Performance Analysis of Unslotted Fiber-Optic Code-Division Multiple-Access (CDMA) Packet Networks

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***Abstract*— This paper examines code-division multiple-access (CDMA) techniques used in unslotted fiber-optic packet networks. Since the inherent properties and signal processing of the conventional communication channels are different from those of the fiber-optic channels, new code sequences must be constructed for fiber-optic applications. In unslotted systems, the exact solution is very difficult to obtain. Therefore, two approximation methods are presented to analyze the performance of such systems. Simulation is performed to verify the accuracy of the results.**

I. Introduction

MOST of the present research on CDMA networks has been restricted to slotted systems. It is easy to analyze slotted systems because the number of other interferers during a slot remains unchanged. An unslotted system is more robust than a slotted system since no coordination among the users is required. However, the performance analysis is much harder because the interference level varies during the reception of a packet. In fact, it varies a bit as shown in Fig. 1. The exact solution is very difficult to obtain. These difficulties stem from two causes. First, as mentioned above, the interference level is different at each bit. We must enumerate all possible states (i.e., the number of interferers in each chip). As the number of interferers and the number of bits of a packet increase, the number of states grows to an intractable value. Second, since the code sequence (i.e., signature) pattern repeats itself in every bit, the bits of a packet are strongly correlated. Even if we can enumerate all possible states, we still cannot get around this dependence problem.

To overcome the above difficulties, we present two approximation methods to evaluate the system performance. These two approximations reduce the original problem which requires finding the correlated bit-error probabilities to the final problem which only requires computing the bit-error probability for a single bit.

The rest of this paper is organized as follows. In Section II, we present some assumptions. In Section III, the performance analysis of a distributed unslotted hybrid frequency-hop/time-hop/on–off keying (FH/TH/OOK) fiber-optic CDMA packet network is presented. We consider the throughput of the network for different packet lengths and number of frequency slots. We assume that a transmitter can always find an idle receiver waiting to receive this packet (i.e., nonpaired off) [1]. In this situation, the number of successful transmissions in the network can be greater than one in a packet duration. In Section IV, some numerical results and a summary are presented. We conclude in Section V.

II. System Assumptions

The goal of our research is to analyze the performance of CDMA packet networks using code sequences with given orthogonality properties. There are several code sequences designed for use in fiber-optic CDMA [2]–[6]. We only consider $(c, d, 1)$ code sequences. The orthogonality properties of these code sequences allow us to extract useful information which is relevant to our analysis.

The format of transmitted signals in typical fiber-optic CDMA communication networks is on–off keying (OOK) [5], [6]. In the OOK format, a "1" bit is transmitted as a pulse signature pattern and a "0" bit is transmitted as an empty bit (i.e., no pulse signature pattern is transmitted). Each bit of a packet is encoded by a specific signature pattern (we call it the time-hopped (TH) pattern in the rest of the paper) which is only one bit long. Therefore, the interference due to other simultaneous transmissions is highly correlated in time-hop/on–off keying (TH/OOK) fiber-optic CDMA communication networks. This means that the probabilities of bit errors in a packet are also highly correlated.

We consider a fully connected distributed topology, in which a transmission by any node is heard by all nodes in the network. In the following analysis, we make the following assumptions.

1) Infinite population assumption. The number of users is sufficiently large such that, with finite total offered traffic, a user is idle with probability close to one. The offered traffic is a Poisson distribution with mean G (packets/packet duration).

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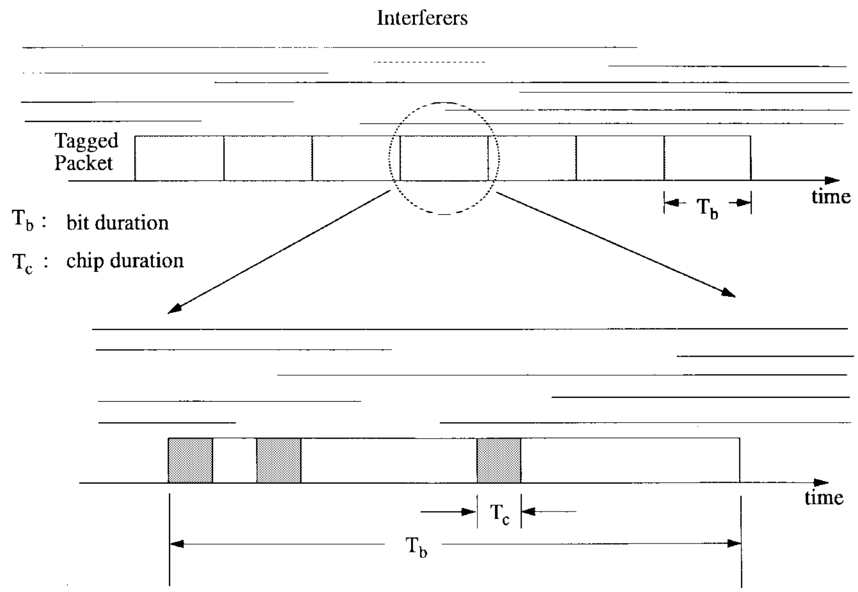

Fig. 1. Interferers overlap with the tagged packet.

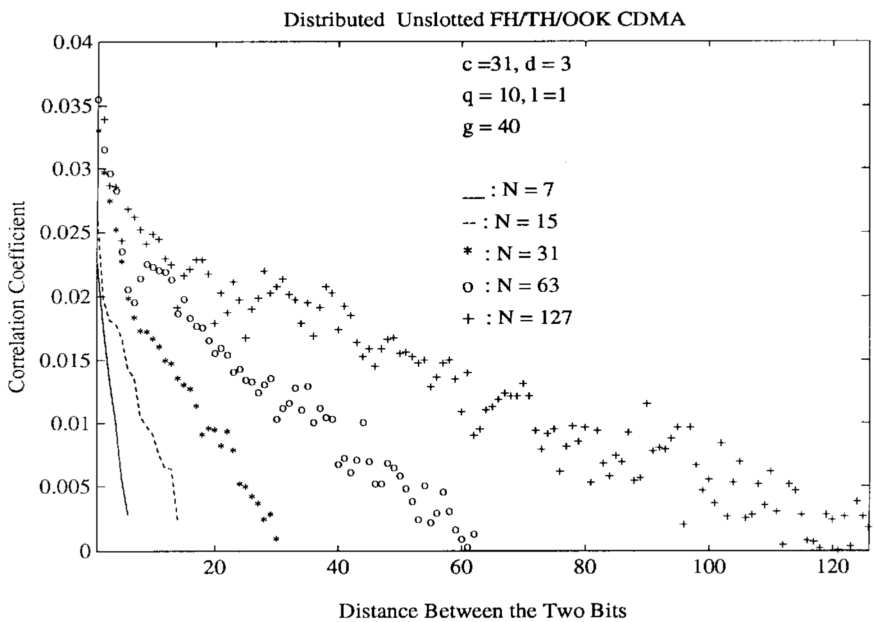

Fig. 2. Correlation coeff. of bit errors versus distance (FH/TH/OOK, $q = 10$).

2) There are l different groups which correspond to l different code sequences used in the network. Each user can come from one of these l groups.
3) Each packet consists of N bits, and takes T seconds to transmit. Each bit is divided into c chips (length of signature pattern), and the number of pulses (weight of signature pattern) in each bit is d.
4) Each node transmits with the same intensity. Without loss of generality, assume that this intensity is unity.
5) A transmission is received with equal light power by all other nodes. Since the loss of the optical fiber is very small, this assumption holds for high-speed local-area network applications.
6) An idle receiver will capture the first packet addressed to it. The capture of this packet is successful only if any other packets are received at least one chip behind the first packet, and any other transmissions during this reception are treated as interference.
7) The analysis presented does not account for thermal noise. Instead, we are primarily concerned with multiple-access interference due to other simultaneous transmissions.
8) A separate and error-free channel is used for acknowledgment. Since the acknowledgment is very short (a few bits), we can implement a very powerful forward error-correcting code to achieve error-free transmission with minimal additional bandwidth.
9) The timing information needed for synchronization is perfect.
10) For simplicity of analysis, the network is assumed to be chip synchronized, namely, the time axis is divided

into chips, and each packet can only be initiated at chip boundaries. With this assumption, our results will be pessimistic [5], [6].

III. Throughput Analysis for Distributed Unslotted Fiber-Optic FH/TH/OOK CDMA Packet Networks

In this section, we employ a hybrid scheme FH/TH/OOK[1] which combines frequency-hop (FH) CDMA and time-hop (TH) CDMA. Each packet is encoded by a two-layered encoding process. The inner layer encoding, which is the same as TH/OOK, uses a TH signature pattern (inner signature) to encode bits. FH signature patterns (outer signature) are used in the outer layer encoding process. We assume that q frequency slots can be chosen. The optical signal from a transmitter is hopped from slot to slot by changing the frequency at certain instants in time called *hop epochs*. The intervals of time between two consecutive hop epochs are called *hop intervals*. We consider only fixed-rate hopping, so all intervals are of the same length $T_{\rm hop}$. Furthermore, we assume that the *hop interval* $T_{\rm hop}$ is equal to the bit interval T_b. Besides, we assume that random frequency-hopping patterns are used. If a user wants to transmit a packet, then in each hop interval, he randomly and independently chooses one of the q available frequency slots to transmit.

The *tagged packet* method is used, namely, we arbitrarily pick a packet as our target packet, and then we evaluate the probability that the tagged packet is correctly received given that other overlapping interferers transmit during the transmission of the tagged packet. In order to find bit-error probabilities of the tagged packet, we assume that an interfering bit arrives at an instant uniformly distributed within the tagged bit. Therefore, the throughput S of the system is given by

$$S = G \cdot \mathrm{Prob}\{\text{the tagged packet is correctly received}\}.$$

Thus, in the following sections, we will find the probability that the tagged packet is correctly received.

A. Approximation Method 1 (Poisson Approximation)

In this section, we want to find the conditions under which the number of errors in a packet can be approximated by a Poisson distribution. As mentioned in the previous section, the bit errors of a packet are strongly dependent under TH/OOK systems. However, the randomizing effect of frequency hopping (FH) will reduce this dependency as q (i.e., the number of frequency slots which corresponds to the processing gain in the FH scheme) increases.

The correlation coefficient is a measure of the correlation of two random variables. Define X_α as an error indicator random variable:

$$X_\alpha = \begin{cases} 1, & \text{if bit } \alpha \text{ is incorrect} \\ 0, & \text{otherwise} \end{cases} \qquad \forall \alpha \in I$$

where $I = \{1, 2, \cdots, N\}$, and N is the number of bits per packet. Given $\alpha, \beta \in I$, and that the distance between bits α and β is $|\alpha - \beta|$, then the correlation coefficient $\rho_{\alpha\beta}$ of X_α and X_β is defined as

$$\rho_{\alpha\beta}(|\alpha - \beta|) \equiv \frac{E[(X_\alpha - E(X_\alpha))(X_\beta - E(X_\beta))]}{\sigma_\alpha \sigma_\beta} \qquad (1)$$

where $\sigma_i^2 = E[(X_i - E(X_i)^2], i = \alpha, \beta$.

Since the exact solution for the correlation coefficient is not easy to obtain, simulation results are shown instead. The Monte Carlo method is used in the simulation where the simulation time is 1 000 000 packet times. For $q = 10$, we plot the correlation coefficient of bit errors versus the distance between them in Fig. 2 for different N with $c = 31$, $d = 3$, $l = 1$, $G = 40$. The correlation coefficient decreases with increasing distance between two bits. The correlation coefficient increases as N increases under the same distance. In Fig. 3, we plot the same curves with the same parameters, except that q is changed from 10 to 30. As expected, the correlation coefficient decreases as q increases. Furthermore, the correlation coefficient decreases as the number of groups l (i.e., number of code sequences) increases (Fig. 4). Because the probability of bit error is dominated by the interference from the same group, this dominance decreases as the number of groups increases. Unfortunately, the upper bound for the correlation coefficient is not easy to obtain. This bound should be a function of q, l, c, and d.

For comparison, the correlation coefficients of bit errors for TH/OOK systems are shown in Fig. 5 for different N with $c = 31$, $d = 3$, $N = 31$, $G = 10$, $l = 1$. The correlation coefficients of TH/OOK systems are much larger than those of FH/TH/OOK systems. This indicates that the Poisson approximation cannot be applied to TH/OOK systems. From the above observation, the correlation coefficients of bit errors between two bits decreases as q increases. Uncorrelatedness does not imply independence. However, the above fact does indicate that the Poisson approximation is suitable as q increases.

1) Bound for Difference Between Actual Distribution and Poisson Distribution: The bound for the difference between the actual distribution and the Poisson distribution is derived by Chen [7], and is modified to a more usable form by Arratia *et al.* [8]. Let I be an arbitrary index set, and for $\alpha \in I$, let X_α be a Bernoulli random variable with $P(X_\alpha = 1) = 1 - P(X_\alpha = 0) = p_\alpha$. Let $W \equiv \Sigma_{\alpha \in I} X_\alpha$ and $\lambda \equiv E\{W\} = \Sigma_{\alpha \in I} p_\alpha$, $\lambda \in (0, \infty)$. For each $\alpha \in I$, suppose we have chosen $B_\alpha \subset I$ with $\alpha \in B_\alpha$; then B_α can be thought of as a *neighborhood of dependence* for α, such that X_α is independent or nearly independent of all of the X_β for $\beta \notin B_\alpha$.

Define three parameters as follows [8]:

$$b_1 \equiv \sum_{\alpha \in I} \sum_{\beta \in B_\alpha} p_\alpha \cdot p_\beta$$

$$b_2 \equiv \sum_{\alpha \in I} \sum_{\alpha \neq \beta \in B_\alpha} E\{X_\alpha X_\beta\}$$

$$b_3' \equiv \sum_{\alpha \in I} E \left| E \left\{ X_\alpha - p_\alpha \;\middle|\; \sum_{\beta \in I - B_\alpha} X_\beta \right\} \right|.$$

[1] Some prefer different names, e.g., multiwavelength, multicolor CDMA.

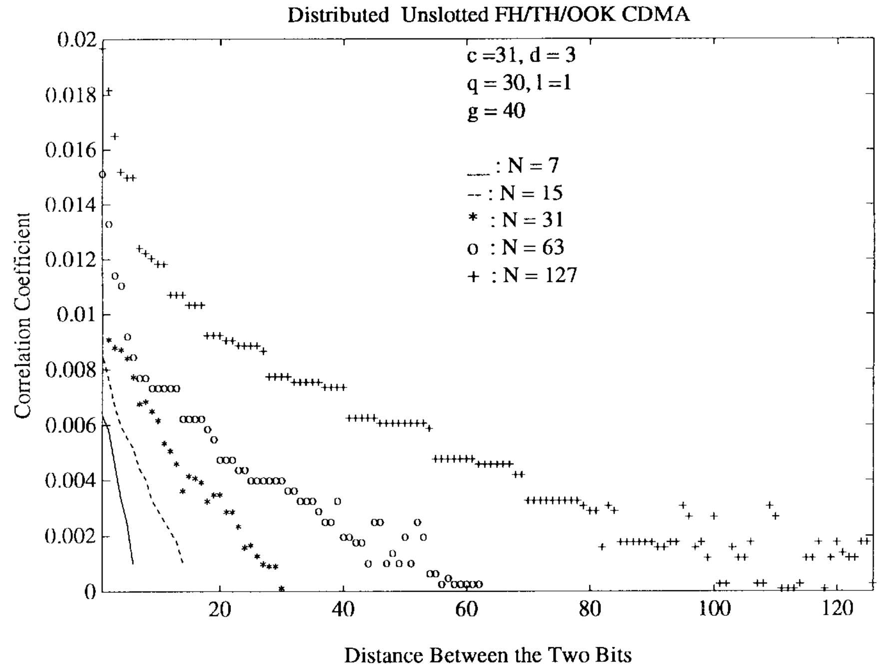

Fig. 3. Correlation coeff. of bit errors versus distance (FH/TH/OOK, $q = 30$).

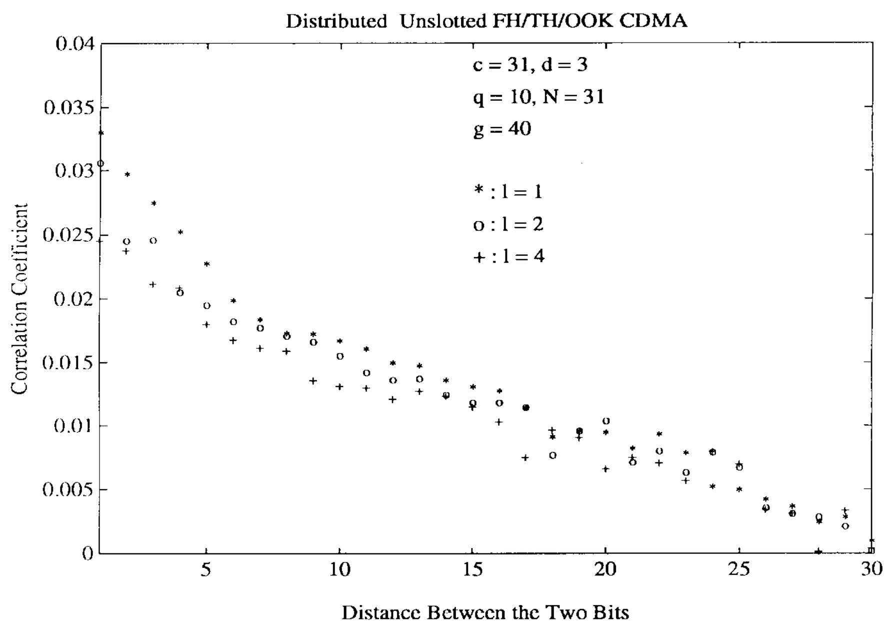

Fig. 4. Correlation coeff. of bit errors versus distance for $l = 1, 2, 4$ (FH/TH/OOK).

Quoting from [8]: "Loosely, b_1 measures the neighborhood size, b_2 measures the expected number of neighbors of a given occurrence and b_3' measures the dependence between an event and the number of occurrences outside its neighborhood."

Let Y be a Poisson random variable with mean λ. $P(Y = k) = (e^{-\lambda}\lambda^k/k!)$, $k = 0, 1, 2, \cdots$. The total variation distance (TVD), which is a measure of the difference between the actual distribution W and the Poisson distribution Y, is given by [8]

$$2 \limsup_{A \subset Z^+} |P(W \in A) - P(Z \in A)| \leq 2\left[(b_1 + b_2)\frac{1 - e^{-\lambda}}{\lambda} + b_3' \cdot \min(1, 1.4\lambda^2)\right]$$

where $Z^+ \equiv \{0, 1, 2, \cdots\}$.

By choosing a neighborhood of size one, i.e., $B_\alpha = \alpha$,

$$b_1 = \sum_{\alpha \in I} \sum_{\beta \in B_\alpha} p_\alpha \cdot p_\beta = N p_\alpha^2 = \frac{\lambda^2}{N}$$

$$b_2 = \sum_{\alpha \in I} \sum_{\alpha \neq \beta \in B_\alpha} E\{X_\alpha X_\beta\} = 0.$$

In the asymptotic case, i.e., $q \to \infty$, following a similar procedure in [9], the error contribution due to b_3' can be neglected. Therefore, the total variation distance can be bounded by (λ/N) as $q \to \infty$.

2) Derivation of p_α and λ: The analysis uses the *tagged packet* technique to compute the probability that the tagged packet is correctly received while other interferers are trans-

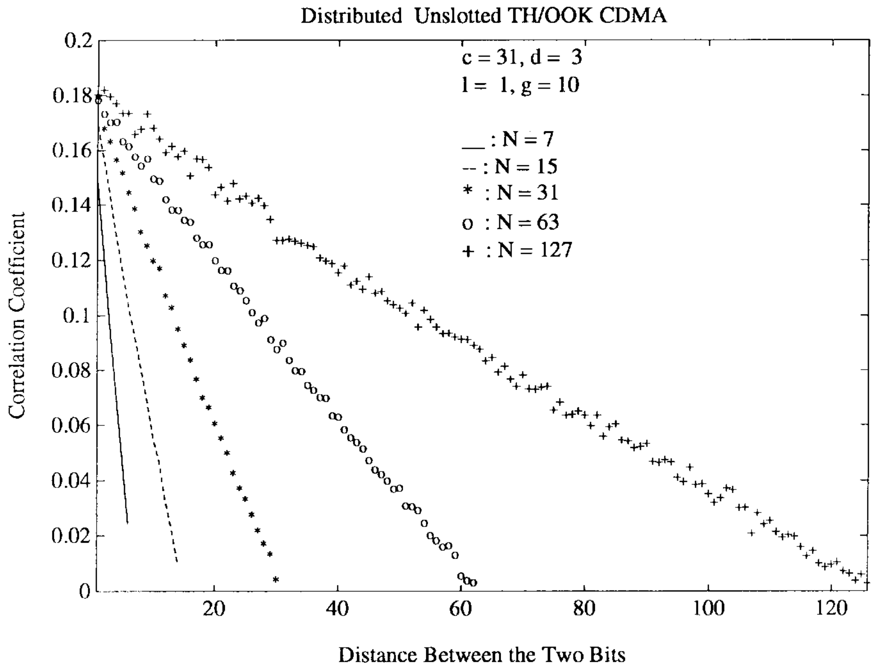

Fig. 5. Correlation coeff. of bit errors versus distance (TH/OOK).

mitting during the transmission of the tagged packet. The error probability of bit α in the tagged packet is p_α, i.e., $P(X_\alpha = 1)$. We assume that there are l groups of code sequences (i.e., signatures) used in the system.

$$\begin{aligned} p_\alpha &= P(X_\alpha = 1) \\ &= \sum_{u=1}^{l} P(X_\alpha = 1 \mid \text{ the tagged packet uses signature } u) \\ &\quad \cdot P(\text{the tagged packet uses signature } u). \end{aligned} \tag{2}$$

For simplicity, we assume that a packet is transmitted using one of l signatures with equal probability, i.e., P(the tagged packet uses signature u) $= 1/l$. Since each signature has the same orthogonality property, $P(X_\alpha = 1 \mid$ the tagged packet uses signature u) is independent of u. Without loss of generality, we assume that the tagged packet uses signature pattern 1, i.e., $u = 1$. Let E be the event that the tagged packet uses signature pattern 1. Therefore,

$$p_\alpha = P(X_\alpha = 1) = P(X_\alpha = 1 \mid E).$$

Assume that bit α is a "0" or "1" bit with probability 1/2. Let Z be the event that bit α is a "0" bit. Since bit error can occur only when bit α is a "0" bit, $P(X_\alpha = 1 \mid E) = \frac{1}{2} P(X_\alpha = 1 \mid Z, E)$.

Since the offered traffic is Poisson with average rate G, the offered traffic due to interferers arriving during the tagged bit α is also Poisson with average rate $G \cdot (N + 1/N)$, where N is the number of bits per packet. Thus,

$$P(X_\alpha = 1 \mid Z, E) = \sum_{i=0}^{\infty} P(X_\alpha = 1 \mid i, Z, E) \cdot P(i \mid Z, E) \tag{3}$$

where $P(i \mid Z, E) = (e^{-\hat{G}} \hat{G}^i / i!)$, $\hat{G} = G \cdot (N + 1/N)$.

For conciseness, we use the following shorthand notation to represent conditional probabilities. Let $P(X_\alpha = 1 \mid i, Z, E)$ represent the error probability of bit α, given i other interferers in the tagged bit α, tagged bit α is a "0" bit, and the tagged packet uses signature 1.

An interferer can hit the tagged bit α if the bit of the interferer overlapping the tagged bit has the same frequency as the tagged bit, and this interfering bit is a "1" bit. We assume that the frequency is randomly chosen by a bit from one of q frequency slots with probability $1/q$. Since the frequency of a bit and the bit being a "0" or "1" bit are independent, the probability for an interferer to hit the tagged bit is $1/2q$. Let j be the number of interferers that can hit the tagged bit α. Therefore, we obtain

$$\begin{aligned} &P(X_\alpha = 1 \mid i, Z, E) \\ &\quad = \sum_{j=0}^{\infty} P(X_\alpha = 1 \mid j, i, Z, E) \cdot P(j \mid i, Z, E) \end{aligned} \tag{4}$$

where $P(j \mid i, Z, E) = B(i, j, (1/2q))$ and $B(n, k, p) \equiv \binom{n}{k} p^k (1-p)^{n-k}$ is the probability mass function of the binomial distribution with parameters n, k, p. Since we assume that interferers can come from one of l groups with equal probability, we have

$$\begin{aligned} &P(X_\alpha = 1 \mid j, i, Z, E) \\ &\quad = \sum_{\underline{j} \in \Sigma_j} P(X_\alpha = 1 \mid \underline{j}, j, i, Z, E) \cdot P(\underline{j} \mid j, i, Z, E) \end{aligned} \tag{5}$$

where

$$\underline{j} \equiv (j_1, j_2, \cdots, j_l)$$

$$j_k \equiv \text{ number of interferers from group } k$$

$$\Sigma_j \equiv \left\{ \underline{j} \,\middle|\, \sum_{k=1}^{l} j_k = j \right\}$$

$$P(\underline{j} \mid j, i, Z, E) = \binom{j}{j_1, j_2, \cdots, j_l} \left(\frac{1}{l}\right)^j.$$

From the orthogonality property of the signatures, interferers from different groups can overlap the tagged signature at only one chip position. However, severe damage occurs when the interferers come from the same group as the tagged signature. If these interferers initiate at the first chip position of the tagged signature, then they will overlap all of the d chip positions of the tagged signature. Let B be the event that an interferer initiates at the first chip position of the tagged signature. Obviously, $P(B) = (1/c)$. We observe that $P(X_\alpha = 1 \mid \underline{j}, j, i, Z, E) = P(X_\alpha = 1 \mid \underline{j}, Z, E)$ because $\underline{j}$ provides all of the information needed to compute the bit-error probability.

Let H be the event that at least one interferer is from the first group (i.e., the same group as the tagged packet). The probability of H is given by $1 - (1 - (1/c))^{j_1}$. Therefore, we have

$$\begin{aligned} P(X_\alpha = 1 \mid \underline{j}, Z, E) &= P(X_\alpha = 1 \mid H, \underline{j}, Z, E) \cdot P(H \mid \underline{j}, Z, E) \\ &\quad + P(X_\alpha = 1 \mid \bar{H}, \underline{j}, Z, E) \cdot P(\bar{H} \mid \underline{j}, Z, E) \qquad (6) \end{aligned}$$

where

$$P(H \mid \underline{j}, Z, E) = 1 - \left(1 - \frac{1}{c}\right)^{j_1}$$

$$P(\bar{H} \mid \underline{j}, Z, E) = \left(1 - \frac{1}{c}\right)^{j_1}$$

$\bar{H}$ is the complement of H.

Since the threshold of the threshold detector is set to d, we obtain

$$P(X_\alpha = 1 \mid H, \underline{j}, Z, E) = 1 \cdot U(j_1 - 1) \qquad (7)$$

$$P(X_\alpha = 1 \mid \bar{H}, \underline{j}, Z, E) = P(I_t \geq d \mid \bar{H}, \underline{j}, Z, E) \qquad (8)$$

where $U(t)$ is the step function of t and I_t is the accumulated interference level at the input of the threshold detector.

Let ξ_k be the accumulated interference level at the input of the threshold detector due to the interferers from group k, where $k \in \{1, 2, \cdots, l\}$; then $I_t = \Sigma_{k=1}^{l} \xi_k$. Therefore, we have

$$P(\xi_k \mid \bar{H}, \underline{j}, Z, E) = B(j_k, \xi_k, P_{vk}) \qquad (9)$$

where

$$P_{vk} = \begin{cases} \dfrac{d(d-1)}{c-1}, & \text{if } k = 1 \\ \dfrac{d^2}{c}, & \text{if } k \neq 1. \end{cases}$$

Since $\{\xi_k, k = 1, 2, \cdots, l\}$ are independent random variables, we have

$$\begin{aligned} P(I_t \mid \bar{H}, \underline{j}, Z, E) &= P(\xi_1 \mid \bar{H}, \underline{j}, Z, E) * P(\xi_2 \mid \bar{H}, \underline{j}, Z, E) \\ &\quad * \cdots * P(\xi_l \mid \bar{H}, \underline{j}, Z, E) \qquad (10) \end{aligned}$$

where $*$ denotes the convolution operator.

Because the number of errors in a packet $W = \sum_{\beta}^{N} X_\beta$, and $X_1, X_2, \cdots, X_N$ have the same probability distribution, we have

$$\lambda = E\{W\} = \sum_{\beta=1}^{N} E\{X_\beta\} = N \cdot p_\alpha \qquad (11)$$

where α is an arbitrary bit of a packet.

Now, we can use a Poisson distribution for a random variable Y with the same mean λ to approximate the original distribution of W, which is the sum of N dependent random variables. Since no error-correcting codes are employed, the probability that the tagged packet is correctly received is given by $e^{-\lambda}$.

B. Approximation Method 2

In the first approximation method, we assume that the number of interferers in each bit is the same. Actually, given the number of interferers for a tagged packet, the number of interferers in each bit is different. Furthermore, some interferers may start transmission during the tagged packet, and some may leave during the tagged packet. In this section, we will consider the distributions of these starting (or final) and leaving (or initial) interferers. This concept is based on the paper of Tarr *et al.* [10]. From these initial and final interferer distributions, we can find the expected value of the bit-error probability for each bit in a packet given the total number of interferers and the number of initial interferers in the tagged packet.

First, we divide I_T total interferers transmitting during the tagged packet into I_i initial interferers and I_f final interferers. Define

$$\begin{aligned} i_j &\equiv \{\text{number of initial interferers in } j\text{th bit of the tagged packet}\} \\ f_j &\equiv \{\text{number of final interferers in } j\text{th bit of the tagged packet}\}. \end{aligned}$$

From [10], we obtain

$$P(i_{j+1} = m - k \mid i_j = m, I_i, I_T) = B\left(m, k, \frac{1}{N - j + 1}\right) \qquad (12)$$

$$\begin{aligned} P(i_{j+1} = k \mid I_i, I_T) &= \sum_{m=0}^{I_i - k} P(i_{j+1} = k \mid i_j = k + m, I_i, I_T) \\ &\quad \cdot P(i_j = k + m \mid I_i, I_T) \qquad (13) \end{aligned}$$

with initial condition $P(i_1 = I_i) = 1$ and

$$P(f_{j+1} = m + k \mid f_j = m, I_i, I_T) = B\left(I_f - m, k, \frac{1}{N - j}\right) \qquad (14)$$

$$\begin{aligned} P(f_{j+1} = k \mid I_i, I_T) &= \sum_{m=0}^{k} P(f_{j+1} = k \mid f_j = k - m, I_i, I_T) \\ &\quad \cdot P(f_j = k - m \mid I_i, I_T) \qquad (15) \end{aligned}$$

with initial condition $P(f_0 = 0) = 1$.

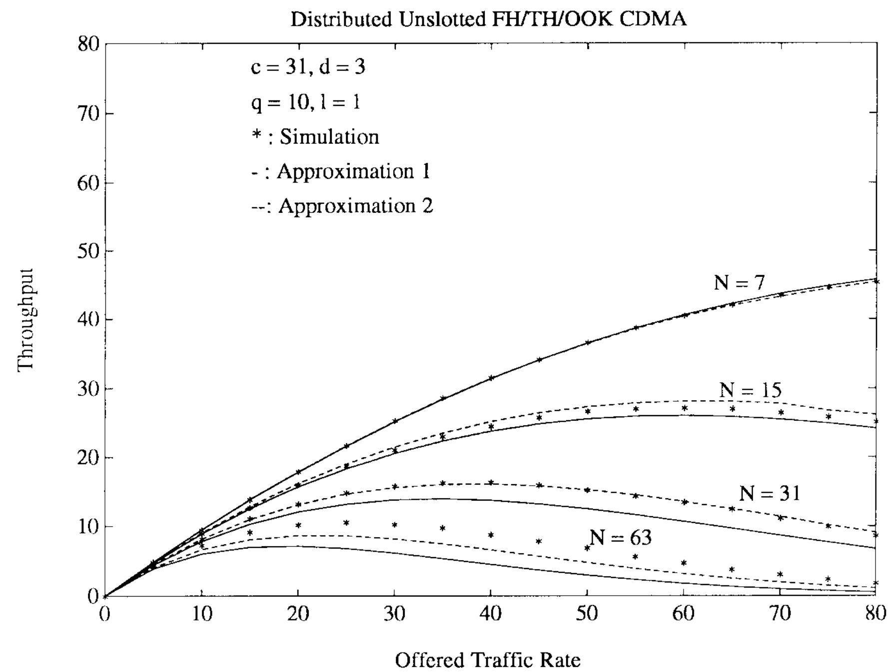

Fig. 6. Throughput versus offered traffic rate for different N's.

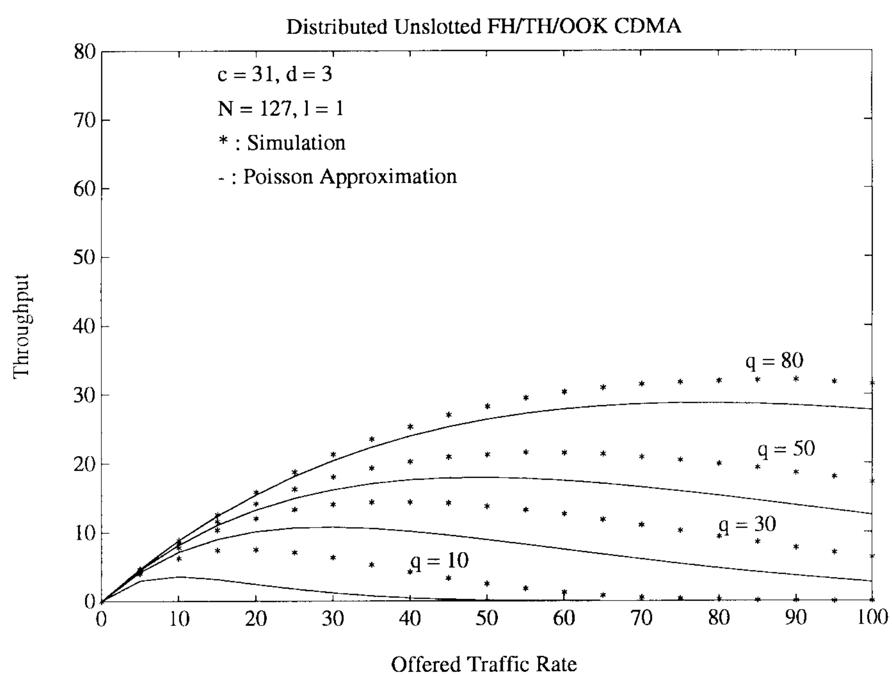

Fig. 7. Throughput versus offered traffic rate for different q's.

Given I_i and I_T, i_j and f_j are independent, then the distribution of the total number of interferers in each bit of the tagged packet is given by

$$P(t_j = k \mid I_i, I_T) = \sum_{m=0}^{k} P(i_j = m \mid I_i, I_T) \cdot P(f_j = k - m \mid I_i, I_T). \quad (16)$$

Therefore, the expected values of bit-error probabilities for each bit j of the tagged packet given I_i and I_T are

$$P(X_j = 1 \mid I_i, I_T) = \sum_{m=0}^{I_T} P(X_j = 1 \mid m, I_i, I_T) \cdot P(t_j = m \mid I_i, I_T) \quad j = 1, 2, \cdots, N. \quad (17)$$

We assume that an interferer is either an initial interferer or a final interferer with equal probability. Besides, to simplify the

TABLE I
SIMULATION VERSUS APPROX. METHODS FOR $c = 31$, $d = 3$, $q =$, $l = 1$

N	Offered Traffic Rate	Total Variation Distance	Throughput		
			Simulation	Approx. 1	Approx. 2
7	10	0.007116	9.46307	9.4509	9.4576
	20	0.010710	17.8795	17.834	17.859
	40	0.011182	31.5429	31.391	31.438
	80	0.015944	45.3963	45.795	45.424
15	10	0.038326	8.96680	8.8601	9.0037
	20	0.059147	15.9999	15.644	16.136
	40	0.065284	24.5253	23.796	25.177
	80	0.031830	25.1205	24.207	26.151
31	10	0.132706	8.19793	7.7871	8.1273
	20	0.176125	13.2019	12.038	13.087
	40	0.187122	16.2987	13.675	16.053
	80	0.099751	8.67253	6.7635	9.1075
63	10	0.326620	7.22800	6.0152	6.6314
	20	0.416490	10.1618	7.1283	8.6366
	40	0.330076	8.78667	4.5157	6.5918
	80	0.230794	1.88533	0.5280	1.1435
127	10	0.708152	6.29087	3.5892	4.4491
	20	0.703230	7.57373	2.4994	3.8277
	40	0.506468	4.34213	0.4924	1.1667
	80	0.405522	0.25173	0.0032	0.0211

TABLE II
SIMULATION VERSUS APPROX. METHODS FOR $c = 31$, $d = 3$, $q = 10$, $l = 2$

N	Offered Traffic Rate	Total Variation Distance	Throughput		
			Simulation	Approx. 1	Approx. 2
7	10	0.004654	9.72903	9.7137	9.7051
	20	0.007358	18.8225	18.783	18.748
	40	0.007408	34.2757	34.231	34.046
	80	0.008014	50.1699	50.369	49.648
15	10	0.022583	9.45553	9.3966	9.4685
	20	0.038316	17.6970	17.483	17.738
	40	0.051789	29.2641	28.650	29.424
	80	0.026816	30.2165	29.685	31.029
31	10	0.079819	9.02083	8.7930	8.9789
	20	0.126407	15.9096	15.146	15.787
	40	0.150857	22.1848	20.069	21.890
	80	0.098028	12.5579	10.310	12.877
63	10	0.225209	8.44247	7.6997	8.0767
	20	0.316332	13.6723	11.368	12.521
	40	0.320163	14.2819	9.8476	12.206
	80	0.215358	3.12267	1.2438	2.3110
127	10	0.521243	7.82963	5.9040	6.5497
	20	0.658456	11.3698	6.4036	7.9312
	40	0.491185	8.02013	2.3711	3.9242
	80	0.376431	0.44586	0.3130	0.0884

analysis, we incorrectly assume that the bit errors among the tagged packet are independent. Since no error-correcting codes are used, the probability that the tagged packet is correctly received is given by

$$P(\text{the tagged packet is correctly received}) = \sum_{I_T=0}^{\infty} \sum_{I_i=0}^{I_T} \prod_{j=1}^{N} (1 - P(X_j = 1 \mid I_i, I_T) \cdot P(I_i \mid I_T)) \cdot P(I_T) \quad (18)$$

where

$$P(I_i \mid I_T) = B\left(I_T, I_i, \frac{1}{2}\right)$$
$$P(I_T) = \frac{e^{-2G}(2G)^{I_T}}{I_T!} \qquad I_T = 0, 1, 2, \cdots.$$

The above "$2G$" is due to the Poisson distributions for the number of initial and final interferers in the tagged packet, each with mean G. We can follow the same technique as in the previous section to derive $P(X_j = 1 \mid I_i, I_T)$, where $j = 1, 2, \cdots, N$.

IV. NUMERICAL EXAMPLES AND SUMMARY

In this section, we discuss some numerical results on the accuracy of these two approximation methods and the throughput performance of unslotted CDMA networks. Since the exact solution is not available, simulation is performed to verify the accuracy of the approximations. Two methods are used to measure the accuracy. In the first method, we measure the total variation distance between approximation 1 (i.e., Poisson approximation) and simulation results on the distribution of the number of errors in a packet. In the second method, we measure the differences in throughput between the simulation results and that due to approximations 1 and 2.

Fig. 6 shows the throughput versus offered traffic rate for different N's, with parameters $c = 31$, $d = 3$, $q = 10$, $l = 1$. For each N, the results of approximation 2 are closer to the simulation results than those of approximation 1. However, the approximation and the simulation results deviate very little as N increases. Table I shows the total variation distance between approximation 1 and the simulation. Besides, they also show the throughputs for the approximations and simulation. The total variation distance increases as N increases, which is not expected. The reason is not obvious. However, from Section III-A, we observe that the correlation coefficient of bit errors increases as N increases for the same distance between 2 bits. This may be because the effect of N dominates the effect of λ on the total variation distance.

As l (i.e., the number of signatures) increases, the approximation methods improve (cf. Tables I–III). This is expected. Since the contribution to the correlation of bit errors stems mainly from the interference of the same signature group, as the number of signature groups increases, the error contribution from the same group decreases. Approximation 2 is better than approximation 1 in most cases. This is because approximation 2 considers the distribution of the number of interferers in each bit. On the other hand, approximation 1 (Poisson

TABLE III
SIMULATION VERSUS APPROX. METHODS FOR $c = 31$, $d = 3$, $q = 10$, $l = 4$

N	Offered Traffic Rate	Total Variation Distance	Throughput		
			Simulation	Approx. 1	Approx. 2
7	10	0.002387	9.85123	9.8480	9.8308
	20	0.004646	19.2963	19.273	19.200
	40	0.004463	35.6961	35.678	35.337
	80	0.014704	51.7461	52.327	50.265
15	10	0.012563	9.70756	9.6770	9.6942
	20	0.022203	18.5912	18.474	18.542
	40	0.036049	31.6908	31.308	31.619
	80	0.030196	32.8581	32.213	33.175
31	10	0.045153	9.46693	9.3440	9.4379
	20	0.077918	17.4486	16.974	17.317
	40	0.101738	25.4938	24.108	25.326
	80	0.093757	14.4706	12.207	14.767
63	10	0.131433	9.11387	8.7119	8.9051
	20	0.206560	15.7576	14.329	15.019
	40	0.257081	17.9031	14.294	16.311
	80	0.192544	3.74773	1.7531	3.0648
127	10	0.314007	8.74747	7.5731	7.9679
	20	0.474125	13.7320	10.212	11.403
	40	0.413457	10.5177	5.0254	6.9765
	80	0.350075	0.53466	0.0036	0.1592

TABLE IV
SIMULATION VERSUS POISSON APPROX. FOR $c = 31$, $d = 3$, $l = 1$, $N = 127$

q	Offered Traffic Rate	Total Variation Distance	Throughput	
			Simulation	Poisson
10	10	0.708152	6.29089	3.5892
	20	0.703230	7.57373	2.4994
	40	0.506468	4.34213	0.4924
	80	0.405522	0.251733	0.0032
30	10	0.208599	7.79068	7.1119
	20	0.279237	12.0940	10.114
	40	0.305409	14.5061	10.152
	80	0.236916	9.40800	4.7837
50	10	0.102966	8.4546	8.1499
	20	0.151033	14.2647	13.287
	40	0.191188	20.3301	17.641
	80	0.177668	20.0067	15.336
80	10	0.047013	8.92707	8.7993
	20	0.077785	15.9370	15.488
	40	0.105190	25.3937	23.991
	80	0.118728	31.9576	28.711

approximation) assumes that the number of interferers in each bit is the same.

Unfortunately, the computation for approximation 2 is very complicated. The CPU time increases exponentially as N

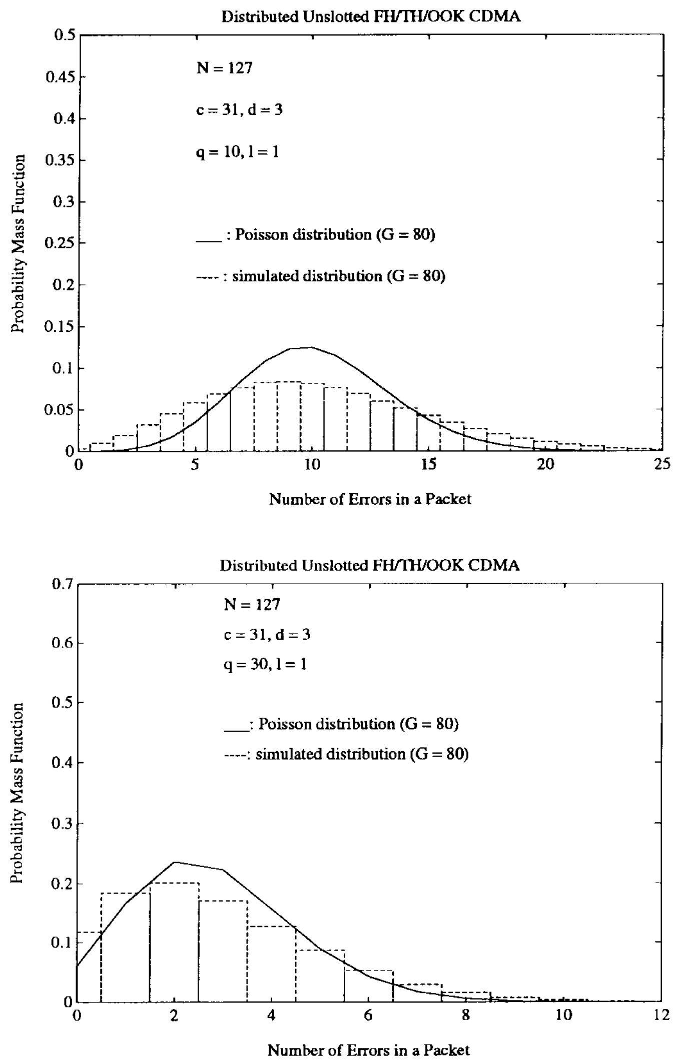

Fig. 8. Probability mass functions for $q = 10$ and 30.

increases. Therefore, approximation 2 is not practical for large N. In our case, for $N = 127$, the CPU time on a SUN SPARC 400 system is about 100 h! However, the computation for approximation 1 (Poisson approximation) is very simple and fast (CPU time of 3 min for the same parameters; the CPU time for the simulation is about 130 h). Furthermore, it does not depend on N. As q increases, the difference between the Poisson approximation and the simulation decreases very quickly, as shown in Fig. 7 and Table IV for the same parameters as above, except that q varies from 10 to 80. Fig. 8 shows the probability mass functions for $q = 10$ and 30, respectively. As q increases, it shows that the Poisson distribution is a good approximation. From the above discussion, approximation 2 can be used in systems with small N, and approximation 1 is suitable for systems with large q and large N. For example, the standard ATM cell is 53 octets, i.e., 424 bits. Its performance can be evaluated using approximation 1.

V. CONCLUSION

In this paper, we have analyzed the performance of unslotted fiber-optic packet networks. In unslotted systems, the exact solution is very difficult to obtain. Therefore, two approximation methods are presented. Furthermore, simulation is performed to verify the accuracy of the results. The numerical results show that approximation 2 can be used in systems with small N, and approximation 1 is suitable for systems with large q and large N.

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