An Efficient Method for Designing Two-Channel PR FIR Filter Banks with Low System Delay

J. S. Mao, S. C. Chan, and K. L. Ho

***Abstract*—In this paper, an efficient method for designing perfect reconstruction (PR) two-channel finite impulse response (FIR) filter banks with low system delay is proposed. It is based on the use of nonlinear-phase FIR function in a structure previously proposed by Phoong *et al.* [1]. The design problem is formulated as a complex polynomial approximation problem and is solved effectively using the Remez exchange algorithm with very low design complexity. Design examples show that filter banks with flexible stopband attenuation and system delay can be readily obtained by the proposed algorithm.**

***Index Terms*—Design method, filter banks, low system delay, perfect reconstruction, Remez exchange algorithm, two-channel.**

I. Introduction

BECAUSE of the low delay requirement in subband coding, subband adaptive filtering, and other applications, there is a growing interest in designing PR filter banks (FBs) with such properties. Conventional methods for designing low delay two-channel PR FB are usually based on numerical optimization [3], [7], which produces FB that is only pseudo-PR. More recently, several factorizations for PR low delay FB were reported [5], [6]. Unfortunately, they usually require unconstrained optimization of highly nonlinear functions, which is easily trapped in local minima and requires considerably high design complexity. Another approach based on linear programming has also been proposed in [4]. The FB so obtained is PR. However, because the FB does not have any structure, it is not robust to coefficient quantization and generally requires higher implementation complexity. In this paper, a class of low delay two-channel FB, using the structure in [1], and [2], is proposed. In particular, we show that the functions $\alpha(z)$ and $\beta(z)$ of the structure in [1] can be chosen as nonlinear-phase FIR functions to realize FB with low system delay and good frequency characteristics. We also show that the design of the proposed FB can be formulated as a complex polynomial approximation problem, which can be solved by the Remez exchange algorithm with very low design complexity. The paper is organized as follows: in Section II, a brief summary of the two-channel structural PR FB proposed in [1] is given together with the basic idea behind the proposed low delay FIR FB and its design procedures. Several design examples are given in Section III, and the conclusions are drawn in Section IV.

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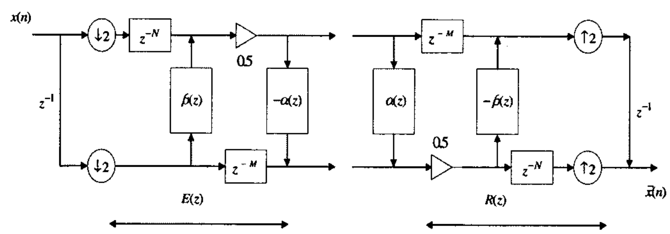

Fig. 1. Two-channel PR filter bank in [1].

II. Proposed Method

A. Structurally Imposed FIR Biorthogonal FBs

Fig. 1 shows the structure of the two-channel PR FB proposed in [1]. The analysis lowpass filter $H_0(z)$ and highpass filter $H_1(z)$ can be shown to be

$$H_0(z) = (z^{-2N} + z^{-1}\beta(z^2))/2,$$

and

$$H_1(z) = -\alpha(z^2)H_0(z) + z^{-2M-1}. \tag{2.1}$$

where $\alpha(z)$ and $\beta(z)$ can be polynomials and rational functions to achieve PR. In [1], they are chosen to be identical, and the delay parameter M is equal to $2N-1$. To design a linear-phase FIR FB with $\alpha(z) \neq \beta(z)$, they can be chosen as type-II linear-phase FIR functions with $M \neq 2N-1$. The lengths of $\beta(z)$ and $\alpha(z)$ should be $N_\beta = 2N$ and $N_\alpha = 2(M-N+1)$, respectively, with a system delay of $2N+2N+1$. For low delay applications, the lengths of $\beta(z)$ and $\alpha(z)$ are therefore constrained by the system parameters M and N, and might not be able to provide the desired stopband attenuation. In this paper, $\beta(z)$ and $\alpha(z)$ are chosen as nonlinear-phase FIR functions. Under the same system delay $2N+2N+1$, N_β, and N_α in low delay FBs are no longer constrained by the parameters N and M as their linear-phase counterparts. Therefore, the stopband performance will significantly be improved by choosing $\beta(z)$ and $\alpha(z)$ to be nonlinear-phase.

B. Design of Analysis Lowpass Filter

Let ω_{p_0} and ω_{s_0} be, respectively, the passband and stopband cutoff frequencies of filter $H_0(z)$. Similarly, let $\omega_{p_1} = \omega_{s_0} = \omega_s$ and $\omega_{s_1} = \omega_{p_0} = \omega_p$ be the passband and stopband cutoff frequencies of $H_1(z)$. To design $\beta(z)$ with nonlinear-phase, let us decompose it into its even and odd parts: $\beta(z) = \beta_e(z) + \beta_o(z)$. When the length of $\beta(z)$ is even, $\beta_e(z)$ and $\beta_o(z)$ are type-II and type-IV linear-phase FIR functions, respectively. Similarly, when the length of $\beta(z)$ is odd, it can be decomposed into a sum of type-I and type-III FIR functions [10]. For simplicity, only the even length case will be considered (details of the odd-length case will be given in [10]). $\beta_e(z)$ and $\beta_o(z)$ can be written as follows:

$$\beta_e(e^{j\omega}) = e^{-j\omega(N_\beta-1)/2}\cos(\omega/2)P_e^{(\beta)}(\cos\omega),$$

and

$$\beta_o(e^{j\omega}) = je^{-j\omega(N_\beta-1)/2}\sin(\omega/2)P_o^{(\beta)}(\cos\omega) \quad (2.2)$$

where $P_e^{(\beta)}(\cos\omega)$ and $P_o^{(\beta)}(\cos\omega)$ are polynomials with order $(N_\beta/2) - 1$. The desired response of $H_0(e^{j\omega})$, $H_d^{(0)}(e^{j\omega})$ is equal to $e^{-j2\omega N}$ for $0 \le \omega \le \omega_p$ and zero otherwise. The error function $E(\omega) = H_0(e^{j\omega}) - H_d^{(0)}(e^{j\omega})$ is then given by

$$E(\omega) = -E^*(\pi-\omega) = e^{-j2\omega N}\left\{e^{-j2\omega N_\Delta}P^{(\beta)}(e^{j\omega}) - 1\right\}/2, \quad \omega \in [0, \pi/2] \quad (2.3)$$

where

$$N_\Delta = N_\beta/2 - N,$$

and

$$P^{(\beta)}(e^{j\omega}) = \cos\omega\cdot P_e^{(\beta)}(\cos(2\omega)) + j\cdot\sin\omega\cdot P_o^{(\beta)}(\cos(2\omega)). \quad (2.4)$$

The desired response of $P^{(\beta)}(e^{j\omega})$ is seen to be $P_d^{(\beta)}(e^{j\omega}) = e^{j2\omega N_\Delta}$, which is recognized as a complex polynomial approximation problem. They can be solved by a variety of methods [8], [9]. For simplicity, the complex polynomial is decomposed into its real and imaginary parts and is solved by two independent real Chebyshev approximations [9]. The desired functions of $P_e^{(\beta)}(\cos(2\omega))$ and $P_o^{(\beta)}(\cos(2\omega))$ are then given by

$$\hat{P}_e^{(\beta)}(\cos(2\omega)) = \cos(2\omega N_\Delta)/\cos\omega$$

and

$$\hat{P}_o^{(\beta)}(\cos(2\omega)) = \sin(2\omega N_\Delta)/\sin\omega, \quad 0 < \omega < \pi/2. \quad (2.5)$$

Writing $x = \cos(2\omega)$, the two Chebyshev approximation problems can be written as

$$a_{k,\,opt} = \arg\min_{a_k}\max\left|W_e(x)(P_e^{(\beta)}(x) - \hat{P}_e^{(\beta)}(x))\right|$$

and

$$b_{k,\,opt} = \arg\min_{b_k}\max\left|W_o(x)(P_o^{(\beta)}(x) - \hat{P}_o^{(\beta)}(x))\right| \quad (2.6)$$

where $a_{k,\,opt}$ and $b_{k,\,opt}$ are, respectively, the coefficients of the polynomials $P_e^{(\beta)}(\cos(2\omega))$ and $P_o^{(\beta)}(\cos(2\omega))$, which minimize the minimax problems in (2.6) in the interval

$$I_x = (-1, \tilde{x}_s] \cup [x_s, 1], \quad \text{with } \tilde{x}_s < x_s = \cos(2\omega_p)$$

and

$$W_e(x) = \cos(0.5\cdot\arccos(x))$$
$$W_o(x) = \sin(0.5\cdot\arccos(x)). \quad (2.7)$$

The interval $(0, \tilde{x}_s]$ is an optional disjoint interval to control the values of $P_e^{(\beta)}(\cos(2\omega))$ and $P_o^{(\beta)}(\cos(2\omega))$ in the transition band of $H_0(e^{j\omega})$. Equation (2.6) can readily be solved

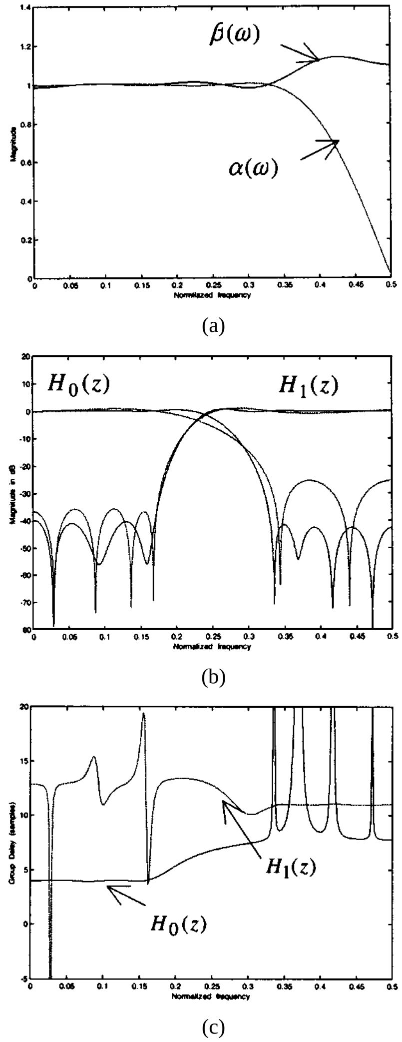

Fig. 2. Example 1. (a) Magnitude responses of $\beta(z)$ and $\alpha(z)$; (b) magnitude responses of analysis filters $H_0(z)$ and $H_1(z)$: low delay (solid lines) and linear-phase (dashed lines); and (c) Group delay responses of the proposed low delay analysis filters.

using the function REMEZ in the signal processing Toolbox of MATLAB.

C. *Design of Analysis Highpass Filter*

Let us assume that $\beta(z)$ is properly designed so that $H_0(z)$ is a reasonably good lowpass filter. From (2.1), it can be seen that the frequency response of $H_1(z)$ depends on $H_0(z)$ and $\alpha(z)$. The error function $E(\omega)$ of $H_1(z)$ is $E(\omega) = e^{-j\omega(2M+1)} - \alpha(e^{2j\omega})H_0(e^{j\omega}) - H_d^{(1)}(e^{j\omega})$, where $H_d^{(1)}(e^{j\omega})$ is the desired response of $H_1(e^{j\omega})$ and is equal to $e^{-j\omega(2M+1)}$ for $\omega_s \le \omega \le \pi$ and zero otherwise. The ideal magnitude response of $\alpha(e^{2j\omega})$ suggests that its value should be close to one, except around $\omega = \pi/2$, where it is even smaller [see Fig. 2(a) for an example]. It then follows from (2.1) that the passband ripple of $H_1(e^{j\omega})$ is approximately equal to the stopband error of $H_0(e^{j\omega})$. This allows us to minimize only the stopband attenuation of $H_1(e^{j\omega})$ using $\alpha(e^{2j\omega})$, instead of minimizing $E(\omega)$ over the pass- and stop-bands and relies on the high stopband attenuation of $H_0(e^{j\omega})$ to achieve a small passband ripple. First of all, let us consider the case where $\alpha(z)$

is an even-length FIR filter with length N_α. The odd length case can be derived similarly and will be described in [10]. Using again the even and odd parts decomposition, $\alpha(z)$ can be expressed as $\alpha_e(z) + \alpha_o(z)$, where

$$\alpha_e(e^{j\omega}) = e^{-j\omega(N_\alpha - 1)/2} \cos(\omega/2) P_e^{(\alpha)}(\cos\omega)$$

and

$$\alpha_o(e^{j\omega}) = je^{-j\omega(N_\alpha - 1)/2} \sin(\omega/2) P_o^{(\alpha)}(\cos\omega) \quad (2.8)$$

where $P_e^{(\alpha)}(\cos\omega)$ and $P_o^{(\alpha)}(\cos\omega)$ are polynomials with order $(N_\alpha/2) - 1$. Let the lowpass filter $H_0(e^{j\omega})$ be written as $H_0(e^{j\omega}) = A(e^{j\omega})e^{-j2\omega N}$, where $A(e^{j\omega})$ is a complex function but is approximately equal to one in [0, ω_p] for $\beta(z)$, with sufficient high order. Substituting (2.8) into the error function $E(\omega)$, one obtains

$$E(\omega) = e^{-j\omega(2M+1)} \left\{1 - e^{-j2\omega M_\Delta} P^{(\alpha)}(e^{j\omega}) \cdot A(e^{j\omega})\right\}, \quad \omega \in [0, \pi/2] \quad (2.9)$$

where $M_\Delta = (N_\alpha/2) + (N - M) - 1$, and $P^{(\alpha)}(e^{j\omega}) = \cos\omega \cdot P_e^{(\alpha)}(\cos(2\omega)) + j \sin\omega \cdot P_o^{(\alpha)}(\cos(2\omega))$. Therefore, the ideal response of $P^{(\alpha)}(e^{j\omega})$ is $[e^{-j2\omega M_\Delta} A(e^{j\omega})]^{-1}$, $\omega \in [0, \pi/2]$. Again, if $P_e^{(\alpha)}(\cos(2\omega))$ and $P_e^{(\alpha)}(\cos(2\omega))$ are used to approximate separately the real and imaginary parts of $P_d^{(\alpha)}(\omega)$, their ideal responses are

$$\hat{P}_e^{(\alpha)}(\cos(2\omega)) = \mathrm{Re}\left(A^*(e^{j\omega})e^{j2\omega M_\Delta}\right) / \left(\left|A(e^{j\omega})\right|^2 \cos\omega\right)$$

and

$$\hat{P}_o^{(\alpha)}(\cos(2\omega)) = \mathrm{Im}\left(A^*(e^{j\omega})e^{j2\omega M_\Delta}\right) / \left(\left|A(e^{j\omega})\right|^2 \sin\omega\right) \quad (2.10)$$

$\omega \in [0, \pi/2]$. Writing $x = \cos(2\omega)$, the problem can be solved using the Chebyshev approximation as in (2.6), with weighting functions

$$\tilde{W}_e(x) = \left(\cos(0.5 \cdot \arccos(x)) \left|A(e^{j0.5 \cdot \arccos(x)})\right|^2\right)$$

and

$$\tilde{W}_o(x) = \left(\sin(0.5 \cdot \arccos(x)) \left|A(e^{j0.5 \cdot \arccos(x)})\right|^2\right). \quad (2.11)$$

They again can be solved by the Remez exchange algorithm. Approximating the real and imaginary parts of $P_d^{(\alpha)}(\omega)$ [or $P_d^{(\beta)}(\omega)$] separately as two real Chebyshev approximations does not necessarily produce an equiripple error function. A reweight technique is therefore used to modify the weighting function as $W_{\delta,k}(\omega) = \tilde{W}_k(\omega)\{\delta_e^2(\omega) + \delta_o^2(\omega)\}^{P/2}$, where $k = e, o$, $\delta_e(\omega)$ and $\delta_o(\omega)$ are the error in approximating the real and imaginary parts of $P_d^{(\alpha)}(\omega)$ (or $P_d^{(\beta)}(\omega)$). From experimental results, $P = 2$ is found to perform better than $P = 1$, and is used in this work. (2.6), (2.7) and (2.10), (2.11) are first solved using the Remez exchange algorithm. After that, it is solved again using the new weighting function. Considerable improvement over the independent Chebyshev approximations can be obtained.

TABLE I
OVERALL PERFORMANCE OF EXAMPLE 1 COMPARED TO OTHER CONVENTIONAL METHODS

Performance Comparison *	Ex. 1 vs. Linear Phase, and Ref. [3, 4]				Ex. 1 vs. Ref. [5]		Ex. 1 vs. Ref. [7]	
	Ex. 1	Ex. 1 (LP)	Ref. [3]	Ref. [4]	Ex. 1 **	Ref. [5]	Ex. 1 **	Ref. [7]
$\omega_{p_0}(\pi)$	0.34	0.34	0.38	0.4	0.24	0.3	0.4	0.4
$\omega_{s_0}(\pi)$	0.66	0.66	0.68	0.7	0.76	0.8	0.6	0.6
$\omega_{p_1}(\pi)$	0.66	0.66	0.68	0.6	0.76	0.7	0.6	0.6
$\omega_{s_1}(\pi)$	0.34	0.34	0.38	0.3	0.24	0.2	0.4	0.4
δ_{s_0} (dB)	42	26	39	45	55	56	30	25
δ_{s_1} (dB)	40	36	39	40	54	57	29	25
PR	Yes	Yes	No	Yes	Yes	Yes	Yes	No
Robust to quantization	Yes	Yes	No	No	Yes	Yes	Yes	No
Filter length (low/high	15/33	15/33	16/16	22/28	15/33	32/32	15/33	28/36
Implementation	9 Mul. 9 Add.	3 Mul. 6 Add.	32 Mul. 30	50 Mul. 48	9 Mul. 9 Add.	11 Mul. 9 Add.	9 Mul. 9 Add.	64 Mul. 62 Add.
Design Complexity	low	low	high	medium	low	High	low	High

III. DESIGN EXAMPLES

Example 1: In this design example, $\beta(z)$ and $\alpha(z)$ are nonlinear-phase FIR functions with lengths $N_\beta = 8$ and $N_\alpha = 10$. The delay parameters N and M are, respectively, 2 and 5. The overall system delay is $n_0 = 15$ samples. Fig. 2 displays the magnitude responses of $\beta(z)$ (solid line), $\alpha(z)$ (dashed line), and the frequency responses of the analysis filters (solid line) designed by the proposed method. To demonstrate the advantage of the proposed low delay FIR FB over the linear-phase FB, a two-channel linear-phase FIR FB with the same system delay was also designed by the proposed Remez exchange algorithm. The delay parameters N and M are still 2 and 5, but the length of $\beta(z)$ and $\alpha(z)$ are shortened to $N_\beta = 4$ and $N_\alpha = 8$, respectively, due to the linear-phase constraint. The magnitude responses of the linear-phase analysis filters are plotted as dashed lines in Fig. 2(b). It is observed that, under the same system delay ($n_0 = 15$) and cutoff frequencies ($\omega_p = 0.34\pi$ and $\omega_s = 0.66\pi$), the stopband attenuation of the low-delay analysis filters are much higher than its linear-phase counterpart: 42 dB versus 26 dB for $H_0(z)$, and 40 dB versus 36 dB for $H_1(z)$. This improvement of stopband attenuation, however, requires additional computations in implementing the FB (the number of variables in the low-delay $\beta(z)$ and $\alpha(z)$ is 18, while that of the linear-phase is 6). It can also be seen from Fig. 2(c) that the proposed low delay FB is approximately linear-phase in their passbands. In order to compare the proposed method to the conventional methods in [3], [4], the cutoff frequencies and system delay are chosen to be the same as the design examples in [3, Fig. 5] and [4, Fig. 3]. The stopband attenuations of the analysis filters in [3] are about 39 dB. The stopband attenuations of $H_0(z)$ and $H_1(z)$ in [4] are, respectively, 45 dB and 40 dB. Therefore, the proposed analysis filters have comparable passband and stopband performance as those in [3] and [4]. By adjusting the cutoff frequencies to ($\omega_p = 0.24\pi$, $\omega_s = 0.76\pi$) and ($\omega_p = 0.4\pi$, $\omega_s = 0.6\pi$), the proposed method can still provide comparable stopband attenuation as that in [5, Fig. 1] (55 dB versus 56 dB), and higher stopband attenuation than that in [7, Fig. 2] (30 dB versus 25 dB). It is also noted that the design and implementation complexities of the proposed method are very low and there is no reconstruction error, which

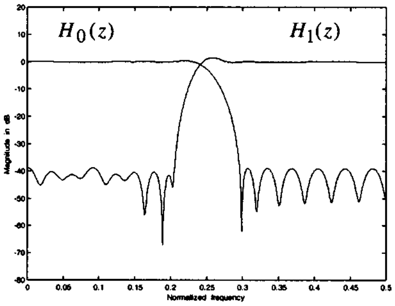

Fig. 3. Magnitude responses of $H_0(z)$ and $H_1(z)$ in Example 2.

is always present in other methods based on constrained nonlinear optimization [3], [7]. Furthermore, as mentioned earlier, the proposed filter bank structure is still PR, even under coefficient quantization, unlike the direct form in [4]. The design complexity of the proposed method is also much lower than the unconstrained nonlinear optimization methods in [5] and [6], thanks to the Remez Exchange algorithm. The overall performance comparison between the proposed method and the conventional low delay filter bank design methods [3]–[5] and [7] are summarized in Table I, where the arithmetic complexity is simply defined as the number of multiplications and additions per sample in implementing the filter bank. This demonstrates the good performance, flexibility, low implementation, and design complexities of the proposed method as compared with conventional methods.

Example 2: In this example, a two-channel low delay FIR filter bank with higher order is designed. The lengths of $\beta(z)$ and $\alpha(z)$ are chosen to be odd numbers, $N_\beta = 13$ and $N_\alpha = 15$. The overall system delay of the filter bank is $n_0 = 23$ samples with $N = 3$ and $M = 8$. Fig. 3 plots the magnitude responses of the analysis filters $H_0(z)$ and $H_1(z)$. It can be seen that the stopband attenuation of $H_0(z)$ and $H_1(z)$ is about 39 dB, and their passband and stopband cutoff frequencies are, respectively, $\omega_p = 0.41\pi$ and $\omega_s = 0.59\pi$. It is also observed that due to the higher system delay, the transition band is sharper than that of Example 1.

IV. Conclusion

An efficient algorithm for designing two-channel low delay PR FIR filter banks is presented. It is based on the use of nonlinear-phase FIR functions in the structure previously proposed by Phoong *et al.* [1]. The design problem is formulated as a polynomial approximation problem and is solved using the Remez exchange algorithm. Design examples show that filter banks with flexible stopband attenuation and system delay can readily be obtained by the proposed algorithm.

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