New Positive Realness Conditions for Uncertain Discrete Descriptor Systems: Analysis and Synthesis

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Abstract—**This paper deals with the problems of positive real (PR) analysis and PR control for uncertain discrete-time descriptor systems. The parameter uncertainties are assumed to be time-invariant norm bounded and appear in both the state and input matrices. A new necessary and sufficient condition for a discrete-time descriptor system to be regular, causal, stable and extended strictly PR (ESPR) is proposed in terms of a strict linear matrix inequality. Based on this, the concepts of strong robust admissibility with ESPR and strong robust admissibilizability with ESPR were introduced. Without any additional assumptions on the system matrices, necessary and sufficient conditions for strong robust admissibility with ESPR and strong robust admissibilizability with ESPR are obtained. Through these results, the problems of PR analysis and PR control are solved. Furthermore, an explicit expression of a desired state feedback controller is also given, which involves no decomposition of the system matrices.**

Index Terms—**Descriptor systems, discrete-time systems, parameter uncertainty, positive real (PR) control, state feedback.**

I. INTRODUCTION

POSITIVE real (PR) control is a research problem of recurring interest in the past years [1], [3], [10]. The study of this problem is motivated by the robust and nonlinear control where a well-known fact is that the positive realness of a certain loop transfer function will guarantee the overall stability of a feedback system if uncertainty or nonlinearity can be characterized by a PR system [12]. By solving a pair of Riccati inequalities, a solution to this problem was given in [10]. When parameter uncertainties appear in a system model, the PR control problem was studied in [14], where state feedback controllers were designed and a Riccati equation approach was developed, while in [7] observer-based dynamic output-feedback controllers were constructed in terms of solutions to certain linear matrix inequalities (LMIs). The corresponding results for discrete-time systems can be found in [8]. It is worth mentioning that the results in [8] were further extended to two-dimensional (2-D) systems in [16].

On the other hand, it is known that descriptor system models are more convenient and natural than state-space models in the description of many practical systems, such as interconnected large-scale systems, economic systems, power systems [2], [5]. Descriptor systems are also referred to as singular systems, implicit systems, generalized state-space systems, differential-algebraic systems or semi-state systems. The study of descriptor systems has received much attention during the past decades, and many results based on the theory of state-space systems have been extended to the area of descriptor systems [2], [5], [6], [19]. Very recently, the positive realness of descriptor systems was studied in [21], where PR lemmas for descriptor systems in both the continuous and discrete contexts have been proposed in terms of generalized algebraic Riccati equations and inequalities. However, the design of controllers that achieve the positive realness of the closed-loop system has not been investigated in [21]. It is worth pointing out that the problem of PR control for discrete descriptor systems is much more difficult than that for state-space systems due to the fact that the Lyapunov-type matrix in discrete-time descriptor systems is indefinite as reported in [21] while in the state-space case the Lyapunov matrix is positive definite. It is also worth mentioning that in [21] parameter uncertainties in descriptor system model have not been considered.

In this paper, we consider the problems of PR analysis and PR control for uncertain discrete-time descriptor systems. The parameter uncertainties are assumed to be time invariant and unknown but norm bounded appearing in both the state and input matrices. In order to overcome the difficulty encountered when designing state feedback controllers for discrete-time descriptor systems, we first present a new necessary and sufficient condition in terms of a strict LMI, which ensures a discrete-time descriptor system to be regular, causal, stable and *extended strictly PR* (ESPR). It should be pointed out that the given strict LMI is more desirable than a nonstrict one in [21] since testing a nonstrict LMI may cause some numerical problems [11]. In order to solve the problems, the concepts of strong robust admissibility with ESPR and strong robust admissibilizability with ESPR were introduced. Without assumptions on the system matrices, necessary and sufficient conditions for strong robust admissibility with ESPR and strong robust admissibilizability with ESPR are derived in terms of a strict LMI and matrix inequalities, respectively. When these matrix inequalities are feasible, a desired state feedback controller can be constructed directly and no decomposition of the system matrices is involved.

A. Notation

Throughout this paper, for real symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive def-

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inite). I is the identity matrix with appropriate dimension. The superscripts "T" and "$*$" represent the transpose and the complex conjugate transpose, respectively. The notation $D_{\text{int}}(0,1)$ is the interior of the unit disk with center at the origin. We use $\lambda(E, A)$ to represent the set $\{z\,|\det(zE - A) = 0\,\}$. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

II. DEFINITIONS AND PROBLEM FORMULATION

Consider an uncertain linear discrete-time descriptor system described by

$$(\Sigma): Ex(k+1)=(A+\Delta A)x(k)+(B_1+\Delta B_1)u(k)+B\omega(k) \tag{1}$$

$$z(k) = Cx(k) + D\omega(k) \tag{2}$$

where $x(k) \in \mathbb{R}^n$ is the state; $u(k) \in \mathbb{R}^m$ is the control input; $\omega(k) \in \mathbb{R}^l$ is the disturbance input; and $z(k) \in \mathbb{R}^q$ is the controlled output. The matrix $E \in \mathbb{R}^{n \times n}$ may be singular; we assume that rank $E = r \leq n$, and $q = l$. A, B, B_1, C and D are known real constant matrices with appropriate dimensions. ΔA and ΔB are time-invariant matrices representing norm-bounded parameter uncertainties, and are assumed to be of the form

$$\begin{bmatrix} \Delta A & \Delta B_1 \end{bmatrix} = MF(\sigma)\begin{bmatrix} N_1 & N_2 \end{bmatrix} \tag{3}$$

where M, N_1, and N_2 are known real constant matrices with appropriate dimensions. The uncertain matrix $F(\sigma)$ satisfies

$$F(\sigma)F(\sigma)^T \leq I \tag{4}$$

and $\sigma \in \Xi$, where Ξ is a compact set in $\mathbb{R}$. Furthermore, it is assumed that given any matrix $F : FF^T \leq I$, there exists a $\sigma \in \Xi$ such that $F = F(\sigma)$. ΔA and ΔB are said to be admissible if both (3) and (4) hold.

Remark 1: It should be pointed out that the structure of the uncertainty with the form (3) and (4) has been widely used when dealing with the problem of robust control for regular and descriptor uncertain systems in both continuous- and discrete-time contexts; see e.g., [13], [20], and the references therein.

The nominal discrete-time descriptor system of (Σ) with $u(k) = 0$ is

$$(\hat{\Sigma}): \qquad Ex(k + 1) = Ax(k) + B\omega(k) \tag{5}$$

$$z(k) = Cx(k) + D\omega(k). \tag{6}$$

For the descriptor discrete-time system $(\hat{\Sigma})$, we introduce the following definition.

Definition 1—[2], [5]:

(I) $(\hat{\Sigma})$ is said to be regular if $\det(zE - A)$ is not identically zero.

(II) $(\hat{\Sigma})$ is said to be causal if the degree of $\det(zE - A)$ is equivalent to rank E.

(III) $(\hat{\Sigma})$ is said to be stable if $\lambda(E, A) \subset D_{\text{int}}(0,1)$.

(IV) $(\hat{\Sigma})$ is said to be admissible if it is regular, causal and stable.

When system $(\hat{\Sigma})$ is regular, the transfer function of this system is given as follows:

$$G(z) = C(zE - A)^{-1}B + D. \tag{7}$$

Throughout this paper, we shall use the following concept of positive realness.

Definition 2—[21]:

(I)   System $(\hat{\Sigma})$ is said to be PR if its transfer function $G(z)$ is analytic in $|z| > 1$ and satisfies $G(z) + G^*(z) \geq 0$ for $|z| > 1$.

(II)   System $(\hat{\Sigma})$ is said to be strictly PR (SPR) if its transfer function $G(z)$ is analytic in $|z| \geq 1$ and satisfies $G(e^{j\theta}) + G^*(e^{j\theta}) > 0$ *for* $\theta \in [0, 2\pi)$.

(III)   System $(\hat{\Sigma})$ is said to be ESPR if it is SPR and $G(\infty) + G(\infty)^T > 0$.

The problems of PR analysis and PR control for the uncertain discrete-time descriptor system (Σ) will be addressed. The purpose of the PR analysis problem is to develop conditions ensuring that the uncertain descriptor system (Σ) with $u(k) = 0$ is admissible and ESPR for all parameter uncertainty ΔA satisfying (3) and (4), while the aim of the PR control problem is to design a state feedback controller for (Σ) such that the resulting closed-loop system is admissible and ESPR.

III. MAIN RESULTS

We first give the following results, which will be used in the derivation of our main results.

Lemma 1: The discrete-time descriptor system $(\hat{\Sigma})$ is admissible if and only if there exist matrices $P > 0$ and Q such that

$$A^TPA - E^TPE + QS^TA + A^TSQ^T < 0 \tag{8}$$

where $S \in \mathbb{R}^{n \times (n-r)}$ is any matrix with full column and satisfies $E^TS = 0$.

Proof: For the proof of Lemma 1, see [15].   □

Lemma 2—[17]: Given any matrices X, Y and Z with appropriate dimensions such that $Y > 0$. Then, we have

$$X^TZ + Z^TX + X^TYX \geq -Z^TY^{-1}Z.$$

Lemma 3—[4], [21]: Consider a state-space system described by

$$x(k + 1) = Ax(k) + B\omega(k) \tag{9}$$

$$z(k) = Cx(k) + D\omega(k). \tag{10}$$

Then, this system is stable and ESPR if and only if there exists a matrix $P > 0$ such that

$$\begin{bmatrix} A^TPA - P & C^T - A^TPB \\ C - B^TPA & -(D^T + D - B^TPB) \end{bmatrix} < 0.$$

Theorem 4: The discrete-time descriptor system $(\hat{\Sigma})$ is admissible and ESPR if and only if there exist matrices $P > 0$ and Q such that the following LMI holds:

$$\begin{bmatrix} A^T P A - E^T P E & C^T - A^T P B \\ C - B^T P A & -\left(D^T + D - B^T P B\right) \end{bmatrix}$$
$$+ \begin{bmatrix} A^T \\ -B^T \end{bmatrix} S Q^T + Q S^T \begin{bmatrix} A^T \\ -B^T \end{bmatrix}^T < 0 \quad (11)$$

where $S \in \mathbb{R}^{n \times (n-r)}$ is any matrix with full column and satisfies $E^T S = 0$.

Proof: For the proof of Lemma 1, see the Appendix . ☐

Remark 2: Theorem 1 provides a necessary and sufficient condition for the discrete-time descriptor system $(\hat{\Sigma})$ to be admissible and ESPR. In the case when $E = I$, that is, the descriptor system $(\hat{\Sigma})$ reduces to a state-space system, Theorem 1 coincides with Lemma 4.2 in [4]. Therefore, Theorem 1 can be regarded as an extension of existing ESPR results for discrete-time state-space systems to descriptor systems.

Remark 3: Note that the condition in (11) is a strict LMI, which is in contrast to those in [21], where nonstrict LMI conditions were given. It should be pointed out that some numerical problems may arise when checking nonstrict LMI conditions. Therefore, the strict LMI condition in (11) is more desirable from the numerical point of view.

Considering Theorem 1, we introduce the following definitions in order to solve the problems of PR analysis and PR control formulated in the previous section.

Definition 3: The uncertain discrete-time descriptor system (Σ) with $u(k) \equiv 0$ is said to be strongly robustly admissible with ESPR if there exist matrices $P > 0$ and Q such that

$$\begin{bmatrix} (A+\Delta A)^T P (A+\Delta A) - E^T P E & C^T - (A+\Delta A)^T P B \\ C - B^T P (A+\Delta A) & -\left(D^T + D - B^T P B\right) \end{bmatrix}$$
$$+ \begin{bmatrix} (A + \Delta A)^T \\ -B^T \end{bmatrix} S Q^T + Q S^T \begin{bmatrix} (A + \Delta A)^T \\ -B^T \end{bmatrix}^T < 0 \quad (12)$$

for all the parameter uncertainty ΔA satisfying (3) and (4), where $S \in \mathbb{R}^{n \times (n-r)}$ is any matrix with full column and satisfies $E^T S = 0$.

Definition 4: The uncertain discrete-time descriptor system (Σ) is said to be strongly robustly admissibilizable with ESPR if

there exists a linear state feedback control law $u(k) = Kx(k)$, $K \in \mathbb{R}^{m \times n}$, such that the closed-loop system is strongly robustly admissible with ESPR in the sense of Definition 3.

In the following, attention will be focused on the development of conditions for strong robust admissibility with ESPR and strong robust admissibilizability with ESPR, respectively. To this end, we need the following result.

Lemma 4: [9] Given matrices Ω, Γ and Ξ of appropriate dimensions and with Ω symmetric, then

$$\Omega + \Gamma F(\sigma) \Xi + (\Gamma F(\sigma) \Xi)^T < 0$$

for all $F(\sigma)$ satisfying $F(\sigma) F(\sigma)^T \leq I$, if and only if there exists a scalar $\epsilon > 0$ such that

$$\Omega + \epsilon \Gamma \Gamma^T + \epsilon^{-1} \Xi^T \Xi < 0.$$

Now, we are in a position to present a necessary and sufficient condition for strong robust admissibility with ESPR.

Theorem 2: The uncertain discrete-time descriptor system (Σ) with $u(k) = 0$ is strongly robustly admissible with ESPR if and only if there exist a scalar $\epsilon > 0$, matrices $P > 0$ and Q such that the following LMI holds:

$$\begin{bmatrix} \Pi_1 + \tilde{\Pi}_1 & \Pi_2 & \Pi_3 M \\ \Pi_2^T & -P & PM \\ M^T \Pi_3^T & M^T P & -\epsilon I \end{bmatrix} < 0 \quad (13)$$

where Π_1, Π_2, and Π_3 are given in (14)–(16), respectively, shown at the bottom of the page, and the matrix $S \in \mathbb{R}^{n \times (n-r)}$ is given in Definition 3.

Proof: By Definition 3, it is easy to see that the unforced discrete-time descriptor system of (Σ) is strongly robustly admissible with ESPR if and only if there exist matrices $P > 0$ and Q such that (12) holds. Applying the Schur complement formula to (12), it can be seen that (12) is equivalent to (17) shown at the bottom of the page. Observe that (17) can be rewritten as

$$\begin{bmatrix} \Pi_1 & \Pi_2 \\ \hline \Pi_2^T & -P \end{bmatrix} + \begin{bmatrix} [\Pi_3 \Delta A \quad 0] & 0 \\ \hline [P \Delta A \quad 0] & 0 \end{bmatrix}$$
$$+ \begin{bmatrix} \begin{bmatrix} \Delta A^T \Pi_3^T \\ 0 \end{bmatrix} & \begin{bmatrix} \Delta A^T P \\ 0 \end{bmatrix} \\ \hline 0 & 0 \end{bmatrix} < 0.$$

$$\Pi_1 = \begin{bmatrix} -E^T P E & C^T - A^T P B \\ C - B^T P A & -\left(D^T + D - B^T P B\right) \end{bmatrix} + \begin{bmatrix} A^T \\ -B^T \end{bmatrix} S Q^T + Q S^T \begin{bmatrix} A^T \\ -B^T \end{bmatrix}^T \quad (14)$$

$$\tilde{\Pi}_1 = \begin{bmatrix} \epsilon N_1^T N_1 & 0 \\ 0 & 0 \end{bmatrix} \quad \Pi_2 = \begin{bmatrix} A^T P \\ 0 \end{bmatrix} \quad (15)$$

$$\Pi_3 = \begin{bmatrix} 0 \\ -B^T P \end{bmatrix} + Q S^T \quad (16)$$

$$\begin{bmatrix} \Pi_1 + \begin{bmatrix} 0 & -\Delta A^T P B \\ -B^T P \Delta A & 0 \end{bmatrix} + \begin{bmatrix} \Delta A^T S Q^T \\ 0 \end{bmatrix} + \begin{bmatrix} \Delta A^T S Q^T \\ 0 \end{bmatrix}^T & \begin{bmatrix} (A+\Delta A)^T P \\ 0 \end{bmatrix} \\ \hline [P(A+\Delta A) \quad 0] & -P \end{bmatrix} < 0. \quad (17)$$

That is

$$\begin{bmatrix} \Pi_1 & \Pi_2 \\ \Pi_2^T & -P \end{bmatrix} + \begin{bmatrix} \Pi_3 M \\ PM \end{bmatrix} F(\sigma) \begin{bmatrix} \tilde{N} & 0 \end{bmatrix}$$
$$+ \left(\begin{bmatrix} \Pi_3 M \\ PM \end{bmatrix} F(\sigma) \begin{bmatrix} \tilde{N} & 0 \end{bmatrix} \right)^T < 0 \quad (18)$$

where

$$\tilde{N} = \begin{bmatrix} N_1 & 0 \end{bmatrix}.$$

By Lemma 4, it can be shown that (18) holds for all $F(\sigma)$ satisfying (4) if and only if there exists a scalar $\epsilon > 0$ such that

$$\begin{bmatrix} \Pi_1 & \Pi_2 \\ \Pi_2^T & -P \end{bmatrix} + \epsilon^{-1} \begin{bmatrix} \Pi_3 M \\ PM \end{bmatrix} \begin{bmatrix} \Pi_3 M \\ PM \end{bmatrix}^T + \epsilon \begin{bmatrix} \tilde{N}^T \\ 0 \end{bmatrix} \begin{bmatrix} \tilde{N}^T \\ 0 \end{bmatrix}^T < 0$$
$$(19)$$

which, by the Schur complement formula, results in the LMI in (13). This completes the proof. □

The following theorem gives the result on the strong robust admissibilizability with ESPR.

Theorem 3: The uncertain discrete-time descriptor system (Σ) is strongly robustly admissibilizable with ESPR if and only if there exist scalars $\epsilon > 0$, $\delta > 0$, matrices $P > 0$, Q_1 and Q_2 such that

$$\mathcal{Z} = P^{-1} - \epsilon^{-1} M M^T > 0 \quad (20)$$
$$\mathcal{X} = D^T + D - B^T P B + Q_2 S^T B + B^T S Q_2^T - \epsilon^{-1} \left(Q_2 S^T - B^T P \right) M \left(I + \epsilon^{-1} M^T \mathcal{Z}^{-1} M \right) M^T \left(S Q_2^T - P B \right) > 0$$
$$(21)$$

and

$$\mathcal{Y}_1 + \mathcal{Y}_2 \mathcal{X}^{-1} \mathcal{Y}_2^T - \left(\Psi_1^T + \mathcal{Y}_2 \mathcal{X}^{-1} \Psi_2^T \right) \Xi^{-1} \left(\Psi_1 + \Psi_2 \mathcal{X}^{-1} \mathcal{Y}_2^T \right) < 0$$
$$(22)$$

where the matrix $S \in \mathbb{R}^{n \times (n-r)}$ is given in Definition 3, and

$$\mathcal{Y}_1 = \epsilon N_1^T N_1 + Q_1 S^T A + A^T S Q_1^T - E^T P E$$
$$+ \epsilon^{-1} Q_1 S^T M M^T S Q_1^T$$
$$+ \left(A^T + \epsilon^{-1} Q_1 S^T M M^T \right) \mathcal{Z}^{-1} \left(A + \epsilon^{-1} M M^T S Q_1^T \right)$$
$$\mathcal{Y}_2 = C^T - Q_1 S^T B + \left(A^T + \epsilon^{-1} Q_1 S^T M M^T \right)$$
$$\times \left(I + \epsilon^{-1} \mathcal{Z}^{-1} M M^T \right) \left(S Q_2^T - P B \right)$$
$$\Psi_1 = \epsilon N_2^T N_1 + B_1^T \mathcal{Z}^{-1} A + B_1^T \left(I + \epsilon^{-1} \mathcal{Z}^{-1} M M^T \right) S Q_1^T$$
$$\Psi_2 = B_1^T \left(I + \epsilon^{-1} \mathcal{Z}^{-1} M M^T \right) \left(S Q_2^T - P B \right)$$
$$\Xi = \epsilon N_2^T N_2 + B_1^T \mathcal{Z}^{-1} B_1 + \Psi_2 \mathcal{X}^{-1} \Psi_2^T + \delta I.$$

In this case, a state feedback control law chosen by

$$u(k) = -\Xi^{-1} \left(\Psi_1 + \Psi_2 \mathcal{X}^{-1} \mathcal{Y}_2^T \right) x(k) \quad (23)$$

will be such that the closed-loop system is strongly robustly admissible with ESPR.

Proof: By Definition 4 and Theorem 2, we have that the uncertain discrete-time descriptor system (Σ) is strongly robustly admissibilizable with ESPR if and only if there exist a scalar $\epsilon > 0$, matrices $P > 0$, Q and $K \in \mathbb{R}^{m \times n}$ such that

$$\begin{bmatrix} \hat{\Pi}_1 + \tilde{\Pi}_1 & \hat{\Pi}_2 & \Pi_3 M \\ \hat{\Pi}_2^T & -P & PM \\ M^T \hat{\Pi}_3^T & M^T P & -\epsilon I \end{bmatrix} < 0 \quad (24)$$

where Π_3 is given in (16), and $\Pi_1 \hat{\Pi}_1$, A_c, and N_c are given in (25)–(27) at the bottom of the page, respectively. Write

$$Q = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}$$

where the partition is compatible with the related matrices. Then, (24) can be rewritten as the last equation at the bottom of the page, which, by the Schur complement formula again,

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$$\Pi_1 = \begin{bmatrix} -E^T P E & C^T - A_c^T P B \\ C - B^T P A_c & -(D^T + D - B^T P B) \end{bmatrix} + \begin{bmatrix} A_c^T \\ -B^T \end{bmatrix} S Q^T + Q S^T \begin{bmatrix} A_c^T \\ -B^T \end{bmatrix}^T \quad (25)$$

$$\tilde{\Pi}_1 = \begin{bmatrix} \epsilon N_c^T N_c & 0 \\ 0 & 0 \end{bmatrix}, \quad \hat{\Pi}_2 = \begin{bmatrix} A_c^T P \\ 0 \end{bmatrix} \quad (26)$$

$$A_c = A + B_1 K \quad N_c = N_1 + N_2 K \quad (27)$$

$$\begin{bmatrix} \epsilon N_c^T N_c + Q_1 S^T A_c + A_c^T S Q_1^T - E^T P E & C^T - A_c^T \left(P B - S Q_2^T \right) - Q_1 S^T B & A_c^T P & Q_1 S^T M \\ C - \left(B^T P - Q_2 S^T \right) A_c - B^T S Q_1^T & -\left(D^T + D - B^T P B + Q_2 S^T B + B^T S Q_2^T \right) & 0 & \left(Q_2 S^T - B^T P \right) M \\ P A_c & 0 & -P & P M \\ M^T S Q_1^T & M^T \left(S Q_2^T - P B \right) & M^T P & -\epsilon I \end{bmatrix} < 0$$

is equivalent to $\mathcal{Z} > 0$, $\mathcal{X} > 0$, and equation (28) at the bottom of the page holds. This, by the Schur complement formula, can be shown to be equivalent to

$$\mathcal{Y}_1 + \mathcal{Y}_2 \mathcal{X}^{-1} \mathcal{Y}_2^T + \left(\Psi_1^T + \mathcal{Y}_2 \mathcal{X}^{-1} \Psi_2^T\right) K + K^T \left(\Psi_1 + \Psi_2 \mathcal{X}^{-1} \mathcal{Y}_2^T\right)$$
$$+ K^T \left(\epsilon N_2^T N_2 + B_1^T \mathcal{Z}^{-1} B_1 + \Psi_2 \mathcal{X}^{-1} \Psi_2^T\right) K < 0. \quad (29)$$

It is easy to see that (29) is satisfied if and only if there exists a scalar $\delta > 0$ such that $\Xi > 0$ and

$$\mathcal{Y}_1 + \mathcal{Y}_2 \mathcal{X}^{-1} \mathcal{Y}_2^T + \left(\Psi_1^T + \mathcal{Y}_2 \mathcal{X}^{-1} \Psi_2^T\right) K$$
$$+ K^T \left(\Psi_1 + \Psi_2 \mathcal{X}^{-1} \mathcal{Y}_2^T\right) + K^T \Xi K < 0.$$

This can be rewritten as

$$\mathcal{Y}_1 + \mathcal{Y}_2 \mathcal{X}^{-1} \mathcal{Y}_2^T - \left(\Psi_1^T + \mathcal{Y}_2 \mathcal{X}^{-1} \Psi_2^T\right) \Xi^{-1} \left(\Psi_1 + \Psi_2 \mathcal{X}^{-1} \mathcal{Y}_2^T\right)$$
$$+ \left[K^T + \left(\Psi_1^T + \mathcal{Y}_2 \mathcal{X}^{-1} \Psi_2^T\right) \Xi^{-1}\right]$$
$$\times \Xi \left[K + \Xi^{-1} \left(\Psi_1 + \Psi_2 \mathcal{X}^{-1} \mathcal{Y}_2^T\right)\right] < 0. \quad (30)$$

Finally, it is easy to see that there exists a matrix K such that (30) holds if and only if (22) holds, and in this case, a suitable K can be chosen as in (23). This completes the proof. $\quad\square$

In the case when there is no parameter uncertainties in the uncertain discrete-time descriptor system (Σ), this system will reduce to

$$(\bar{\Sigma}): \qquad Ex(k+1) = Ax(k) + B_1 u(k) + B\omega(k)$$
$$z(k) = Cx(k) + D\omega(k)$$

then, by Theorem 3, we have the following result.

Corollary 1: Consider the discrete-time descriptor system $(\bar{\Sigma})$. Then, there exists a state feedback controller such that the closed-loop system is admissible and ESPR if and only if there exist a scalar $\delta > 0$, matrices $P > 0$ and Q such that

$$\mathcal{V} = D^T + D - B^T PB + Q_2 S^T B + B^T S Q_2 > 0 \quad (31)$$

and

$$\mathcal{W} - \mathcal{U}^T \mathcal{T}^{-1} \mathcal{U} < 0 \quad (32)$$

where the matrix $S \in \mathbb{R}^{n \times (n-r)}$ is given in Definition 3, and

$$\mathcal{W} = Q_1 S^T A + A^T S Q_1^T - E^T PE + A^T PA$$
$$+ \left[C^T - Q_1 S^T B + A^T \left(S Q_2^T - PB\right)\right]$$
$$\times \mathcal{V}^{-1} \left[C - B^T S Q_1^T + \left(Q_2 S^T - B^T P\right) A\right]$$
$$\mathcal{U} = B_1^T S Q_1^T + B_1^T PA + B_1^T \left(S Q_2^T - PB\right)$$
$$\times \mathcal{V}^{-1} \left[C - B^T S Q_1^T + \left(Q_2 S^T - B^T P\right) A\right]$$
$$\mathcal{T} = B_1^T \left(P + \mathcal{V}^{-1}\right) B_1 + \delta I.$$

In this case, a suitable state feedback control law can be chosen by

$$u(k) = -\mathcal{T}^{-1} \mathcal{U} x(k).$$

Remark 4: Corollary 1 provides a necessary and sufficient condition for the existence of state feedback controllers ensuring admissibility and ESPR of the closed-loop system. It is noted that the design of a desired state feedback controller involves no decomposition of the system matrices, which is due to the use of Theorem 1 where the positive definite matrix P is introduced. Usually, in the synthesis of discrete-time descriptor systems, decompositions of system matrices will be involved because of the indefiniteness of the Lyapunov-type matrix [18], [20]; this will make the design procedure indirect and relatively complicated. It is also worth pointing out that in the context of discrete-time descriptor systems, the indefiniteness of the Lyapunov-type matrix may make it difficult to obtain necessary and sufficient conditions for the existence of state feedback controllers guaranteeing some performance of the closed-loop system [18], [20]. Considering the above, the results in Corollary 1 is elegant from a mathematical point of view.

IV. NUMERICAL EXAMPLE

In this section, we give an example to demonstrate the effectiveness of the proposed method.

Consider an uncertain discrete-time descriptor system in (1) and (2) with parameters as follows:

$$E = \begin{bmatrix} 1 & 3 & 0.5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 5.04 & 10.2 & 2.1 \\ 2 & -1.5 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -1 & -0.3 & 1 \\ 0 & 1 & 0 \\ 0.6 & -0.1 & 0.8 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.5 & 1 \\ 1 & 0 \\ 1 & -1 \end{bmatrix}$$

$$M = \begin{bmatrix} 0.1 \\ 0.1 \\ -0.2 \end{bmatrix}$$

$$C = \begin{bmatrix} -1.56 & 0.8 & -3.3 \\ 2.3 & 2.5 & -2 \end{bmatrix}$$

$$D = \begin{bmatrix} 5 & -1 \\ 0.3 & 8.5 \end{bmatrix}$$

$$N_1 = \begin{bmatrix} 0.38 & 0.6 & 0.4 \end{bmatrix}$$

$$N_2 = \begin{bmatrix} -0.1 & 0.1 & 0 \end{bmatrix}.$$

$$\begin{bmatrix} \mathcal{Y}_1 + \Psi_1^T K + K^T \Psi_1 + K^T \left(\epsilon N_2^T N_2 + B_1^T \mathcal{Z}^{-1} B_1\right) K & \mathcal{Y}_2 + K^T \Psi_2 \\ \mathcal{Y}_2^T + \Psi_2^T K & -\mathcal{X} \end{bmatrix} < 0 \quad (28)$$

It is supposed that the uncertain matrix is given as $F(\sigma) = \sin(\sigma)$. Then, it can be verified that the nominal discrete descriptor system is neither admissible nor ESPR. Now, to solve the PR control problem, we choose

$$S = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Then, it can be checked that

$$P = \begin{bmatrix} 3 & 0.5 & 1 \\ 0.5 & 2 & -0.8 \\ 1 & -0.8 & 5 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} -0.8 & 1.64 \\ 0.5 & 1.4 \\ -2.5 & -1.7 \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} 0.2 & 0 \\ 0.1 & -0.1 \end{bmatrix}$$

$$\delta = 0.0121$$

$$\epsilon = 1.1232$$

satisfy the matrix inequalities in (20)–(22). Therefore, by Theorem 3, we have that the PR control problem is solvable, and a desired state feedback control law can be chosen as

$$u(k) = \begin{bmatrix} 2.5947 & 5.1464 & 1.9953 \\ -1.8499 & 1.0858 & 0.3834 \\ -2.8844 & -4.3098 & -0.7825 \end{bmatrix} x(k).$$

V. Conclusion

The problems of PR analysis and PR control for discrete-time descriptor systems with parameter uncertainties in both the state and input matrices have been studied. In terms of a strict LMI, a new necessary and sufficient condition for a discrete-time descriptor system to be regular, causal, stable and ESPR have been proposed. The problems are solved via the notions of strong robust admissibility with ESPR and strong robust admissibilizability with ESPR. Without any additional assumptions on the system matrices, necessary and sufficient conditions for strong robust admissibility with ESPR and strong robust admissibilizability with ESPR have been obtained in terms of a strict LMI and matrix inequalities, respectively. An explicit construction procedure to obtain a desired state feedback control law has also been given, which involves no decomposition of the system matrices.

Appendix
Proof of Theorem 1

A. Sufficiency

Assume that the LMI in (11) is satisfied. We first show the admissibility of the system $(\hat{\Sigma})$. To this end, we write

$$Q = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}$$

where $Q_1 \in \mathbb{R}^{n \times (n-r)}$ and $Q_2 \in \mathbb{R}^{l \times (n-r)}$. Then, the LMI in (11) can be rewritten as (33) shown at the bottom of the page, which implies

$$A^T P A - E^T P E + A^T S Q_1^T + Q_1 S^T A < 0.$$

Therefore, by Lemma 1, we have that system $(\hat{\Sigma})$ is admissible. Next, we show that under the condition of the theorem, the discrete-time descriptor system $(\hat{\Sigma})$ is ESPR. By the Schur complement formula, it follows from (33) that

$$A^T P A - E^T P E + A^T S Q_1^T + Q_1 S^T A + \Omega < 0 \quad (34)$$

where

$$\Omega = \left(C^T - A^T P B + A^T S Q_2^T - Q_1 S^T B \right) \\ \times U^{-1} \left(C - B^T P A + Q_2 S^T A - B^T S Q_1^T \right) \quad (35)$$

$$U = D^T + D - B^T P B + B^T S Q_2^T + Q_2 S^T B > 0. \quad (36)$$

Then, it is easy to see that (34) implies that there exists a matrix $W > 0$ such that

$$A^T P A - E^T P E + A^T S Q_1^T + Q_1 S^T A + \Omega + W < 0. \quad (37)$$

Let

$$\Phi(j\theta) = e^{j\theta} E - A.$$

Then, recalling that the system $(\hat{\Sigma})$ is admissible, we have that $\Phi(j\theta)$ is nonsingular for all $\theta \in [0, 2\pi)$. Pre- and post-multiplying (37) by $B^T \Phi(-j\theta)^{-T}$ and $\Phi(j\theta)^{-1} B$, respectively, yield

$$B^T \Phi(-j\theta)^{-T} \left[A^T P A - E^T P E + A^T S Q_1^T + Q_1 S^T A \right] \\ \times \Phi(j\theta)^{-1} B + B^T \Phi(-j\theta)^{-T} (\Omega + W) \Phi(j\theta)^{-1} B \leq 0 \quad (38)$$

which implies

$$-B^T \Phi(-j\theta)^{-T} \left[A^T P A - E^T P E + A^T S Q_1^T + Q_1 S^T A \right] \\ \times \Phi(j\theta)^{-1} B \geq B^T \Phi(-j\theta)^{-T} (\Omega + W) \Phi(j\theta)^{-1} B. \quad (39)$$

$$\begin{bmatrix} A^T P A - E^T P E + A^T S Q_1^T + Q_1 S^T A & C^T - A^T P B + A^T S Q_2^T - Q_1 S^T B \\ C - B^T P A + Q_2 S^T A - B^T S Q_1^T & -\left(D^T + D - B^T P B + B^T S Q_2^T + Q_2 S^T B \right) \end{bmatrix} < 0 \quad (33)$$

Now, by some simple algebraic manipulations, it can be verified that

$$
\begin{aligned}
&B^T P B + B^T \Phi(-j\theta)^{-T} \left(A^T P + Q_1 S^T\right) B \\
&\quad + B^T \left(A^T P + Q_1 S^T\right)^T \Phi(j\theta)^{-1} B \\
&= -B^T \Phi(-j\theta)^{-T} \left(A^T P A - E^T P E + A^T S Q_1^T\right. \\
&\quad \left. + Q_1 S^T A\right) \Phi(j\theta)^{-1} B
\end{aligned}
\tag{40}
$$

where the relationship $E^T S = 0$ is used. From (38) and (40), it is easy to show that for all $\theta \in [0, 2\pi)$

$$
\begin{aligned}
&B^T P B + B^T \Phi(-j\theta)^{-T} \left(A^T P + Q_1 S^T\right) B \\
&\quad + B^T \left(A^T P + Q_1 S^T\right)^T \Phi(j\theta)^{-1} B \\
&\geq B^T \Phi(-j\theta)^{-T} (\Omega + W) \Phi(j\theta)^{-1} B.
\end{aligned}
\tag{41}
$$

Note that for all $\theta \in [0, 2\pi)$

$$
\begin{aligned}
&G(e^{j\theta}) + G(e^{j\theta})^* \\
&= D + D^T + C\Phi(j\theta)^{-1} B + \left(C\Phi(j\theta)^{-1} B\right)^* \\
&= U + B^T P B + \left(C - B^T P A + Q_2 S^T A - B^T S Q_1^T\right) \Phi(j\theta)^{-1} B \\
&\quad + B^T \Phi(-j\theta)^{-T} \left(C^T - A^T P B + A^T S Q_2^T - Q_1 S^T B\right) \\
&\quad + \left[\left(B^T P A - Q_2 S^T A + B^T S Q_1^T\right) \Phi(j\theta)^{-1} - Q_2 S^T\right] B \\
&\quad + B^T \left[\Phi(-j\theta)^{-T} \left(A^T P B - A^T S Q_2^T + Q_1 S^T B\right) - S Q_2^T\right] \\
&= U + \left(C - B^T P A + Q_2 S^T A - B^T S Q_1^T\right) \Phi(j\theta)^{-1} B \\
&\quad + B^T \Phi(-j\theta)^{-T} \left(C^T - A^T P B + A^T S Q_2^T - Q_1 S^T B\right) \\
&\quad + B^T P B + B^T \Phi(-j\theta)^{-T} \left(A^T P + Q_1 S^T\right) B + B^T \left(A^T P\right. \\
&\quad \left. + Q_1 S^T\right)^T \Phi(j\theta)^{-1} B
\end{aligned}
\tag{42}
$$

where U is given in (35). Then, by (41) and (42), we have that for all $\theta \in [0, 2\pi)$

$$
\begin{aligned}
&G(e^{j\theta}) + G(e^{j\theta})^* \\
&\geq U + \left(C - B^T P A + Q_2 S^T A - B^T S Q_1^T\right) \Phi(j\theta)^{-1} B \\
&\quad + B^T \Phi(-j\theta)^{-T} \left(C^T - A^T P B + A^T S Q_2^T - Q_1 S^T B\right) \\
&\quad + B^T \Phi(-j\theta)^{-T} (\Omega + W) \Phi(j\theta)^{-1} B.
\end{aligned}
$$

Noting $(\Omega + W) > 0$ and using Lemma 2, we obtain

$$
\begin{aligned}
&G(e^{j\theta}) + G(e^{j\theta})^* \geq U - \left(C - B^T P A + Q_2 S^T A - B^T S Q_1^T\right) \\
&\quad \times (\Omega + W)^{-1} \left(C^T - A^T P B + A^T S Q_2^T - Q_1 S^T B\right).
\end{aligned}
\tag{43}
$$

Observe that

$$
\begin{aligned}
&\Omega + W - \left(C^T - A^T P B + A^T S Q_2^T - Q_1 S^T B\right) U^{-1} \\
&\quad \times \left(C - B^T P A + Q_2 S^T A - B^T S Q_1^T\right) = W > 0
\end{aligned}
$$

which, by the Schur complement formula, implies (44) shown at the bottom of the page. By the Schur complement formula again, it follows from (44) that

$$
\begin{aligned}
&U - \left(C - B^T P A + Q_2 S^T A - B^T S Q_1^T\right) (\Omega + W)^{-1} \\
&\quad \times \left(C^T - A^T P B + A^T S Q_2^T - Q_1 S^T B\right) > 0.
\end{aligned}
\tag{45}
$$

Therefore, from (43) and (45), we have that for all $\theta \in [0, 2\pi)$

$$
G(e^{j\theta}) + G^*(e^{j\theta}) > 0.
\tag{46}
$$

On the other hand, since $(\hat{\Sigma})$ is admissible, we can find two nonsingular matrices $\tilde{M}_1$ and $\tilde{N}_1$ such that [2]

$$
\tilde{M}_1 E \tilde{N}_1 = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \quad \tilde{M}_1 A \tilde{N}_1 = \begin{bmatrix} \tilde{A}_1 & 0 \\ 0 & I \end{bmatrix}.
\tag{47}
$$

In this case, the matrix S satisfying $E^T S = 0$ can be chosen as

$$
S = \tilde{M}_1^T \begin{bmatrix} 0 \\ I \end{bmatrix} \tilde{H}_1
$$

where $\tilde{H}_1$ is any nonsingular matrix. Now, write

$$
\begin{aligned}
C\tilde{N}_1 &= [\tilde{C}_1 \quad \tilde{C}_2] \\
\tilde{M}_1 B &= \begin{bmatrix} \tilde{B}_1 \\ \tilde{B}_2 \end{bmatrix} \\
\tilde{M}_1^{-T} P \tilde{M}_1^{-1} &= \begin{bmatrix} \tilde{P}_1 & \tilde{P}_2 \\ \tilde{P}_2^T & \tilde{P}_3 \end{bmatrix} \\
\tilde{N}_1^T Q_1 \tilde{H}_1^T &= \begin{bmatrix} \tilde{Q}_{11} \\ \tilde{Q}_{21} \end{bmatrix} \\
Q_2 \tilde{H}_1^T &= \tilde{Q}_2
\end{aligned}
$$

where the partition is compatible with that of $\tilde{M}_1 E \tilde{N}_1$ and $\tilde{M}_1 A \tilde{N}_1$ in (47). Then, it is easy to see that

$$
\tilde{P}_1 > 0.
\tag{48}
$$

Pre- and post-multiplying (33) by $\mathrm{diag}\left(\tilde{N}_1^T, I\right)$ and $\mathrm{diag}\left(\tilde{N}_1, I\right)$, respectively and then using the above notations, we have (49) shown at the bottom of the page. Where

$$
\begin{bmatrix}
* & C - B^T P A + Q_2 S^T A - B^T S Q_1^T \\
C^T - A^T P B + A^T S Q_2^T - Q_1 S^T B & \Omega + W
\end{bmatrix} > 0
\tag{44}
$$

$$
\begin{bmatrix}
* & * & * \\
* & \tilde{P}_3 + \tilde{Q}_{21} + \tilde{Q}_{21}^T & \tilde{C}_2^T - \tilde{P}_2^T \tilde{B}_1 - \tilde{P}_3 \tilde{B}_2 + \tilde{Q}_2^T - \tilde{Q}_{21} \tilde{B}_2 \\
* & \tilde{C}_2 - \tilde{B}_1^T \tilde{P}_2 - \tilde{B}_2^T \tilde{P}_3 + \tilde{Q}_2 - \tilde{B}_2^T \tilde{Q}_{21}^T & \mathcal{V}
\end{bmatrix} < 0
\tag{49}
$$

$*$ represents matrices that will not be used in the following discussion, and

$$\mathcal{V} = -D^T - D + \tilde{B}_1^T \tilde{P}_1 \tilde{B}_1 + \tilde{B}_2^T \tilde{P}_2^T \tilde{B}_1$$
$$+ \tilde{B}_1^T \tilde{P}_2 \tilde{B}_2 + \tilde{B}_2^T \tilde{P}_3 \tilde{B}_2 - \tilde{B}_2^T \tilde{Q}_2^T - \tilde{Q}_2 \tilde{B}_2.$$

Pre- and post-multiplying (49) by

$$\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & \tilde{B}_2^T & I \end{bmatrix}$$

and its transpose, respectively, and then noting the 3–3 block, we obtain

$$-D^T - D + \tilde{C}_2 \tilde{B}_2 + \tilde{B}_2^T \tilde{C}_2^T + \tilde{B}_1^T \tilde{P}_1 \tilde{B}_1 < 0.$$

This together with (48) gives

$$D - \tilde{C}_2 \tilde{B}_2 + D^T + \tilde{B}_2^T \tilde{C}_2^T > 0$$

which together with

$$G(z) = C(zE - A)^{-1}B = \tilde{C}_1(zI - \tilde{A}_1)^{-1}\tilde{B}_1 - \tilde{C}_2\tilde{B}_2 + D$$

implies

$$G(\infty) + G(\infty)^T > 0.$$

Considering this and (46), it easy to see that the discrete-time descriptor system $(\hat{\Sigma})$ is ESPR.

B. Necessity

Suppose that the discrete-time descriptor system $(\hat{\Sigma})$ is admissible and ESPR. Then, there exist two nonsingular matrices $\mathcal{M}$ and $\mathcal{N}$ such that [2]

$$E = \mathcal{M} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \mathcal{N} \quad A = \mathcal{M} \begin{bmatrix} \mathcal{A} & 0 \\ 0 & I \end{bmatrix} \mathcal{N}. \tag{50}$$

Then, it is easy to see that the matrix S satisfying $E^T S = 0$ can be written as

$$S = \mathcal{M}^{-T} \begin{bmatrix} 0 \\ I \end{bmatrix} \mathcal{H} \tag{51}$$

where $\mathcal{H}$ is any nonsingular matrix. Write

$$B = \mathcal{M} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \quad C = [C_1 \quad C_2] \mathcal{N}$$

where the partition is compatible with that of A. Note that

$$G(z) = C_1 (zI - \mathcal{A})^{-1} B_1 + D - C_2 B_2.$$

Considering the system $(\hat{\Sigma})$ is ESPR, by Lemma 3, it follows that there exists a matrix $\tilde{P} > 0$ such that

$$\begin{bmatrix} \mathcal{A}^T \tilde{P} \mathcal{A} - \tilde{P} & C_1^T - \mathcal{A}^T \tilde{P} B_1 \\ C_1 - B_1^T \tilde{P} \mathcal{A} & -\left(D^T + D - C_2 B_2 - B_2^T C_2^T - B_1^T \tilde{P} B_1\right) \end{bmatrix} < 0. \tag{52}$$

Let

$$Q_1 = 0 \quad Q_2 = -I \quad Q_3 = -C_2 + B_2^T.$$

Then, by (52), it is easy to see that we obtain the first equation of the bottom of the page. Pre- and post-multiplying this inequality by

$$\begin{bmatrix} I & 0 & 0 \\ 0 & 0 & I \\ 0 & I & -B_2^T \end{bmatrix}$$

and its transpose, respectively, we obtain (53) shown at the bottom of the page. Set

$$\mathcal{P} = \mathcal{M}^{-T} \begin{bmatrix} \tilde{P} & 0 \\ 0 & I \end{bmatrix} \mathcal{M} \quad \mathcal{Q} = \begin{bmatrix} \mathcal{N} & 0 \\ \hline 0 & I \end{bmatrix}^T \begin{bmatrix} Q_1 \\ Q_2 \\ \hline Q_3 \end{bmatrix} \mathcal{H}^{-T}. \tag{54}$$

Then, pre- and post-multiplying (53) by

$$\begin{bmatrix} \mathcal{N} & 0 \\ \hline 0 & I \end{bmatrix}^T$$

and its transpose, we can write

$$\begin{bmatrix} A^T \mathcal{P} A - E^T \mathcal{P} E & C^T - A^T \mathcal{P} B \\ C - B^T \mathcal{P} A & -\left(D^T + D - B^T \mathcal{P} B\right) \end{bmatrix}$$
$$+ \begin{bmatrix} A^T \\ -B^T \end{bmatrix} S \mathcal{Q}^T + \mathcal{Q} S^T \begin{bmatrix} A^T \\ -B^T \end{bmatrix}^T < 0. \tag{55}$$

That is, the matrices $\mathcal{P}$ and $\mathcal{Q}$ given in (54) satisfy (55). This completes the proof. $\qquad\square$

$$\begin{bmatrix} \mathcal{A}^T \tilde{P} \mathcal{A} - \tilde{P} & C_1^T - \mathcal{A}^T \tilde{P} B_1 & Q_1 \\ C_1 - B_1^T \tilde{P} \mathcal{A} & -\left(D^T + D - C_2 B_2 - B_2^T C_2^T - B_1^T \tilde{P} B_1\right) & C_2 + Q_3 + B_2^T Q_2 \\ Q_1^T & C_2^T + Q_3^T + Q_2^T B_2 & I + Q_2 + Q_2^T \end{bmatrix} < 0$$

$$\left[\begin{array}{cc|c} \mathcal{A}^T \tilde{P} \mathcal{A} - \tilde{P} & Q_1 & C_1^T - \mathcal{A}^T \tilde{P} B_1 - Q_1 B_2 \\ Q_1^T & I + Q_2 + Q_2^T & C_2^T - B_2 + Q_3^T - Q_2 B_2 \\ \hline C_1 - B_1^T \tilde{P} \mathcal{A} - B_2^T Q_1^T & C_2 - B_2^T + Q_3 - B_2^T Q_2^T & -D^T - D + B_1^T \tilde{P} B_1 + B_2^T B_2 - B_2^T Q_3^T - Q_3 B_2 \end{array}\right] < 0 \tag{53}$$

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