it converges to the $(n-3)$-dimensional subspace of the eigenvectors with the $n-3$ largest eigenvalue magnitudes, and so on. Eventually, the trajectory converges to a one-dimensional attractor, spanned by the eigenvector of matrix $A'_i(X^*_i, \psi^*_i)$ with the maximal eigenvalue magnitude, and finally to the zero-dimensional vertex of the limit cycle which corresponds to the setup change under consideration.

V. Conclusion

The dynamics of the one-machine n-part-type setup scheduling problem have been studied analytically. The study is conducted in the phase space of both state and costate variables. Making use of necessary optimal setup change conditions allows us to derive an operator which maps a point in the phase space into another point of the same space along the solution of both state and costate differential equations. Expressed analytically, the operator maps the optimal switching surface out of a specific setup to itself and therefore provides insight into the optimal behavior of the system and proves various properties of the optimal schedule including the existence of attractors driving its dynamics.

Different procedures for numerical construction of the switching surfaces in X-space can be suggested on the basis of our results. For example, one can collect points that lie close to the switching surface in the manner described below and then spline them to build its approximation. The collection of those points is obtained by applying the operator A_i^{-1} to the points that belong to the hyperplane which approximates the switching surface in the vicinity of the limit cycle. Since this hyperplane and operator A_i^{-1} are expressed analytically, the procedure may locate points that are arbitrarily close to the switching surface for a particular setup. This and other ideas for numerical development of the presented approach constitute interesting objectives of future research.

References

[1] M. Caramanis, A. Sharifnia, J. Hu, and S. Gershwin, "Development of a science base for planning and scheduling manufacturing systems," in *Proc. NSF Design and Manufacturing Systems Conf.*, Austin, TX, 1991, pp. 27–40.

[2] C. Chase, J. Serrano, and P. Ramadge, "Periodicity and chaos from switched flow systems: Contrasting examples of discretely controlled continuous systems," *IEEE Trans. Automat. Contr.*, vol. 38, pp. 70–83, 1993.

[3] G. Dobson, "The economic lot scheduling problem: Achieving feasibility using time-varying lot sizes," *Ops. Res.*, vol. 35, pp. 764–771, 1987.

[4] M. Elhafsi and S. Bai, "Transient and steady-state analysis of a manufacturing system with setup changes," *J. Global Optim.*, vol. 8, pp. 349–378, 1996.

[5] J. Hu and M. Caramanis, "Dynamic set-up scheduling of flexible manufacturing systems: Design and stability of near optimal general round robin policies," in *Discrete Event Systems*, IMA vol. in Math. and Appl. Series, P. R. Kumar and P. P. Varaiya, Eds. New York: Springer-Verlag, 1995, pp. 73–104.

[6] C. Humes, Jr., L. O. Brandao, and M. P. Garcia, "A mixed dynamics approach for linear corridor policy: A revisitation of dynamic setup scheduling and flow control in manufacturing systems," *Discrete Event Dynamic Syst.*, vol. 5, pp. 59–82, 1995.

[7] J. G. Kimemia and S. B. Gershwin, "An algorithm for the computer control of a flexible manufacturing system," *IIE Trans.*, vol. 15, no. 4, pp. 353–362, 1983.

[8] E. Khmelnitsky, K. Kogan, and O. Maimon, "A maximum principle based method for scheduling in a flexible manufacturing system," *Discrete Event Dynamic Syst.*, vol. 5, pp. 343–355, 1995.

[9] E. Khmelnitsky and K. Kogan, "Necessary optimality conditions for a generalized problem of production scheduling," *Optimal Contr. Appl. Methods*, vol. 15, pp. 215–222, 1994.

[10] F. L. Lewis, *Optimal Control*, Wiley, 1988.

[11] A. Sharifnia, M. Caramanis, and S. Gershwin, "Dynamic set-up scheduling and flow control in manufacturing systems," *Discrete Event Dynamic Syst.*, vol. 1, pp. 149–175, 1991.

[12] G.-X. Yu and P. Vakili, "Periodic and chaotic dynamics of a switched-server system under corridor policies," *IEEE Trans. Automat. Contr.*, vol. 15, Apr. 1996.

Reliable Control Using Redundant Controllers

Guang-Hong Yang, Si-Ying Zhang, James Lam, and Jianliang Wang

***Abstract*—This paper presents a methodology for the design of reliable control systems by using multiple identical controllers to a given plant. The resulting closed-loop control system is reliable in the sense that it provides guaranteed internal stability and H_∞ performance (in terms of disturbance attenuation), not only when all controllers are operational but also when some controller outages (sensor and/or actuator) occur. A numerical example is given to illustrate the proposed design procedures.**

***Index Terms*—Algebraic Riccati equations, control system design, reliable control.**

I. Introduction

Recently, the design problems of reliable centralized and decentralized control systems achieving various reliability goals have been treated by several authors; see [1]–[4] and references therein. One such goal is the reliable stabilization problem for a given plant. Vidyasagar and Viswanadham [1] discuss the reliable stabilization of a plant by two controllers summed together by means of factorization methods and given any stabilizing controller for a plant, a procedure of designing a second stabilizing controller such that the sum of the two controllers also stabilizes the plant. Gundes and Kabuli [2] investigate the reliable stabilization problem for two-channel decentralized control systems and present reliable decentralized controller design methods for strongly stabilizable plants. Another reliability goal is to provide guaranteed system performance. Veillette *et al.* [3] present a new methodology for the design of reliable centralized and decentralized control systems by using the algebraic Riccati equation approach, where the resulting designs provide guaranteed closed-loop stability and H_∞ performance not only when all control components are operating, but also in case of some admissible control component failures.

In [4], Siljak investigated reliability of control structures using more than one controller for a given plant, which is a natural way to introduce redundancy into a control scheme for enhancing reliability. In this paper, we consider the reliable multicontroller design problem in the special case where all the control channels are identical.

Manuscript received June 6, 1995; revised January 29, 1996 and July 30, 1996. This work was supported in part by the Chinese Natural Science Foundation.

G.-H. Yang was with the Department of Automatic Control, Northeastern University, Shenyang, Liaoning, 110006 China. He is now with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798 (egyang@ntu.edu.sg).

S.-Y. Zhang is with the Department of Automatic Control, Northeastern University, Shenyang, Liaoning, 110006 China.

J. Lam is with the Department of Mechanical Engineering, The University of Hong Kong, Hong Kong.

J. Wang is with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798.

Publisher Item Identifier S 0018-9286(98)08422-0.

II. Problem Formulation

Consider the linear time-invariant plant with q identical control channels described by

$$\dot{x} = Ax + \sum_{i=1}^{q} Bu_i + Gw_0 \quad (q > 1) \tag{1}$$

$$y_i = Cx + w_i, \qquad i = 1, \cdots, q \tag{2}$$

$$z = \begin{bmatrix} x^T & H^T & u_1^T & \cdots & u_q^T \end{bmatrix}^T \tag{3}$$

where $x \in R^n$ is the state, $y_i \, (i = 1, \cdots, q)$ are the measured outputs, z is an output to be regulated, $w_i \, (i = 0, 1, \cdots, q)$ are the square-integrable disturbances, and $u_i \, (i = 1, \cdots, q)$ are the control inputs.

The problem is to design q identical controllers for the plant, where the ith controller uses the measurement y_i to generate the control u_i. The q controllers are described by

$$\dot{\xi}_i = A_c \xi_i + Ly_i \tag{4}$$

$$u_i = K\xi_i, \qquad i = 1, \cdots, q \tag{5}$$

where K is the feedback gain, L is the observer gain, K_w is the disturbance estimate gain, and

$$A_c = A + BK + GK_w - LC. \tag{6}$$

Applying the q controllers of (4) and (5) to the plant of (1)–(3), the resulting closed-loop system is as follows:

$$\dot{x}_q = A_q x_q + G_q w_q \tag{7}$$

$$z = H_q x_q \tag{8}$$

where $x_q = [x^T \ \ \xi_1^T \ \ \cdots \ \ \xi_q^T]^T$, $w_q = [w_0^T \ \ w_1^T \ \ \cdots \ \ w_q^T]^T$

$$A_q = \begin{bmatrix} A & BK & BK & \cdots & BK \\ LC & A_c & 0 & \cdots & 0 \\ LC & 0 & A_c & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ LC & 0 & 0 & \cdots & A_c \end{bmatrix} \tag{9}$$

$$G_q = \operatorname{diag}\left[G \quad \underbrace{L \ L \ \cdots \ L}_{q} \right] \tag{10}$$

$$H_q = \operatorname{diag}\left[H \quad \underbrace{K \ K \ \cdots \ K}_{q} \right]. \tag{11}$$

The failure of a controller is modeled as the measurement outage $(y_i = 0)$ or the control input outage $(u_i = 0)$. The design objective is to select the feedback gain K, the observer gain L, and the disturbance estimate gain K_w so that for any p controller failures $(0 \leq p \leq q - 1)$, the resulting closed-loop system is internally stable and the H_∞-norm of the closed-loop transfer function matrix is bounded by some prescribed constant $\alpha > 0$.

Remark 2.1: It should be noted that the closed-loop system of (7) and (8) is very similar to the state model of symmetrically interconnected systems discussed in [5] and [6]. By the results in [5], A_q is Hurwitz if and only if both $\left[\begin{smallmatrix} A & qBK \\ LC & A_c \end{smallmatrix}\right]$ and A_c are Hurwitz. So, all controllers of (4) and (5) must be guaranteed to be open-loop stable in the design procedure in order to ensure closed-loop internal stability and system performance. In other words, the system must be strongly stabilizable. This necessary condition for the above reliable control problem is due to the symmetry in the closed-loop system.

The problem formulation given above in (1)–(6) has the following characteristics. First, identical channels (with identical sensors and identical actuators) are used to improve the reliability of the closed-loop system. This is motivated by the common practice in (e.g., aircraft) industry to use identical sensors, actuators, subsystems and/or channels to prove high reliability [9, Sec. 5.4]. Second, the redundant channels are introduced in a pure *passive* way [9, Sec. 3.4]. Namely, there is no control system reconfiguration involved when any of the allowable outages occurs. The resulting controller provides guaranteed internal stability and system performance (in the sense of H_∞ disturbance attenuation) not only when all control channels are operating correctly, but also when some control channels experience breakdowns/outages. This formulation is related to the multimodel approach (simultaneous stabilization) of [7] and [8] but is different from the *active* approaches of redundancy of fault detection, fault location, and fault recovery [9]–[12]. In the active approaches, system malfunction has to be allowed for a finite amount of time to facilitate fault detection, location, isolation, and recovery. But such a malfunction does not exist in the passive approach proposed here in this paper, making our method suitable for applications where even a temporary malfunction is not allowed.

The next section will present a design procedure for the reliable controller design problem by using the algebraic Riccati equation approach.

III. Main Results

Let $E = \{1, 2, \cdots, q\}$ denote the set of the q controllers of (4) and (5) subject to failures. The problem is to compute a control law which guarantees closed-loop stability and an H_∞-norm bound in spite of controller failures corresponding to any proper subset $e \subset E$. By the symmetry of the matrix A_q, we may assume that $e = \{r+1, r+2, ..., q\} = e_o \cup e_c$ with $r \geq 1$, where $e_o = \{j : y_j = 0, j = r+1, \cdots, q\}$, and $e_c = \{j : u_j = 0, j = r+1, \cdots, q\}$. Let

$$w_{eo} = \begin{bmatrix} w_0^T & w_1^T & \cdots & w_r^T & t_{r+1} w_{r+1}^T & \cdots & t_q w_q^T \end{bmatrix}^T \tag{12}$$

$$z_{eo} = \begin{bmatrix} x^T & H^T & u_1^T & \cdots & u_r^T & p_{r+1} u_{r+1}^T & \cdots & p_q u_q^T \end{bmatrix}^T \tag{13}$$

where

$$t_j = \begin{cases} 0, & j \in e_o \\ 1, & j \neq e_o \end{cases} \qquad p_j = \begin{cases} 0, & j \in e_c \\ 1, & j \neq e_c. \end{cases} \tag{14}$$

When the controller failures corresponding to the subset $e = e_o \cup e_c$ occur, the closed-loop system matrices then take the form

$$A_{qe} = \begin{bmatrix} A & BK & \cdots & BK & t_{r+1}BK & \cdots & t_q BK \\ LC & A_c & \cdots & 0 & 0 & \cdots & 0 \\ \cdot & \cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\ LC & 0 & \cdots & A_c & 0 & \cdots & 0 \\ p_{r+1}LC & 0 & \cdots & 0 & A_c & \cdots & 0 \\ \cdot & \cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\ p_q LC & 0 & \cdots & 0 & 0 & \cdots & A_c \end{bmatrix} \tag{15}$$

$$G_{qeo} = \operatorname{diag}\left[G \quad \underbrace{L \ \cdots \ L}_{r} \quad t_{r+1}L \quad \cdots \quad t_q L \right] \tag{16}$$

$$H_{qec} = \operatorname{diag}\left[H \quad \underbrace{K \ \cdots \ K}_{r} \quad p_{r+1}K \quad \cdots \quad p_q K \right] \tag{17}$$

where $A_c = A + BK + GK_w - LC$.

The result given in the following theorem presents a procedure for output feedback controller design to guarantee that A_{qe} is Hurwitz and that $T_e(s) = H_{qec}(sI - A_{qe})^{-1} G_{qeo}$, the transfer function matrix from w_{eo} to z_{ec}, satisfies $\|T_e\|_\infty \leq \alpha$.

Theorem 3.1: Let (A, H) be a detectable pair and α be a positive constant. Suppose

$$K = -B^T X_0, \qquad K_w = \frac{1}{\alpha^2} G^T X_0 \tag{18}$$

where $X_0 \geq 0$ is symmetric and satisfies the state-feedback design algebraic Riccati equation

$$A^T X_0 + X_0 A + \frac{1}{\alpha^2} X_0 G G^T X_0 - X_0 B B^T X_0 + H^T H + (q-1)\alpha^2 C^T C = 0 \tag{19}$$

with $A + BK + GK_w$ Hurwitz. Suppose also

$$L = q(I - Y X_0/\alpha^2)^{-1} Y C^T \tag{20}$$

where $Y > 0$ is symmetric and satisfies the observer design algebraic Riccati equation

$$A_1 Y + Y A_1^T + \frac{1}{\alpha^2} Y H^T H Y - Y C^T C Y + G G^T + \frac{2q}{\alpha^2} Y K^T K Y + \left(\frac{3}{2}q - 1\right)\alpha^2 B B^T = 0 \tag{21}$$

and where $A_1 = A - qBB^T X_0$, and $\rho\{Y X_0\} < \alpha^2$. Then, for controller failures corresponding to any proper subset $e \subset E$, the closed-loop system is asymptotically stable and $\|T_e\|_\infty \leq \alpha$. Furthermore, all controllers are open-loop stable (A_c Hurwitz).

The following preliminaries will be used in the proof of Theorem 3.1.

Lemma 3.2 [3]: Let $T(s) = H_0(sI - F)^{-1} G_0$, with (F, H_0) a detectable pair. If there exists a real matrix $X \geq 0$ and a positive scalar α such that

$$F^T X + X F + \frac{1}{\alpha^2} X G_0 G_0^T X + H_0^T H_0 \leq 0 \tag{22}$$

then F is Hurwitz and $T(s)$ satisfies $\|T\|_\infty \leq \alpha$.

Consider the matrix $T(n, s) \in R^{(s+1)n \times (s+1)n}$ given by $T(n, 1) = \mathrm{diag}[I_n \quad I_n]$

$$T(n,s) = \begin{bmatrix} I_n & 0 & 0 & 0 & \cdots & 0 \\ 0 & I_n & -I_n & -I_n & \cdots & -I_n \\ 0 & I_n & I_n & 0 & \cdots & 0 \\ 0 & I_n & 0 & I_n & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & I_n & 0 & 0 & \cdots & I_n \end{bmatrix} \quad (s > 1)$$

where I_n is an $n \times n$ identity matrix. Let the matrix $T \in R^{(q+1)n \times (q+1)n}$ be defined as follows:

$$T = T(0) \quad T(1) \quad \cdots \quad T(q-1) \tag{23}$$

where

$$T(i) = \mathrm{diag}\Big[\underbrace{I_n \ \cdots \ I_n}_{i} \quad T(n, q-i)\Big], \qquad i = 0, 1, \ldots, q-1.$$

Lemma 3.3: Let $F \in R^{(q+1)n \times (q+1)n}$ be given by

$$F = \begin{bmatrix} f_{00} & f_{01} & f_{01} & \cdots & f_{01} \\ f_{10} & f_{11} & 0 & \cdots & 0 \\ f_{10} & 0 & f_{11} & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ f_{10} & 0 & 0 & \cdots & f_{11} \end{bmatrix}$$

where $f_{00}, f_{11} \in R^{n \times n}$. Then the following equalities hold.

3.3-1) $T^{-1} F T = \mathrm{diag}[f_1, f_{11}, \cdots, f_{11}]$, where

$$f_1 = \begin{bmatrix} f_{00} & q f_{01} \\ f_{10} & f_{11} \end{bmatrix}.$$

3.3-2) $T^T F T = \mathrm{diag}[f_2, q(q-1) f_{11}, \cdots, 6 f_{11}, 2 f_{11}]$, where

$$f_2 = \begin{bmatrix} f_{00} & q f_{01} \\ q f_{10} & q f_{11} \end{bmatrix}.$$

3.3-3) $T^{-1} F (T^{-1})^T = \mathrm{diag}[f_3, \frac{1}{q(q-1)} f_{11}, \cdots, \frac{1}{3 \times 2} f_{11}, \frac{1}{2 \times 1} f_{11}]$, where

$$f_3 = \begin{bmatrix} f_{00} & \frac{1}{q} f_{01} \\ \frac{1}{q} f_{10} & \frac{1}{q} f_{11} \end{bmatrix}.$$

Lemma 3.4: Let

$$G_+ = [G \quad \sqrt{q-1}\,\alpha B], \qquad H_+ = [H^T \quad \sqrt{q-1}\,\alpha C^T]^T. \tag{24}$$

Then, under the assumptions of Theorem 3.1

$$A_q^T X + X A_q + \frac{1}{\alpha^2} X G_{q+} G_{q+}^T X + H_{q+}^T H_{q+} \leq 0 \tag{25}$$

where A_q is given by (9)

$$G_{q+} = \mathrm{diag}\Big[G_+ \quad \underbrace{L \ L \ \cdots \ L}_{q}\Big] \tag{26}$$

$$H_{q+} = \mathrm{diag}\Big[H_+ \quad \underbrace{K \ K \ \cdots \ K}_{q}\Big] \tag{27}$$

$$X = \begin{bmatrix} X_0 + X_1 & -\frac{1}{q}X_1 & -\frac{1}{q}X_1 & \cdots & -\frac{1}{q}X_1 \\ -\frac{1}{q}X_1 & \frac{1}{q}X_1 & 0 & \cdots & 0 \\ -\frac{1}{q}X_1 & 0 & \frac{1}{q}X_1 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ -\frac{1}{q}X_1 & 0 & 0 & \cdots & \frac{1}{q}X_1 \end{bmatrix} \tag{28}$$

with $X_1 = (\alpha^2 Y^{-1} - X_0)^{-1}$, where X_0 and Y are as given in Theorem 3.1.

Proof: By Lemma 3.3, (9), and (26)–(28), we have

$$\begin{aligned} T^T &\left(A_q^T X + X A_q + \frac{1}{\alpha^2} X G_{q+} G_{q+}^T + H_{q+}^T H_{q+}\right) T \\ &= T^T A_q^T (T^T)^{-1} T^T X T + T^T X T T^{-1} A_q T \\ &\quad + \frac{1}{\alpha^2} T^T X T T^{-1} G_{q+} G_{q+}^T (T^{-1})^T T^T X T + T^T H_{q+}^T H_{q+} T \\ &= \mathrm{diag}[\triangle_1 \quad q(q-1)\triangle_2 \quad \cdots \quad 6\triangle_2 \quad 2\triangle_2] \end{aligned} \tag{29}$$

where

$$\triangle_1 = A_t^T X_t + X_t A_t + \frac{1}{\alpha^2} X_t G_t G_t^T X_t + H_t^T H_t \tag{30}$$

with

$$A_t = \begin{bmatrix} A & qBK \\ LC & A_c \end{bmatrix}, \qquad X_t = \begin{bmatrix} X_0 + X_1 & -X_1 \\ -X_1 & X_1 \end{bmatrix}$$

$$G_t = \mathrm{diag}\left[G_+ \quad \frac{1}{\sqrt{q}} L\right], \qquad H_t = \mathrm{diag}[H_+ \quad \sqrt{q} K]$$

$$\triangle_2 = A_c^T X_1 + X_1 A_c + \frac{1}{\alpha^2 q} X_1 L L^T X_1 + q K^T K. \tag{31}$$

In the following, we shall show that $\triangle_1 \leq 0$ and $\triangle_2 \leq 0$. Let

$$M_t = \begin{bmatrix} I & 0 \\ I & I \end{bmatrix}.$$

Then, from (18), (19), and (24)

$$M_t^T \triangle_1 M_t = \begin{bmatrix} U_1 & 0 \\ 0 & U_2 \end{bmatrix} \tag{32}$$

where

$$U_1 = A^T X_0 + X_0 A + \frac{1}{\alpha^2} X_0 G_+ G_+^T X_0 - q X_0 B B^T X_0 + H_+^T H_+ = 0 \tag{33}$$

$$U_2 = \left(A + \frac{1}{\alpha^2} G_+ G_+^T X_0 - LC\right)^T X_1 + X_1 \left(A + \frac{1}{\alpha^2} G_+ G_+^T X_0 - LC\right) + \frac{1}{\alpha^2 q} X_1 L L^T X_1 + \frac{1}{\alpha^2} X_1 G_+ G_+^T X_1 + q K^T K. \tag{34}$$

By (20) and (28), we have

$$L = q\alpha^2 X_1^{-1} C^T, \qquad X_1 + X_0 = \alpha^2 Y^{-1}. \tag{35}$$

Thus, from (21), it follows that

$$\begin{aligned} U_2 =& (X_1 + X_0)A + A^T(X_1 + X_0) \\ &+ \frac{1}{\alpha^2}(X_1 + X_0) G_+ G_+^T (X_1 + X_0) + H_+^T H_+ - q\alpha^2 C^T C \\ &+ \left(\frac{1}{\alpha\sqrt{q}} X_1 L - \sqrt{q}\alpha C^T\right)\left(\frac{1}{\alpha\sqrt{q}} L^T X_1 - \sqrt{q}\alpha C\right) \\ =& \alpha^2 Y^{-1}\Big[AY + YA^T + \frac{1}{\alpha^2} Y H^T H Y - Y C^T C Y \\ &+ GG^T + (q-1)\alpha^2 B B^T\Big] Y^{-1} \\ =& \alpha^2 Y^{-1}\Big[A_1 Y + Y A_1^T + \frac{1}{\alpha^2} Y H^T H Y - Y C^T C Y \\ &+ GG^T + (q-1)\alpha^2 B B^T - qBKY - qYK^T B^T\Big] Y^{-1} \\ \le& \alpha^2 Y^{-1}\Big[A_1 Y + Y A_1^T + \frac{1}{\alpha^2} Y H^T H Y - Y C^T C Y \\ &+ GG^T + \frac{2q}{\alpha^2} Y K^T K Y + \left(\frac{3}{2}q - 1\right)\alpha^2 B B^T\Big] Y^{-1} \\ =& 0. \end{aligned}$$

Hence

$$\triangle_1 = \left(M_t^T\right)^{-1} \begin{bmatrix} U_1 & 0 \\ 0 & U_2 \end{bmatrix} M_t^{-1} \le 0. \tag{36}$$

Similarly, from (19), (21), (24), (34), and (35)

$$\begin{aligned} \triangle_2 =& X_1 A_1 + A_1^T X_1 + \frac{1}{\alpha^2} X_1 G_+ G_+^T X_0 \\ &+ \frac{1}{\alpha^2} X_0 G_+ G_+^T X_1 - X_1 L C - C^T L^T X_1 \\ &+ \frac{1}{\alpha^2 q} X_1 L L^T X_1 + q K^T K \\ =& X_1 A_1 + A_1^T X_1 + \frac{1}{\alpha^2} X_1 G_+ G_+^T X_0 + \frac{1}{\alpha^2} X_0 G_+ G_+^T X_1 \\ &- X_1 L C - C^T L^T X_1 + \frac{1}{\alpha^2 q} X_1 L L^T X_1 \\ &+ X_0 A_1 + A_1^T X_0 + \frac{1}{\alpha^2} X_0 G_+ G_+^T X_0 \\ &+ H_+^T H_+ - q X_0 B K - q K^T B^T X_0 \\ \le& (X_1 + X_0) A_1 + A_1^T (X_1 + X_0) + \frac{1}{\alpha^2}(X_1 + X_0) G_+ \\ &\times G_+^T (X_1 + X_0) + H^T H - C^T C + 2q X_0 B B^T X_0 \\ =& \alpha^2 Y^{-1}\Big[A_1 Y + Y A_1^T + \frac{1}{\alpha^2} Y H^T H Y - Y C^T C Y \\ &+ GG^T + \frac{2q}{\alpha^2} Y K^T K Y + (q-1)\alpha^2 B B^T\Big] Y^{-1} \\ \le& 0. \end{aligned} \tag{37}$$

From (29), (36), and (37), and a nonsingularity of the matrix of (23), it follows that inequality (25) is true. □

Lemma 3.5: Under the assumptions of Theorem 3.1, the matrix $A_c = A + BK + GK_w - LC$ is Hurwitz.

Proof: From (31) and (37), it follows:

$$X_1^{-1}(A + BK + GK_w - LC)^T + (A + BK + GK_w - LC) X_1^{-1} + \frac{1}{\alpha^2 q} L L^T \le X_1^{-1} \triangle_2 X_1^{-1} \le 0. \tag{38}$$

Let $v \neq 0$ satisfy $(A + BK + GK_w - LC)^T v = \lambda v$. Then (38) gives $2\mathrm{Re}(\lambda) v^* X_1^{-1} v + \frac{1}{\alpha^2 q} v^* L L^T v \le 0$, and it implies that $\mathrm{Re}(\lambda) \le 0$. If $\mathrm{Re}(\lambda) = 0$, then $L^T v = 0$. Thus, $(A + BK + GK_w)^T v = (A + BK + GK_w - LC)^T v = \lambda v$. Since $A + BK + GK_w$ is Hurwitz, it follows that $\mathrm{Re}(\lambda) < 0$, which is in contradiction with $\mathrm{Re}(\lambda) = 0$. Hence, if $\mathrm{Re}(\lambda) < 0$, it further implies that $A + BK + GK_w - LC$ is Hurwitz. □

Proof of Theorem 3.1: Let $e = \{r+1, r+2, \cdots, q\} = e_o \cup e_c (r \ge 1)$ correspond to a subset of controllers subject to outages. From (9)–(11) and (15)–(17), we have

$$\begin{aligned} A_{qe} &= A_q - B_{ec} K_{ec} - L_{eo} C_{eo} \\ H_{qec} &= H_q - K_{ec} \\ G_{qeo} &= G_q - L_{eo} L_{eo}^T \end{aligned}$$

where

$$B_{ec} = \begin{bmatrix} 0 & \cdots & 0 & (1 - t_{r+1})B & \cdots & (1 - t_q)B \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ \cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\ 0 & \cdots & 0 & \cdot & \cdots & 0 \end{bmatrix}$$

$$C_{eo}^T = \begin{bmatrix} 0 & \cdots & 0 & (1 - p_{r+1})C^T & \cdots & (1 - p_q)C^T \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ \cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\ 0 & \cdots & 0 & \cdot & \cdots & 0 \end{bmatrix}$$

$$K_{ec} = \mathrm{diag}[0 \quad \cdots \quad 0 \quad (1 - t_{r+1})K \quad \cdots \quad (1 - t_q)K]$$

$$L_{eo} = \mathrm{diag}[0 \quad \cdots \quad 0 \quad (1 - p_{r+1})L \quad \cdots \quad (1 - p_q)L].$$

By Lemma 3.2, this closed-loop system with sensor and/or actuator outages is internally stable and has an H_∞ disturbance attenuation of $\alpha > 0$ if

$$\Delta_{qe} \triangleq A_{qe}^T X + X A_{qe} + \frac{1}{\alpha^2} X G_{qeo} G_{qeo}^T X + H_{qec}^T H_{qec} \le 0. \tag{39}$$

It is easy to see that

$$\begin{aligned} H_{qec}^T H_{qec} &= H_q^T H_q - K_{ec}^T K_{ec} \\ G_{qeo} G_{qeo}^T &= G_q G_q^T - L_{eo} L_{eo}^T \\ B_{ec} B_{ec}^T &\le \mathrm{diag}[(q-1)BB^T \quad 0 \quad \cdots \quad 0] \\ C_{eo}^T C_{ec} &\le \mathrm{diag}[(q-1)C^T C \quad 0 \quad \cdots \quad 0]. \end{aligned}$$

Then by Lemma 3.4 and (26)–(28), we have that

$$\begin{aligned} \Delta_{qe} =& A_q^T X + X A_q + \frac{1}{\alpha^2} X G_q G_q^T X + H_q^T H_q \\ &- K_{ec}^T B_{ec}^T X - C_{eo}^T L_{eo}^T X - X B_{ec} K_{ec} - X L_{eo} C_{eo} \\ &- \frac{1}{\alpha^2} X L_{eo} L_{eo}^T X - K_{ec}^T K_{ec} \\ =& A_q^T X + X A_q + \frac{1}{\alpha^2} X G_{q+} G_{q+}^T X + H_{q+}^T H_{q+} \\ &- X \mathrm{diag}[(q-1)BB^T \quad 0 \quad \cdots \quad 0] X \\ &- \mathrm{diag}[(q-1)C^T C \quad 0 \quad \cdots \quad 0] - K_{ec}^T B_{ec}^T X - X B_{ec} K_{ec} \\ &- K_{ec}^T K_{ec} - X L_{eo} C_{eo} - C_{eo}^T L_{eo}^T X - \frac{1}{\alpha^2} X L_{eo} L_{eo}^T X \\ \le& -\left(\frac{1}{\alpha} X L_{eo} + \alpha C_{eo}^T\right)\left(\frac{1}{\alpha} L_{eo}^T X + \alpha C_{eo}\right) \\ &- \left(X B_{ec} + K_{ec}^T\right)\left(B_{ec}^T X + K_{ec}\right) \le 0. \end{aligned}$$

Provided that (A_{qe}, H_{qec}) is a detectable pair, Lemma 3.2 guarantees that A_{qe} is Hurwitz and that $T_e(s) = H_{qec}(sI - A_{qe})^{-1}G_{qeo}$, the transfer function matrix from w_{eo} to z_{ec}, satisfies $\|T_e\|_\infty \le \alpha$. To prove detectability of (A_{qe}, H_{qec}), assume that $v^T = (v_1^T, v_2^T) \neq 0$ satisfies $A_{qe}v = \lambda v$ and $H_{qec}v = 0$. Then $Av_1 = \lambda v_1$ and $Hv_1 = 0$. From detectability of (A, H), it follows that either $\mathrm{Re}(\lambda) < 0$ or $v_1 = 0$. If $v_1 = 0$, then $A_{qe}v = \lambda v$ gives $(A + BK + GK_w - LC)v_2 = \lambda v_2$. By Lemma 3.5, we have that $\mathrm{Re}(\lambda) < 0$. Thus, the proof of Theorem 3.1 is completed. □

For the decentralized reliable control problem, the outage of all sensors in a control channel is the same as the outage of all actuators in that control channel. As the system under consideration is single-input/single-output (SISO) in each control channel, a controller failure can be modeled as either an actuator outage or a sensor outage in that channel. By modeling controller failures only as actuator outages in the corresponding control channels and by using similar arguments as in Theorem 3.1, we have the following design procedure.

Corollary 3.6: Let (A, H) be a detectable pair and α be a positive constant. Suppose that K and K_w are as given in (18) with $X_0 \ge 0$ being symmetric and satisfying the following state-feedback design algebraic Riccati equation:

$$A^T X_0 + X_0 A + \frac{1}{\alpha^2} X_0 G G^T X_0 - X_0 B B^T X_0 + H^T H = 0 \quad (40)$$

and with $A + BK + GK_w$ Hurwitz. Suppose also that L is as given in (20) with $Y > 0$ being symmetric and satisfying the observer design algebraic Riccati equation

$$A_1 Y + Y A_1^T + \frac{1}{\alpha^2} Y H^T H Y - qYC^TCY + GG^T + \frac{2q}{\alpha^2} Y K^T K Y + \left(\frac{3}{2}q - 1\right)\alpha^2 B B^T = 0 \quad (41)$$

where $A_1 = A - qBB^T X_0$ and $\rho\{Y \ X_0\} < \alpha^2$. Then, for controller failures corresponding to any proper subset $e \subset E$, the closed-loop system is asymptotically stable and $\|T_e\|_\infty \le \alpha$. □

Similarly, by modeling controller failures only as sensor outages in the corresponding control channels, we have the following design procedure.

Corollary 3.7: Let (A, H) be a detectable pair and α be a positive constant. Suppose that K and K_w are as given in (18) with $X_0 \ge 0$ being symmetric and satisfying the state-feedback design algebraic Riccati equation

$$A^T X_0 + X_0 A + \frac{1}{\alpha^2} X_0 G G^T X_0 - qX_0 B B^T X_0 + H^T H + (q-1)\alpha^2 C^T C = 0 \quad (42)$$

with $A + BK + GK_w$ Hurwitz. Suppose also that L is as given in (20) with $Y > 0$ being symmetric and satisfying the observer design algebraic Riccati equation

$$A_1 Y + Y A_1^T + \frac{1}{\alpha^2} Y H^T H Y - YC^TCY + GG^T + \frac{2q}{\alpha^2} Y K^T K Y + \frac{q}{2}\alpha^2 B B^T = 0 \quad (43)$$

with $A_1 = A - qBB^T X_0$ and $\rho\{Y \ X_0\} < \alpha^2$. Then, for controller failures corresponding to any proper subset $e \subset E$, the closed-loop system is asymptotically stable, and $\|T_e\|_\infty \le \alpha$. □

It can be seen easily that the controller designs in Corollaries 3.6 and 3.7 are less conservative than that in Theorem 3.1. The proofs for the above two corollaries are quite similar to that of Theorem 3.1 and are hence omitted.

Remark 3.8: It should be noted that a solution to the above reliable controller design problem could be derived directly from the work of Veillette *et al.* [3]. This is done by setting all the B's and C's equal in their design equations for reliable decentralized control, where the decentralized design for the case of redundant controllers is achieved with a combined observer-design equation of dimensions $qn \times qn$,[1] which can be reduced to an algebraic Riccati-like equation (ARLE) of dimension $2n \times 2n$ by using a method similar to that of Lemma 3.4.

Similar to the approach of Veillette *et al.* to reliable control [3], sensor and/or actuator failures (outages) are also treated as plant uncertainty in our approach here. However, by using the symmetry in the closed-loop system, our design approach in this paper involves two $n \times n$ design equations only, as opposed to the higher order design equations of Veillette *et al.* [3] (order $2n \times 2n$). The design given by Theorem 3.1 requires only a standard algebraic Riccati equation (ARE) of dimensions $n \times n$ for the observer design. Hence computational procedure in Theorem 3.1 is much simpler than that of Veillette *et al.* Furthermore, the redundant controllers themselves are automatically guaranteed to be stable. The design method in [3] for decentralized reliable control (for actuator outages) guarantees only that some of the controllers are open-loop stable, unless more complicated design equations are used. In the context of our (decentralized) design problem here, all controllers are guaranteed to be open-loop stable. This is equivalent to any one of the decentralized controllers being open-loop stable because all controllers are assumed identical here.

Remark 3.9: For simplicity, only results for SISO systems are given. The generalization to multi-input/multi-output (MIMO) systems is straightforward, except that the notations will get complicated. But in the MIMO case, Corollaries 3.6 and 3.7 may only apply to actuator outages and sensor outages, respectively.

IV. An Example

Now we look at an example to illustrate the design procedure given in the previous section. The plant is of the form (1)–(3) and has two identical control channels $(q = 2)$. The plant matrices are given as follows:

$$A = \begin{bmatrix} -2 & 1 & 1 & 1 \\ 3 & 0 & 0 & 2 \\ -1 & 0 & -2 & -3 \\ -2 & -1 & 2 & -1 \end{bmatrix}, \quad G = \begin{bmatrix} \frac{1}{2} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$H = \begin{bmatrix} 0 & 0 & \frac{1}{2} & 0 \end{bmatrix}, \quad C = [1 \quad 0 \quad 0 \quad 0], \quad q = 2.$$

It is easy to check that the open-loop system is unstable, and (A, H) is a completely observable pair, and hence, detectable. By solving the ARE's in (40) and (41) in Corollaries 3.6, we have an output feedback controller Σ_a of the form (4) and (5) with

$$A_c = \begin{bmatrix} -261.5469 & 1.0223 & 1.0153 & 1.0097 \\ -651.5013 & 0 & 0 & 2.0000 \\ -300.5297 & 0 & -2.0000 & -3.0000 \\ 337.4229 & -2.0018 & 1.3165 & -1.4850 \end{bmatrix}$$

$$L = \begin{bmatrix} 259.5615 \\ 654.5013 \\ 299.5297 \\ -340.0691 \end{bmatrix}$$

$$K = [-0.6462 \quad -1.0018 \quad -0.6835 \quad -0.4850].$$

[1]This high-order design equation is similar to but not the same as the algebraic Riccati equation (ARE) and it is referred to as algebraic Riccati-like equation (ARLE).

TABLE I
RELIABLE CONTROLLER DESIGN RESULTS

Outages Designed for Controller	Actuator Outage Σ_a	Sensor Outage Σ_s
Design α	4.08	4.39
No Outage α_o	3.50	4.23
Controller Failure α_c	3.32	3.98

Similarly, by solving the ARE's in (42) and (43) in Corollaries 3.7, we have an output feedback controller Σ_s of the form (4) and (5) with

$$A_c = \begin{bmatrix} -23.9136 & 1.0316 & 1.0271 & 1.0169 \\ -21.1069 & 0 & 0 & 2.0000 \\ -53.4086 & 0 & -2.0000 & -3.0000 \\ 36.2765 & -2.1497 & 1.1114 & -1.5931 \end{bmatrix}$$

$$L = \begin{bmatrix} 21.9833 \\ 24.1069 \\ 52.4086 \\ -39.5770 \end{bmatrix}$$

$$K = [-1.3005 \quad -1.1497 \quad -0.8886 \quad -0.5931].$$

Both of these two controllers Σ_a and Σ_s can provide internal stability and guaranteed disturbance attenuation for the closed-loop system not only when both control channels are operational but also when any of these two control channels experiences an outage.

The design results are given in Table I. The two values of the closed-loop disturbance attenuation are computed for each of the two controllers. Namely:

α_o: when there is no outage;

α_c: when there is a controller failure.

The "Design α" in Table I is the value of α used in solving the two corresponding design equations.

The actual achievable values of α (namely α_o and α_c) for the closed-loop system are all less than and quite close to the value of α for which the design equations have solutions and the conditions in the Corollaries are satisfied. This indicates that degree of conservativeness in the design method is not very severe.

From Table I, it would seem that the actual system performance would be better when some controller failure occurs, contrary to the desirable property of graceful degradation of performance. This is so, however, because a controller failure (modeled as an actuator outage and/or sensor outage) effectively eliminates one column and/or one row of the closed-loop transfer function matrix. This is similar to an observation made in [3].

ACKNOWLEDGMENT

The authors wish to thank the reviewers for many useful suggestions on the initial manuscript of the present work.

REFERENCES

[1] M. Vidyasagar and N. Viswanadham, "Reliable stabilization using a multicontroller configuration," *Automatica*, vol. 21, no. 5, pp. 599–602, 1985.

[2] A. N. Gundes and M. G. Kabuli, "Reliable decentralized control," in *Proc. American Control Conf.*, Baltimore, MD, pp. 3359–3363.

[3] R. J. Veillette, J. V. Medanic, and W. R. Perkins, "Design of reliable control systems," *IEEE Trans. Automat. Contr.*, vol. 37, pp. 290–304, 1992.

[4] D. D. Siljak, "Reliable control using multiple control systems," *Int. J. Contr.*, vol. 31, no. 2, pp. 303–329, 1980.

[5] C. S. Araujo and J. C. Castro, "Application of power system stabilisers in a plant with identical units," *Proc. Inst. Elec. Eng.*, vol. 138, pt. C, no. 1, pp. 11–18, 1991.

[6] K. M. Sundareshan and R. M. Elbanna, "Qualitative analysis and decentralized controller synthesis for a class of large-scale systems with symmetrically interconnected subsystems," *Automatica*, vol. 27, no. 2, pp. 383–388, 1991.

[7] J. Ackermann, "Multi-model approaches to robust control system design," in *Uncertainty and Control: Proceedings of an International Seminar Organized by DFVLR, Bonn, Germany, May 1985*, J. Ackermann, Ed. Berlin, Germany: Springer-Verlag, 1986.

[8] ——, *Sampled-Data Control Systems*. Berlin, Germany: Springer-Verlag, 1985.

[9] B. W. Johnson, *Design and Analysis of Fault Tolerant Digital Systems*. Reading, MA: Addison-Wesley, 1989.

[10] H. E. Rauch, "Autonomous control reconfiguration," *IEEE Contr. Syst.*, vol. 15, no. 6, pp. 37–48, 1995.

[11] M. Kinnaert, R. Hanus, and P. Arte, "Fault detection and isolation for unstable linear systems," *IEEE Trans. Automat. Contr.*, vol. 40, pp. 740–742, Apr. 1995.

[12] C.-C. Tsui, "A general failure detection, isolation and accommodation system with model uncertainty and measurement noise," *IEEE Trans. Automat. Contr.*, vol. 39, pp. 2318–2321, Nov. 1994.

Design of Performance Robustness for Uncertain Linear Systems with State and Control Delays

J. S. Luo, P. P. J. van den Bosch, S. Weiland, and A. Goldenberge

***Abstract*—The linear systems considered in this paper are subject to uncertain perturbations of norm-bounded time-varying parameters and multiple time delays in system state and control. The time delays are uncertain, independent of each other, and allowed to be time-varying. The integral quadratic cost criterion is employed to measure system performance. Using solutions of Lyapunov and Riccati equations, a linear state feedback control law is proposed to stabilize the perturbed system and to guarantee an upper bound of system performance, which is applicable to arbitrary time delays.**

***Index Terms*— Algebraic Riccati equation, delay effects, linear–quadratic control, Lyapunov matrix equation, robustness, stability, uncertain systems.**

I. INTRODUCTION

The problem of stabilizing uncertain systems with time-varying and bounded parametric uncertainties has attracted a considerable amount of interest in recent years. Among different approaches, Lyapunov and Riccati equation descriptions of uncertainty are important ways to deal with the problem. Based on linear optimal control theory with quadratic cost criteria and using Lyapunov stability theory, many methods have been proposed for finding a linear state feedback law

Manuscript received April 22, 1996.

J. S. Luo is with the Bombardier-DeHaviland, Canada.

P. P. J. van den Bosch and S. Weiland are with the Measurement and Control Group, Department of Electrical Engineering, Eindhoven University of Technology, 5600 MB Eindhoven, The Netherlands (e-mail: P.P.J.v.d.Bosch@ele.tue.nl).

A. Goldenberge is with the Robotics and Automation Laboratory, University of Toronto, Toronto, Canada.

Publisher Item Identifier S 0018-9286(98)07533-3.