

Bifurcations Associated with Sub-Synchronous Resonance

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Abstract— This paper describes a set of results of detecting nonlinear phenomena appearing in a turbine generator power system with series-capacitor compensation. The analysis was based on the Floquet theory as well as the Hopf Bifurcation theorem. After the first Hopf bifurcation, the stable limit cycle bifurcates to a stable torus and an unstable limit cycle which connects to a stable limit cycle by a supercritical torus bifurcation. The stable limit cycle joins with an unstable limit cycle at a cyclic fold bifurcation. This unstable limit cycle is connected to the second Hopf. It has been also numerically demonstrated that such a strange sequence of periodic orbits is created by a q-axis damper winding.

Keywords— Bifurcation, Floquet multiplier, Nonlinear analysis, Subsynchronous resonance, Series capacitor compensation, Power system dynamics

I. INTRODUCTION

We study the phenomenon of SSR (Sub-Synchronous Resonance) in models of turbine-generator systems with series-capacitor compensation. The usual analysis of SSR is based on frequency domain methods using linear models such as in [2]. Some controllers using electronically switched transmission line compensators, which are collectively known as FACTS (Flexible Alternating Current Transmission Systems), have been designed also using frequency domain analysis and proven to be effective in mitigating the deleterious effects of SSR in [4] and [5]. However, studies in [6] and [7] indicate that because of the stable limit cycle with small magnitude existing after the Hopf bifurcation, the range of instability of torsional oscillations is narrower than the one which is predicted by an eigenanalysis method.

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This paper presents the analysis of limit cycle based on the Floquet theory as well as the Hopf bifurcation theorem. Section 2 contains some mathematical preliminaries. The 12th order turbine-generator model is described in Section 3. Results of numerical studies using AUTO94 [8] are given in Section 4. It should be noted that the **torus bifurcation**, documented in Section 4, has not been reported in SSR studies such as [4], [5], [6] or [7]. The result shows a surprising similarity to the 2 machine system with nonlinear dynamics [18].

II. MATHEMATICAL PRELIMINARIES

A. Supercritical and Subcritical Hopf

The following discussion is adapted from [9], [10], [15], [17]. Any parameterized two-dimensional system with an equilibrium at 0 and purely imaginary eigenvalues, can be cast by a smooth change of coordinates into the polar form

$$\begin{aligned}\dot{r} &= (d\gamma + ar^2)r \\ \dot{\theta} &= \omega + c\gamma + br^2.\end{aligned}$$

In general, the equilibrium need not be at the origin. If $d \neq 0$, there is a bifurcation at $\gamma = 0$ since the origin is stable for $d\gamma < 0$ and unstable for $d\gamma > 0$. Since the first equation does not depend on θ , we see immediately that there are periodic orbits, $r = \text{constant} \neq 0$. If $a \neq 0$ and $d \neq 0$ these solutions lie along the parabola $d\gamma + ar^2 = 0$. The Hopf bifurcation theorem says that this situation holds in general. The following hypotheses are needed:

1. The system $\dot{x} = f(x, \gamma)$ has an isolated equilibrium at $x_0(\gamma)$.
2. $f(x, \gamma)$ is C^r ($r \geq 4$).
3. The Jacobian $\partial f(x_0(\gamma), \gamma)/\partial x$ possesses a pair of complex conjugate, simple eigenvalues

$$\lambda(\gamma) = \alpha(\gamma) + j\omega(\gamma), \quad \bar{\lambda}(\gamma) = \alpha(\gamma) - j\omega(\gamma)$$

4. At the critical value

$$\alpha(\gamma_0) = 0, \quad \omega_0 := \omega(\gamma_0) > 0, \quad \text{and} \quad \alpha'(\gamma_0) > 0$$

Besides $\pm j\omega_0$, the other eigenvalues of the critical Jacobian $\partial f(x(\gamma_0), \gamma_0)/\partial x$ have strictly negative real part.

The following *existence* and *uniqueness* conditions are from the Hopf bifurcation theorem in [15].

Existence. There is $\epsilon_H > 0$ and a C^{r-1} function

$$\gamma(\epsilon) = \gamma_0 + \gamma_2 \epsilon^2 + O(\epsilon^3)$$

and for each $0 < \epsilon < \epsilon_H$ there is a nonconstant periodic solution $x_\epsilon(t)$ near the equilibrium $x_0(\gamma)$ for the parameter value $\gamma = \gamma(\epsilon)$. The period of x_ϵ is a C^{r-1} function

$$T(\epsilon) = 2\pi\omega_0^{-1}[1 + T_2\epsilon^2] + O(\epsilon^3)$$

and its amplitude grows as $O(\epsilon)$.

Uniqueness. If $\gamma_2 \neq 0$ then there is $\epsilon_0 > 0$ such that for each $0 < \epsilon < \epsilon_0$, x_ϵ is the only periodic orbit for $\gamma = \gamma(\epsilon)$ near $x_0(\gamma(\epsilon))$.

Stability. Exactly one of the characteristic exponents of $x_\epsilon(t)$ approaches 0 as $\epsilon \rightarrow 0$, and it is given by the real C^{r-1} function

$$\beta(\epsilon) = \beta_2 \epsilon^2 + O(\epsilon^3)$$

The relationship

$$\beta_2 = -2\alpha'(\gamma_0)\gamma_2$$

holds. Moreover the periodic solution $x_\epsilon(t)$ is orbitally asymptotically stable with asymptotic phase if $\beta(\epsilon) < 0$ but is unstable if $\beta(\epsilon) > 0$. The bifurcation is said to be *supercritical* in the former case and *subcritical* in the latter. For the two dimensional case, the formula for β_2 is given by

$$\begin{aligned} 8\beta_2 &= \frac{1}{\omega_0} \{f_{11}^1(f_{11}^2 - f_{12}^1) + f_{22}^2(f_{12}^2 - f_{22}^1) \\ &+ (f_{11}^2 f_{12}^2 - f_{12}^1 f_{22}^1)\} \\ &+ (f_{111}^1 + f_{122}^1 + f_{112}^1 + f_{222}^2) \end{aligned}$$

Here

$$\begin{aligned} f &= (f^1, f^2), \\ f_{pq}^i &= \frac{\partial^2 f^i}{\partial x_p \partial x_q}(0, \gamma_0), \\ f_{pqr}^i &= \frac{\partial^3 f^i}{\partial x_p \partial x_q \partial x_r}(0, \gamma_0) \end{aligned}$$

In the general case, one uses the center manifold theorem and perturbation analysis with multiple time scales to reduce an n dimensional system satisfying the hypotheses to a two-dimensional model.

B. Cyclic Fold and Torus Bifurcations

Consider a differential equation $\dot{x} = f(x)$, $x \in \mathbb{R}^n$. Denote its solution starting at a point x by $\phi_t(x) = \phi(t, x)$. The map $\phi : (t, x) \rightarrow \phi(t, x)$ is called the flow of the vector field $f(x)$. Let Γ be a periodic orbit of period T of

the flow ϕ . We first take a *local* cross section $S \subset \mathbb{R}^n$, of dimensional $n - 1$, such that (i) the flow ϕ is everywhere transverse to S (This means the inner product $\langle f(x), n(x) \rangle \neq 0$ for all $x \in S$, where $n(x)$ is the unit normal to S at x), and (ii) S intersects Γ at a *unique* point p . Let $\mathcal{U} \subset S$ be a neighbourhood of p . For $q \in \mathcal{U}$ the *first return* or *Poincaré* map $\mathcal{P} : \mathcal{U} \rightarrow S$ is defined by

$$\mathcal{P}(q) := \phi_{\tau(q)}(q) = \phi(\tau(q), q)$$

where $\tau(q)$ is the time it takes for the orbit based at q to first return to S . Note that $\tau(p) = T$ and $\tau(q) \rightarrow T$ as $q \rightarrow p$. Also p is a fixed point of \mathcal{P} . In local coordinates, \mathcal{P} is a map from $\mathcal{U} \subset \mathbb{R}^{n-1}$ to \mathbb{R}^{n-1} , so $D\mathcal{P}(p)$ has $n - 1$ eigenvalues. The eigenvalues of $D\mathcal{P}(p)$ are the same as $n - 1$ eigenvalues of $D\phi_T(p)$, and are called the *Floquet multipliers* (or characteristic multipliers) associated with the periodic orbit Γ .

Suppose the system has a periodic orbit for a parameter value μ . Denote the Floquet multipliers by $\lambda_1(\mu), \dots, \lambda_{n-1}(\mu)$. If the periodic solution is stable, then $\lambda_1(\mu), \dots, \lambda_{n-1}(\mu)$ are all inside the unit circle. The multipliers are functions of the parameter μ . As μ is varied, some of them may cross the unit circle at a critical value μ_0 . A multiplier crossing the unit circle is called a *critical multiplier*. Several types of branching may occur depending on where a multiplier or a pair of complex conjugate multipliers exits the unit circle. There are three possible cases. In the numerical study, we **only** observed **Case 1** and **Case 3**.

1. **Cyclic Fold:** Only one multiplier crosses the unit circle along the positive real axis with $\lambda_i(\mu_0) = 1$ for some $1 \leq i \leq n - 1$.
2. **Period Doubling:** Only one multiplier crosses the unit circle along the negative real axis with $\lambda_i(\mu_0) = -1$ for some $1 \leq i \leq n - 1$.
3. **Torus:** Only one pair of complex conjugate multipliers crosses the unit circle with $|\lambda_i(\mu_0)| = 1$, $\text{Im}(\lambda_i(\mu_0)) = -\text{Im}(\overline{\lambda_i(\mu_0)}) > 0$, and $\frac{d}{d\mu} |\lambda_i(\mu_0)| \neq 0$ for some $1 \leq i \leq n - 1$.

III. THE 12TH ORDER TURBINE-GENERATOR MODEL

A. System Description

The configuration of the single-machine to infinite bus power system used in this study is shown in Fig.1.

The notation used here is standard [1], [2], [3] and will not be elaborated further; ω_0 is the frequency of the voltage at the infinite bus, d and q refer to the direct and quadrature axes. The 12 variables are armature winding flux (Φ_d, Φ_q), field winding flux (Ψ_f), damper winding flux ($\Psi_{kd}, \Psi_{kq1}, \Psi_{kq2}$), absolute angle of the generator (θ_1), relative angle of turbine (λ_2), angular velocities of

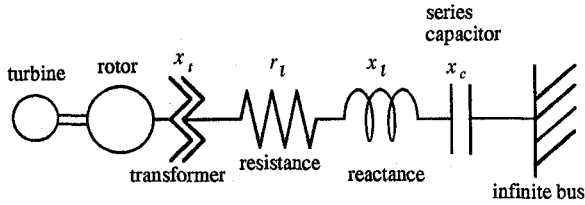


Fig. 1. Configuration of power system for study.

the generator and turbine (ω_1, ω_2) and voltages of the series capacitor (V_{cd}, V_{cq}), where $\lambda_2 = \theta_2 - \theta_1$ and the absolute angle of the turbine is θ_2 . Note that, the 12th order model has only 1 torsional mode, no AVR and no governor. This model is the simplest system that can exhibit subsynchronous resonance.

$$\begin{aligned}
 \dot{\theta}_1 &= \omega_1 - 1 \\
 \dot{\lambda}_2 &= \omega_2 - \omega_1 \\
 M_1 \omega_0 \dot{\omega}_1 &= -D_{11}(\omega_1 - 1) - D_{12}(\omega_1 - \omega_2) \\
 &\quad + K_{12}\lambda_2 - (\Phi_d i_q - \Phi_q i_d) \\
 M_2 \omega_0 \dot{\omega}_2 &= -D_{22}(\omega_2 - 1) - D_{12}(\omega_2 - \omega_1) \\
 &\quad - K_{12}\lambda_2 + T_{m2} \\
 \dot{\Phi}_d &= +\omega_1 \Phi_q + R_A i_d + V_{cd} + E_s \sin(\theta_1) \\
 \dot{\Phi}_q &= -\omega_1 \Phi_d + R_A i_q + V_{cq} + E_s \cos(\theta_1) \\
 \dot{\Psi}_f &= -R_F i_f + E_{FD} \\
 \dot{\Psi}_{kd} &= -R_{kd} i_{kd} \\
 \dot{\Psi}_{kq1} &= -R_{kq1} i_{kq1} \\
 \dot{\Psi}_{kq2} &= -R_{kq2} i_{kq2} \\
 \dot{V}_{cd} &= +\omega_1 V_{cq} + X_c i_d \\
 \dot{V}_{cq} &= -\omega_1 V_{cd} + X_c i_q
 \end{aligned}$$

$\Phi_d = \Psi_d - x_d i_d$ and $\Phi_q = \Psi_q - x_q i_q$. i_d, i_f and i_{kd} are linear combinations of Φ_d, Ψ_f and Ψ_{kd} . i_q, i_{kq1} and i_{kq2} are linear combinations of Φ_q, Ψ_{kq1} and Ψ_{kq2} . τ is the time variable and $\tau = \omega_0 t$ is the time scale. X_c is the negative reactance due to series-capacitor compensation.

B. System Constants

Two sets of system constants are provided; (A) A model with a long distance power transmission line, where it has only one damper winding in q-axis. (B) A model cited from the IEEE first benchmark model [19], [3] except that it has only one turbine, no AVR and no governor. System constants are shown in Table 1.

C. Nomenclature for Eigenanalysis

(A) **LC modes** are associated with eigenvalues due to $\Phi_d, \Phi_q, V_{cd}, V_{cq}$ dynamics. The name "LC" is derived from

TABLE I
SYSTEM CONSTANTS

model A			
$x_{ad} = 1.350$	$x_d = 1.600$	$x_f = 1.440$	$x_{kd} = 1.380$
$x_{aq} = 1.350$	$x_q = 1.600$	$x_{kq} = 1.370$	
$r_f = 0.0006$	$r_{kd} = 0.0062$	$r_{kq} = 0.0124$	
$r_a = 0.0018$			
$M_1 = 5.57$	$M_2 = 2.21$	$D_{11} = 0.5$	$D_{22} = 0.5$
$D_{12} = 1.2$	$K_{12} = 25.0$		
$x_t = 0.197$	$x_l = 1.593$	$r_l = 0.04626$	
model B			
$x_{ad} = 1.660$	$x_d = 1.790$	$x_f = 1.700$	$x_{kd} = 1.666$
$x_{aq} = 1.580$	$x_q = 1.710$	$x_{kq1} = 1.695$	$x_{kq2} = 1.825$
$r_f = 0.001$	$r_{kd} = 0.0037$	$r_{kq1} = 0.0053$	$r_{kq2} = 0.0182$
$r_a = 0.0015$			
$M_1 = 3.9827$	$M_2 = 1.8054$	$D_{11} = 0.5$	$D_{22} = 0.5$
$D_{12} = 0.6$	$K_{12} = 150.0$		
$x_t = 0.14$	$x_l = 0.56$	$r_l = 0.0315$	

the 'inductive' nature of the Φ_d, Φ_q dynamics and the 'capacitive' nature of the V_{cd}, V_{cq} dynamics. The 4 eigenvalues form 2 complex conjugate pairs that coincide at $X_c = 0$. As X_c is increased the eigenvalues separate. The eigenvalues that move away from the real axis are called *super-synchronous* and the pair that moves toward the real axis are called *sub-synchronous*. For values of X_c close to zero, the LC modes have a frequency of about 360 rad/sec. Of primary interest is the movement of the *sub-synchronous* eigenvalues since these interact with the eigenvalues of the *torsional* modes. Little work has been done on the *super-synchronous* eigenvalues but it is important to note that if these eigenvalues move to a very high ω value, then there could be possible interaction with the high frequency behavior of thyristor switches. Some examples of such harmonic induced switching time bifurcations can be found in [16].

(B) **Torsional mode** eigenvalues arise out of the variables λ_2, ω_2 associated with turbine dynamics. The torsional modes have a frequency of about 100 rad/sec.

(C) **EM (Electro-Mechanical) modes** are associated with the variables θ_1, ω_1 associated with generator dynamics. The frequency is about 6 rad/sec.

(D) **Damper windings** eigenvalues are associated with the $\Psi_{kd}, \Psi_{kq1}, \Psi_{kq2}$ variables. These eigenvalues have no imaginary components in this case.

(E) **Field winding eigenvalue**, from variable Ψ_f , has no imaginary component since there is no AVR. The Ψ_f eigenvalue is about 10^{-2} smaller than the damper eigenvalues.

IV. NONLINEAR ANALYSIS OF SSR BY AUTO94

A. Interaction between LC and torsional modes

The interaction of the subsynchronous LC mode and the torsional mode leads to 3 Hopf bifurcations and Fig.2 shows the downward movement of the LC eigenvalue as

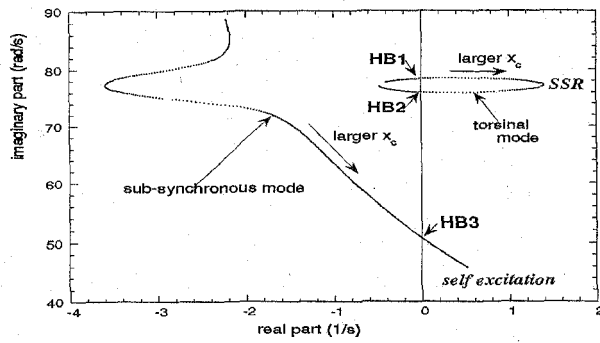
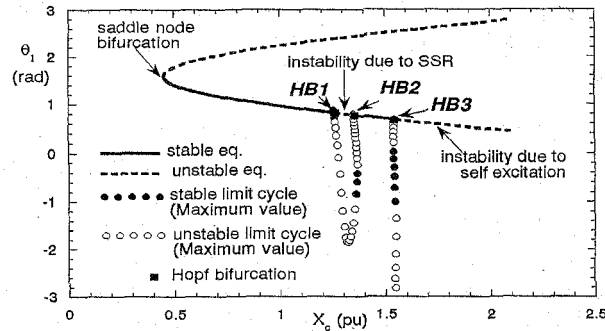
Fig. 2. behavior of eigenvalues w.r.t. X_c (model A).

Fig. 3. Bifurcation diagram (model A).

X_c is increased in model A. When the LC eigenvalue nears the ω location of the torsional eigenvalue, the latter moves to the right. Eventually, the torsional eigenvalue moves across the $j\omega$ axis. This signifies a Hopf bifurcation. In Fig.2, this is labeled *HB1*. As the LC moves further downward, it causes the torsional eigenvalue to return to the left half plane. This is the second Hopf bifurcation. In Fig.2, this is labeled *HB2*. After *HB2* the torsional dynamics remain stable. However, on further increase of X_c , the LC eigenvalue crosses the $j\omega$ axis to the right half plane and the system has no stable equilibrium points. In Fig.2, this is labeled *HB3*.

B. The Other Modes: EM, Damper and Field

The plot in Fig.2 does not show the EM, damper and field winding eigenvalues because these remain in the left half plane. Compare these results to the Hopf bifurcation case studies, where it is the EM mode that is the primary cause of nonlinear oscillations [11], [12], [13], [14].

C. The Main Bifurcation Diagram

The main bifurcation diagrams are shown in Fig.3 for model A and in Fig.4 for model B. It is observed that both figures are qualitatively same. Fig.5 is a magnification of the periodic orbit in Fig.3 detailing the sequence of supercritical Hopf, torus bifurcations, cyclic-fold and

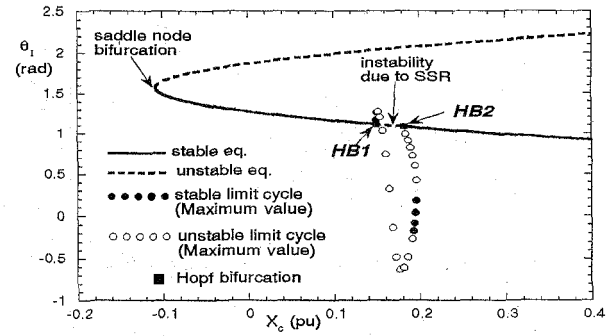


Fig. 4. Bifurcation diagram (model B).

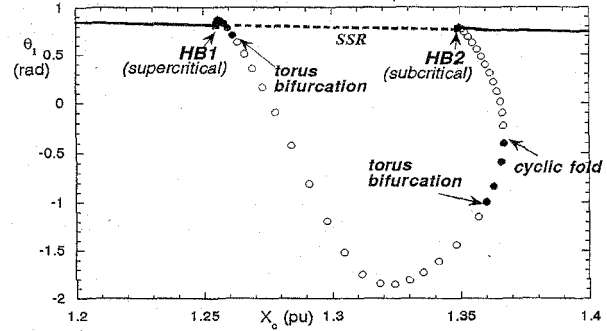


Fig. 5. Magnification of the periodic orbit (model A).

subcritical Hopf bifurcations. Fig.5 shows a surprising similarity to the 4 variable (2 machine) system with load dynamics [18]. The following comparisons can be drawn:

(I) The similarity is that the stable equilibrium branch undergoes an approximately identical sequence of bifurcations namely: Subcritical Hopf \rightarrow Cyclic Fold \rightarrow Stable Limit Cycle \rightarrow Attractor \rightarrow No Attractors \rightarrow Attractor \rightarrow Stable Limit Cycle \rightarrow Supercritical Hopf.

(II) One difference is that the bifurcations for the 2 machine system are **load-induced**, whereas the bifurcations for SSR occur by the interaction between LC and torsional eigenvalues.

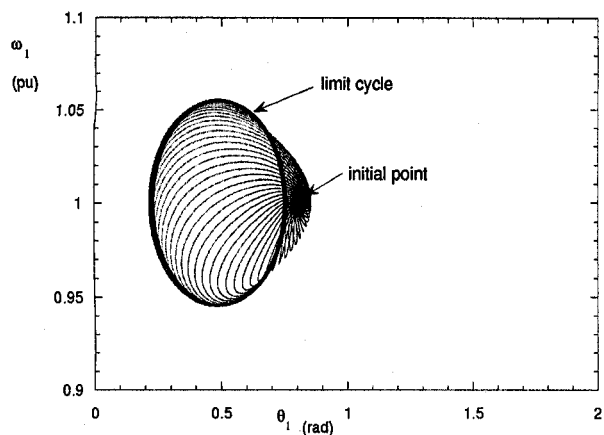
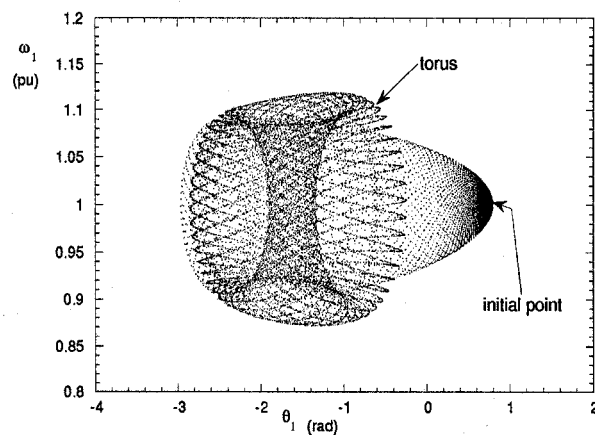
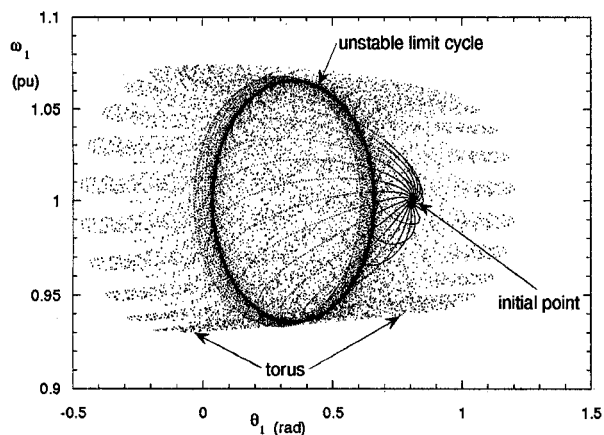
(III) Another difference is that in the 2 machine system the attractor was chaotic, whereas the attractor is a stable **torus** in the SSR system.

(IV) Whereas, the destruction of the chaotic attractor of the 2 machine system via a boundary crisis has been proved, it is still not clear how the window of *no stable attractors* for the SSR system is established.

(V) Finally, the region of instability, beyond the Hopf bifurcation point labeled *HB3* in Fig.2 does **not** exist for the 2 machine system.

D. Detail of Periodic Orbits

Fig.6 shows the supercritical Hopf in $\theta_1-\omega_1$ space for $X_c = 1.260$ of model A. After the supercritical Hopf

Fig. 6. Limit cycle at $X_c = 1.260$.Fig. 8. Torus orbit at $X_c = 1.344$.Fig. 7. Torus orbit at $X_c = 1.2625$.

bifurcation and before the cyclic fold there are bifurcations to a stable (invariant) torus at $X_c = 1.2621$ and $X_c = 1.3599$. In Figs. 7 and 8 torus orbits at $X_c = 1.2625$ and $X_c = 1.344$ are shown in θ_1 - ω_1 space, respectively.

E. Dependence of System Parameters

Among system parameters damper windings have almost same speed in response with the torsional mode. To verify some dependence of system parameters on the periodic orbits models with only one damper winding (a 1-q damper model and a 1-d damper model) were provided. Figs. 9 and 10 show the respective bifurcation diagrams. Moreover, the value of damping coefficient D_{12} , which has direct relation to the stability of torsional mode, has been varied. Figure 11 shows the corresponding results. Models with at least one damper winding in q-axis produce a branch of periodic orbits which connect the two Hopf bifurcation points. This periodic orbit surrounds the branch of unstable equilibrium points due to the SSR. Also, the damping coefficient D_{12} plays an important role to de-

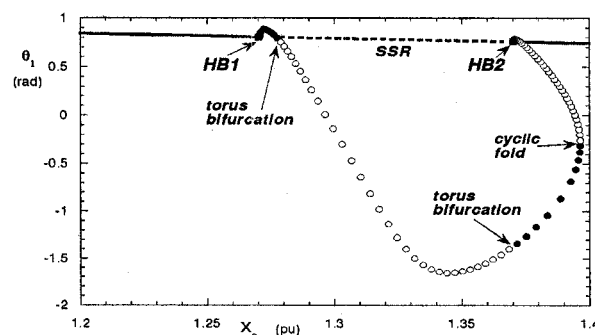


Fig. 9. Bifurcation diagram of 1-q damper model.

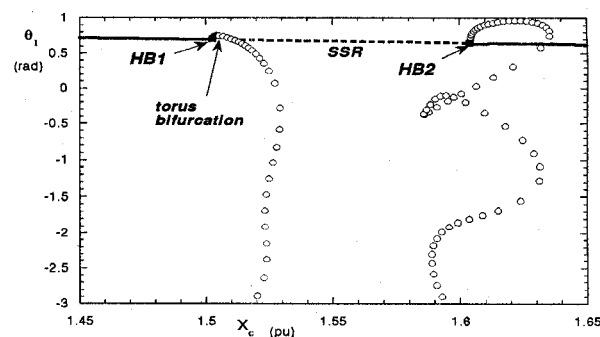


Fig. 10. Bifurcation diagram of 1-d damper model.

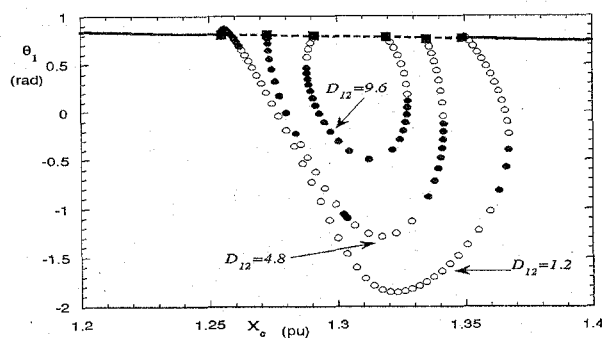


Fig. 11. Bifurcation diagram for different values of D_{12} .

termine the characteristics of the periodic orbits. A large value of D_{12} makes the periodic orbits attractive when the model has at least one damper winding in q-axis. Note that completely same results are obtained for the model B (the IEEE first benchmark model).

V. CONCLUSIONS

In this paper some strange nonlinear attractors appearing in the system with SSR phenomena has been described. In the bifurcation diagram there is a branch corresponding to periodic orbits which connects the two Hopf bifurcation points. In this branch, a torus bifurcation as well as cyclic fold have been detected. It has been observed that a damper winding in the q-axis produces this sequence of bifurcations. The damping coefficient in the turbine-rotor shaft system makes the periodic orbits attractive.

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