Shot noise of spin current

Baigeng Wang,[1,2] Jian Wang,[1,3,*] and Hong Guo[4]
[1]*Department of Physics, The University of Hong Kong, Pokfulam Road, Hong Kong, China*
[2]*National Laboratory of Solid State Microstructures and Department of Physics, Nanjing University, Nanjing, People's Republic of China*
[3]*Key Laboratory of Materials Physics, Institute of Solid State Physics, Chinese Academy of Sciences, Hefei, Anhui, China*
[4]*Department of Physics, McGill University, Montreal, Quebec, Canada H3A 2T8*
(Received 17 December 2003; published 5 April 2004)

We report a theoretical investigation of shot noise of spin current without an accompanying charge current. For a two-probe spin pump, both cross- and autocorrelations are needed to characterize the noise. The corresponding Fano factors measure the spin unit of the quasiparticles in the spin current. The shot noise also detects open channels for spin transport, and can have qualitatively different behavior compared with the shot noise of charge current.

DOI: 10.1103/PhysRevB.69.153301 PACS number(s): 73.23.Ad, 72.10.Bg, 73.40.Gk

Due to the particle nature of carriers, the fluctuation of charge current gives rise to the notion of shot noise.[1] The correlation properties of current are important because they provide further information for transport in addition to that contained in the *average* current.[2] An example is the determination of quasiparticle charge in the fractional quantum Hall effect by measuring both the power of shot noise $S=2Q\overline{I_e}$ and the average electric current $\overline{I_e}$, deducing[3] $Q=e/3$. Similarly, for charge-current correlation in normal-superconductor tunnel junctions,[4] measurements[6] of shot noise determines $Q=2e$, the charge of Cooper pairs. A useful quantity in analyzing shot noise is the Fano factor $F\equiv S/(2e\overline{I_e})$, which detects open quantum channels in a device.[1] For a normal system, the current correlation between different probes (cross correlation) is negative for fermions and positive for bosons.[2,5]

So far all theoretical and experimental attention has been devoted to correlations of charge current. In this paper, we theoretically study the correlation of *spin current* in the absence of charge current. Experimentally, the flow of a pure spin-current without an accompanying charge current has been realized very recently[7] in a semiconductor heterostructure, by pushing spin-up electrons to move in one direction and an equal number of spin-down electrons to move in the opposite direction. Thereby the net charge current vanishes because $I_e=e(I_\uparrow+I_\downarrow)=0$, and a finite spin current results because $I_s=s(I_\uparrow-I_\downarrow)\neq 0$. Here $(I_\uparrow, I_\downarrow)$ are the electron current for spin-up and spin-down channels, respectively, and s is the spin of carriers. The possibility of flowing pure spin current without charge current has also been investigated theoretically.[8–11] In this work we show that the shot noise of spin current contains additional physics to that of the average spin current itself. In particular, we found that by measuring the Fano factor of a spin current, it is possible to determine the spin unit that is transported. This is useful not only from a spintronics practical point of view, but also from a fundamental science perspective: determining the granularity of the spin unit in quantum transport is important in strongly correlated phenomena and quantum information processing.[12] We also found that the Fano factor helps to determine "open" spin transport channels of a device. Furthermore, because spin current may not be conserved, its shot noise has a more complicated behavior than that of charge current.

To investigate shot noise of spin current, we need a device which generates it. We adopt the spin pump device analyzed in Ref. 10 for this purpose. The operation principle[10] of the spin pump is summarized in Fig. 1(a). Briefly, it consists of a quantum scattering region in contact with two leads having the same electrochemical potential μ. For simplicity, we consider a single spin-degenerate level ϵ in the device and neglect electron-electron interaction. An external *rotating* magnetic field, say counterclockwise, $\mathbf{B}(t)=B_o[\sin\theta\cos\omega t\,\mathbf{i}+\sin\theta\sin\omega t\,\mathbf{j}+\cos\theta\mathbf{k}]$, is applied, and its z component provides a Zeeman splitting so that $\epsilon_\uparrow\equiv\epsilon+B_o\cos\theta$ and $\epsilon_\downarrow\equiv\epsilon-B_o\cos\theta$. Here the spin index $\sigma\equiv\pm 1\equiv\uparrow\downarrow$ (and $\bar{\sigma}\equiv-\sigma$). By adjusting a gate voltage v_g to tune the energy level ϵ, the energy diagram of Fig. 1(a) is established: $\epsilon_\downarrow<\mu<\epsilon_\uparrow$. A spin-down electron can now flow into the scattering region because $\epsilon_\downarrow<\mu$. It then absorbs a photon, flips its spin due to the time-dependent counterclockwise rotating field, and makes a transition to the $\epsilon_\uparrow$ level from which it flows out of the scattering region because $\epsilon_\uparrow>\mu$. Therefore, spin-down electrons flow toward the device while spin-up electrons flow away from it, hence in the leads we have opposite motion of the spin-up/down electrons. This way, a pure spin current is flowing away from the device in the absence of charge current. More details of the spin pump are found in Ref. 10, but here we investigate the shot noise of the spin current.

The spin pump is described by the following Hamiltonian:[10]

$$
\begin{aligned}
H=&\sum_{k,\sigma,\alpha}\epsilon_k C^\dagger_{k\alpha\sigma}C_{k\alpha\sigma}+\sum_\sigma[\epsilon+\sigma B_0\cos\theta]d^\dagger_\sigma d_\sigma\\
&+\gamma[\exp(-i\omega t)d^\dagger_\uparrow d_\downarrow+\text{H.c.}]\\
&+\sum_{k,\sigma,\alpha}[T_{k\alpha}C^\dagger_{k\alpha\sigma}d_\sigma+\text{H.c.}].
\end{aligned}
\tag{1}
$$

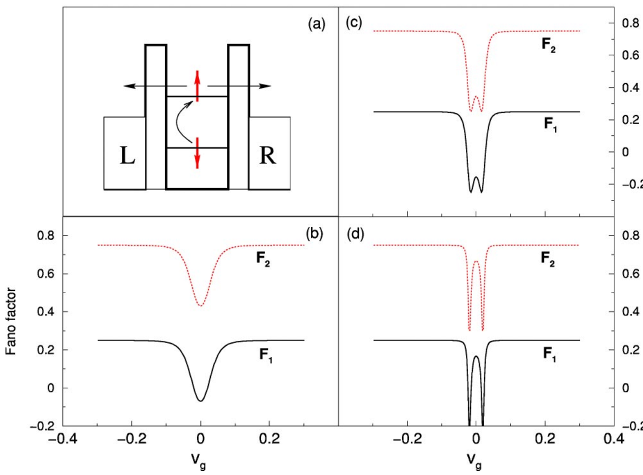

FIG. 1. (a) A schematic illustration of the operation principle of the spin pump which generates a spin current without an accompanying charge current inside the two leads. (b,c,d) are Fano factors of adiabatic limit vs gate voltage v_g for different coupling strength Γ_L (symmetric system), with parameter $\gamma=0.02$. (b) $\Gamma_L=0.16$; (c) $\Gamma_L=0.05$, and (d) $\Gamma_L=0.017$.

The first term stands for noninteracting electrons in lead $\alpha=L,R$ and $C^\dagger_{k\alpha\sigma}$ is the creation operator. The same chemical potential μ is set for both leads. The second term describes the scattering region of the spin pump, which is characterized by energy level ϵ and spin σ. The term proportional to γ is the applied rotating magnetic field discussed above, which provides the driving force for the spin current.[10] The last term gives tunneling between the leads and the scattering region with tunneling matrix $T_{k\alpha}$.

To analyze the noise spectrum of spin current, we define a spin-dependent particle current operator ($\hbar=1$),

$$\hat{J}_{\alpha,\sigma}\equiv\sum_k \frac{d[C^\dagger_{k\alpha\sigma}C_{k\alpha\sigma}]}{dt}=-i\sum_k [T_{k\alpha}C^\dagger_{k\alpha\sigma}d_\sigma-\text{H.c.}], \tag{2}$$

where the second equal sign is obtained by applying the Heisenberg equation of motion using Hamiltonian (1). Then the charge-current operator is obtained from Eq. (2) as $\hat{I}_{\alpha,e}=e\Sigma_\sigma \hat{J}_{\alpha,\sigma}$, and the spin-current operator is $\hat{I}_{\alpha,s}=s(\hat{J}_{\alpha,\uparrow}-\hat{J}_{\alpha,\downarrow})$. We define the following correlation between spin-dependent particle currents in leads α,β:

$$S^{\sigma\sigma'}_{\alpha\beta}\equiv\langle\Delta J_{\alpha\sigma}(t_1)\Delta J_{\beta\sigma'}(t_2)\rangle, \tag{3}$$

where $\Delta J_{\alpha\sigma}(t)\equiv[\hat{J}_{\alpha\sigma}(t)-\overline{J}_{\alpha\sigma}]$ and $\overline{J}_{\alpha\sigma}\equiv\langle\hat{J}_{\alpha\sigma}\rangle$. Here $\langle\cdots\rangle$ denotes both statistical and quantum averages on the nonequilibrium state. The noise spectra of spin current can be obtained from the quantity $S^{\sigma\sigma'}_{\alpha\beta}$. For example, the cross correlation is given by $S_{\text{spin},1}(\omega)\equiv s^2\langle(\Delta J_{L\uparrow}-\Delta J_{L\downarrow})(\Delta J_{R\uparrow}-\Delta J_{R\downarrow})\rangle=s^2(S^{\uparrow\uparrow}_{LR}+S^{\downarrow\downarrow}_{LR}-S^{\uparrow\downarrow}_{LR}-S^{\downarrow\uparrow}_{LR})$, and the autocorrelation is $S_{\text{spin},2}(\omega)\equiv s^2\langle(\Delta J_{L\uparrow}-\Delta J_{L\downarrow})(\Delta J_{L\uparrow}-\Delta J_{L\downarrow})\rangle=s^2(S^{\uparrow\uparrow}_{LL}+S^{\downarrow\downarrow}_{LL}-S^{\uparrow\downarrow}_{LL}-S^{\downarrow\uparrow}_{LL})$. We calculate $S^{\sigma\sigma'}_{\alpha\beta}$ using the standard Keldysh nonequilibrium Green's function (NEGF) formalism.[13] Briefly, after substituting Eq. (2) into Eq. (3), the NEGF $G^<(t_2,t_1)\equiv i\langle C^\dagger_{k\alpha\sigma}(t_2)d_\sigma(t_1)\rangle$ is calculated via the Keldysh equation $\mathbf{G}^<=\mathbf{G}^r\mathbf{\Sigma}^<\mathbf{G}^a$, where $\mathbf{G}^{r,a}$ is the retarded/advanced Green's function.[13] The self-energy $\mathbf{\Sigma}^<=\Sigma_\alpha i f_\alpha\Gamma_\alpha$, where f_α is the Fermi distribution of lead α and Γ_α is the linewidth of the coupling between the device and lead α.[13] In other words, $S^{\sigma\sigma'}_{\alpha\beta}$ can be written in a general form in terms of the Green's functions of a problem.

For the spin-pump Hamiltonian (1), the Green's functions were already derived *exactly* before:[10] $G^r_{\sigma\bar{\sigma}}(E,E')=2\pi\delta(E-E'+\bar{\sigma}\omega)G^r_{\sigma\bar{\sigma}}(E)$ with

$$G^r_{\sigma\bar{\sigma}}(E)=\frac{\gamma}{(E-\epsilon+i\Gamma/2)(E-\epsilon+\sigma\omega+i\Gamma/2)-\gamma^2}, \tag{4}$$

where $\Gamma\equiv\Gamma_L+\Gamma_R$.

For shot noise of spin current, we need to distinguish between cross correlation (between left and right leads) and autocorrelation (between the same lead). The reason is because a spin current may not be conserved due to spin-flip mechanisms in a device. This is different from that of the charge current of a two-probe system where these two correlations only differ by a sign due to charge conservation. The cross correlation of spin current is found to be

$$S_{\text{spin},1}(\omega)=s^2\int\frac{dE}{2\pi}f_\downarrow(1-f_\uparrow)\Gamma_L\Gamma_R[|G^r_{\uparrow\downarrow}|^2+|G^r_{\downarrow\uparrow}|^2 -2\Gamma^2(|G^r_{\uparrow\downarrow}|^4+|G^r_{\downarrow\uparrow}|^4)] \tag{5}$$

and the autocorrelation is found to be

$$S_{\text{spin},2}(\omega)=s^2\int\frac{dE}{2\pi}f_\downarrow(1-f_\uparrow) \times\{2[-\Gamma_L^2\Gamma^2(|G^r_{\uparrow\downarrow}|^4+|G^r_{\downarrow\uparrow}|^4)+\Gamma_L\Gamma(|G^r_{\uparrow\downarrow}|^2 +|G^r_{\downarrow\uparrow}|^2)]-\Gamma_L\Gamma_R(|G^r_{\uparrow\downarrow}|^2+|G^r_{\downarrow\uparrow}|^2)\}. \tag{6}$$

Here $G^r_{\sigma\sigma'}$ is given by Eq. (4), and $f_\uparrow=f_\uparrow(E)$ and $f_\downarrow=f_\downarrow(E-\omega)$ are the Fermi distribution functions.

Expressions (5), (6) are valid for arbitrary frequency ω. At low temperature and in the adiabatic limit $\omega\to 0$, using Eq. (4), Eqs. (5), (6) are reduced to

$$S_{\text{spin},1}=\frac{\omega s^2}{\pi}(1-A)\left(T_o-\frac{2T_o^2}{A}\right), \tag{7}$$

$$S_{\text{spin},2}=\frac{\omega s^2}{\pi}[(1+A)T_o-2T_o^2], \tag{8}$$

where $A\equiv\Gamma_L/\Gamma<1$ gives a measure of the device symmetry, and the quantity T_o is defined as

$$T_o\equiv\frac{\Gamma_L\Gamma\gamma^2}{\left(\epsilon^2+\frac{\Gamma^2}{4}-\gamma^2\right)^2+\gamma^2\Gamma^2}. \tag{9}$$

In fact, T_o is the transmission coefficient of the spin pump for a single spin channel[10] at Fermi level $E_F=\mu=0$. These results are exact for the rotating angle $\theta\to 0$. If $\theta\neq 0$, the same expression is true except that ϵ^2 in Eq. (9) is replaced by $\epsilon_\uparrow \epsilon_\downarrow$. Hence, the cross correlation $S_{\mathrm{spin},1}$ can be positive, negative, and even zero, depending on a number of parameters, namely the gate voltage which controls the energy level position ϵ, the linewidth function Γ, the symmetry of the device (parameter A), and the external magnetic-field strength γ. Of course, the autocorrelation is always positive definite because $T_o \leqslant A$ [from Eq. (9)]. When the cross correlation is zero, the autocorrelation has a value $s^2\omega\Gamma_L/(2\pi\Gamma)$. We now discuss the properties of the shot noise of spin current.

First, we discuss the Fano factors of shot noise which reveal the transported spin unit. For spin current, two Fano factors must be defined: $F_1 = S_{\mathrm{spin},1}/2I_s$ and $F_2 = S_{\mathrm{spin},2}/2I_s$. The spin current generated by the spin pump is given by the transmission coefficient T_o, $I_s = 2(\omega s/2\pi)T_o$, where the factor 2 is due to the two spin channels. Therefore, the Fano factors are

$$F_1 = \frac{s}{2}(1-A)\left(1-\frac{2T_o}{A}\right), \tag{10}$$

$$F_2 = \frac{s}{2}(1+A-2T_o). \tag{11}$$

This gives a very interesting prediction for devices having small transparency to transport ($T_o \ll 1$): $F_1 \approx (s/2)(1-A)$ and $F_2 \approx (s/2)(1+A)$. For symmetric device $A=1/2$, hence $F_1 \approx s/4$ while $F_2 \approx 3s/4$. In other words, there is a "universal" regime where both F_1 and F_2 give the transferred spin s but their ratio is a universal number 3. This is a useful result because it provides a practical way to measure the quasiparticle spin of the spin current. For asymmetric devices, the ratio $F_1/F_2 \approx (1-A)/(1+A)$ which depends on, and therefore can be used to measure, the relative coupling strength Γ_L/Γ. For the spin pump, the small transparency limit ($T_o \ll 1$ limit) can be achieved by tuning the gate voltage v_g such that the energy level $|\epsilon| \sim |ev_g|$ is much greater that the linewidth Γ. Figures 1(b)–1(d) plot the Fano factors in unit of transferred spin s versus gate voltage v_g. The two dips in the Fano factors are due to resonance transmission,[14] but away from the resonance is the universal regime.

Next, we found that the shot noise itself has interesting and complicated behavior, as shown in Fig. 2 for the cross correlation $S_{\mathrm{spin},1}$ of Eq. (7). Because of the spin-flip mechanism, both spin-up and spin-down electrons contribute to spin current. The cross correlation between spin-up electrons is found to be negative definite, and the same is true for spin-down electrons, but it is positive definite between the spin-up and the spin-down electrons. The competition between these contributions gives rise to either a positive or a negative overall cross correlation. Such a complicated behavior is qualitatively different from the correlation in charge current. Interestingly, as one varies the ratio $\Gamma/(2\gamma)$, different line shapes are possible for the cross correlation Eq. (7): (i) strong coupling: $\Gamma/2 > (2+\sqrt{3})\gamma$, the cross correlation is positive definite with a broad peak at $\epsilon=0$ (inset of Fig. 2); (ii) as Γ is decreased such that $\gamma < \Gamma/2 < (2+\sqrt{3})\gamma$, this positive peak at $\epsilon=0$ changes to a local minimum sandwiched between a double peak structure[14] (solid line in Fig. 2); (iii) as Γ is decreased further, $\gamma > \Gamma/2 > (2-\sqrt{3})\gamma$, a third peak emerges at $\epsilon=0$ (dotted line); (iv) finally, in the weak-coupling regime $\Gamma/2 < (2-\sqrt{3})\gamma$, the third peak splits (dashed line in Fig. 2). Despite this rather complicated line shape, the main conclusion is that cross correlation of spin current has large reductions at resonance transmission, hence it can be useful in detecting open channels of spin-current transport. This is similar to the sub-Poissonian shot noise of charge current, which is useful in detecting open transmission channels.[15]

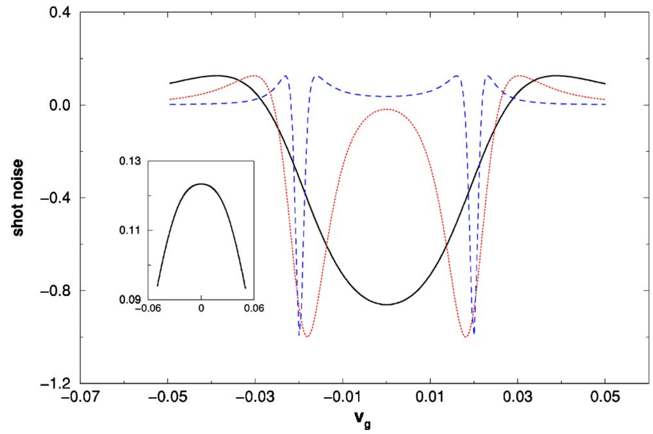

FIG. 2. The cross correlation $S_{\mathrm{spin},1}$ (adiabatic limit) vs v_g for different coupling strengths. Solid line, $\Gamma=0.05$; dotted line, $\Gamma=0.017$; dashed line, $\Gamma=0.004$. Inset, $S_{\mathrm{spin},1}$ vs v_g when $\Gamma=0.16$. Here field strength $\gamma=0.02$, $\Gamma_L=\Gamma_R$, and the unit of $S_{\mathrm{spin},1}$ is $\omega/(16\pi)$.

Equations (5) and (6) together with Eq. (4) allow the investigation of spin-current correlation in the nonadiabatic regime ($\omega \neq 0$), plotted in Fig. 3. As a function of frequency, the shot noise shows an oscillatory behavior between positive and negative values due to photon-assisted processes. Interestingly, for $\omega \neq 0$, the noise becomes asymmetric with respect to the gate voltage, as shown in the inset of Fig. 3 for three different frequencies. By increasing ω from $\omega=0.01$ (solid line) to $\omega=0.02$ (dotted line) and finally to $\omega=0.03$, the cross correlation changes from largely negative values to completely positive definite ones.

Although the net charge current is identically zero, there is still a shot noise of charge current due to the opposite flow of spin-up and spin-down electrons. This shot noise (cross correlation) can be easily calculated,

$$\begin{aligned} S_e = \langle \Delta I_L \Delta I_R \rangle &= q^2 \sum_{\sigma\sigma'} S_{LR}^{\sigma\sigma'} \\ &= -q^2 \Gamma_L \Gamma_R \int \frac{dE}{2\pi} [|G^r_{\uparrow\downarrow}|^2 + |G^r_{\downarrow\uparrow}|^2] f_\downarrow (1-f_\uparrow) \\ &= -\frac{q^2\omega}{2\pi} 2(1-A)T_o, \end{aligned} \tag{12}$$

where the last equality is true for the adiabatic limit at low temperatures and T_o is given by Eq. (9). As expected, S_e is always negative. The autocorrelation is obtained by setting

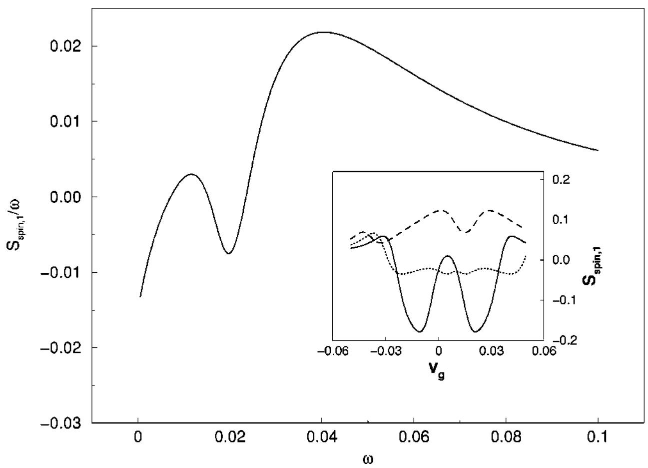

FIG. 3. $S_{\mathrm{spin},1}/\omega$ [unit $1/(4\pi)$] vs frequency ω. Other parameters are $v_g=0.007$, $\Gamma=0.016$, $\gamma=0.02$. Inset: $S_{\mathrm{spin},1}$ vs v_g at different frequencies with $\Gamma=0.017$: $\omega=0.01$ (solid line); $\omega=0.02$ (dotted line); $\omega=0.03$ (dashed line); the unit of $S_{\mathrm{spin},1}$ is $\omega/(4\pi)$ with $\omega=0.005$.

$\alpha=\beta=\mathrm{L}$ in $S_{\alpha\beta}^{\sigma\sigma'}$, and it is straightforward to show $\langle\Delta I_L\Delta I_L\rangle=-\langle\Delta I_L\Delta I_R\rangle$. This is the expected result for charge current which is a conserved quantity, i.e., $I_L+I_R=0$. Experimentally,[16] the quantum partition noise of a point contact has already been observed in the absence of bias voltage so that the average charge current is identically zero.

Finally, we show that the adiabatic limit shot noise of spin current can also be obtained from a scattering matrix approach[1] for symmetric devices ($A=\frac{1}{2}$). Setting $\omega/2\pi=1$ to be the unit, the autocorrelation of spin-up/down electron channels is simply $S_{LL}^{\uparrow\uparrow}=S_{LL}^{\downarrow\downarrow}=T_o(1-T_o)$, which is the well-known expression for shot noise of charge current.[1] Next, for the spin pump, electrons in the left lead are not correlated to those in the right lead with the same spin polarization (e.g., $\langle I_L^{\uparrow}I_R^{\uparrow}\rangle=0$), and one obtains $S_{LR}^{\uparrow\uparrow}=S_{LR}^{\downarrow\downarrow}=-T_o^2$. Finally, electrons in the left lead do correlate with those in the right lead having opposite spin—due to the spin flips of the spin pump—hence we need to calculate a quantity $u\equiv S_{LL}^{\uparrow\downarrow}+S_{LL}^{\downarrow\uparrow}=S_{LR}^{\uparrow\downarrow}+S_{LR}^{\downarrow\uparrow}$, where the second equality is true for a symmetric device. The quantity u can be derived from the shot noise of charge current: autocorrelation is $S_{e,1}=S_{LL}^{\uparrow\uparrow}+S_{LL}^{\downarrow\downarrow}+u=2T_o(1-T_o)+u$ and cross correlation is $S_{e,2}=S_{LR}^{\uparrow\uparrow}+S_{LR}^{\downarrow\downarrow}+u=-2T_o^2+u$. Because $S_{e,1}=-S_{e,2}$ for charge current, $u=2T_o^2-T_o$. We therefore obtain, for spin current, $S_{\mathrm{spin},1}=s^2(S_{LR}^{\uparrow\uparrow}+S_{LR}^{\downarrow\downarrow}-u)=s^2(T_o-4T_o^2)$ in units of $\omega/2\pi$, which agrees with Eq. (7). Also, $S_{\mathrm{spin},2}=s^2(S_{LL}^{\uparrow\downarrow}+S_{LL}^{\downarrow\uparrow}-u)=s^2(3T_o-4T_o^2)$, in agreement with Eq. (8). This simple analysis, of course, cannot be applied to finite frequency.

In summary, we have analyzed the shot noise of spin current without an accompanying charge current. We apply the theory to a spin pump device and derived exact expressions for the shot-noise spectra. Both cross and autocorrelations are necessary in order to characterize spin-current noise. The corresponding Fano factors F_1 and F_2 have an interesting universal limit away from resonance transmission, $F_1\approx s/4$ and $F_2\approx 3s/4$, so that by measuring the shot noise one can determine the spin unit of the quasiparticle that is transported. It is also found that shot noise detects open transport channels by having a resonance behavior.

ACKNOWLEDGMENTS

We gratefully acknowledge support by a RGC grant from the SAR Government of Hong Kong under Grant No. HKU 7113/02P and from NSERC of Canada and FCAR of Quebec (H.G).

*Electronic address: jianwang@hkusub.hku.hk

[1] For recent reviews, see, for example, Ya.M. Blanter and M. Büttiker, Phys. Rep. **336**, 1 (2000).
[2] M. Büttiker, Phys. Rev. B **46**, 12 485 (1992).
[3] L. Saminadayar *et al.*, Phys. Rev. Lett. **79**, 2526 (1997).
[4] M.P. Anantram and S. Datta, Phys. Rev. B **53**, 16 390 (1996); P. Samuelsson and M. Büttiker, Phys. Rev. Lett. **89**, 046601 (2002); B.G. Wang and J. Wang, Phys. Rev. B **67**, 014509 (2003).
[5] M. Henny *et al.*, Science **284**, 296 (1999).
[6] F. Lefloch *et al.*, Phys. Rev. Lett. **90**, 067002 (2003).
[7] M.J. Stevens *et al.*, Phys. Rev. Lett. **90**, 136603 (2003); J. Hübner *et al.*, *ibid.* **90**, 216601 (2003).
[8] A. Brataas *et al.*, Phys. Rev. B **66**, 060404 (2002).
[9] P. Sharma and C. Chamon, Phys. Rev. Lett. **87**, 096401 (2001); E.R. Mucciolo *et al.*, *ibid.* **89**, 146802 (2002).
[10] B.G. Wang *et al.*, Phys. Rev. B **67**, 092408 (2003).
[11] Q.-f. Sun *et al.*, Phys. Rev. Lett. **90**, 258301 (2003).
[12] G. Burkard *et al.*, Phys. Rev. B **61**, 16 303 (2000).
[13] A.P. Jauho *et al.*, Phys. Rev. B **50**, 5528 (1994).
[14] The reason that there are two resonant states is because when field γ is turned on, the spin degeneracy of the original state ϵ is broken. Two resonant states are then found at $\epsilon\pm\gamma-i\Gamma/2$ with level spacing 2γ and half-width $\Gamma/2$. Depending on the ranges of parameters γ relative to Γ, the two resonance states may or may not overlap, resulting in one or two dips in the noise.
[15] A.H. Steinback *et al.*, Phys. Rev. Lett. **76**, 3806 (1996).
[16] L.-H. Reydellet *et al.*, Phys. Rev. Lett. **90**, 176803 (2003).