On the Multiple-Access Capacity of Multitone-CDMA Communications

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Abstract—This letter derives a bit-error-probability expression for an asynchronous multitone-code-division multiple access (CDMA) system and shows that the multiple-access (MA) capacity of this system is identical to that of a conventional (single-carrier) direct-sequence spread-spectrum system under conditions of equal bandwidth and equal bit rate. It is also shown that using more subcarriers to increase the processing gain does not increase the MA capacity.

Index Terms—Multiple-access capacity, multitone-CDMA, OFDM.

I. INTRODUCTION

MULTITONE code-division multiple access (MT-CDMA) technique [1] combines both orthogonal frequency division multiplexing (OFDM) and direct-sequence spread-spectrum (DSSS) technique in such a way that, by increasing the number of subcarriers used, the processing gain of a MT-CDMA system can be made substantially greater than that of a conventional (single-carrier) DSSS system under the same-bandwidth condition. It has been expected that this greater processing gain could lead to a greater reduction in the multiple-access interference (MAI) so that a MT-CDMA system could support a larger number of users than a conventional DSSS system [2]. However, based on the bit error probability derived in Section II, we show in Section III that, under equal-bandwidth and equal-bit-rate conditions, the multiple-access (MA) capacity of an asynchronous MT-CDMA system is identical to that of a conventional DSSS system and does not increase by an increase of the number of subcarriers used. Despite the MA capacity is not increased, the greater processing gain of a MT-CDMA signal does have other advantages such as more effective suppression of the narrowband interference. The result of this letter thus indicates that, for an *effective* utilization of the greater processing gain, system designers should focus on these advantages rather than on increasing the MA capacity.

II. SYSTEM MODEL AND ERROR PERFORMANCE

Consider a MT-CDMA system with K users transmitting their signals at equal power and equal bit rate. At the transmitter, the incoming bit stream having a rate of M/T_b bits per second is first serial-to-parallel converted into M lower-rate bit substreams each of which has a bit duration of T_b seconds. Assume that rectangular symbol waveform and binary phase shift keying are used. The M substreams are, respectively, modulated onto M orthogonal subcarriers that are equally spaced by f_Δ Hz. Orthogonality among subcarriers is maintained by setting $f_\Delta = 1/T_b$. The data-modulated subcarriers are multiplexed and the resultant multiplexed signal is spread by a spreading sequence of length N. The chip duration, T_c, is given by $T_c = T_b/N$. The spread signal is then transmitted over the channel. Note that in the special case of $M = 1$, the MT-CDMA system is essentially a conventional DSSS system.

The received signal is the sum of K transmitted signals corrupted by additive white Gaussian noise (AWGN) having a two-sided power spectral density $N_0/2$. It is assumed that the receiver is intended to detect the Lth-user signal. The complex envelope of the received signal is given by

$$r(t) = n(t) + \sum_{k=1}^{K} \sqrt{2P_b} e^{j\theta_k} \sum_{i=-\infty}^{\infty} \sum_{m=0}^{M-1} b_{i,m}^{(k)} e^{j2\pi m f_\Delta(t-\tau_k)}$$
$$\cdot u_k(t - iT_b - \tau_k) \qquad (1)$$

where $n(t)$ is the baseband-equivalent AWGN; P_b is the signal power per bit (so that MP_b is the transmitted signal power); θ_k is the random phase introduced by the kth-user transmitter; $b_{i,m}^{(k)} \in \{+1, -1\}$ is the ith bit transmitted on the mth subcarrier of the kth-user signal; τ_k is the delay of the kth-user signal relative to the Lth-user signal; and

$$u_k(t) = \sum_{n=0}^{N-1} a_n^{(k)} \psi(t - nT_c) \qquad (2)$$

is the kth-user spectral spreading waveform. In (2), $(a_0^{(k)}, a_1^{(k)}, \cdots, a_{N-1}^{(k)})$ is the kth user spreading sequence satisfying $|a_n^{(k)}| = 1$, and $\psi(t)$ is the chip waveform given by $\psi(t) = 1$ for $t \in [0, T_c)$ and $\psi(t) = 0$ otherwise. Asynchronous signal transmission is considered so that τ_L is given by $\tau_L = 0$ and τ_k's, $k \neq L$, are modeled as mutually independent random variables uniformly distributed over $[0, T_b)$. The received signal is processed by a bank of M matched filters each of which detects the bits transmitted on a particular subcarrier.

Assume that coherent detection is used and $b_{\tilde{i},\tilde{m}}^{(L)}$ is to be detected. The desired matched-filter output, $Z_{\tilde{i},\tilde{m}}$, is given by

$$Z_{\tilde{i},\tilde{m}} = \frac{1}{\sqrt{2P_b T_b}} \int_{\tilde{i}T_b}^{(\tilde{i}+1)T_b} r(t) u_L^*(t-\tilde{i}T_b) e^{-j2\pi\tilde{m}f_\Delta t} e^{-j\theta_L} \, dt. \tag{3}$$

The decision variable for $b_{\tilde{i},\tilde{m}}^{(L)}$ is the sign of the real part of $Z_{\tilde{i},\tilde{m}}$. Although we use M matched filters to process $r(t)$ for theoretical modeling, in practical implementations efficient DFT algorithms are often employed [3]. Let $E_b = P_b T_b$ denote the bit energy. Substituting (1) into (3) gives

$$Z_{\tilde{i},\tilde{m}} = \eta + b_{\tilde{i},\tilde{m}}^{(L)} + \sum_{k=1,\,k\neq L}^{K} J_k \tag{4}$$

where η is the zero-mean complex Gaussian noise with a variance $E\{\eta\eta^*\} = (E_b/N_0)^{-1}$, and

$$J_k = \frac{1}{N} e^{j(\theta_k-\theta_L)} \sum_{m=0}^{M-1} e^{-j2\pi m f_\Delta \tau_k} e^{j2\pi(m-\tilde{m})f_\Delta \tilde{i}T_b}$$
$$\cdot \left[b_{\tilde{i}-1,m}^{(k)} S_{k,L}((m-\tilde{m})f_\Delta) + b_{\tilde{i},m}^{(k)} \hat{S}_{k,L}((m-\tilde{m})f_\Delta) \right] \tag{5}$$

is the MAI due to the kth user. In (5),

$$S_{k,L}(\mu) = T_c^{-1} \int_0^{\tau_k} e^{j2\pi\mu t} u_k(t+T_b-\tau_k) u_L^*(t) \, dt \tag{6}$$

and

$$\hat{S}_{k,L}(\mu) = T_c^{-1} \int_{\tau_k}^{T_b} e^{j2\pi\mu t} u_k(t-\tau_k) u_L^*(t) \, dt. \tag{7}$$

Let

$$R(\tau;\mu) = T_c^{-1} \int_0^\tau e^{j2\pi\mu t} \, dt, \tag{8}$$

$$\hat{R}(\tau;\mu) = T_c^{-1} \int_\tau^{T_c} e^{j2\pi\mu t} \, dt \tag{9}$$

and

$$C_{k,L}(q;\xi)$$
$$= \begin{cases} \displaystyle\sum_{i=0}^{N-1-q} a_i^{(k)} a_{i+q}^{(L)*} e^{j2\pi(i+q)\xi}, & 0 \le q \le N-1 \\ \displaystyle\sum_{i=-q}^{N-1} a_i^{(k)} a_{i+q}^{(L)*} e^{j2\pi(i+q)\xi}, & -(N-1) \le q < 0 \\ 0, & |q| \ge N. \end{cases} \tag{10}$$

Let $w_k \in \{0, 1, \cdots, N-1\}$ and $\hat{\tau}_k \in [0, T_c)$ be the quotient and remainder, respectively, when τ_k is divided by T_c. Note that $\tau_k = w_k T_c + \hat{\tau}_k$. Substituting (2) into (6) and (7) followed by some algebraic manipulations, we get

$$S_{k,L}(\mu) = R(\hat{\tau}_k;\mu) \cdot C_{k,L}(-(N-w_k-1);\mu T_c)$$
$$+ \hat{R}(\hat{\tau}_k;\mu) \cdot C_{k,L}(-(N-w_k);\mu T_c) \tag{11}$$

$$\hat{S}_{k,L}(\mu) = R(\hat{\tau}_k;\mu) \cdot C_{k,L}(w_k+1;\mu T_c)$$
$$+ \hat{R}(\hat{\tau}_k;\mu) \cdot C_{k,L}(w_k;\mu T_c). \tag{12}$$

We apply the Gaussian approximation on the MAI in the computation of the bit error probability, P_e. This approximation is justified for a moderate value of K. It follows from (4) that

$$P_e = Q\left(\left\{ \left(\frac{2E_b}{N_0} \right)^{-1} + \frac{1}{2} \sum_{k=1,\,k\neq L}^{K} \text{var}\{J_k\} \right\}^{-1/2} \right) \tag{13}$$

where $Q(x) = (2\pi)^{-1/2} \int_x^\infty e^{-t^2/2} \, dt$ is the standard Q function, and

$$\text{var}\{J_k\} = \frac{1}{N^2} \sum_{m=0}^{M-1} \Big[V_1(V_4+V_6) + V_2(V_5+V_7)$$
$$+ 2\,\text{Re}(V_3 V_8 + V_3 V_9) \Big]. \tag{14}$$

In (14)

$V_1 = E\left\{ |R(\hat{\tau}_k;\mu)|^2 \right\}$

$V_2 = E\left\{ |\hat{R}(\hat{\tau}_k;\mu)|^2 \right\}$

$V_3 = E\{ R(\hat{\tau}_k;\mu) \cdot \hat{R}^*(\hat{\tau}_k;\mu) \},$

$V_4 = E\left\{ |C_{k,L}(-(N-w_k-1);\mu T_c)|^2 \right\}$

$V_5 = E\left\{ |C_{k,L}(-(N-w_k);\mu T_c)|^2 \right\}$

$V_6 = E\left\{ |C_{k,L}(w_k+1;\mu T_c)|^2 \right\}$

$V_7 = E\left\{ |C_{k,L}(w_k;\mu T_c)|^2 \right\}$

$V_8 = E\{ C_{k,L}(-(N-w_k-1);\mu T_c) C_{k,L}^*(-(N-w_k);\mu T_c) \}$

$V_9 = E\{ C_{k,L}(w_k+1;\mu T_c) \cdot C_{k,L}^*(w_k;\mu T_c) \}$

where $\mu = (m-\tilde{m})f_\Delta$ and $E\{\cdot\}$ is the expectation taken over random variables $\hat{\tau}_k$ and w_k. Notice that P_e is dependent on the choice of $\tilde{m}$.

III. ANALYSIS OF THE MA CAPACITY

The MA capacity may be defined as the number of users that can be supported at a given error performance. We are interested in comparing the MA capacity of a MT-CDMA system and that of a conventional DSSS system under conditions of equal bit transmission rate and equal bandwidth. This comparison is made easy since a MT-CDMA system reduces to a conventional DSSS system in the special case of $M = 1$. We are also interested in comparing MA capacities for MT-CDMA systems using different values of M. We want to find out whether the MA capacity could be increased by an increase in the processing gain through the use of more subcarriers.

We assume that the MT-CDMA system and the corresponding conventional DSSS system have the same chip duration T_c. As a result, the bandwidth of a MT-CDMA signal is slightly greater than the signal bandwidth required for a conventional DSSS system. The difference in their bandwidths is small (see [1]) so that the equal-bandwidth condition can be considered satisfied. As M subcarriers are used in the MT-CDMA system, under equal-bit-rate condition the bit duration T_b of the MT-CDMA system is required to be M times of that of the conventional DSSS system. Hence, the spreading-code length N of the MT-CDMA system should also be M times of that of the corresponding conventional DSSS system. The increase in the spreading-code length constitutes

TABLE I

MAXIMUM AND MINIMUM VALUES OF I FOR VARIOUS COMBINATIONS
OF M AND N UNDER THE CONDITION OF SAME N/M

	I	
N/M	min.	max.
16/1	0.6667	0.6667
32/2	0.6660	0.6660
64/4	0.6655	0.6662
128/8	0.6653	0.6662
256/16	0.6651	0.6662
512/32	0.6650	0.6662
1024/64	0.6650	0.6662
128/1	0.6667	0.6667
256/2	0.6667	0.6667
512/4	0.6667	0.6667
1024/8	0.6667	0.6667

an increase in the processing gain of a MT-CDMA system. Define the *basic processing gain* N_{ba} by

$$N_{ba} = \frac{N}{M} \tag{15}$$

which is a constant invariant to the choice of M. Note that N_{ba} is also the spreading-code length of the conventional DSSS system. Since M bits are transmitted concurrently, the power per bit P_b for a MT-CDMA system is required to be $1/M$ times of that of the conventional DSSS system. The net result of an M-fold increase in the bit duration and an M-fold reduction in the power per bit is that the bit energy values are the same for both MT-CDMA and conventional DSSS systems. Thus, we can compare the two systems at the same E_b/N_0 value.

Some insight can be obtained by considering random binary sequences as spreading sequences. It can be shown that, for random binary sequences and $k \neq L$,

$$E\left\{|C_{k,L}(q; \mu T_c)|^2\right\} = \begin{cases} N - |q| & |q| < N \\ 0 & |q| \geq N \end{cases} \tag{16}$$

$$E\{C_{k,L}(q_1; \mu T_c) \cdot C_{k,L}^*(q_2; \mu T_c)\} = 0 \qquad \text{for } q_1 \neq q_2. \tag{17}$$

It follows that $\text{var}\{J_k\} = I/N_{ba}$ where

$$I = \frac{1}{M} \sum_{m=0}^{M-1} \left[E\left\{|R(\hat{\tau}_k; (m - \tilde{m})f_\Delta)|^2\right\} + E\left\{|\hat{R}(\hat{\tau}_k; (m - \tilde{m})f_\Delta)|^2\right\} \right]. \tag{18}$$

Table I lists the maximum and minimum values of I obtained over $\tilde{m} \in \{0, 1, \cdots, M - 1\}$ for various combinations of M and N under the condition of same N_{ba}. It is apparent that $I \approx 2/3$ for all combinations. Therefore, P_e in (13) can be closely approximated by

$$P_e \approx Q\left(\left\{ \left(\frac{2E_b}{N_0} \right)^{-1} + \frac{K-1}{3N_{ba}} \right\}^{-1/2} \right). \tag{19}$$

Equation (19) is identical to the one obtained by Pursley [4] for computing the error probability of a conventional DSSS system. It follows that the MA capacity of a MT-CDMA system is the same as that of a conventional DSSS system when asynchronous signal transmission and random sequences are used. Also notice that P_e for a MT-CDMA system depends only on the basic processing gain. Thus, using a larger number of subcarriers to increase the processing gain of the system does not reduce P_e and hence does not increase the MA capacity.

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