T - \Lambda^T & \overline{\tau}_1 A^T Z & YM_1 \\ G_1^T & J_2 & G_4 & G_5 & U(B_1, B_2) & \overline{\tau}_1 W_1 & L^T & \overline{\tau}_1 A^T Z & U(M_1, M_2) \\ G_2^T & G_4^T & J_3 & \epsilon N_2^T N_3 & 0 & \overline{\tau}_1 W_2 & L_{d1}^T & \overline{\tau}_1 A_{d1}^T Z & 0 \\ G_3^T & G_5^T & \epsilon N_3^T N_2 & J_4 & 0 & 0 & 0 & \overline{\tau}_1 A_{d2}^T Z & 0 \\ B_1^T Y & U(B_1, B_2)^T & 0 & 0 & -\gamma^2 I & 0 & 0 & \overline{\tau}_1 B_1^T Z & 0 \\ \overline{\tau}_1 W_1^T & \overline{\tau}_1 W_1^T & \overline{\tau}_1 W_2^T & 0 & 0 & -\overline{\tau}_1 Z & 0 & 0 & 0 \\ L - \Lambda & L & L_{d1} & 0 & 0 & 0 & -I & 0 & 0 \\ \overline{\tau}_1 ZA & \overline{\tau}_1 ZA & \overline{\tau}_1 ZA_{d1} & \overline{\tau}_1 ZA_{d2} & \overline{\tau}_1 ZB_1 & 0 & 0 & -\overline{\tau}_1 Z & \overline{\tau}_1 ZM_1 \\ M_1^T Y & U(M_1, M_2)^T & 0 & 0 & 0 & 0 & 0 & \overline{\tau}_1 M_1^T Z & -\epsilon I \end{bmatrix} < 0 \tag{46}$$

Proof: By (47), it is easy to see that $I - XY^{-1}$ is nonsingular. Therefore, there always exist nonsingular matrices S_1 and S_2 such that (58) holds. Similar to [24], we define

$$\Pi_1 = \begin{bmatrix} \bar{Y} & I \\ S_2^T & 0 \end{bmatrix}, \quad \Pi_2 = \begin{bmatrix} I & X \\ 0 & S_1^T \end{bmatrix} \tag{59}$$

where

$$\bar{Y} = Y^{-1} > 0.$$

Let

$$P = \Pi_2 \Pi_1^{-1}. \tag{60}$$

Then, by (47) and (58), it can be verified that $P > 0$. Now, pre- and post-multiplying the LMI in (46) by diag$(\bar{Y}, I, I, I, I, I, I, I, I)$, we have (61), shown at the bottom of the page, where Υ_1, Υ_2, Υ_3, $\tilde{N}_1$, $\tilde{N}_2$, $\tilde{N}_3$, and $\tilde{M}_1$ are given in (21)–(25), respectively, and A_f, B_f, and C_f are given in (57). Finally, pre- and post-multiplying the LMI in (61) by diag$(\Pi_1^{-T}, I, I, I, I, I, I, I)$ and its transpose, respectively, and then using Theorem 1, we obtain the desired result. □

Remark 3: Theorem 2 provides a sufficient condition for the solvability of the robust H_∞ filtering problem for uncertain distributed delay systems. Since the condition in (46) and (47) depends on the size of the delays, it will be less conservative than the existing delay-independent ones in [24]. It is also worth pointing out that a desired H_∞ filter can be obtained by solving the LMIs in (46) and (47), which can be implemented by using standard numerical algorithms [2]. The results in Theorem 2 can be extended to multiple delay case along a similar line as in the derivation of Theorem 2.

IV. Illustrative Example

Consider the uncertain distributed delay system (Σ) with parameters as follows:

$$A = \begin{bmatrix} -1.8 & 0.5 \\ 0.3 & -2.2 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} -3.6 & 1.5 \\ 1.2 & -2.8 \end{bmatrix}$$
$$A_{d2} = \begin{bmatrix} 0 & 0.5 \\ 0.2 & -0.7 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -0.6 \\ 0 \end{bmatrix}$$
$$C = [-0.8 \quad 0.2], \quad C_{d1} = [0.6 \quad 0]$$
$$C_{d2} = [-0.2 \quad 0.8], \quad B_2 = 0.5$$
$$L = [0.2 \quad 0.1], \quad L_{d1} = [0 \quad 0]$$
$$M_1 = \begin{bmatrix} 0.1 \\ -0.2 \end{bmatrix}, \quad M_2 = 0.1$$
$$N_1 = [-0.1 \quad 0], \quad N_2 = [0.1 \quad -0.2]$$
$$N_3 = [0.2 \quad -0.1].$$

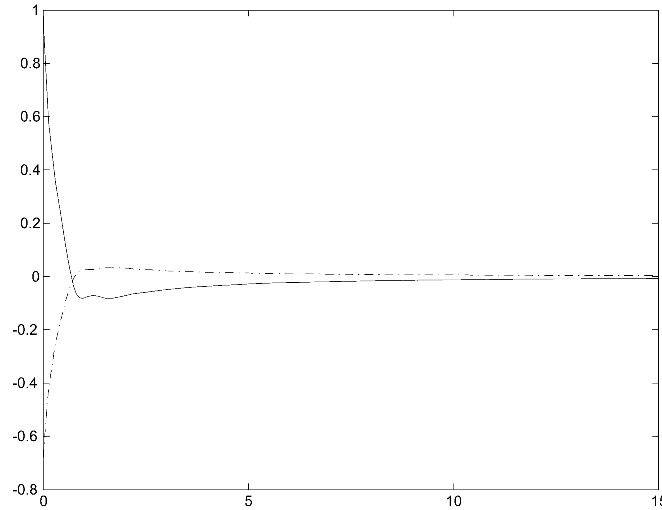

Fig. 1. State response of $\hat{x}_1(t)$ (—) and $\hat{x}_2(t)$ (--).

In this example, we suppose $\tau_2 = 0.8$, and the H_∞ performance level $\gamma = 0.6$. Then, it can be verified that the conditions in [24] are not satisfied for this system, which implies that the conditions in [24] fail to conclude whether or not there exist H_∞ filters for this system. However, by Theorem 2 in this paper, it can be calculated that for all $0 < \tau_1 \le 0.3558$, there exist robust H_∞ filters. As an example, we assume $\tau_1 = 0.2$. In this case, we use the Matlab LMI Control Toolbox to solve the LMIs in (46) and (47) and choose

$$S_1 = \begin{bmatrix} -1.2 & 0.6 \\ 0.2 & 1.5 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 0.4458 & -0.1717 \\ -0.0758 & -0.2657 \end{bmatrix}. \tag{62}$$

It can be seen that the matrices S_1 and S_2 chosen in (62) satisfy (58). Therefore, according to Theorem 2, a desired H_∞ filter can be computed as

$$\dot{\hat{x}}(t) = \begin{bmatrix} -3.5576 & -1.1022 \\ -0.1576 & -3.2383 \end{bmatrix} \hat{x}(t) + \begin{bmatrix} -1.1812 \\ 0.4863 \end{bmatrix} y(t)$$
$$\hat{z}(t) = [0.2310 \quad -0.1644]\, \hat{x}(t).$$

The simulation result of the state response of the designed filter is given in Fig. 1, where the initial condition is $[1 \quad -0.7]^T$, and the exogenous disturbance input $\omega(t)$ is given as

$$\omega(t) = \frac{1}{0.5 + 1.2t}, \quad t \ge 0.$$

Fig. 2 is the simulation result of the error response of $z(t) - \hat{z}(t)$. From these simulation results, it can be seen that the designed H_∞ filter satisfies the specified requirements.

$$\begin{bmatrix}
\Pi_1^T(\Upsilon_1+\epsilon\tilde{N}_1^T\tilde{N}_1) & \Pi_1^T(\Upsilon_2+\epsilon\tilde{N}_1^T\tilde{N}_2) & \Pi_1^T(PA_{cd2}+\epsilon\tilde{N}_1^T\tilde{N}_3) & \Pi_1^T PB_c & \overline{\tau}_1\Pi_1^T H^T W_1 & \Pi_1^T L_c^T & \overline{\tau}_1\Pi_1^T A_c^T H^T Z & \Pi_1^T P\tilde{M}_1 \\
(\Upsilon_2^T+\epsilon\tilde{N}_2^T\tilde{N}_1)\Pi_1 & \Upsilon_3+\epsilon\tilde{N}_2^T\tilde{N}_2 & \epsilon\tilde{N}_2^T\tilde{N}_3 & 0 & \overline{\tau}_1 W_2 & L_{cd1}^T & \overline{\tau}_1 A_{cd1}^T H^T Z & 0 \\
(A_{cd2}^T P+\epsilon\tilde{N}_3^T\tilde{N}_1)\Pi_1 & \epsilon\tilde{N}_3^T\tilde{N}_2 & \epsilon\tilde{N}_3^T\tilde{N}_3-Q_2 & 0 & 0 & 0 & \overline{\tau}_1 A_{cd2}^T H^T Z & 0 \\
B_c^T P\Pi_1 & 0 & 0 & -\gamma^2 I & 0 & 0 & \overline{\tau}_1 B_c^T H^T Z & 0 \\
\overline{\tau}_1 W_1^T H\Pi_1 & \overline{\tau}_1 W_2^T & 0 & 0 & -\overline{\tau}_1 Z & 0 & 0 & 0 \\
L_c\Pi_1 & L_{cd1} & 0 & 0 & 0 & -I & 0 & 0 \\
\overline{\tau}_1 ZHA_c\Pi_1 & \overline{\tau}_1 ZHA_{cd1} & \overline{\tau}_1 ZHA_{cd2} & \overline{\tau}_1 ZHB_c & 0 & 0 & -\overline{\tau}_1 Z & \overline{\tau}_1 ZH\tilde{M}_1 \\
\tilde{M}_1^T P\Pi_1 & 0 & 0 & 0 & 0 & 0 & \overline{\tau}_1\tilde{M}_1^T H^T Z & -\epsilon I
\end{bmatrix} < 0 \tag{61}$$

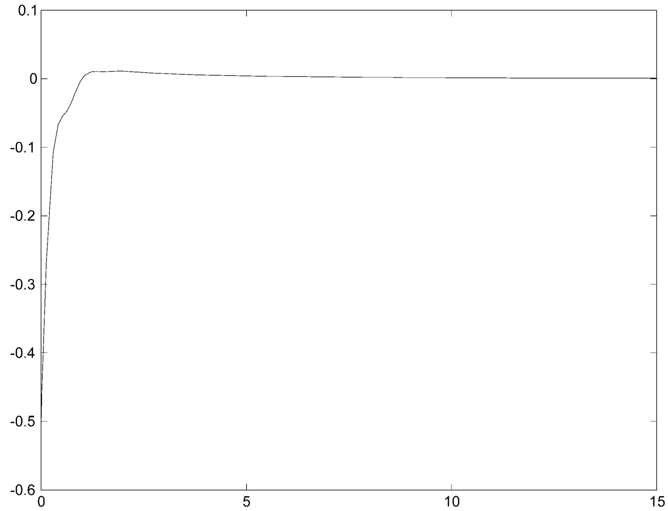

Fig. 2. Error response $\tilde{z}(t)$.

V. CONCLUSION

In this paper, the problem of robust H_∞ filtering for linear uncertain systems with both discrete and distributed delays has been studied. A delay-dependent approach for the design of general full-order filters has been developed. The designed filter guarantees asymptotic stability and a prescribed H_∞ performance level of the error system for all admissible uncertainties. The derived condition is less conservative than existing ones in the literature, which has been demonstrated by an illustrative example.

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