Robust H_∞ Control for Uncertain Discrete-Time-Delay Fuzzy Systems Via Output Feedback Controllers

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***Abstract*—This paper investigates the problem of robust output feedback H_∞ control for a class of uncertain discrete-time fuzzy systems with time delays. The state–space Takagi–Sugeno fuzzy model with time delays and norm-bounded parameter uncertainties is adopted. The purpose is the design of a full-order fuzzy dynamic output feedback controller which ensures the robust asymptotic stability of the closed-loop system and guarantees an H_∞ norm bound constraint on disturbance attenuation for all admissible uncertainties. In terms of linear matrix inequalities (LMIs), a sufficient condition for the solvability of this problem is presented. Explicit expressions of a desired output feedback controller are proposed when the given LMIs are feasible. The effectiveness and the applicability of the proposed design approach are demonstrated by applying this to the problem of robust H_∞ control for a class of uncertain nonlinear discrete delay systems.**

***Index Terms*—Discrete systems, linear matrix inequality (LMI), output feedback, robust H_∞ control, Takagi–Sugeno (T–S) fuzzy models, time-delay systems, uncertain systems.**

I. Introduction

As an alternative method to conventional control approach for complex control systems, fuzzy logic control has received much attention in the past decades. It has been shown that fuzzy logic control is one of the most useful techniques for utilizing the qualitative knowledge of a system to design controllers. A great number of industrial applications via fuzzy logic control have been reported [14], [15], [28]. Among various model-based fuzzy control approaches, the method based on the Takagi–Sugeno (T–S) fuzzy model has become popular today, which gives a simple and effective way to control complex nonlinear systems. The main features of this approach are as follows: first, a nonlinear system is represented by a T–S fuzzy model, in which local dynamics in different state space regions are represented by linear models. Then, the overall model of the system is achieved by a fuzzy "blending" of these fuzzy models. Based on this, the control design can be carried out by the so-called parallel distributed compensation (PDC) scheme. Applications of such a fuzzy control scheme can be found in [10], [21], and [23].

Recently, stability analysis of T–S fuzzy control systems has been investigated, and several stability criteria have been proposed; see, e.g., [18] and [19]. When parameter uncertainties appear, the problems of robust stability analysis and robust stabilization for fuzzy systems have been studied. For example, by a linear matrix inequality (LMI) approach, some robust stability results were presented in [17] in the continuous case; based on these, robust fuzzy stabilizing controllers were constructed via the PDC scheme. The corresponding results for discrete case can be found in [11]. Very recently, the robust H_∞ control problem for fuzzy systems described by T–S fuzzy model has been addressed. By the LMI approach, sufficient conditions for the solvability of this issue were proposed in [4] and [12] for the discrete and continuous cases, respectively. It should be pointed out that in both [4] and [12], state feedback controllers were designed under the assumption that all state variables are available.

On the other hand, it is well-known that time delay arises quite naturally in propagation phenomena, population dynamics or engineering systems such as chemical processes, long transmission lines in pneumatic systems [8]. Many results on estimation and control issues related to time-delay systems have been proposed [8], [9], [25], [26]. Recently, fuzzy systems with time delays have been introduced in [3] and [5], where several stability analysis results were given via different approaches, and stabilizing controllers were also designed. When delays are time-varying, the stability results for fuzzy delay systems were given in [27]. It is noted that in [3], [5], and [27], no parameter uncertainties were taken into account. In the case when parameter uncertainties arise and not all of the states are available directly, the robust output feedback H_∞ control problem for fuzzy systems with time delays was discussed in [13]. In terms of solutions to a certain LMI, an output feedback controller was designed in [13]. These results were further applied to a class of nonlinear delay systems. It is worth noting that the results in [13] were obtained in the context of continuous T–S fuzzy systems. For discrete fuzzy systems with time delays and parameter uncertainties subject to that all state variables are not available, however, the problem of robust H_∞ control via output feedback controllers is still open and remains unsolved, which motivates the present study.

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In this paper, we are concerned with the problem of robust output feedback H_∞ control for a class of discrete fuzzy systems with parameter uncertainties and time delays. The T–S fuzzy model is adopted for fuzzy modeling of a discrete uncertain nonlinear systems with time delays. The parameter uncertainties are assumed to be time varying but norm bounded. The purpose is the design of a full-order fuzzy dynamic output feedback controller such that the resulting closed-loop system is robustly asymptotically stable while satisfying a prescribed H_∞ performance level irrespective of the parameter uncertainties. Sufficient conditions for the solvability of this problem are obtained in terms of LMIs. A desired output feedback controller can be constructed by using standard numerical algorithms to solve these given LMIs [2], and no tuning of parameters is required.

Notation: Throughout this paper, for real symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive–semidefinite (respectively, positive–definite). I is an identity matrix with appropriate dimension. $\mathbb{N}$ is the set of natural numbers. $l_2[0,\infty)$ refers to the space of square summable infinite vector sequences. $\|\cdot\|_2$ stands for the usual $l_2[0,\infty)$ norm. The notation M^T represents the transpose of the matrix M. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

II. Problem Formulation

The T–S fuzzy dynamic model is described by fuzzy IF–THEN rules, which locally represent linear input–output relations of nonlinear systems. Similar to [16], a discrete-time T–S fuzzy model with time delays and parameter uncertainties can be described by

Plant Rule i: IF $s_1(k)$ is μ_{i1} and $s_2(k)$ is μ_{i2} and $\cdots$ and $s_g(k)$ is μ_{ig}, THEN

$$x(k+1) = [A_i + \Delta A_i(k)]x(k) + [A_{di} + \Delta A_{di}(k)] \times x(k-\tau) + [B_i + \Delta B_i(k)]u(k) + D_{1i}\omega(k) \quad (1)$$

$$y(k) = C_i x(k) + C_{di} x(k-\tau) + D_{2i}\omega(k) \quad (2)$$

$$z(k) = E_i x(k) + E_{di} x(k-\tau) + G_i u(k) \quad (3)$$

$$x(k) = \phi(k) \quad \forall k \in [-\tau, 0], \quad i = 1, 2, \ldots, r \quad (4)$$

where μ_{ij} is the fuzzy set and r is the number of IF-THEN rules; $x(k) \in \mathbb{R}^n$ is the state; $u(k) \in \mathbb{R}^m$ is the control input; $y(k) \in \mathbb{R}^s$ is the measured output; $z(k) \in \mathbb{R}^q$ is the controlled output; $\omega(k) \in \mathbb{R}^p$ is the disturbance input which is assumed to belong to $l_2[0,\infty)$; $\tau > 0$ is an integer representing the time delay of the fuzzy system; $s_1(k), s_2(k), \ldots, s_g(k)$ are the premise variables. Throughout this paper, it is assumed that the premise variables do not depend on the input variables $u(k)$ explicitly. $A_i, A_{di}, B_i, C_i, C_{di}, D_{1i}, D_{2i}, E_i$, and G_i are known real constant matrices; $\Delta A_i(k), \Delta A_{di}(k)$ and $\Delta B_i(k)$ are real-valued unknown matrices representing time-varying parameter uncertainties, and are assumed to be of the form

$$[\Delta A_i(k) \quad \Delta A_{di}(k) \quad \Delta B_i(k)] = M_i F_i(k) [N_{1i} \quad N_{2i} \quad N_{3i}], \qquad i = 1, 2, \ldots, r \quad (5)$$

where M_i, N_{1i}, N_{2i}, and N_{3i} are known real constant matrices and $F_i(\cdot) : \mathbb{N} \rightarrow \mathbb{R}^{l_1 \times l_2}$ are unknown time-varying matrix function satisfying

$$F_i(k)^T F_i(k) \leq I \qquad \forall k. \quad (6)$$

The parameter uncertainties $\Delta A_i(k), \Delta A_{di}(k)$ and $\Delta B_i(k)$ are said to be admissible if both (5) and (6) hold. It is worth mentioning that interval bounded parameters can also be used to describe uncertain systems. In the discrete-time case, interval model control and applications can be found in [1], [29], and the references therein.

Given a pair $(x(k), u(k))$, the final output of the fuzzy system is inferred as follows:

$$x(k+1) = \sum_{i=1}^{r} h_i(s(k))\{[A_i + \Delta A_i(k)]x(k) + [A_{di} + \Delta A_{di}(k)]x(k-\tau) + [B_i + \Delta B_i(k)]u(k) + D_{1i}\omega(k)\} \quad (7)$$

$$y(k) = \sum_{i=1}^{r} h_i(s(k))[C_i x(k) + C_{di} x(k-\tau) + D_{2i}\omega(k)] \quad (8)$$

$$z(k) = \sum_{i=1}^{r} h_i(s(k))[E_i x(k) + E_{di} x(k-\tau) + G_i u(k)] \quad (9)$$

where

$$h_i(s(k)) = \frac{\varpi_i(s(k))}{\sum_{j=1}^{r} \varpi_j(s(k))} \qquad \varpi_i(s(k)) = \prod_{j=1}^{g} \mu_{ij}(s_j(k))$$

$$s(k) = [s_1(k) \quad s_2(k) \quad \cdots \quad s_g(k)]$$

in which $\mu_{ij}(s_j(k))$ is the grade of membership of $s_j(k)$ in μ_{ij}. Then, it can be seen that

$$\varpi_i(s(k)) \geq 0, \qquad i = 1, 2, \ldots, r$$

$$\sum_{j=1}^{r} \varpi_j(s(k)) > 0$$

for all k. Therefore, for all k

$$h_i(s(k)) \geq 0, \qquad i = 1, 2, \ldots, r \quad (10)$$

$$\sum_{j=1}^{r} h_j(s(k)) = 1. \quad (11)$$

Now, by the parallel distributed compensation (PDC) technique, we consider the following full-order fuzzy dynamic output feedback controller for the fuzzy system (7)–(9):

Control Rule i: IF $s_1(k)$ is μ_{i1} and $s_2(k)$ is μ_{i2} and ... and $s_g(k)$ is μ_{ig}, THEN

$$\hat{x}(k+1) = A_{Ki}\hat{x}(k) + B_{Ki} y(k) \quad (12)$$

$$u(k) = C_{Ki}\hat{x}(k), \qquad i = 1, 2, \ldots, r \quad (13)$$

where $\hat{x}(k) \in \mathbb{R}^n$ is the controller state, A_{Ki}, B_{Ki}, and C_{Ki} are matrices to be determined later. Then, the overall fuzzy output feedback controller is given by

$$\hat{x}(k+1) = \sum_{i=1}^{r} h_i(s(k)) [A_{Ki}\hat{x}(k) + B_{Ki} y(k)] \quad (14)$$

$$u(k) = \sum_{i=1}^{r} h_i(s(k)) C_{Ki}\hat{x}(k). \quad (15)$$

From (7)–(9), (14), and (15), the closed-loop system can be obtained as

$$\xi(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(s(k))h_j(s(k))[A_{cij}(k)\xi(k) + A_{cdij}(k)H\xi(k-\tau) + D_{cij}\omega(k)] \quad (16)$$

$$z(k) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(s(k))h_j(s(k))[E_{cij}\xi(k) + E_{cdi}H\xi(k-\tau)] \quad (17)$$

where

$$\xi(k) = \begin{bmatrix} x(k) \\ \hat{x}(k) \end{bmatrix}$$

and

$$A_{cij}(k) = A_{cij} + \Delta A_{cij}(k)$$
$$A_{cdij}(k) = A_{cdij} + \Delta A_{cdij}(k) \quad (18)$$
$$A_{cij} = \begin{bmatrix} A_i & B_iC_{Kj} \\ B_{Kj}C_i & A_{Kj} \end{bmatrix}$$
$$\Delta A_{cij}(k) = \begin{bmatrix} \Delta A_i(k) & \Delta B_i(k)C_{Kj} \\ 0 & 0 \end{bmatrix} \quad (19)$$
$$A_{cdij} = \begin{bmatrix} A_{di} \\ B_{Kj}C_{di} \end{bmatrix} \quad \Delta A_{cdij}(k) = \begin{bmatrix} \Delta A_{di}(k) \\ 0 \end{bmatrix}$$
$$D_{cij} = \begin{bmatrix} D_{1i} \\ B_{Kj}D_{2i} \end{bmatrix} \quad (20)$$
$$E_{cij} = [E_i \quad G_iC_{Kj}] \quad E_{cdi} = E_{di} \quad H = [I \quad 0]. \quad (21)$$

Then, the robust fuzzy H_∞ control problem to be addressed in this paper can be formulated as follows: given an uncertain fuzzy system (7)–(9) and a scalar $\gamma > 0$, determine an output feedback fuzzy controller in the form of (12) and (13) such that

R1) The closed-loop system in (16) and (17) is robustly asymptotically stable when $\omega(k) = 0$.

R2) Under zero-initial condition, the controlled output z satisfies

$$\|z\|_2 < \gamma\|\omega\|_2 \quad (22)$$

for any nonzero $\omega \in l_2$ and all admissible uncertainties.

III. Main Results

In this section, an LMI approach will be developed to solve the problem of robust output feedback H_∞ control of uncertain discrete delay fuzzy systems formulated in the previous section. We first give the following results which will be used in the proof of our main results.

Lemma 1 [24]: Given any matrices $\mathcal{X}, \mathcal{Y}$, and $\mathcal{Z}$ with appropriate dimensions such that $\mathcal{Y} > 0$. Then, we have

$$\mathcal{X}^T\mathcal{Z} + \mathcal{Z}^T\mathcal{X} \le \mathcal{X}^T\mathcal{Y}\mathcal{X} + \mathcal{Z}^T\mathcal{Y}^{-1}\mathcal{Z}.$$

Lemma 2 [22]: Let $\mathcal{A}, \mathcal{D}, \mathcal{S}, \mathcal{W}$, and F be real matrices of appropriate dimensions such that $\mathcal{W} > 0$ and $F^TF \le I$. Then, for any scalar $\epsilon > 0$ such that $\mathcal{W} - \epsilon\mathcal{D}\mathcal{D}^T > 0$, we have

$$(\mathcal{A} + \mathcal{D}F\mathcal{S})^T\mathcal{W}^{-1}(\mathcal{A} + \mathcal{D}F\mathcal{S}) \le \mathcal{A}^T(\mathcal{W} - \epsilon\mathcal{D}\mathcal{D}^T)^{-1}\mathcal{A} + \epsilon^{-1}\mathcal{S}^T\mathcal{S}.$$

Theorem 1: The uncertain system in (16) and (17) is robustly asymptotically stable and (22) is satisfied if there exist matrices $P > 0$, and $Q > 0$, and scalars $\epsilon_{ij} > 0, 1 \le i \le j \le r$, such that the matrix inequalities shown in (23) and (24) at the bottom of the page, hold, where

$$\tilde{N}_{ij} = [N_{1i} \quad N_{3i}C_{Kj}] \quad \tilde{M}_i = \begin{bmatrix} M_i^T & 0 \end{bmatrix}^T$$
$$\tilde{M}_{ij} = [\tilde{M}_i \quad \tilde{M}_j]. \quad (25)$$

Proof: Under the conditions of the theorem, we first establish the robust asymptotic stability of the system in (16). To this end, we consider (16) with $\omega(k) = 0$; that is

$$\xi(k+1) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(s(k))h_j(s(k)) \times [A_{cij}(k)\xi(k) + A_{cdij}(k)H\xi(k-\tau)]. \quad (26)$$

Choose a Lyapunov function candidate for the system in (26) as follows:

$$V(\xi(k)) = \xi(k)^TP\xi(k) + \sum_{i=k-\tau}^{k-1} \xi(i)^TH^TQH\xi(i). \quad (27)$$

$$\begin{bmatrix} H^TQH - P & 0 & 0 & A_{cii}^T & \tilde{N}_{ii}^T & E_{cii}^T \\ 0 & -Q & 0 & A_{cdii}^T & N_{2i}^T & E_{cdii}^T \\ 0 & 0 & -\gamma^2 I & D_{cii}^T & 0 & 0 \\ A_{cii} & A_{cdii} & D_{cii} & \epsilon_{ii}\tilde{M}_i\tilde{M}_i^T - P^{-1} & 0 & 0 \\ \tilde{N}_{ii} & N_{2i} & 0 & 0 & -\epsilon_{ii}I & 0 \\ E_{cii} & E_{cdii} & 0 & 0 & 0 & -I \end{bmatrix} < 0, \qquad 1 \le i \le r \quad (23)$$

and

$$\begin{bmatrix} 4H^TQH - 4P & 0 & 0 & A_{cij}^T + A_{cji}^T & \tilde{N}_{ij}^T & \tilde{N}_{ji}^T & E_{cij}^T + E_{cji}^T \\ 0 & -4Q & 0 & A_{cdij}^T + A_{cdji}^T & N_{2i}^T & N_{2j}^T & E_{cdij}^T + E_{cdji}^T \\ 0 & 0 & -4\gamma^2 I & D_{cij}^T + D_{cji}^T & 0 & 0 & 0 \\ A_{cij} + A_{cji} & A_{cdij} + A_{cdji} & D_{cij} + D_{cji} & \epsilon_{ij}\tilde{M}_{ij}\tilde{M}_{ij}^T - P^{-1} & 0 & 0 & 0 \\ \tilde{N}_{ij} & N_{2i} & 0 & 0 & -\epsilon_{ij}I & 0 & 0 \\ \tilde{N}_{ji} & N_{2j} & 0 & 0 & 0 & -\epsilon_{ij}I & 0 \\ E_{cij} + E_{cji} & E_{cdij} + E_{cdji} & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0, \quad 1 \le i < j \le r \quad (24)$$

Then it can be verified that

$$\begin{aligned}
&\Delta V(\xi(k)) \\
&\quad = V(\xi(k+1)) - V(\xi(k)) \\
&\quad = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{u=1}^{r}\sum_{v=1}^{r} h_i(s(k))h_j(s(k))h_u(s(k))h_v(s(k)) \\
&\qquad \times [A_{cij}(k)\xi(k) + A_{cdij}(k)H\xi(k-\tau)]^T P \\
&\qquad \times [A_{cuv}(k)\xi(k) + A_{cduv}(k)H\xi(k-\tau)] \\
&\qquad + \xi(k)^T(H^TQH - P)\xi(k) \\
&\qquad - \xi(k-\tau)^T H^T QH\xi(k-\tau) \\
&\quad = \frac{1}{4}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{u=1}^{r}\sum_{v=1}^{r} h_i(s(k))h_j(s(k))h_u(s(k))h_v(s(k)) \\
&\qquad \times \{[A_{cij}(k) + A_{cji}(k)]\xi(k) \\
&\qquad + [A_{cdij}(k) + A_{cdji}(k)]H\xi(k-\tau)\}^T \\
&\qquad \times P\{[A_{cuv}(k) + A_{cvu}(k)]\xi(k) \\
&\qquad + [A_{cduv}(k) + A_{cdvu}(k)]H\xi(k-\tau)\} \\
&\qquad + \xi(k)^T(H^TQH - P)\xi(k) \\
&\qquad - \xi(k-\tau)^T H^T QH\xi(k-\tau).
\end{aligned} \tag{28}$$

Using Lemma 1 and noting (11), we have

$$\begin{aligned}
&\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{u=1}^{r}\sum_{v=1}^{r} h_i(s(k))h_j(s(k))h_u(s(k))h_v(s(k)) \\
&\quad \times \{[A_{cij}(k) + A_{cji}(k)]\xi(k) \\
&\quad + [A_{cdij}(k) + A_{cdji}(k)]H\xi(k-\tau)\}^T \\
&\quad \times P\{[A_{cuv}(k) + A_{cvu}(k)]\xi(k) \\
&\quad + [A_{cduv}(k) + A_{cdvu}(k)]H\xi(k-\tau)\} \\
&\quad \le \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(s(k))h_j(s(k))\{[A_{cij}(k) + A_{cji}(k)]\xi(k) \\
&\quad + [A_{cdij}(k) + A_{cdji}(k)]H\xi(k-\tau)\}^T \\
&\quad \times P\{[A_{cij}(k) + A_{cji}(k)]\xi(k) \\
&\quad + [A_{cdij}(k) + A_{cdji}(k)]H\xi(k-\tau)\}.
\end{aligned}$$

This together with (28) and the relationship

$$x(k-\tau) = H\xi(k-\tau) \tag{29}$$

implies

$$\begin{aligned}
\Delta V(\xi(k)) &\le \frac{1}{4}\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(s(k))h_j(s(k))\alpha(k)^T[\mathcal{M}_{cij}(k) \\
&\quad + \mathcal{M}_{cji}(k)]^T P[\mathcal{M}_{cij}(k) + \mathcal{M}_{cji}(k)]\alpha(k) \\
&\quad + \xi(k)^T(H^TQH - P)\xi(k) \\
&\quad - x(k-\tau)^T Qx(k-\tau) \\
&= \sum_{i=1}^{r} h_i^2(s(k))\alpha(k)^T[\mathcal{M}_{cii}(k)^T \\
&\quad \times P\mathcal{M}_{cii}(k) + Z_c]\alpha(k) \\
&\quad + \frac{1}{2}\sum_{i,j=1,i<j}^{r} h_i(s(k))h_j(s(k)) \\
&\quad \times \alpha(k)^T\{[\mathcal{M}_{cij}(k) + \mathcal{M}_{cji}(k)]^T \\
&\quad \times P[\mathcal{M}_{cij}(k) + \mathcal{M}_{cji}(k)] + 4Z_c\}\alpha(k)
\end{aligned} \tag{30}$$

where

$$\begin{aligned}
\alpha(k) &= [\xi(k)^T \quad x(k-\tau)^T]^T \\
\mathcal{M}_{cij}(k) &= [A_{cij}(k) \quad A_{cdij}(k)]. \\
Z_c &= \begin{bmatrix} H^TQH - P & 0 \\ 0 & -Q \end{bmatrix}.
\end{aligned}$$

Now, from (24), it is easy to see that for $1 \le i < j \le r$, (31), as shown at the bottom of the page, holds. Set

$$\begin{aligned}
\mathcal{M}_{cij} &= [A_{cij} \quad A_{cdij}] \quad N_{cij} = [\tilde{N}_{ij} \quad N_{2i}] \\
\mathcal{N}_{ij} &= \begin{bmatrix} N_{cij} \\ N_{cji} \end{bmatrix}.
\end{aligned} \tag{32}$$

for $1 \le i \le j \le r$. Then, applying the Schur complement to (31) gives

$$P^{-1} - \epsilon_{ij}\tilde{M}_{ij}\tilde{M}_{ij}^T > 0 \tag{33}$$

and

$$\begin{aligned}
&(\mathcal{M}_{cij} + \mathcal{M}_{cji})^T \left(P^{-1} - \epsilon_{ij}\tilde{M}_{ij}\tilde{M}_{ij}^T\right)^{-1} \\
&\quad \times (\mathcal{M}_{cij} + \mathcal{M}_{cji}) + \epsilon_{ij}^{-1}\mathcal{N}_{ij}^T\mathcal{N}_{ij} + 4Z_c < 0
\end{aligned} \tag{34}$$

for $1 \le i < j \le r$. Considering (33) and using Lemma 2, we have that for $1 \le i < j \le r$

$$\begin{aligned}
&[\mathcal{M}_{cij}(k) + \mathcal{M}_{cji}(k)]^T P[\mathcal{M}_{cij}(k) + \mathcal{M}_{cji}(k)] \\
&\quad = [\mathcal{M}_{cij} + \mathcal{M}_{cji} + \tilde{M}_{ij}F_{ij}(k)\mathcal{N}_{ij}]^T \\
&\qquad \times P[\mathcal{M}_{cij} + \mathcal{M}_{cji} + \tilde{M}_{ij}F_{ij}(k)\mathcal{N}_{ij}] \\
&\quad \le (\mathcal{M}_{cij} + \mathcal{M}_{cji})^T \left(P^{-1} - \epsilon_{ij}\tilde{M}_{ij}\tilde{M}_{ij}^T\right)^{-1} \\
&\qquad \times (\mathcal{M}_{cij} + \mathcal{M}_{cji}) + \epsilon_{ij}^{-1}\mathcal{N}_{ij}^T\mathcal{N}_{ij}
\end{aligned} \tag{35}$$

where

$$F_{ij}(k) = \begin{bmatrix} F_i(k) & 0 \\ 0 & F_j(k) \end{bmatrix}.$$

Then, it follows from (34) and (35) that for $1 \le i < j \le r$

$$\begin{aligned}
&[\mathcal{M}_{cij}(k) + \mathcal{M}_{cji}(k)]^T \\
&\qquad \times P[\mathcal{M}_{cij}(k) + \mathcal{M}_{cji}(k)] + 4Z_c < 0.
\end{aligned} \tag{36}$$

$$\begin{bmatrix}
4H^TQH - 4P & 0 & A_{cij}^T + A_{cji}^T & \tilde{N}_{ij}^T & \tilde{N}_{ji}^T \\
0 & -4Q & A_{cdij}^T + A_{cdji}^T & N_{2i}^T & N_{2j}^T \\
A_{cij} + A_{cji} & A_{cdij} + A_{cdji} & \epsilon_{ij}\tilde{M}_{ij}\tilde{M}_{ij}^T - P^{-1} & 0 & 0 \\
\tilde{N}_{ij} & N_{2i} & 0 & -\epsilon_{ij}I & 0 \\
\tilde{N}_{ji} & N_{2j} & 0 & 0 & -\epsilon_{ij}I
\end{bmatrix} < 0. \tag{31}$$

On the other hand, from (23), we have

$$\begin{bmatrix} H^TQH-P & 0 & A_{cii}^T & \tilde{N}_{1ii}^T \\ 0 & -Q & A_{cdii}^T & N_{2i}^T \\ A_{cii} & A_{cdii} & \epsilon_{ii}\tilde{M}_i\tilde{M}_i^T - P^{-1} & 0 \\ \tilde{N}_{1ii} & N_{2i} & 0 & -\epsilon_{ii}I \end{bmatrix} < 0, \quad 1 \le i \le r \tag{37}$$

which, by the Schur complement, implies

$$P^{-1} - \epsilon_{ii}\tilde{M}_i\tilde{M}_i^T > 0 \tag{38}$$

and

$$\mathcal{M}_{cii}^T\left(P^{-1} - \epsilon_{ii}\tilde{M}_i\tilde{M}_i^T\right)^{-1}\mathcal{M}_{cii} + \epsilon_{ii}^{-1}N_{cii}^TN_{cii} + Z_c < 0 \tag{39}$$

where $\mathcal{M}_{cii}$ and N_{cii} are given in (32). Taking into account (38) and using Lemma 2 again, we have

$$\begin{aligned} &\mathcal{M}_{cii}(k)^TP\mathcal{M}_{cii}(k) \\ &\quad = [\mathcal{M}_{cii} + \tilde{M}_iF_i(k)N_{cii}]^TP[\mathcal{M}_{cii} + \tilde{M}_iF_i(k)N_{cii}] \\ &\quad \le \mathcal{M}_{cii}^T\left(P^{-1} - \epsilon_{ii}\tilde{M}_i\tilde{M}_i^T\right)^{-1}\mathcal{M}_{cii} + \epsilon_{ii}^{-1}N_{cii}^TN_{cii} \end{aligned} \tag{40}$$

for $1 \le i \le r$. Then, by (39) and (40), it can be established that for $1 \le i \le r$

$$\mathcal{M}_{cii}(k)^TP\mathcal{M}_{cii}(k) + Z_c < 0. \tag{41}$$

Then, it follows from (30), (36), and (41) that

$$\Delta V(\xi(k)) < 0$$

for all $\alpha(k) \neq 0$. Hence, the uncertain system (16) with $\omega(k) = 0$ is robustly asymptotically stable.

Next, we show that for any nonzero $\omega \in l_2$ the uncertain system in (16) and (17) satisfies (22) under zero initial condition. To this end, we introduce

$$J_N = \sum_{k=0}^{N}[z(k)^Tz(k) - \gamma^2\omega(k)^T\omega(k)] \tag{42}$$

where the scalar $N \in \mathbb{N}$. Noting the zero initial condition and (16), (17), and (29), we can deduce

$$\begin{aligned} J_N &= \sum_{k=0}^{N}\{z(k)^Tz(k) - \gamma^2\omega(k)^T\omega(k) \\ &\quad + [V(\xi(k+1)) - V(\xi(k))]\} - V(\xi(N+1)) \\ &\le \sum_{k=0}^{N}\{z(k)^Tz(k) - \gamma^2\omega(k)^T\omega(k) \\ &\quad + [V(\xi(k+1)) - V(\xi(k))]\} \\ &= \sum_{k=0}^{N}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{u=1}^{r}\sum_{v=1}^{r}\right. \\ &\quad h_i(s(k))h_j(s(k))h_u(s(k))h_v(s(k)) \\ &\quad \times ([E_{cij}\xi(k) + E_{cdi}x(k-\tau)]^T \\ &\quad \times [E_{cuv}\xi(k) + E_{cdu}x(k-\tau)] \\ &\quad + [A_{cij}(k)\xi(k) + A_{cdij}(k)x(k-\tau) + D_{cij}\omega(k)]^T \\ &\quad \times P[A_{cuv}(k)\xi(k) + A_{cduv}(k)x(k-\tau) + D_{cuv}\omega(k)]) \\ &\quad + \xi(k)^T(H^TQH - P)\xi(k) \\ &\quad \left. - x(k-\tau)^TQx(k-\tau) - \gamma^2\omega(k)^T\omega(k)\right\} \end{aligned} \tag{43}$$

where $V(\xi(k))$ is given in (27). By Lemma 1, we have

$$\begin{aligned} &\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{u=1}^{r}\sum_{v=1}^{r}h_i(s(k))h_j(s(k))h_u(s(k))h_v(s(k)) \\ &\quad \times [E_{cij}\xi(k) + E_{cdi}x(k-\tau)]^T[E_{cuv}\xi(k) + E_{cdu}x(k-\tau)] \\ &\quad = \frac{1}{4}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{u=1}^{r}\sum_{v=1}^{r}h_i(s(k))h_j(s(k))h_u(s(k))h_v(s(k)) \\ &\qquad \times [(E_{cij} + E_{cji})\xi(k) + (E_{cdi} + E_{cdj})x(k-\tau)]^T \\ &\qquad \times [(E_{cuv} + E_{cvu})\xi(k) + (E_{cdu} + E_{cdv})x(k-\tau)] \\ &\quad \le \frac{1}{4}\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(s(k))h_j(s(k))[(E_{cij} + E_{cji})\xi(k) \\ &\qquad + (E_{cdi} + E_{cdj})x(k-\tau)]^T \\ &\qquad \times [(E_{cij} + E_{cji})\xi(k) + (E_{cdi} + E_{cdj})x(k-\tau)] \end{aligned}$$

and

$$\begin{aligned} &\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{u=1}^{r}\sum_{v=1}^{r}h_i(s(k))h_j(s(k))h_u(s(k))h_v(s(k)) \\ &\quad \times [A_{cij}(k)\xi(k) + A_{cdij}(k)x(k-\tau) + D_{cij}\omega(k)]^T \\ &\quad \times P[A_{cuv}(k)\xi(k) + A_{cduv}(k)x(k-\tau) + D_{cuv}\omega(k)] \\ &\quad = \frac{1}{4}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{u=1}^{r}\sum_{v=1}^{r}h_i(s(k))h_j(s(k))h_u(s(k))h_v(s(k)) \\ &\qquad \times \{[A_{cij}(k) + A_{cji}(k)]\xi(k) + [A_{cdij}(k) \\ &\qquad + A_{cdji}(k)]x(k-\tau) + [D_{cij} + D_{cji}]\omega(k)\}^T \\ &\qquad \times P\{[A_{cuv}(k) + A_{cvu}(k)]\xi(k) + [A_{cduv}(k) \\ &\qquad + A_{cdvu}(k)]x(k-\tau) + [D_{cuv} + D_{cvu}]\omega(k)\} \\ &\quad \le \frac{1}{4}\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(s(k))h_j(s(k)) \\ &\qquad \times \{[A_{cij}(k) + A_{cji}(k)]\xi(k) + [A_{cdij}(k) \\ &\qquad + A_{cdji}(k)]x(k-\tau) + [D_{cij} + D_{cji}]\omega(k)\}^T \\ &\qquad \times P\{[A_{cij}(k) + A_{cji}(k)]\xi(k) + [A_{cdij}(k) \\ &\qquad + A_{cdji}(k)]x(k-\tau) + [D_{cij} + D_{cji}]\omega(k)\}. \end{aligned}$$

Therefore

$$\begin{aligned} J_N \le \sum_{k=0}^{N}\frac{1}{4}&\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(s(k))h_j(s(k))\right. \\ &\times ([(E_{cij} + E_{cji})\xi(k) + (E_{cdi} + E_{cdj})x(k-\tau)]^T \\ &\times [(E_{cij} + E_{cji})\xi(k) + (E_{cdi} + E_{cdj})x(k-\tau)] \\ &+ \{[A_{cij}(k) + A_{cji}(k)]\xi(k) + [A_{cdij}(k) \\ &+ A_{cdji}(k)]x(k-\tau) + [D_{cij} + D_{cji}]\omega(k)\} \\ &\times P\{[A_{cij}(k) + A_{cji}(k)]\xi(k) + [A_{cdij}(k) \\ &+ A_{cdji}(k)]x(k-\tau) + [D_{cij} + D_{cji}]\omega(k)\}) \\ &+ \xi(k)^T(4H^TQH - 4P)\xi(k) \\ &\left. - 4x(k-\tau)^TQx(k-\tau) - 4\gamma^2\omega(k)^T\omega(k)\right\} \end{aligned}$$

$$
\begin{aligned}
&= \sum_{k=0}^{N}\sum_{i=1}^{r} h_i^2(s(k))\beta(k)^T \\
&\quad \times \left[\Upsilon_{cii}^T\Upsilon_{cii} + \Gamma_{cii}^T(k)P\Gamma_{cii}(k) + \tilde{Z}_c\right]\beta(k) \\
&\quad + \frac{1}{2}\sum_{k=0}^{N}\sum_{i,j=1,i<j}^{r} h_i(s(k))h_j(s(k))\beta(k)^T \\
&\quad \times \{(\Upsilon_{cij}+\Upsilon_{cjj})^T(\Upsilon_{cij}+\Upsilon_{cjj}) + [\Gamma_{cij}(k) \\
&\quad + \Gamma_{cji}(k)]^T P[\Gamma_{cij}(k)+\Gamma_{cji}(k)] + 4\tilde{Z}_c\}\beta(k). \qquad (44)
\end{aligned}
$$

where

$$
\begin{aligned}
\beta(k) &= [\xi(k)^T \quad x(k-\tau)^T \quad \omega(k)^T]^T \\
\Upsilon_{cij} &= [E_{cij} \quad E_{cdi} \quad 0] \\
\Gamma_{cij}(k) &= [A_{cij}(k) \quad A_{cdij}(k) \quad D_{cij}] \\
\tilde{Z}_c &= \begin{bmatrix} H^TQH-P & 0 & 0 \\ 0 & -Q & 0 \\ 0 & 0 & -\gamma^2 \end{bmatrix}.
\end{aligned}
$$

Using (23) and (24), and following a similar way to the derivation of (36) and (41), we can obtain

$$
\begin{aligned}
&(\Upsilon_{cij}+\Upsilon_{cjj})^T(\Upsilon_{cij}+\Upsilon_{cjj}) + [\Gamma_{cij}(k) \\
&\quad + \Gamma_{cji}(k)]^T P[\Gamma_{cij}(k)+\Gamma_{cji}(k)] + 4\tilde{Z}_c < 0 \qquad (45)
\end{aligned}
$$

and

$$
\Upsilon_{cii}^T\Upsilon_{cii} + \Gamma_{cii}^T(k)P\Gamma_{cii}(k) + \tilde{Z}_c < 0. \qquad (46)
$$

Therefore, the inequality in (44) together with (45) and (46) gives that for any $\beta(k) \neq 0$

$$
J_N < 0 \qquad (47)
$$

which implies $\|z\|_2 < \gamma\|\omega\|_2$ for any nonzero $\omega \in l_2$. This completes the proof. □

Now, we are in a position to give the main result on the solvability of the robust output feedback H_∞ control problem.

Theorem 2: Consider the uncertain discrete delay fuzzy system (1)–(4). Given a scalar $\gamma > 0$, then there exists a full-order fuzzy dynamic output feedback controller (12) and (13) such that the requirements (R1) and (R2) are satisfied if there exist matrices $X > 0, Y > 0, \Omega_i, \Phi_i$, and $\Psi_i, 1 \le i \le r$, such that the following LMIs:

$$
\begin{bmatrix}
-J_1 & 0 & H_{1ii}^T & 0 & H_{2ii}^T & H_{3ii}^T & \tilde{H}^T \\
0 & -J_2 & H_{5ii}^T & 0 & 0 & 0 & 0 \\
H_{1ii} & H_{5ii} & -J_3 & H_{6i}^T & 0 & 0 & 0 \\
0 & 0 & H_{6i} & -J_{4i} & 0 & 0 & 0 \\
H_{2ii} & 0 & 0 & 0 & -J_{5ii} & 0 & 0 \\
H_{3ii} & 0 & 0 & 0 & 0 & -I & 0 \\
\tilde{H} & 0 & 0 & 0 & 0 & 0 & -J_6
\end{bmatrix} < 0 \qquad (48)
$$

for $1 \le i \le r$ and (49), as shown at the bottom of the page, for $1 \le i < j \le r$, hold for some given matrix $Q > 0$ and scalars $\epsilon_{ij} > 0, 1 \le i \le j \le r$, where

$$
\begin{aligned}
J_1 &= \begin{bmatrix} Y & I & 0 \\ I & X & 0 \\ 0 & 0 & Q \end{bmatrix} \quad J_2 = \gamma^2 I \quad J_3 = \begin{bmatrix} Y & I \\ I & X \end{bmatrix} \\
J_{4ij} &= \mathrm{diag}\left(\epsilon_{ij}^{-1}I, \epsilon_{ij}^{-1}I\right) \quad J_{4i} = \epsilon_{ii}^{-1}I \\
J_{5ij} &= \epsilon_{ij}I \quad J_6 = Q^{-1} \\
H_{1ij} &= \begin{bmatrix} A_iY + B_i\Psi_j & A_i & A_{di} \\ \Omega_i & XA_i + \Phi_jC_i & XA_{di} + \Phi_jC_{di} \end{bmatrix} \\
H_{2ij} &= [N_{1i}Y + N_{3i}\Psi_j \quad N_{1i} \quad N_{2i}] \\
H_{3ij} &= [E_iY + G_i\Psi_j \quad E_i \quad E_{di}] \\
H_{4ij} &= \begin{bmatrix} \Psi_j - \Psi_i & 0 & 0 \\ (C_i - C_j)Y & 0 & 0 \end{bmatrix} \\
H_{5ij} &= \begin{bmatrix} D_{1i} \\ XD_{1i} + \Phi_jD_{2i} \end{bmatrix} \\
H_{6ij} &= \begin{bmatrix} M_i^T & M_i^TX \\ M_j^T & M_j^TX \end{bmatrix} \quad H_{6i} = \begin{bmatrix} M_i^T & M_i^TX \end{bmatrix} \\
H_{7ij} &= \begin{bmatrix} 0 & 0 \\ X(B_i - B_j) & \Phi_j - \Phi_i \end{bmatrix} \quad \tilde{H} = [Y \quad I \quad 0].
\end{aligned}
$$

Furthermore, a desired robust dynamic output feedback H_∞ controller is given in the form of (14) and (15) with parameters as follows:

$$
A_{Ki} = S^{-1}\left(\Omega_i - XA_iY - XB_i\Psi_i - \Phi_iC_iY\right)W^{-T} \qquad (50)
$$

$$
B_{Ki} = S^{-1}\Phi_i \quad C_{Ki} = \Psi_iW^{-T}, \qquad 1 \le i \le r \qquad (51)
$$

$$
\begin{bmatrix}
-4J_1 & 0 & H_{1ij}^T + H_{1ji}^T & 0 & H_{2ij}^T & H_{2ji}^T & H_{3ij}^T + H_{3ji}^T & \tilde{H}^T & 0 & H_{4ij}^T \\
0 & -4J_2 & H_{5ij}^T + H_{5ji}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
H_{1ij} + H_{1ji} & H_{5ij} + H_{5ji} & -J_3 & H_{6ij}^T & 0 & 0 & 0 & 0 & H_{7ij} & 0 \\
0 & 0 & H_{6ij} & -J_{4ij} & 0 & 0 & 0 & 0 & 0 & 0 \\
H_{2ij} & 0 & 0 & 0 & -J_{5ij} & 0 & 0 & 0 & 0 & 0 \\
H_{2ji} & 0 & 0 & 0 & 0 & -J_{5ij} & 0 & 0 & 0 & 0 \\
H_{3ij} + H_{3ji} & 0 & 0 & 0 & 0 & 0 & -I & 0 & 0 & 0 \\
\tilde{H} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4}J_6 & 0 & 0 \\
0 & 0 & H_{7ij}^T & 0 & 0 & 0 & 0 & 0 & -I & 0 \\
H_{4ij} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I
\end{bmatrix} < 0 \qquad (49)
$$

where S and W are any nonsingular matrices satisfying

$$SW^T = I - XY. \tag{52}$$

Proof: Under the conditions of the theorem, we first show that there always exist nonsingular matrices S and W such that (52) is satisfied. To this end, we note that (48) implies

$$\begin{bmatrix} -Y & -I \\ -I & -X \end{bmatrix} < 0$$

which, by the Schur complement formula, gives that $X - Y^{-1} > 0$, therefore $I - XY$ is nonsingular. This ensures that there always exist nonsingular matrices S and W such that (52) is satisfied. Now, we introduce the following nonsingular matrices

$$\Pi_1 = \begin{bmatrix} Y & I \\ W^T & 0 \end{bmatrix} \quad \Pi_2 = \begin{bmatrix} I & X \\ 0 & S^T \end{bmatrix}. \tag{53}$$

Let

$$\hat{P} = \Pi_2 \Pi_1^{-1}. \tag{54}$$

Then

$$\hat{P} = \begin{bmatrix} X & S \\ S^T & \Xi \end{bmatrix}$$

where

$$\Xi = W^{-1} Y (X - Y^{-1}) Y W^{-T} > 0. \tag{55}$$

Observe that

$$\Xi - S^T X^{-1} S = S^T (YX - I)^{-1} \times (Y - X^{-1})(XY - I)^{-1} S > 0. \tag{56}$$

$$\begin{bmatrix} -4J_1 + H_{4ij}^T H_{4ij} & 0 & H_{1ij}^T + H_{1ji}^T & 0 & H_{2ij}^T & H_{2ji}^T & H_{3ij}^T + H_{3ji}^T & \tilde{H}^T \\ 0 & -4J_2 & H_{5ij}^T + H_{5ji}^T & 0 & 0 & 0 & 0 & 0 \\ H_{1ij} + H_{1ji} & H_{5ij} + H_{5ji} & -J_3 + H_{7ij} H_{7ij}^T & H_{6ij}^T & 0 & 0 & 0 & 0 \\ 0 & 0 & H_{6ij} & -J_{4ij} & 0 & 0 & 0 & 0 \\ H_{2ij} & 0 & 0 & 0 & -J_{5ij} & 0 & 0 & 0 \\ H_{2ji} & 0 & 0 & 0 & 0 & -J_{5ij} & 0 & 0 \\ H_{3ij} + H_{3ji} & 0 & 0 & 0 & 0 & 0 & -I & 0 \\ \tilde{H} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} J_6 \end{bmatrix} < 0. \tag{57}$$

$$\begin{aligned} &\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & (\Psi_j - \Psi_i)^T (B_i - B_j)^T X \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ X(B_i - B_j)(\Psi_j - \Psi_i) & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ &\le \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ X(B_i - B_j) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ X(B_i - B_j) \end{bmatrix}^T + \begin{bmatrix} (\Psi_j - \Psi_i)^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} (\Psi_j - \Psi_i)^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \end{aligned} \tag{58}$$

and

$$\begin{aligned} &\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & Y(C_i - C_j)^T (\Phi_j - \Phi_i)^T \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ (\Phi_j - \Phi_i)(C_i - C_j) Y & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ &\le \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \Phi_j - \Phi_i \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \Phi_j - \Phi_i \end{bmatrix}^T + \begin{bmatrix} Y(C_i - C_j)^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} Y(C_i - C_j)^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T. \end{aligned} \tag{59}$$

Then, we have $\hat{P} > 0$. By applying the Schur complement formula to (49), we have that for $1 \le i < j \le r$, (57), as shown at the bottom of the previous page, holds. By Lemma 1, it can be deduced that for $1 \le i < j \le r$, (58) and (59), as shown at the bottom of the previous page, hold. Then, it follows from (57)–(59) that (60), as shown at the bottom of the page, holds, where

$$\mathcal{G}_{ij} = H_{1ij} + H_{1ji} + \mathcal{Z}_{ij}$$

$$\mathcal{Z}_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ X(B_i - B_j)(\Psi_j - \Psi_i) + (\Phi_j - \Phi_i)(C_i - C_j)Y & 0 & 0 \end{bmatrix}.$$

Using the relationship

$$\begin{aligned} &(XA_iY + XB_i\Psi_i + \Phi_iC_iY) \\ &\quad + (XA_jY + XB_j\Psi_j + \Phi_jC_jY) \\ &\quad + X(B_i - B_j)(\Psi_j - \Psi_i) + (\Phi_j - \Phi_i)(C_i - C_j)Y \\ &\qquad = (XA_iY + XB_i\Psi_j + \Phi_jC_iY) \\ &\qquad\quad + (XA_jY + XB_j\Psi_i + \Phi_iC_jY) \end{aligned}$$

and considering the notations in (19)–(21) with A_{Ki}, B_{Ki}, and C_{Ki} for $1 \le i \le r$ in (50) and (51), we can verify that the matrix inequality in (60) can be rewritten as shown in the inequality at the bottom of the page. Pre- and postmultiplying this inequality by $\mathrm{diag}(\Pi_1^{-T}, I, I, \Pi_2^{-T}, I, I, I, I, I, I)$ and its transpose, respectively, result in the first equation shown at the bottom of the next page, which, by the Schur complement, is equivalent to (61), as shown at the bottom of the next page, for $1 \le i < j \le r$. Following a similar line as in the derivation of (61) and using (48), we can obtain that for $1 \le i \le r$, (62), as shown at the bottom of the next page, holds. Considering (61) and (62) and applying Theorem 1, we conclude that with the controller parameters in (50) and (51) the closed-loop system (16) and (17) is robustly asymptotically stable and (22) is satisfied. □

Remark 1: Theorem 2 provides a sufficient condition for the solvability of the robust H_∞ output feedback control problem for uncertain discrete time-delay fuzzy systems. We note that (48) and (49) are LMIs in X, Y, Φ_i, Φ_i and $\Psi_i, 1 \le i \le r$, when $Q > 0$ and $\epsilon_{ij} > 0, 1 \le i \le j \le r$, are given. In this case, these LMIs can be solved efficiently by resorting to some standard

$$\begin{bmatrix} -4J_1 & 0 & \mathcal{G}_{ij}^T & 0 & H_{2ij}^T & H_{2ji}^T & H_{3ij}^T + H_{3ji}^T & \tilde{H}^T \\ 0 & -4J_2 & H_{5ij}^T + H_{5ji}^T & 0 & 0 & 0 & 0 & 0 \\ \mathcal{G}_{ij} & H_{5ij} + H_{5ji} & -J_3 & H_{6ij}^T & 0 & 0 & 0 & 0 \\ 0 & 0 & H_{6ij} & -J_{4ij} & 0 & 0 & 0 & 0 \\ H_{2ij} & 0 & 0 & 0 & -J_{5ij} & 0 & 0 & 0 \\ H_{2ji} & 0 & 0 & 0 & 0 & -J_{5ij} & 0 & 0 \\ H_{3ij} + H_{3ji} & 0 & 0 & 0 & 0 & 0 & -I & 0 \\ \tilde{H} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4}J_6 \end{bmatrix} < 0 \qquad (60)$$

$$\left[\begin{array}{cccccccccc} -4\Pi_1^T\hat{P}\Pi_1 & 0 & 0 & \Pi_1^T\left(A_{cij}^T + A_{cji}^T\right)\Pi_2 & 0 & 0 & \Pi_1^T\tilde{N}_{ij}^T & \Pi_1^T\tilde{N}_{ji}^T & \Pi_1^T\left(E_{cij}^T + E_{cji}^T\right) & \Pi_1^TH^T \\ 0 & -4Q & 0 & \left(A_{cdij}^T + A_{cdji}^T\right)\Pi_2 & 0 & 0 & N_{2i}^T & N_{2j}^T & 0 & 0 \\ 0 & 0 & -4\gamma^2 I & \left(D_{cij}^T + D_{cji}^T\right)\Pi_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ \Pi_2^T\left(A_{cij} + A_{cji}\right)\Pi_1 & \Pi_2^T\left(A_{cdij} + A_{cdji}\right) & \Pi_2^T\left(D_{cij} + D_{cji}\right) & -\Pi_2^T\hat{P}^{-1}\Pi_2 & \Pi_2^T\tilde{M}_i & \Pi_2^T\tilde{M}_j & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{M}_i^T\Pi_2 & -\epsilon_{ij}^{-1}I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{M}_j^T\Pi_2 & 0 & -\epsilon_{ij}^{-1}I & 0 & 0 & 0 & 0 \\ \tilde{N}_{ij}\Pi_1 & N_{2i} & 0 & 0 & 0 & 0 & -\epsilon_{ij}I & 0 & 0 & 0 \\ \tilde{N}_{ji}\Pi_1 & N_{2j} & 0 & 0 & 0 & 0 & 0 & -\epsilon_{ij}I & 0 & 0 \\ \left(E_{cij} + E_{cji}\right)\Pi_1 & E_{cdij} + E_{cdji} & 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 \\ H\Pi_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4}Q^{-1} \end{array}\right] < 0$$

numerical algorithms, which involves no tuning of parameters [2]. In the case when the parameters $Q > 0$ and $\epsilon_{ij} > 0, 1 \leq i \leq j \leq r$, are not fixed, it can be seen that (48) and (49) are not LMIs with respect to the parameters $Q > 0$ and $\epsilon_{ij} > 0, 1 \leq i \leq j \leq r$, since these parameters appear in (48) and (49) in a nonlinear fashion, which is sometimes encountered when dealing with the output feedback control problem for time-delay systems with or without parameter uncertainties; see, e.g., [6], [7], [9]. In order to cast the output H_∞ control problem in this paper into an LMI framework, we therefore fix the parameters $Q > 0$ and $\epsilon_{ij} > 0, 1 \leq i \leq j \leq r$; such an approach was also adopted in [6], [7], [9].

Remark 2: It is worth pointing out that the result in Theorem 2 can be readily extended to the case with multiple delays. It is also noted that the result in Theorem 2 are independent of the delay size; therefore, Theorem 2 can be applicable to the case when no *a priori* knowledge about the size of the time delay is available.

IV. Numerical Example

In this section, we will apply the proposed method to design a fuzzy dynamic controller for an uncertain nonlinear discrete delay system. The uncertain discrete nonlinear time-delay system is described as follows:

$$\begin{aligned} x_1(k+1) &= -x_1(k)^2 + 0.3x_2(k) + 0.1x_1(k-2)^2 \\ &\quad - 0.2x_1(k-2)x_2(k-2) \\ &\quad + u_1(k) + x_1(k)\omega(k) + 0.1c_1(k)x_1(k)x_2(k) \\ &\quad + 0.1c_2(k)x_2(k-2) \\ x_2(k+1) &= 0.1x_1(k) + x_2(k) + 0.1c_1(k)x_2(k) + 0.5u_2(k) \\ y(k) &= 0.6x_1(k) + 0.1\omega(k) \\ z(k) &= 0.1x_1(k)^2 + 0.2x_2(k-2) \end{aligned}$$

where $c_1(k)$ and $c_2(k)$ are uncertain parameters satisfying

$$c_1(k) \in [-0.2, 0.2] \quad c_2(k) \in [-0.1, 0.1].$$

$$\begin{bmatrix} -4\hat{P} & 0 & 0 & A_{cij}^T + A_{cji}^T & 0 & 0 & \tilde{N}_{ij}^T & \tilde{N}_{ji}^T & E_{cij}^T + E_{cji}^T & H^T \\ 0 & -4Q & 0 & A_{cdij}^T + A_{cdji}^T & 0 & 0 & N_{2i}^T & N_{2j}^T & E_{cdij}^T + E_{cdji}^T & 0 \\ 0 & 0 & -4\gamma^2 I & D_{cij}^T + D_{cji}^T & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{cij} + A_{cji} & A_{cdij} + A_{cdji} & D_{cij} + D_{cji} & -\hat{P}^{-1} & \tilde{M}_i & \tilde{M}_j & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{M}_i^T & -\epsilon_{ij}^{-1} I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{M}_j^T & 0 & -\epsilon_{ij}^{-1} I & 0 & 0 & 0 & 0 \\ \tilde{N}_{ij} & N_{2i} & 0 & 0 & 0 & 0 & -\epsilon_{ij} I & 0 & 0 & 0 \\ \tilde{N}_{ji} & N_{2j} & 0 & 0 & 0 & 0 & 0 & -\epsilon_{ij} I & 0 & 0 \\ E_{cij} + E_{cji} & E_{cdij} + E_{cdji} & 0 & 0 & 0 & 0 & 0 & 0 & -I & 0 \\ H & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4}Q^{-1} \end{bmatrix} < 0$$

$$\begin{bmatrix} 4H^TQH - 4\hat{P} & 0 & 0 & A_{cij}^T + A_{cji}^T & \tilde{N}_{ij}^T & \tilde{N}_{ji}^T & E_{cij}^T + E_{cji}^T \\ 0 & -4Q & 0 & A_{cdij}^T + A_{cdji}^T & N_{2i}^T & N_{2j}^T & E_{cdij}^T + E_{cdji}^T \\ 0 & 0 & -4\gamma^2 I & D_{cij}^T + D_{cji}^T & 0 & 0 & 0 \\ A_{cij} + A_{cji} & A_{cdij} + A_{cdji} & D_{cij} + D_{cji} & \epsilon_{ij}\tilde{M}_{ij}\tilde{M}_{ij}^T - \hat{P}^{-1} & 0 & 0 & 0 \\ \tilde{N}_{ij} & N_{2i} & 0 & 0 & -\epsilon_{ij} I & 0 & 0 \\ \tilde{N}_{ji} & N_{2j} & 0 & 0 & 0 & -\epsilon_{ij} I & 0 \\ E_{cij} + E_{cji} & E_{cdij} + E_{cdji} & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0 \quad (61)$$

$$\begin{bmatrix} H^TQH - \hat{P} & 0 & 0 & A_{cii}^T & \tilde{N}_{ii}^T & E_{cii}^T \\ 0 & -Q & 0 & A_{cdii}^T & N_{2i}^T & E_{cdii}^T \\ 0 & 0 & -\gamma^2 I & D_{cii}^T & 0 & 0 \\ A_{cii} & A_{cdii} & D_{cii} & \epsilon_{ii}\tilde{M}_i\tilde{M}_i^T - \hat{P}^{-1} & 0 & 0 \\ \tilde{N}_{ii} & N_{2i} & 0 & 0 & -\epsilon_{ii} I & 0 \\ E_{cii} & E_{cdii} & 0 & 0 & 0 & -I \end{bmatrix} < 0 \quad (62)$$

Similar to [20], we assume that $x_1(k) \in [-0.5, 0.5]$, and select the membership functions as

$$\mu_{11}(x_1(k)) = \frac{1}{2}(1 - 2x_1(k)) \quad \mu_{21}(x_1(k)) = \frac{1}{2}(1 + 2x_1(k)).$$

Then, the nonlinear time-delay system can be represented by the following uncertain time-delay T–S model:

Plant Rule 1: IF $x_1(k)$ is μ_{11} THEN

$$\begin{aligned} x_1(k+1) &= [A_1 + \Delta A_1(k)]x(k) \\ &\quad + A_{d1}x(k-2) + B_1u(k) \\ &\quad + D_{11}\omega(k) \\ y(k) &= C_1x(k) + D_{21}\omega(k) \\ z(k) &= E_1x(k) + E_{d1}x(k-2) \end{aligned}$$

and

Plant Rule 2: IF $x_1(k)$ is μ_{21} THEN

$$\begin{aligned} x_1(k+1) &= [A_2 + \Delta A_2(k)]x(k) \\ &\quad + A_{d2}x(k-2) + B_2u(k) \\ &\quad + D_{12}\omega(k) \\ y(k) &= C_2x(k) + D_{22}\omega(k) \\ z(k) &= E_2x(k) + E_{d2}x(k-2) \end{aligned}$$

where

$$\begin{aligned} &A_1 = \begin{bmatrix} 0.5 & 0.3 \\ 0.1 & 1 \end{bmatrix} \quad A_{d1} = \begin{bmatrix} -0.05 & 0.1 \\ 0 & 0 \end{bmatrix} \\ &B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix} \quad D_{11} = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix} \\ &C_1 = [0.6 \quad 0] \quad D_{21} = 0.1 \\ &E_1 = [-0.05 \quad 0] \quad E_{d1} = [0 \quad 0.2] \\ &A_2 = \begin{bmatrix} -0.5 & 0.3 \\ 0.1 & 1 \end{bmatrix} \quad A_{d2} = \begin{bmatrix} 0.05 & -0.1 \\ 0 & 0 \end{bmatrix} \\ &B_2 = B_1 \quad D_{12} = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} \\ &C_2 = C_1 \quad D_{22} = D_{21} \quad E_2 = [0.05 \quad 0] \quad E_{d2} = E_{d1} \end{aligned}$$

and $\Delta A_1(k)$ and $\Delta A_2(k)$ can be represented in the form of (5) and (6) with

$$\begin{aligned} &M_1 = \begin{bmatrix} 0.05 & 0.1 \\ 0.1 & 0 \end{bmatrix} \quad N_{11} = \begin{bmatrix} 0 & 0.2 \\ 0 & 0 \end{bmatrix} \\ &N_{21} = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix} \\ &M_2 = M_1 \quad N_{12} = N_{11} \quad N_{22} = N_{21}. \end{aligned}$$

In this example, we choose the H_∞ performance level $\gamma = 1.8$. In order to design a fuzzy H_∞ output feedback controller for the T–S model, we first choose

$$\epsilon_{11} = 10 \quad \epsilon_{12} = 0.2 \quad \epsilon_{22} = 1 \quad Q = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

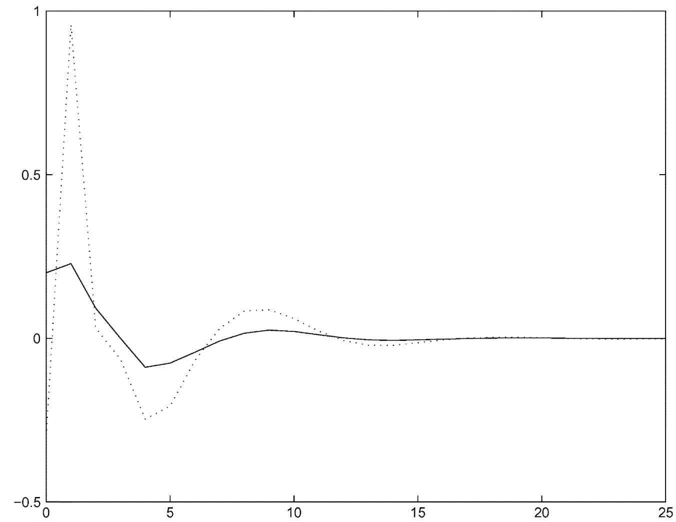

Fig. 1. State response of $x_1(k)$ (—) and $x_2(k)$ ($\cdots$).

Then, using the Matlab LMI Control Toolbox to solve the LMIs in (48) and (49), we obtain the solution as follows:

$$\begin{aligned} &X = \begin{bmatrix} 2.0126 & -0.7162 \\ -0.7162 & 1.2392 \end{bmatrix} \quad Y = \begin{bmatrix} 3.2210 & 0.2170 \\ 0.2170 & 3.9002 \end{bmatrix} \\ &\Omega_1 = \begin{bmatrix} 0.2384 & 0.1137 \\ -0.0989 & 0.3758 \end{bmatrix} \\ &\Omega_2 = \begin{bmatrix} -0.0997 & -0.4315 \\ -0.0040 & 0.3465 \end{bmatrix} \quad \Phi_1 = \begin{bmatrix} -0.2762 \\ -1.1165 \end{bmatrix} \\ &\Phi_2 = \begin{bmatrix} 0.8337 \\ -1.4828 \end{bmatrix} \\ &\Psi_1 = \begin{bmatrix} -0.6124 & -1.2324 \\ -0.9050 & -7.1605 \end{bmatrix} \\ &\Psi_2 = \begin{bmatrix} 0.6401 & -1.2887 \\ -0.8744 & -7.1387 \end{bmatrix}. \end{aligned}$$

Now, choose

$$S = \begin{bmatrix} 1 & 1 \\ 8 & -5 \end{bmatrix} \quad W = \begin{bmatrix} -1.8922 & -3.4350 \\ 0.6235 & 1.7331 \end{bmatrix}. \tag{63}$$

It is easy to verify that the matrices S and W in (63) satisfy the equality in (52); therefore, by Theorem 2, a desired fuzzy output feedback controller can be constructed as in (12) and (13) with

$$\begin{aligned} &A_{K1} = \begin{bmatrix} 0.0666 & 0.0487 \\ 1.0122 & -0.2629 \end{bmatrix} \\ &A_{K2} = \begin{bmatrix} -0.5355 & 0.2276 \\ -0.2573 & 0.1555 \end{bmatrix} \\ &B_{K1} = \begin{bmatrix} -0.1921 \\ -0.0841 \end{bmatrix} \quad B_{K2} = \begin{bmatrix} 0.2066 \\ 0.6271 \end{bmatrix} \\ &C_{K1} = \begin{bmatrix} 4.6541 & -2.3854 \\ 22.9993 & -12.4055 \end{bmatrix} \\ &C_{K2} = \begin{bmatrix} 2.9160 & -1.7926 \\ 22.8867 & -12.3524 \end{bmatrix}. \end{aligned}$$

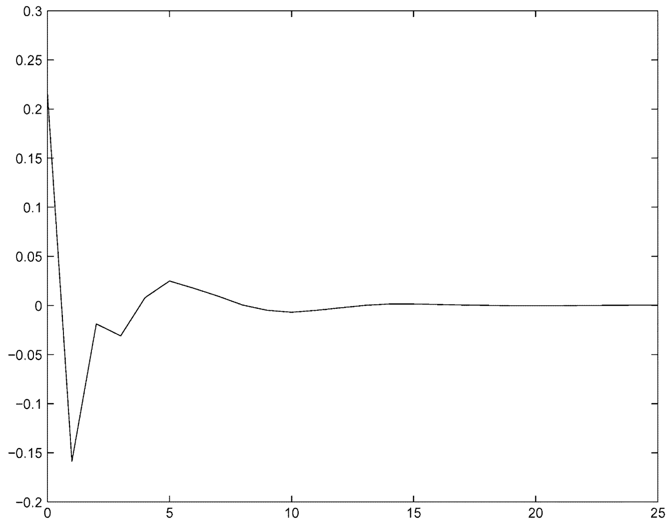

Fig. 2. Control input $u(k)$.

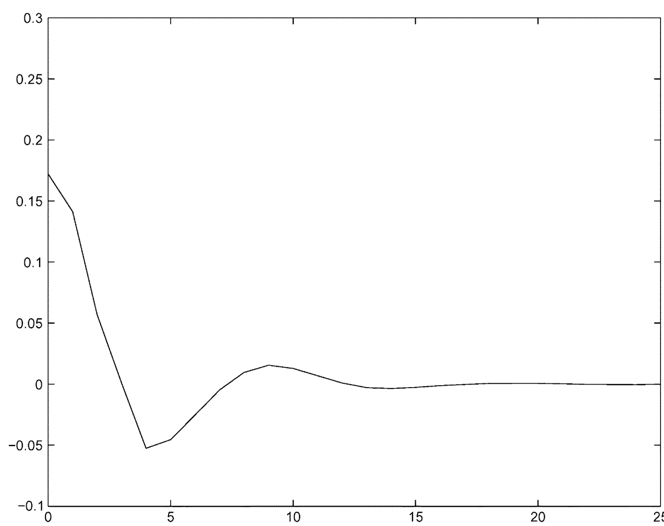

Fig. 3. Measured output $y(k)$.

With the aforementioned fuzzy controller, the simulation results of the state response of the nonlinear system are given in Fig. 1, where the initial conditions are set as

$$x(-2) = [0.5 \quad -0.2]^T \quad x(-1) = [0.1 \quad 0.4]^T$$
$$x(0) = [0.2 \quad -0.3]^T$$

and the exogenous disturbance input $\omega(k) \in l_2[0, \infty)$ is defined as

$$\omega(k) = \frac{r}{1 + 15(k+1)}$$

where r is a random number taken from a uniform distribution over $[0, 2]$, and $c_1(k) = 0.2\cos(k)$ and $c_2(k) = 0.1\sin(k)$. Fig. 2 shows the control input, while Figs. 3 and 4 present the corresponding measured output and the controlled output, respectively. From these simulation results, it can be seen the designed fuzzy output feedback controller ensures the robust asymptotic stability of the closed-loop system and guarantees a prescribed H_∞ performance level.

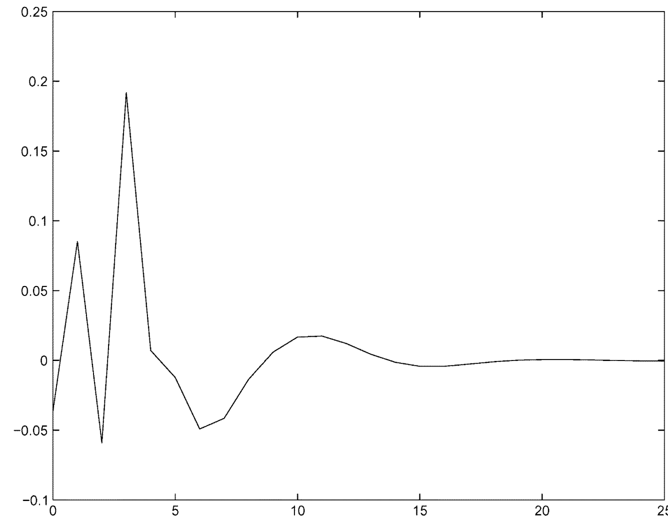

Fig. 4. Controlled output $z(k)$.

V. Conclusion

The problem of robust output feedback H_∞ control for uncertain discrete T–S fuzzy systems with time-varying norm-bounded parameter uncertainties and time delays has been studied. A sufficient condition for the existence of a full-order fuzzy dynamic output feedback controller, which robustly stabilizes the uncertain system and guarantees a prescribed level on disturbance attenuation, has been obtained. The design approach has been applied to the problem of robust H_∞ control of a class of nonlinear discrete delay systems, and the simulation results have showed the effectiveness of the proposed approach.

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