# Estimating the Number of Errors In a System Using a Martingale Approach

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Key Words — Martingale difference, Removal experiment, Time-dependent failure intensity, Weight function, Zero-mean martingale

Summary & Conclusions — A new, efficient procedure estimates the number of errors in a system. A known number of seeded errors are inserted into a system. The failure intensities of the seeded and real errors are allowed to be different and time dependent. When an error is detected during the test, it is removed from the system. The testing process is observed for a fixed amount of time  $\tau$ . Martingale theory is used to derive a class of estimators for the number of seeded errors in a continuous time setting. Some of the estimators and their associated standard deviations have explicit expressions. An optimal estimator among the class of estimators is obtained. A simulation study assesses the performance of the proposed estimators.

#### 1. INTRODUCTION

Mills [15] proposed a capture-recapture sampling method which allows estimation of the number of errors in a system by randomly inserting a known number of errors and then testing the system for both inserted & indigenous errors. Duran & Wiorkowski [5], and Yip & Fong [21] derived maximum likelihood estimates of the indigenous errors and showed methods to obtain s-confidence limits. They assumed that the seeded errors behave as if randomly selected from the distribution of possible real errors in the system. The error seeding method was also discussed in [2, 16].

Martingale theory is used here for an alternative estimation procedure which allows different failure intensities for the seeded & real errors. In order to avoid the identifiability problem, a known constant proportionality is assumed between the seeded & real errors. This problem can be related to a removal experiment for a closed population of a certain type of animal in wildlife studies. Several authors have addressed this problem [4, 6, 9 - 11, 13, 14]. Martingale theory is applied in reliability studies in [12, 19].

Martingale theory is used to derive a class of estimators for the population size of the real errors. Some of the estimators and their associated standard deviations have explicit expressions. An optimal estimator among the class estimators for the number of real errors is obtained.

Acronyms & Abbreviations 1

RMSE (square) root of mean square deviation

StdDev standard deviation ZMM zero-mean martingale.

#### Notation

 $\mathfrak{F}$ , history of the process during [0,t]

ν number of real errors in the system (parameter of interest)

D number of seeded errors (known)

 $\lambda_t$ ,  $\beta_t$  failure intensity for [real, seeded] errors

 $U_{u-}$ ,  $M_{u-}$  number of [real, seeded] errors detected/removed in [0, u)

 $U_t$ ,  $M_t$  number of [real, seeded] errors detected/removed in [0, t]

 $\mathfrak{U}_t$ ,  $\mathfrak{M}_t$  zero-mean martingales

 $\Re_u$  see (3)

 $\lambda_t/\beta_t$ , a known constant

 $\text{Av}_1(\hat{v})$ ,  $\text{SD}_1(\hat{v})$  [average, StdDev] of the 500 simulated values  $\stackrel{d}{=}$  distributed as.

Other, standard notation is given in "Information for Readers & Authors" at the rear of each issue.

#### Assumptions

- 1. A known number of errors, D, is seeded in the system at the beginning of the experiment.
  - 2.  $\lambda_t & \beta_t$  may be time dependent.
  - 3.  $\theta$  is a known constant; it need not be 1.
- 4. The same failure intensity is applied to each type of error in the system.
- 5a. Errors are removed (without introducing new errors or affecting old errors) immediately after detection.
- 5b. Errors are detected/removed one at a time. (This allows a continuous time formulation.)
  - 6.  $U_{u-}$ ,  $M_{u-}$  are measurable w.r.t.  $\mathfrak{F}_{u-}$ .

# 2. CONTINUOUS-TIME MARTINGALE

The methods here are based on results for continuous martingales and follow from the work of [1]. The appendix informally explains martingales. For this paper, ZMM is a stochastic process  $\{\mathfrak{M}_i; t \geq 0\}$  such that:

$$E\{\mathfrak{M}_0\} = 0,$$

 $E\{|\mathfrak{M}_t|\} < \infty$ , for all  $t \geq 0$ ,

 $<sup>^{</sup>I}$ The singular & plural of an acronym or abbreviation are always spelled the same.

$$\mathbb{E}\{\mathfrak{M}_{t+h}|\mathfrak{F}_t\} = \mathfrak{M}_t$$
, for all  $t \geq 0$ ,  $h > 0$ .

We can construct ZMM with respect to the counting processes  $U_t \& M_{t^*}$ . The following 2 statements are true:

$$dU_t | \mathfrak{F}_{t-} \stackrel{d}{=} \operatorname{binm}(\lambda_t \cdot dt, \nu - U_{t-})$$

$$dM_t|F_{t-} \stackrel{d}{=} \operatorname{binm}(\beta_t \cdot dt, D - M_{t-}).$$

By the Doob-Meyer decomposition, (1) & (2) are ZMM with respect to the increasing family of  $\sigma$ -fields or filtration  $\{\mathfrak{F}_t, t \geq 0\}$ 

$$\mathfrak{U}_t = U_t - \int_0^t \lambda_u \cdot (\nu - U_{u-}) \ du, \tag{1}$$

$$\mathfrak{M}_{t} = M_{t} - \int_{0}^{t} \beta_{u} \cdot (D - M_{u-}) du.$$
 (2)

In the presence of  $\lambda_t$ , identifiability problems occur when we want to estimate  $\nu$  by (1) only. Obviously, the information of  $\nu$  &  $\lambda_t$  in (1) is confounded. The extra effort by inserting D errors into the system in the beginning of the testing provides an extra equation to estimate  $\nu$ . However an identifiability problem can still occur with an extra unknown parameter  $\beta$ ; thus assumption 3 is needed. Sacks & Chiang [18] and Wolter [21] made a similar assumption of constant proportionality between two intensities in a competing risk model and a capture-recapture experiment respectively.

We now use the ZMM of (1) & (2) to get an estimating equation which can generate a class of estimators for the population size of real errors,  $\nu$ . Define a martingale-difference:

$$d\mathfrak{R}_{u} = (D - M_{u-}) \cdot dU_{u} - \theta \cdot (\nu - U_{u-}) \cdot dM_{u}. \tag{3a}$$

Then  $E\{d\Re_{u}|\Im_{u_{-}}\}=0$ .

Notation

$$\begin{array}{lll} \textit{W}_{\textit{u}-} & \text{a measurable function w.r.t. } \mathfrak{F}_{\textit{u}-} \\ \mathfrak{R}_{\textit{t}}^* & \int_0^{\textit{t}} \textit{W}_{\textit{u}-} \cdot (\textit{D}-\textit{M}_{\textit{u}-}) \ d\textit{U}_{\textit{u}} - \theta \cdot \int_0^{\textit{t}} \textit{W}_{\textit{u}-} \cdot (\textit{v}-\textit{U}_{\textit{u}-}) \ d\textit{M}_{\textit{u}}. \end{array} \tag{3b}$$

$$\Re^* = \{\Re_t^*, t \ge 0\}$$
, is a ZMM.

Equate (3b) to zero and evaluate it at time  $\tau$ ; then a class of estimators for  $\nu$  is obtained:

$$\hat{v}_w = \left[ \int_0^\tau W_{u-} \cdot (D - M_{u-}) \ dU_u + \theta \cdot \int_0^\tau W_{u-} \cdot U_{u-} \ dM_u \right]$$

$$\int \left[\theta \cdot \int_0^\tau W_{u-} dM_u\right],\tag{4}$$

which depends on the choice of  $W_{u-}$ . The conditional variance of  $\Re$  , is:

$$\operatorname{Var}\{\mathfrak{R}_{\tau}^{*}\} = \operatorname{E}\left\{\int_{0}^{\tau} W_{u-}^{2} \cdot (D - M_{u-})^{2} dU_{u} + \theta^{2} \cdot \int_{0}^{\tau} W_{u-}^{2} \cdot (\nu - U_{u-})^{2} dM_{u}\right\}.$$

The terms for variance follow from a standard result, eg, [3]; the covariance is zero by virtue of the orthogonality result of martingales since  $dM_u$  &  $dU_u$  cannot jump simultaneously. Use a result from [17]:

$$\Re_t^*/\sqrt{\Re_t^*} \to N(0,1)$$
 as  $\nu,D \to \infty$ .

(1) StdDev
$$\{\hat{\nu}_w\} = \left[ \int_0^{\tau} W_{u^-}^2 \cdot (D - M_{u^-})^2 dU_u + \theta^2 \right]$$

$$\cdot \int_{0}^{\tau} W_{u-}^{2} \cdot (\hat{\nu}_{w} - U_{u-})^{2} dM_{u} \bigg]^{\frac{1}{2}} \bigg/ \bigg[ \theta \cdot \int_{0}^{\tau} W_{u-} dM_{u} \bigg]. \tag{5}$$

Any  $W_{u_{-}}$  which is measurable w.r.t.  $\mathfrak{F}_{u_{-}}$  can be used. Consider some choices of  $W_{u_{-}}$  which gives explicit expression for  $\nu$ .

Choice 1: 
$$W_{u-}=1$$

$$\hat{v}_1 = \left[ \int_0^{\tau} (D - M_{u-}) \ dU_u + \theta \cdot \int_0^{\tau} U_{u-} \ dM_u \right] / [\theta \cdot M_{\tau}], \quad (6)$$

$$StdDev\{\hat{\nu}_1\} = \left[ \int_0^\tau (D - M_{u-})^2 dU_u + \theta^2 \right]$$

$$\cdot \int_0^\tau (\nu_1 - U_{u-})^2 dM_u \bigg]^{\frac{1}{2}} \bigg/ \bigg[ \theta \cdot M_\tau \bigg]. \tag{7}$$

Choice 2: Quasi-Score Wu-

 $W_{u-} = (d\Re_u)/(d\langle \Re \rangle_u)$ 

An optimal estimating function within such a class of martingale estimating functions in (3) is discussed in [7, 8]. The best choice, *quasi-score*, is:

$$d\mathfrak{R}_{u} = \mathbb{E}\{d\mathfrak{R}_{u}/d\nu|\mathfrak{F}_{u-}\} = \mathbb{E}\{\theta \cdot dM_{u}|\mathfrak{F}_{u-}\}$$

$$= \theta \cdot (D - M_{u-}) \cdot \beta_{u} \cdot du$$

$$d\langle \mathfrak{R} \rangle_{u} = \mathbb{E}\{(d\mathfrak{R}_{u})^{2}|\mathfrak{F}_{u-}\}$$

$$= \mathbb{E}\{(D - M_{u-})^{2} \cdot dU_{u} + \theta^{2} \cdot (\nu - U_{u-})^{2} \cdot dM_{u}|\mathfrak{F}_{u-}\}$$

$$= (D - M_{u-})^{2} \cdot (\nu - U_{u-}) \cdot \lambda_{u} du + \theta^{2} \cdot (\nu - U_{u-})^{2} \cdot (D - M_{u-}) \cdot \beta_{u} \cdot du.$$

Hence the optimal weight corresponding to (3) is:

$$W_{\nu_{-}}^{*} = [(\nu - U_{\nu_{-}}) \cdot [(D - M_{\nu_{-}}) + (\nu - U_{\nu_{-}}) \cdot \theta]]^{-1}.$$

Accordingly, the optimal estimating equation (gives the tightest • Trials 4, 6, 7 examine the effect of  $\nu$  on the performance when s-confidence limits for  $\hat{v}$ ) is, using (9a):

$$\Re_{t}^{*} = \int_{0}^{t} W_{u_{-}}^{*} \cdot (D - M_{u_{-}}) \ dU_{u} - \theta \cdot \int_{0}^{t} W_{u_{-}}^{*} \cdot (\nu - U_{u_{-}}) \ dM_{u}.$$
(9b)

 $\hat{\nu}^*$  is the solution of (9b). An explicit expression is not available; an iterative procedure is required. From (5),

StdDev
$$\{\hat{p}\} = \left[ \int_0^t (D - M_{u-})^2 / \psi_{u-}^2 dU_u + \theta^2 \right]$$

$$\cdot \int_0^t (\hat{v} - U_{u-})^2 / \psi_{u-}^2 dM_u \bigg] \bigg/ \bigg[ \theta \cdot \int_0^t dM_u / \psi_{u-} \bigg]$$

$$\psi_{u-} \equiv (\hat{\nu}^* - U_{u-}) \cdot [(D - M_{u-}) + (\hat{\nu}^* - U_{u-}) \cdot \theta]$$

## 3. SIMULATION

A Monte Carlo simulation was performed to evaluate the performance of  $\hat{\nu}_1$  from (7) and  $\hat{\nu}^*$  from (9b). Various values of  $\theta$  have been used. An arbitrary stopping time could be used for  $\tau$ . Here we assume that the stopping time is determined by the removed proportion of the seeded errors. We investigate the effects of  $\theta$ , the stopping time, and the proportion of seeded errors placed in the system, on the performance of the estimators.

The simulation results are in two tables. Table 1 lists all the sets of parameter values in the simulation study. Table 2 lists the results of the 7 trials in table 1. There were 500 repetitions for each trial.

#### Notation

 $Av_2(SD_1(\hat{\nu}))$  average  $StdDev\{\hat{\nu}\}$  of the 500 simulated trials RMSE $\{\hat{v}\}$  [Bias $(\hat{v})^2$  + SD<sub>1</sub> $(\hat{v})^2$ ] <sup>1/2</sup>

Bias  $\{\hat{\nu}\}$   $\hat{\nu} - \nu$ 

Coverage proportion of the estimates between the 95% s-confidence limits

proportion of seeded errors removed: the stopping criterion.

The statistics computed were:

 $Av_1(\hat{\nu})$ ,  $SD_1(\hat{\nu})$ ,  $Av_2(SD_1(\hat{\nu}))$ , RMSE, Coverage.

- Trials 1 3 examine the role of P and the performance of the estimators when  $\nu$ , D,  $\theta$  are kept the same. The estimators should perform better when P is large since more information has been collected.
- Trials 2, 4, 5 examine the effect of  $\theta$  when  $\nu$ , D, P are kept the same. When  $\theta > 1$ , the performance of the estimators should improve since more real errors are removed.

D, P are kept the same.

TABLE 1 Description of the Trials

Trial	ν	D	$D/\nu$	θ	P
1	400	50	0.125	0.5	0.3
2	400	50	0.125	0.5	0.5
3	400	50	0.125	0.5	0.9
4	400	50	0.125	1.0	0.5
5	400	50	0.125	2.0	0.5
6	100	50	0.5	1.0	0.5
7	1000	50	0.05	1.0	0.5

Table 2. Simulation Results<sup>†</sup> for the Trials in Table 1 [The top row is for  $\hat{\nu}_1$ ; the bottom row is for  $\hat{\nu}^*$ ]

Trial	ν	D	$Av_1(\hat{\nu})$	$SD_1(\hat{\nu})$	$Av_2(SD_1(\hat{\nu}))$	RMSE	Coverage
1	400	50	402.30	104.05	192.10	216.94	1.00
			397.69	102.08	103.87	139.78	0.95
2	400	50	403.52	72.94	140.26	156.85	1.00
			399.10	70.86	74.18	98.90	0.99
3	400	50	402.38	50.14	87.63	99.98	1.00
			396.10	39.85	36.91	52.07	0.92
4	400	50	401.23	66.46	64.66	88.80	0.95
			393.27	61.51	58.04	81.60	0.93
5	400	50	400.29	53.38	36.88	60.27	0.80
			388.54	40.54	35.69	52.04	0.91
6	100	50	99.32	18.59	18.52	25.19	0.94
			96.82	16.21	16.47	22.56	0.96
7	1000	50	999.31	153.94	154.28	209.65	0.95
			928.02	88.66	129.19	165.83	0.95

<sup>†</sup>The number of significant figures is not intended to imply any accuracy in the estimates, but to illustrate the arithmetic.

- Trials 1-3 confirmed that when P is large: the performance of  $\hat{\nu}_1$  &  $\hat{\nu}^*$  improve, and the RMSE & SD( $\hat{\nu}$ ) decrease appreciably.
- Trials 2, 4, 5 showed that the performance of  $\hat{\nu}_1$  &  $\hat{\nu}^*$  improve when  $\theta > 1$ .
- Trials 4, 6, 7 showed that:  $\hat{\nu}^*$  underestimates  $\nu$  when  $\nu$  is large.

For all trials.

- $\hat{\nu}_1$  is satisfactory though RMSE( $\hat{\nu}_1$ ) > RMSE( $\hat{\nu}^*$ ),
- Coverage( $\hat{\nu}_1$ ) is approx 80% 100%, Coverage( $\hat{\nu}^*$ ) is approx 91% - 99%,
- performance of  $\hat{v}^*$  is uniformly better than  $\hat{v}_1$ .
- $\hat{v}_1$  is easier to compute and can be used as an initial value to search for  $\hat{\nu}^*$ .

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#### APPENDIX

Counting Process, Martingales, and Stochastic Integrals

 $N=\{N_i; t\in \Im\}$  is a counting process if it begins at 0 and increases only by integer-valued jumps, where  $\Im=[0,\infty)$ . The observed process can include one or more counting processes, such as the process counting the number that have failed and the process counting the number that have been censored (lost to follow-up). The increasing family of histories  $\Im=\{\Im_i; t\in \Im\}$  is a filtration.

A process  $\mathfrak{M} = {\mathfrak{M}_t; t \in \mathfrak{I}}$  is a *martingale* (with respect to  $\mathfrak{F}$ ) if, for all  $t \in \mathfrak{I}$ :

$$E\{|\mathfrak{M}_{t}|\} < \infty,$$

$$E\{\mathfrak{M}_{t+s}|\mathfrak{F}_t\}=\mathfrak{M}_t \text{ for all } s\in\mathfrak{I}.$$

A consequence of the martingale property is that  $E\{\mathfrak{M}_t\} = E\{\mathfrak{M}_0\}$  for all  $t \in \mathfrak{I}$ . For a ZMM,  $E\{\mathfrak{M}_0\} = 0$ .

This appendix introduce some properties of martingales which are useful to reliability analysts and gives results for continuous-time martingales in the context of independent continuous failures times. These results are most relevant to applications in this paper. These are not always the most general results. A more rigorous approach is in [3].

Notation

 $N_t$  number of the *n* failure times falling in (0,t] $Y_s$   $n-N_{s-}$ : number still at risk just prior to time *s*.

Let  $N_0 = 0$ . Partition the interval (0,t] into many very small increments. Then write  $N_t$  as  $\int_0^t dN_s$ . We are concerned with the conditional distribution of  $dN_s$ , given  $\mathfrak{F}_{s-}$ . This conditional distribution is binomial:

$$dN_s | \mathfrak{F}_{s-} \stackrel{d}{=} \operatorname{binm}(\alpha_s \cdot ds, Y_s)$$

Most time increments contain 0 failures and a small fraction of the increments contain 1 failure; the probability of an increment containing more than 1 failure is negligible. As,

$$E\{dN_s | \mathfrak{F}_{s-}\} = Y_s \cdot \alpha_s \cdot ds$$
, then,

$$d\mathfrak{M}_{s} = dN_{s} - Y_{s} \cdot \alpha_{s} \cdot ds$$

can be treated as a martingale-difference. It follows that the process,

$$\mathfrak{M} = {\mathfrak{M}_t; t \in \mathfrak{I}}, \text{ specified by,}$$

$$\mathfrak{M}_t = \int_0^t d\mathfrak{M}_s = N_t - \int_0^t Y_s \cdot \alpha_s \ ds,$$

is a ZMM.

Let  $W = \{W_i; t \in \Im\}$  be any process such that  $W_{s-}$  is determined by  $\Re_{s-}$  for each  $s \ge 0$ , then each,

$$W_{s-} \cdot d\mathfrak{M}_{s}$$

is also a martingale-difference. It follows that M\* specified by,

$$\mathfrak{M}_{t}^{*} = \int_{0}^{t} W_{s-} d\mathfrak{M}_{s} = \int_{0}^{t} W_{s-} dN_{s} - \int_{0}^{t} W_{s-} \cdot Y_{s} \cdot \alpha_{s} ds,$$

is a ZMM. This property is very useful for generating estimating equations. In a similar manner, using various choices of W, one obtains estimating equations for other quantities.

The next step is to associate a standard deviation with such an estimate.

$$\begin{aligned} &\operatorname{Var}\{W_{s-}\cdot d\mathfrak{M}_{s}\} = \operatorname{E}\{\operatorname{Var}\{W_{s-}\cdot d\mathfrak{M}_{s}|\ \mathfrak{F}_{s-}\}\} + \operatorname{Var}\{\operatorname{E}\{W_{s-}\cdot d\mathfrak{M}_{s}|\ \mathfrak{F}_{s-}\}\} \\ &\quad \cdot d\mathfrak{M}_{s}|\ \mathfrak{F}_{s-}\}\} \\ &\quad = \operatorname{E}\{W_{s-}^{2}\cdot\operatorname{Var}\{d\mathfrak{M}_{s}|\mathfrak{F}_{s-}\}\} = \operatorname{E}\{W_{s-}^{2}\cdot\operatorname{Var}\{dN_{s}|\mathfrak{F}_{s-}\}\} \\ &\quad = \operatorname{E}\{W_{s-}^{2}\cdot Y_{s}\cdot\alpha_{s}\cdot ds\cdot(1-\alpha_{s}\cdot ds)\} \approx \operatorname{E}\{W_{s-}^{2}\cdot Y_{s}\cdot\alpha_{s}\cdot ds\}. \end{aligned}$$

Hence,

$$\operatorname{Var}\{\mathfrak{M}_{t}^{*}\} = \operatorname{E}\left\{\int_{0}^{t} \operatorname{W}_{s-}^{2} \cdot Y_{s} \cdot \alpha_{s} \ ds\right\} = \operatorname{E}\left\{\int_{0}^{t} \operatorname{W}_{s-}^{2} \ dN_{s}\right\}.$$

A central limit theorem also applies, indicating that inference can be made by using the s-normal distribution when the amount of data is sufficiently large [3].

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