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Enhancing Preservice Teachers' Use of Dialogic Teaching and Dynamic Visualizations in Mathematics Classes: Bridging the Knowing–Doing Gap

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Abstract

Talking productively with students and sufficiently integrating technology into mathematics classrooms have long been regarded as two hurdles for mathematics teachers. To enhance preservice mathematics teachers' dialogic teaching skills and integration of GeoGebra-scaffolded dynamic visualizations, this study proposed and examined the effectiveness of a video-based professional development (PD) approach supported by a digital platform called Classroom Discourse Analyzer. Adopting the QUAL-quan method, one preservice teacher was selected as a representative case. The results showed that the PD approach effectively improved the preservice teacher's declarative knowledge and teaching practice of using lower-order talk moves. The preservice teacher's self-awareness and self-reflection on dialogic teaching informed her future practices. Furthermore, the preservice teacher was able to integrate GeoGebra-scaffolded dynamic visualizations into the instructions with different pedagogical decisions, reflecting how she reacted to student errors and the affordances and constraints of dynamic visualizations. This study suggests that the theoretically robust PD approach can serve as pioneering work in simultaneously promoting dialogic teaching and GeoGebra-scaffolded dynamic visualizations among preservice mathematics teachers. It also demonstrates the potential of integrating digital technologies to design hybrid PD programs to enhance preservice teachers' self-reflection and facilitate improvement in their future teaching practices.

Keywords Dialogic teaching · Dynamic visualization · Preservice mathematics teachers · Video-based professional development

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Introduction

Engaging students in mathematical discourse has proved to be beneficial for meaning-making and knowledge construction in mathematics classrooms (Hundeland et al., 2020; Planas & Schütte, 2018; Sfard, 2012). Mathematical discourse can be characterized either in an authoritative pattern where teachers dominate the discourse (Aguiar et al., 2010; Scott et al., 2006) or in a dialogic pattern where the classroom talk is more student-centered (Alexander, 2018; Bossér & Lindahl, 2021). Though realizing the need to move beyond the teacher-dominant discourse, it remains challenging for teachers to effectively integrate dialogic teaching in their teaching instructions (Chen, Chan, et al., 2020; Michaels et al., 2008; Resnick et al., 2018). According to Alexander (2018), dialogic teaching is a pedagogy to open up students' speaking and listening. It includes an interlocking set of permissive repertoires through which teachers, guided by principles, energize their own and their students' talk. Indeed, researchers have conceptualized principles to scaffold dialogic teaching. For example, Michaels et al. (2008) and Resnick et al. (2010) introduced the eight principles such as "say more," i.e., the teacher invites a student to elaborate on their ideas, in the academically productive talk (APT) framework to facilitate dialogic teaching.

In addition, mathematics teachers are expected to be digitally literate in teaching according to the skills required in 21st-century classrooms (Tondeur et al., 2017; Urbani et al., 2017). Among the relevant technology-supported tools, dynamic visualization tools are useful in mathematics teaching and learning, as they support the visualization of abstract mathematical concepts, norms, or reasoning procedures (Young, 2017). Mathematics teachers can be well equipped with skills in using dynamic visualizations if they want to make their teaching interactive and technologically supported. Regarding preservice teachers, because of their lack of teaching experience and difficulty integrating educational technologies into their teaching (Angeli & Valanides, 2009; Nolan et al., 2015), they may encounter more challenges than experienced teachers during their teaching practice. Therefore, high-quality professional development (PD) programs are important for preservice mathematics teachers (PMTs) to be better prepared for their future teaching careers. However, there is a lack of evidence regarding PD programs for PMTs that focus on both dialogic teaching and the integration of dynamic visualization tools. To address this gap, this case study was conducted to investigate how a video-based PD program can support PMTs' ability to implement dialogic teaching and dynamic visualizations in mathematics classrooms during their teaching practicum.

Theoretical Framework

Dialogic Teaching

Effective dialogue between teachers and students is crucial in the classroom because it facilitates meaningful classroom teaching (Kathard et al., 2015). The scholarly discourse surrounding the tension between authoritative teaching and dialogic teaching has been ongoing for a considerable period (Aguiar et al., 2010; Scott et al., 2006). There remains a lack of consensus among researchers regarding the precise definitions of authoritative and dialogic teaching (Kim & Wilkinson, 2019). For instance, Scott et al. (2006) conducted an examination of student questioning within small groups led by in-service science teachers in Brazilian secondary school classrooms. They emphasized that a teacher-centered classroom environment typifies authoritative teaching, characterized by teachers dominating classroom discourse and giving minimal attention to students' contributions (Scott et al., 2006). Authoritative teaching frequently adheres to a format known as the initiation-reply-evaluation (IRE) model, as described by Cazden (1988) and Mehan (1979). Within the IRE framework, teachers often commence interactions by posing questions or soliciting responses from students (initiation); students then provide their responses to the teacher's prompt (reply); and finally, the teacher assesses the students' replies (evaluation). Additionally, Aguiar et al. (2010) expanded on this concept by proposing four distinctive authoritative interaction patterns: the IRE model, a closed chain of interactions involving sequences of I-R-P-R-P-R…E, where prompts (P) from the teacher elicit further responses, open chains of interactions lacking the final evaluation of the closed chain, and the Question-and-Answer pattern where teachers respond to questions raised by students.

In contrast, dialogic teaching leverages the power of talk to enhance students' thinking, learning, and problem-solving (Alexander, 2018; Kim & Wilkinson, 2019). It is a more student-centered pedagogical approach and values different voices and ideas (Aguiar et al., 2010; Bossér & Lindahl, 2021). A recent examination by Bossér and Lindahl (2021) explored the process of constructing meaning in secondary science classrooms in Sweden. They highlighted that, in contrast to the IRE-driven authoritative teaching, dialogic teaching embraced a range of viewpoints and emphasized the importance of acknowledging student contributions. To address the limitations of authoritative teaching and promote dialogic teaching, Michaels et al. (2008) and Resnick et al. (2010) introduced the APT framework. This framework features eight guiding principles known as *talk moves*, organized into four objectives: elaboration (encouraging students to expand on their ideas) with talk moves of "say more" and "revoice"; reasoning (prompting students to strengthen their reasoning) with talk moves "press for reasoning" and "challenge"; listening (urging students to listen to one another actively) with the talk move "restate"; and thinking with others (encouraging students to engage in active thinking with others) with talk moves including "agree/disagree," "add on," and "explain others." The eight APT talk moves are detailed in Table 1. For example, instead of simply evaluating the correctness of a student's response, teachers can encourage other students to assess the reasoning behind the answer using the "agree/disagree" talk move and asking, "Do you agree/disagree with her idea?" It should be noted that, in the current study, "revoice" broadly refers to a teacher repeating/rephrasing what a student has said. This definition differs slightly from the one put forward by Michaels, Resnick, and colleagues, where "revoice" involves teachers providing students with

Table 1 Definitions and examples of the eight APT talk moves

Talk move	Definition	Example
Say more	The teacher invites a student to elaborate on their ideas	"Can you give us a specific example of that?"
Revoice	The teacher repeats/rephrases what a student has just said	"Let me repeat what you just said…" "Were you saying…?"
Press for reasoning	The teacher invites a student to demonstrate their reasoning	"Why do you think that?"
Challenge	The teacher challenges a student's idea or puts forward a counterexample	"Does this always hold true?" "What if the denominator equals zero?"
Restate	The teacher asks a student to repeat or rephrase the ideas of another student	"Can you repeat what he just said?"
Agree/disagree	The teacher encourages a student to assess another student's reasoning by agreeing or disagreeing	"Do you agree/disagree with her idea?"
Add on	The teacher encourages a student to expand on another student's views	"Who can add an example to demonstrate this idea?"
Explain other	The teacher prompts a student to explain their understanding of another student's statement	"Why do you think she mentioned this?"

Note. Adapted from Michaels et al. (2008) and Resnick et al. (2010)

additional opportunities to verify their ideas. The original definition may pose additional cognitive challenges for PMTs, as it might appear like "say more" and "press for reasoning," and such similarities may cause confusion for PMTs when they first embark on their teaching practice (Nolan et al., 2015). The eight APT talk moves are widely acknowledged as essential promoters of academic learning and student engagement across various subjects and educational levels (Michaels et al., 2008; Tao & Chen, 2023). Despite recognizing the significance of dialogic teaching, many teachers encounter difficulties in effectively implementing it. Both in-service and preservice teachers in PD programs regard dialogic teaching as a new avenue (Hauk et al., 2023). The structured APT talk moves offer teachers specific tools to address these obstacles and facilitate a transition from rote recitation to more sophisticated reasoning and evidence-based dialogues (Michaels et al., 2008; Resnick et al., 2010).

Prior research has demonstrated that dialogic teaching is linked to teachers' professional noticing and responsiveness. Teacher noticing pertains to what teachers observe during instructional interactions, how they make sense of the observed phenomena, and how they utilize this information to guide their instructional decisions (van Es et al., 2017). This serves as the foundation for dialogic teaching. When teachers are attuned to the contributions and needs of students in a dialogic environment, they can better facilitate students' meaningful discussions. Like teacher noticing, responsiveness is another fundamental aspect of dialogic teaching and refers to how much a teacher acknowledges, solicits, takes up, or elaborates on student thinking during the discourse (Bishop, 2021; Jaber, 2021). In contrast to the low cognitive engagement seen in monologic patterns such as IRE (Mehan & Cazden, 2015), Bishop (2021) suggested that when teachers engage in dialogic teaching by being responsive to the cognitive demands that students are capable of, it can enhance their academic performance.

Besides, the potential benefits of dialogic teaching for preservice teachers in the PD programs have been acknowledged. By incorporating dialogic teaching methods, preservice teachers can foster more interactive and engaging classroom environments that encourage student participation and discussion (Gomez Marchant et al., 2021; Lee & Kim, 2016). Nama et al. (2023) pointed out that dialogic teaching incorporates a recursive and co-constructive approach; it can promote students' investigation, justification, and evaluation of their mathematics meaning-making. Consequently, equipping preservice teachers with training in dialogic teaching within PD programs can enhance their readiness and proficiency in comprehending their future students' cognitive processes during instructional practices (Gomez Marchant et al., 2021; Lee & Kim, 2016). Lee and Kim (2016) also highlighted that introducing dialogic teaching during preservice training can have a lasting impact on teachers' practice, as they carry forward the skills and understanding developed during their PD programs. By understanding the importance of the effects of dialogic teaching for preservice mathematics teachers, we can provide more effective PD programs to facilitate pre-service teachers' dialogic teaching skills.

Additionally, as highlighted by Kim and Wilkinson (2019), culture encompasses both a disposition towards knowledge and understanding, as well as social interactions and relationships with others. Research findings from Brazil and Sweden indicated a scarcity of dialogic teaching practices in science classrooms, evident in both group and whole-class discussions (Bossér & Lindahl, 2021; Scott et al., 2006). Similarly, this issue is prevalent in East Asian mathematics classrooms, where the culture of collectivism often fosters authoritative teaching approaches (Leung, 2001), leading to students being more inclined to remain silent or respond collectively during mathematics lessons (Xu & Clarke, 2013, 2019).

Consequently, this study aims to support PMTs in developing dialogic teaching skills based on the structured talk moves outlined in the APT framework during their teaching practicum.

Dynamic Visualization

Dynamic visualization refers to a series of interactions with diverse visual representations (Battista et al., 2018). In contrast with static forms such as non-interactive images or diagrams, dynamic visualizations are characterized by their step-by-step and sequential changes (Wu et al., 2015). A synthesis of 46 studies by Castro-Alonso et al. (2019) revealed that dynamic visualizations have a small but significant advantage over static ones ($g = 0.23$). For instance, Hsu and Hsu (2025) conducted a quasi-experimental study with 244 fifth-grade students in Taiwan to investigate their learning of geometric area concepts using either dynamic or static representations. They found that students using dynamic representations that, for example, allowed them to elongate the base in a fixed area and observe the changes in a parallelogram experienced lower extraneous cognitive load than those using static representations. Similarly, dynamic visualizations are more effective for learning the concept of function in secondary schools than static representations (Kohen et al., 2022; Rolfes et al., 2020). Both studies emphasized that dynamic visualizations enable students to observe immediate effects on graphs and equations after adjusting the parameters and starting a function animation that runs automatically from beginning to end, which can enhance problem-solving skills. In contrast, static visualizations lack these benefits.

A wide range of dynamic geometric systems have been developed to facilitate dynamic visualizations in mathematics learning. Studies have extensively investigated the relationship between the use of dynamic geometric systems and mathematics learning (for reviews, see Li & Ma, 2010; Young, 2017; Zhang et al., 2025). Young (2017) conducted a second-order meta-analysis over the past 30 years and found that technology-enhanced instruction had a positive impact on student achievement in mathematics. This finding is consistent with that of Li and Ma (2010), who found that integrating computer technologies such as dynamic geometric systems into a constructivist approach was associated with better mathematics learning outcomes for K-12 students. A recent meta-analysis conducted by Zhang et al. (2025) specifically examined the use of GeoGebra as a teaching tool for geometry and calculus. The results showed that using GeoGebra-supported dynamic visualizations for a duration of four weeks with fewer than 50 participants was more likely to have a significant impact on student achievement in mathematics than other conditions.

Considering that dynamic visualizations have been extensively adopted in mathematics teaching and learning (e.g., Battista et al., 2018; Young, 2017), it calls for the need to PD programs on technology-enhanced instruction in mathematics education (Benning et al., 2023; Bennison & Goos, 2010; Thurm et al., 2024). A survey of secondary mathematics teachers in Australia showed that although teachers recognize the benefits of technology for student learning, time constraints and limited resources hinder their integration of technology (Bennison & Goos, 2010). Similarly, Benning et al. (2023) found that a year-long GeoGebra-mediated PD program in Ghana led to changes in teachers' pedagogical beliefs, affective attitudes, and competence towards technology, but more administrative support, ongoing training, and technology resources are necessary. Recent research from the perspectives of PD facilitators suggests that gaps exist in teachers' skills and knowledge in integrating digital mathematics tools, and the collectivist and hierarchical nature of Chinese culture may influence their preference for ready-made materials and technical skills training, which could lead Chinese teachers to seek more technical training and prefer standardized and procedural PD content on technology use (Thurm et al., 2024).

In sum, realizing the benefits of dynamic visualizations in mathematics teaching and learning, PD programs are needed to support PMTs for their professional learning in effectively integrating technology into their teaching practices.

Research Question

Given the need to improve PMTs' ability to facilitate dialogic teaching and integrate dynamic visualizations, this study designed and implemented a six-month PD program to develop teachers' dialogic teaching skills and GeoGebra-scaffolded dynamic visualizations. The PD program consisted of two pivotal components: (1) dialogic teaching based on the structured APT talk moves and (2) dynamic visualizations supported by GeoGebra (Fig. 1). Accordingly, the following research question was examined:

> **RQ:** Does the video-based PD program lead to changes to PMTs' ability to use dialogic teaching and dynamic visualizations? If so, what changes?

Method

PD Design

The six-month PD program was implemented in two phases. This study specifically centers on the first phase, which was conducted from February to August

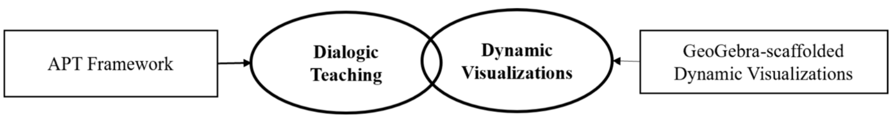


Fig. 1 The PD framework of this study

2023. The first phase was a small-scale project that involved seven participants who were research postgraduate students enrolled in a master's program for mathematics education at a normal university in southwest China. All participants were in their second year of the three-year master's program, which required them to take courses focused on mathematics education, complete a semester-long teaching practicum in their second year, and write a dissertation while passing the defense under the guidance of their university supervisors. During their teaching practicum, the seven participants voluntarily took part in the PD program.

At the beginning of the PD program, it required participants to attend five workshop sessions before starting their school-based teaching practicum. The first three sessions, lasting 60 min each, focused on APT-based dialogic teaching, covering the introduction of eight talk moves and their pedagogical implications. Participants engaged in hands-on activities to examine their declarative knowledge of dialogic teaching and prepare for subsequent coding tests. The last two sessions, lasting 90 min each, centered on GeoGebra-supported dynamic visualizations, covering theoretical and practical aspects of using the software GeoGebra in mathematics teaching and learning. GeoGebra is a dynamic visualization software package that is open-source and freely available (https://www.geogebra.org/). GeoGebra's capabilities include integrating geometry, algebra, and calculus, and it can efficiently handle the relationships among points, lines, polygons, circles, and functions using simple tools and commands. By manipulating the embedded interactive elements, such as moving the slider, inputting a value, or clicking the button, users can actively engage with the dynamic visualization provided by GeoGebra (Zhang et al., 2025).

During their teaching practicum, PMTs were encouraged to video record two lessons, namely pre-intervention and post-intervention lessons, and submit them to the research team within a week of recording. In addition, PMTs were required to complete two coding tests (Coding Tests 1 and 2) to assess their declarative knowledge of APT talk moves. The tests are introduced in the following section. A week after submitting the pre-intervention video recording, the research team provided PMTs with feedback using a digital platform called Classroom Discourse Analyzer (CDA, Chen & Chan, 2022). PMTs were advised to reflect on their pre-intervention teaching practices on CDA and prepare the post-intervention lesson based on the feedback received. According to Chen and Chan (2022), the CDA can help with teachers' reflections and learning in the video-based PD program, and it can visualize the dialogic teaching practices in multiple manners; for example, the CDA is equipped with various sources, including original videos, transcripts, and the visualization of APT moves in a bubble chart, to effectively capture the intricate nature of classroom discourse data. The bubble chart of APT moves can be clicked on by teachers to find a specific discourse talk excerpt. Once an excerpt of interest is located, teachers can access the corresponding video and transcripts to gain insights into their teaching and student learning, facilitating reflection and improvement.

Moreover, PMTs were requested to complete a survey about their epistemological beliefs (Conley et al., 2004) and technological pedagogical content knowledge (TPACK, see Santos & Castro, 2021) at the beginning and end of the PD program. The surveys are described in the following section. The students were also asked to complete a survey about their perceptions of discursive engagement with peers in class and PMTs' dialogic teaching (Chen, Zhang, et al., 2020) at the end of the PD program. At the conclusion of the PD program, PMTs, their university supervisor, and some students were invited to participate in individual interviews. The PD workflow is illustrated in Fig. 2.

Case Description: Participant and Setting

Ms. Lau (pseudonym) was purposively selected as a representative case among the seven participants for several reasons. Firstly, she was the only participant who strictly adhered to the PD workflow illustrated in Fig. 2. Other participants reported having no opportunity or only one chance to record their lessons on video. Secondly, Ms. Lau possessed some prior experience in classroom-based research, and her research topic centered on mathematics teacher classroom behavior, including dialogic teaching. She reported that participating in the PD program was highly relevant to her dissertation. Thirdly, Ms. Lau was the most active participating PMT and demonstrated strong motivation and a positive attitude throughout the six-month PD program.

Ms. Lau conducted her teaching practicum in lower secondary Grade 8 mathematics classrooms at a newly founded school. Most of the recruited mathematics teachers had prior experience teaching in other schools. The students in Grade 8 were approximately 13 to 14 years old. According to the mathematics curriculum standards for lower secondary education (Grades 7 to 9) in China, students are required to study geometric figures such as points, lines, and polygons. The curriculum aims to develop students' understanding of the basic properties and relationships among

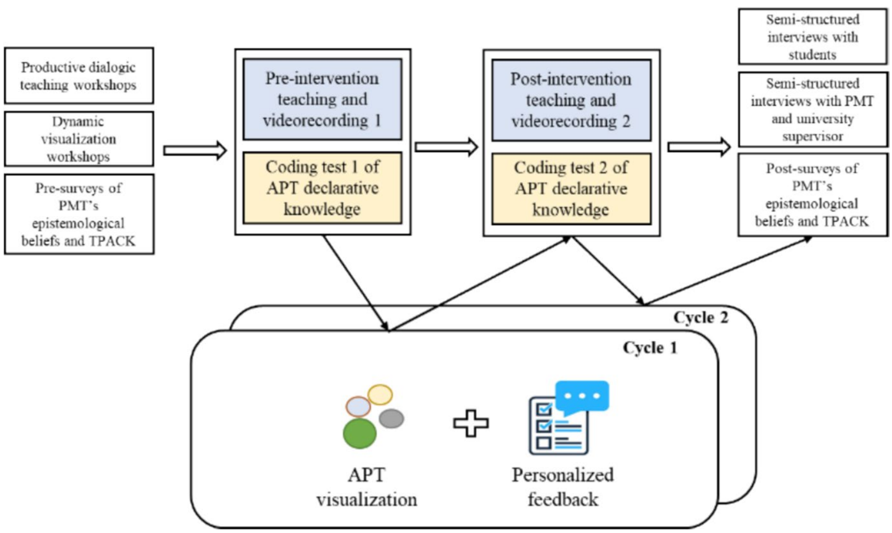


Fig. 2 The PD workflow. *Note*. TPACK: technological pedagogical content knowledge (Santos & Castro, 2021)

these figures, which in turn enhances their spatial intuition and abstract reasoning abilities (Ministry of Education, 2022). Teachers are also encouraged to incorporate technology in their teaching to help students visualize abstract mathematical concepts and construct knowledge. The use of digital resources is also recommended to facilitate self-directed learning and overcome the limitations of traditional mathematics education (Ministry of Education, 2022).

During her Grade 8 teaching practicum, Ms. Lau observed approximately 60 lessons taught by other teachers. She noted that few teachers utilized GeoGebra-supported dynamic visualizations in their daily instructions and preferred traditional chalk and blackboard teaching methods. Ms. Lau also reported that there were no prior courses provided or taken regarding technology-assisted mathematics teaching and learning during her master's program. She taught herself the basics of another dynamic geometric software called Geometer's Sketchpad, and it was her first exposure to the use of GeoGebra-supported dynamic visualizations in teaching practices. Similarly, her students were unfamiliar with virtual manipulatives such as GeoGebra. Table 2 presents the demographic profile of Ms. Lau and her students.

Data Collection and Analysis

Both quantitative and qualitative data were collected to answer the research question. The data came from two observational lessons, two coding tests, pre- and post-surveys to examine Ms. Lau's epistemological beliefs and TPACK, post-surveys to investigate her students' perceptions of discursive engagement with peers in class and her dialogic teaching. In addition, individual semi-structured interviews were conducted with Ms. Lau, her university supervisor, and her students. Data triangulation, involving the synthesis of multiple data sources, was employed to enhance the validity and credibility of the research findings (Merriam & Tisdell, 2016). Informed consent was obtained from participants involved in the study.

Table 3 presents the data collection and analysis for this study. Specifically, Ms. Lau was asked to video record her two teaching lessons on two geometry topics (i.e., *from parallelogram to rectangle* for the pre-intervention lesson and *similar triangles* for the post-intervention lesson). The pre-intervention lesson was taught in early May, and the post-intervention lesson was taught one month later. These video

Table 2 Demographic profile of Ms. Lau and her students

Education level	Year-2 research postgraduate student in China
Assigned school	A newly founded secondary school
Teaching grade	Grade 8 (lower secondary)
Prior training in dynamic visualization tools	No courses taken before. Limited use on the Geometer's Sketchpad. New to GeoGebra
Epistemological beliefs	A constructivist perspective and cognitive attitude of discovery learning toward truths in mathematics
Student characteristics	Active in class, interacted with teacher and other students, supportive of peers
Topics of teaching	• Pre-intervention lesson: *From parallelogram to rectangle* • Post-intervention lesson: *Similar triangles*

Table 3 Data collection and analysis for this study

	Data source	Data type	Data analysis
Dialogic teaching	Ms. Lau's 2 lesson recordings	qualitative	discourse analysis
	Ms. Lau's 2 coding tests	quantitative	descriptive statistics
	post surveys for Ms. Lau's 40 students	quantitative	descriptive statistics
	interview with Ms. Lau	qualitative	thematic analysis
	interview with Ms. Lau's supervisor	qualitative	thematic analysis
	interviews with Ms. Lau's 4 students	qualitative	thematic analysis
Dynamic visualization	Ms. Lau's 2 lesson recordings	qualitative	video analysis
	pre- and post-epistemological belief survey and the TPACK survey	quantitative	descriptive statistics
	interview with Ms. Lau	qualitative	thematic analysis
	interview with Ms. Lau's supervisor	qualitative	thematic analysis
	interviews with Ms. Lau's 4 students	qualitative	thematic analysis

recordings were then transcribed verbatim, and discourse analysis was employed to identify and count the frequencies of Ms. Lau's APT talk moves (Table 1) with the platform CDA. Additionally, the recordings were watched multiple times to locate Ms. Lau's use of dynamic visualizations.

In addition, Ms. Lau was asked to complete two coding tests (Coding Tests 1 and 2) to assess her declarative knowledge of APT talk moves. She watched two video clips of authentic mathematics lessons and was required to identify the APT talk moves used. These clips were chosen and coded by the research team based on the APT framework before the PD program. Both clips were discourse-rich in mathematical concepts to ensure coding test reliability. Ms. Lau's correctness in the tests was counted, and the accuracy rate was calculated.

Pre- and post-survey data were collected to examine Ms. Lau's epistemological beliefs about mathematics and her procedural knowledge of integrating technology into mathematics classrooms. The survey of epistemological beliefs used a 5-point Likert scale ranging from "1 = strongly disagree" to "5 = strongly agree"; a sample item is "Everybody has to believe what mathematicians say" (Conley et al., 2004). The TPACK survey used a 4-point Likert scale ranging from "1 = needs a lot of additional knowledge" to "4 = has strong knowledge"; a sample item is "The student teacher knows how to use Information and Communication Technologies (ICTs), such as GeoGebra, as a tool for sharing ideas and thinking together in mathematics" (Santos & Castro, 2021). Regarding Ms. Lau's students ($n = 40$), the post-surveys included items related to student-perceived discursive engagement with others in class (e.g., "In mathematics class, I discuss with my classmates to learn more about the subject matter") and PMTs' dialogic teaching (e.g., "My mathematics teacher asks questions to test our understanding of what was taught"), rated using a 4-point Likert scale ranging from "1 = rarely" to "4 = frequently" (Chen, Zhang, et al., 2020). All these survey data were analyzed using descriptive statistics.

A semi-structured interview was conducted with Ms. Lau to investigate her practices and perceptions of dialogic teaching and dynamic visualizations in mathematics classrooms. The main interview questions included "Were there any differences in your design of dialogic teaching between the first and second lessons?" and "Will you use GeoGebra in your future teaching and why?" Additionally, an individual semi-structured interview was conducted with Ms. Lau's university supervisor, who was responsible for inspecting her teaching performance. The main interview questions included "How was Ms. Lau's teaching performance?" and "What factors could lead to the success or failure of Ms. Lau's teaching performance?". Besides, individual interviews were conducted with 4 of Ms. Lau's students who had experienced her teaching. The interview questions were focused on evaluating Ms. Lau's implementation of dialogic teaching and dynamic visualizations and included examples such as "What stood out most in your memory about how Ms. Lau guided you to answer questions?" and "Do you believe that the GeoGebra-supported dynamic visualization Ms. Lau used in class helped you to better understand mathematical concepts? If so, how?".

The interview data was analyzed using thematic analysis (Braun & Clarke, 2022). The purpose of this analysis was to supplement and provide contextual insights into the findings related to the teacher's knowledge and practical skills in implementing dialogic teaching and dynamic visualization. The themes included "the teacher's understanding of dialogic teaching," "the teacher's understanding of dynamic visualization," "the challenges in implementing dialogic teaching," etc. While the interview data was analyzed thematically, the themes were not reported in isolation or discussed theme by theme. Instead, the insights derived from the analysis were integrated into the broader narrative to support and enrich the interpretation of the other data sources, such as lesson observations and coding tests. This approach was adopted to align with the study's overarching aim of providing a comprehensive understanding of the teacher's development in dialogic teaching and dynamic visualization rather than focusing exclusively on qualitative themes. By using the interview data as supplementary evidence, we were able to triangulate findings and offer a richer, more nuanced interpretation of the effectiveness of the PD program.

Findings

Ms. Lau's Dialogic Teaching

Declarative Knowledge of Dialogic Teaching

Regarding Ms. Lau's declarative knowledge of dialogic teaching based on the accuracy rate of her two coding tests, the results of Coding Test 1 indicated that Ms. Lau identified all instances of the "say more" talk move with a 100% accuracy rate. She also identified the "revoice" talk move with a 92% accuracy rate and the "press for reasoning" talk move with an 86% accuracy rate. In addition, the results of Coding Test 2 showed that Ms. Lau identified four types of talk moves (i.e., say more, revoice, add on, and explain other) with a 100% accuracy rate. Furthermore, she identified the "press for reasoning" talk move with an 80% accuracy rate. Overall, these findings suggest that Ms. Lau demonstrated solid declarative knowledge of the talk moves in the APT framework. The accuracy rates of Ms. Lau's coding tests are presented in Table 4.

Table 4 Miss Lau's performance in Coding Test 1 and Coding Test 2

Coding Test 1					
APT talk move	say more	revoice	press for reasoning		
Accuracy rate	100%	92%	86%		
Coding Test 2					
APT talk move	say more	revoice	press for reasoning	add on	explain other
Accuracy rate	100%	100%	80%	100%	100%

Table 5 Ms. Lau's dialogic teaching performance in her pre- and post-intervention lessons

APT talk move	Pre-intervention lesson		Post-intervention lesson	
	frequency	percentage	frequency	percentage
Say more	1	1.47%	7	7.78%
Revoice	3	4.41%	13	14.44%
Press for reasoning	1	1.47%	2	2.22%
Challenge	0	0.00%	0	0.00%
Restate	0	0.00%	0	0.00%
Agree/disagree	2	2.94%	0	0.00%
Add on	1	1.47%	0	0.00%
Explain other	0	0.00%	0	0.00%

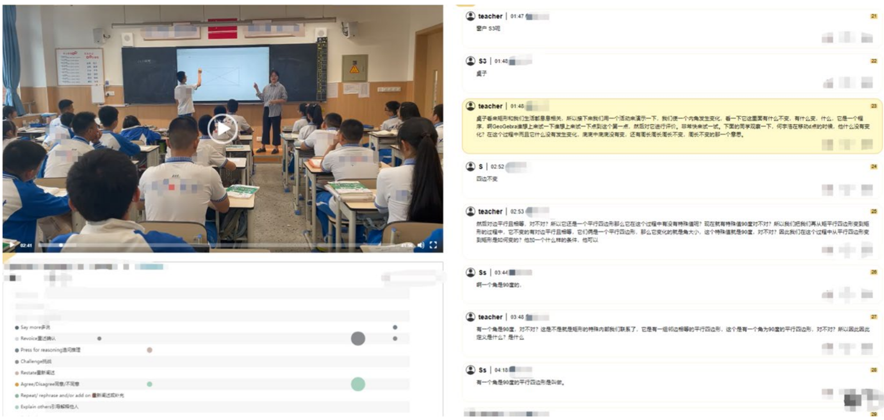
Practice of Dialogic Teaching

The research team analyzed the video recordings and utilized CDA to offer Ms. Lau feedback on her pre- and post-intervention lessons one week after the submission. CDA enables teachers to reflect on their dialogic teaching in multiple ways and representations, such as words, turns, and the frequencies of APT talk moves (Chen & Chan, 2022). Ms. Lau was instructed to review the CDA feedback on her pre-intervention lesson as part of her preparation for the post-intervention lesson. In Ms. Lau's two teaching videos, the results showed that she spoke a total of 3,514 words in 68 turns during the pre-intervention lesson, while in the post-intervention lesson, she spoke 4,924 words in 90 turns. The frequency and percentage of the eight APT talk moves can be found in Table 5. The findings indicated that Ms. Lau predominantly employed three talk moves: say more, revoice, and press for reasoning. Notably, compared to the pre-intervention lesson, the frequency of the "say more" talk move increased from 1 (1.47%) to 7 (7.78%), while the frequency of the "revoice" talk move rose from 3 (4.41%) to 13 (14.44%) in the post-intervention lesson. The "press for reasoning" talk move experienced a slight increase from 1 (1.47%) to 2 (2.22%). However, Ms. Lau did not utilize the challenge, restate, and explain other talk moves in either the pre- or post-intervention lesson. Additionally, the "agree/disagree" and "add on" talk moves decreased from 2 (2.94%) and 1 (1.47%), respectively, to 0 in the post-intervention lesson.

Ms. Lau believed that the PD program effectively supported her in implementing dialogic teaching by providing practical tools and strategies, such as the APT bubble charts (Fig. 3). In the interview, she reflected on how resources like video recordings and workshop materials helped her integrate dialogic teaching into her lesson plans:

> When I developed my lesson plan, I referred to the notes from our workshops. I was thinking about how to add those talk moves to my classroom practice. That's why my use of APT in my second recording [post-intervention lesson] is much higher than in the first recording.

Ms. Lau explained that her use of APT talk moves was both conscious and unconscious. She consciously applied APT talk moves in her classroom. However, she also believed that classroom interactions evolved as a natural process, stating, "I didn't plan



(a) Screenshot of CDA in the pre-intervention lesson

(b) Screenshot of CDA in the post-intervention lesson

Fig. 3 Dialogic teaching in Ms. Lau's classes. *Note*. CDA: Classroom Discourse Analyzer; Top left: video recording panel; Bottom left: APT bubble plot; Right: Transcription in Chinese some APT [talk moves] in advance, but I asked them [students] based on their answers." However, Ms. Lau did not use all her planned APT talk moves in class, as she explained: "I was afraid that time was running out and I wouldn't be able to cover all of the teaching content." Ms. Lau also mentioned that she noticed an increase in the use of lower-order APT talk moves, such as "say more" and "revoice," but felt that she did not use many higher-order moves. Despite these challenges, Ms. Lau still believed that "APT helps organize classroom interactions" and allowed teachers to "check if other students understand the teaching content" and "engage learners to talk more" in future practices.

For her students, the two-part post-survey results showed their perceptions of Ms. Lau's dialogic teaching (Cronbach's $\alpha = 0.61$) and discursive engagement with others (Cronbach's $\alpha = 0.84$). The part-one survey asked questions such as "My mathematics teacher asks us to explain our thoughts in class" ($Mean = 3.90$; $SD = 0.30$) and "My mathematics teacher asks us to listen to one another in class" ($Mean = 3.80$; $SD = 0.56$), which demonstrated students perceived Ms. Lau's extensive implementation of dialogic teaching strategies in class. In a similar vein, responses to questions such as " In mathematics class, I discuss with my classmates to learn about the subject matter" ($Mean = 3.78$; $SD = 0.48$), "In mathematics class, I listen to my classmates when they speak" ($Mean = 3.85$; $SD = 0.36$) in part-two questionnaire showed the sufficient opportunities students perceived to exchange ideas with others.

During the interviews, the students also praised Ms. Lau's overall use of APT talk moves. Her frequent use of APT talk moves helped them "memorize knowledge for the long term." Some talk moves were used as scaffolding to either "lead us [students] step by step to understand the concepts" or "help us [students] to reorganize our logic when expressing why we think this way."

Ms. Lau's Integration of Dynamic Visualizations

Procedural Knowledge of Dynamic Visualizations

The present study utilized GeoGebra, a freely available open-source software, to provide scaffolding for dynamic visualizations in mathematics instruction. The results of the TPACK surveys showed that Ms. Lau faced challenges, such as inaccessible resources, in integrating technologies (i.e., GeoGebra) to support dynamic visualizations in mathematics classes from the beginning of the PD program. As she mentioned during the interview, "Before the training workshops, I had no clue where I could find dynamic visualization resources." In comparison, the post-survey results showed clear improvements in her procedural knowledge, enabling her to effectively integrate dynamic visualizations into her teaching by the end of the PD program.

Despite these improvements, some challenges remained, such as the time spent planning lessons and the skills required to integrate dynamic visualizations. As Ms. Lau explained in her interview: "I am willing to use dynamic visualizations only if I have enough time to prepare or I am doing a demo lesson, because attempting to integrate technologies increases my lesson preparation time. … I am still not competent enough to use complicated tools [GeoGebra]."

Practice of Dynamic Visualizations

According to Ms. Lau's two teaching videos, she incorporated GeoGebra-scaffolded dynamic visualizations into her two lessons twice. Figure 3 presents two screenshots of her pre- and post-intervention lessons. During the pre-intervention lesson, Ms. Lau focused on the topic of transforming a parallelogram into a rectangle. As shown in Fig. 3(a), Ms. Lau asked a student to manipulate the dynamic visualization. The student was asked to move Point D to change the angle of the parallelogram, while the other students were asked to observe what remained unchanged. The dynamic visualization used in this lesson is presented in Fig. 4(a).

Excerpt 1 is a transcription of the conversation that took place in the pre-intervention lesson, transcribed from Chinese (i.e., the original medium of instruction). In this excerpt, Ms. Lau initiated the discussion by inviting individual students to provide examples of rectangles in daily life. She merely echoed students' brief responses, such as "*Windows, and S3?*" (Turn 5), following a "Ms. Lau-S1-Ms. Lau-S2-Ms. Lau-S3" pattern. During her explanation and when requesting the volunteer student (S4) to present the first dynamic visualization, Ms. Lau primarily called upon the entire class for a choral response by asking, "*What remains unchanged?*" (Turn 7). The teacher followed a "Ms. Lau-Students" pattern for the most part in the subsequent discussion. However, she seldom engaged students with previous responses and encouraged them to continue their discussion.

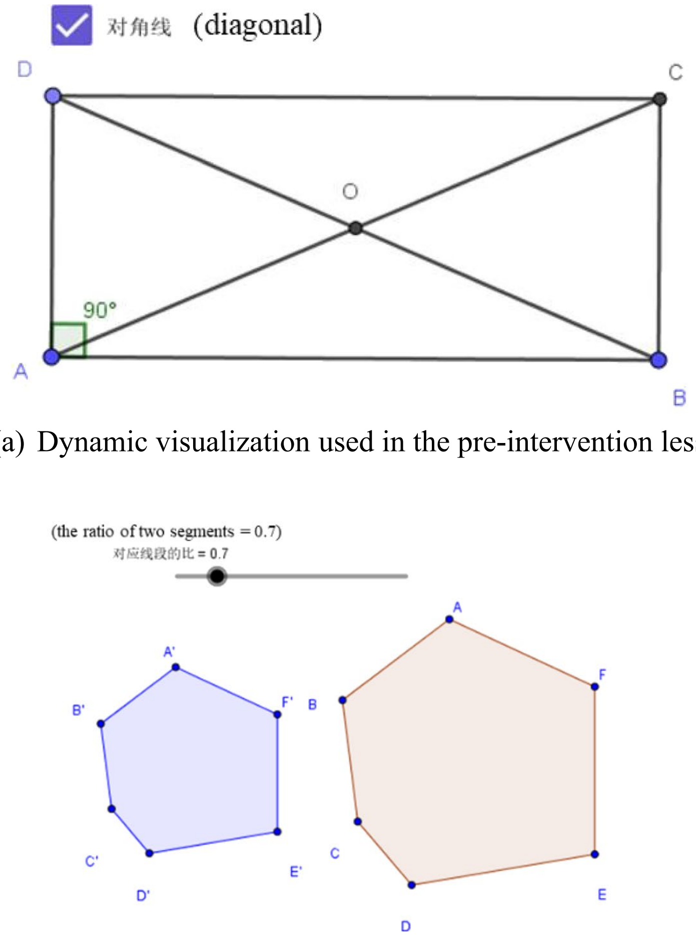
Excerpt 1

Ms. Lau's Dialogic Interaction in Her Pre-Intervention Lesson

1	Ms. Lau	Can anyone give some examples of rectangles from our daily lives? (*Moving her arm*)
2	S1	The blackboard
3	Ms. Lau	The blackboard, what else? S2. (*Moving her arm*)
4	S2	Windows
5	Ms. Lau	Windows, and S3?
6	S3	Tables
7	Ms. Lau	Tables. It seems that rectangles are relevant to our lives, so let's demonstrate this with an activity in which we perform an interior angle change (*Clicking the slider with one finger*) and see (*Dragging the slider with one finger to enlarge the rectangle*) what stays the same and what changes (*Dragging the slider again with one finger to change the size of the rectangle and its position*). I would like a volunteer to help me with GeoGebra. Who wants to give it a try? Click on Point D, then move it. Who wants to come here and try, S4? For the other students, observe what doesn't change (*Moving her arm*) when S4 moves Point D. When S4 moves Point D, what remains unchanged?
8	Students	The base

In the post-intervention lesson, Ms. Lau shifted her focus to the topic of similar triangles. Figure 3(b) illustrates how Ms. Lau introduced a fundamental concept: the ratio of two segments in a hexagon. The dynamic visualization she used in this lesson is shown in Fig. 4(b).

Excerpt 2 presents a transcription of a classroom discussion that occurred during the post-intervention lesson, where she utilized GeoGebra-supported dynamic visualization to guide the whole-class exploration of the size relationship between two hexagons. In contrast to the pre-intervention lesson described in excerpt 1, where student engagement was lacking in previous responses, Ms. Lau took charge of operating GeoGebra in this lesson instead of assigning a student to manipulate it. Upon displaying two static geometric figures to the students, Ms. Lau proceeded



(a) Dynamic visualization used in the pre-intervention lesson

(b) Dynamic visualization used in the post-intervention lesson

Fig. 4 Two GeoGebra-scaffolded dynamic visualizations used in Ms. Lau's lessons to demonstrate by using GeoGebra to adjust the sizes of the hexagons, as depicted in Figs. 3(b) and 4(b), by dragging the slider to enlarge and shrink them. In this excerpt, Ms. Lau not only focused on soliciting choral responses from the students but also provided concise feedback. For example, she initiated the discussion by asking the entire class, "*What do you find that changes in this process?*" (Turn 1). Following the students' responses, she reiterated their observations as feedback by stating, "*The area changes, and the length of the side also changes, right?*" (Turn 3) while dynamically visualizing the process using the slider. Additionally, Ms. Lau acknowledged the students' response, stating, "*Yes, the corresponding sides are proportional…*" (Turn 5), after prompting the class to elaborate further on the relationship between the sizes. Despite adhering to the "Ms. Lau-Students" pattern, she emphasized fostering continuity and consistency in encouraging students to delve deeper into the problem with the aid of GeoGebra.

Excerpt 2

Ms. Lau's Interaction with Students when Using Dynamic Visualization in Post-Intervention Lesson

1	Ms. Lau	…Students, think about it (*Opening GeoGebra*), then how to describe the size relationship between the two figures? These are the two figures we just saw; how do we describe them? Let's look at this picture; we move this (*Dragging the slider*), and this large hexagon can be seen as this small hexagon, enlarged, right? (*Looking at the students while operating GeoGebra*) Then, this small hexagon is the result of shrinking the large hexagon (*Looking at the students while dragging the slider*). When they overlap, then let's look at this animated picture. What do you find that changes in this process? (*Dragging the slider*)
2	Students	The length of the side changes, and the area changes
3	Ms. Lau	The area changes, and the length of the side also changes, right? So, how do we describe the relationship between these two hexagons? (*Dragging the slider*) How to describe it? The relationship between the sides is relatively close, so how should we describe it? (*Looking at the students*)
4	Students	Corresponding sides
5	Ms. Lau	Yes, the corresponding sides are proportional, so when we study similar figures, we must first learn the proportion of line segments…

After illustrating the proportionality of line segments using GeoGebra, Ms. Lau presented an exercise to the class: "What is the ratio between two segments $AB = 6$ cm and $CD = 4$ cm? And what if $AB = 8$ cm and $CD = 2$ dm?" This exercise aimed to stimulate a whole-class discussion centered on the question: "Does the ratio of line segment lengths depend on the units of measurement used?".

Excerpt 3 provides a detailed account of Ms. Lau's dialogic teaching during this discussion. She emphasized the importance of both individual student contributions and collective class participation, which are fundamental aspects of dialogic teaching (Aguiar et al., 2010; Bossér & Lindahl, 2021). In addition, Ms. Lau employed various talk moves from the APT framework to encourage students to expand on and clarify their ideas. She initiated the discussion by prompting students to raise their hands to share their thoughts on whether the choice of units influences the ratio of two segments. When student S5 expressed the view that units were indeed relevant, Ms. Lau encouraged him to elaborate using "say more" talk move, "*It is related, how is it related?*" (Turn 1). Following S5's explanation, Ms. Lau repeated and paraphrased his response using a "revoice" talk move, "*It may need to be converted, which means that it is compared under the same unit length, right?*" (Turn 3). Furthermore, when another student, S6, expressed the contrary belief that unit choice did not affect the ratio of the segments, Ms. Lau invited him to provide further clarification by inviting him to "say more". After S6's response, Ms. Lau not only affirmed his contribution by stating, "*Yes, correct, very good.*" (Turn 5) but also implemented the talk move "revoice" to continue, "*…no matter what unit it is, its ratio is the same…*" (Turn 5). She then redirected the discussion to the entire class, seeking a collective response regarding the initial exercise by inquiring, "*Then, do I need to bring a unit for AB and CD?*" (Turn 5) Finally, Ms. Lau evaluated the class response and concluded that while calculating the ratio of two line segments requires consistent units, the resulting ratio is a positive number without units.

Excerpt 3

Ms. Lau's Dialogic Teaching Practice in Her Post-Intervention Lesson

1	Ms. Lau	Let's think about it. If the ratio of the lengths of two line segments is related to the units they use, raise your hands. Do the rest of the students think it is irrelevant? It is related, how is it related?
2	S5	If it is related, it may need to be converted
3	Ms. Lau	It may need to be converted, which means that it is compared under the same unit length, right? Then, … who didn't raise their hands? Who thinks it is irrelevant? Does anyone think it is?
4	S6	Because I think that its unit length is, that is, it is proportional, the lengths of the two line segments are still a ratio no matter what unit length is used to compare them
5	Ms. Lau	Yes, correct, very good. It is under the same unit; no matter what unit it is, its ratio is the same, so it is both related and irrelevant, right? Do you understand? Then, do I need to bring a unit for AB and CD?
6	Students	No need
7	Ms. Lau	Yes, yes, no need. It is a ratio, and a ratio is data without a unit…

Discussion

The present study proposed a video-based PD approach aimed at simultaneously supporting PMTs' implementation of APT-based dialogic teaching and integrating GeoGebra-scaffolded dynamic visualizations into their teaching practicum. This case study contributes to research on the use of digital platforms, e.g., CDA, in PD programs and sheds light on how the use of technologies can bridge the gap between "knowing" and "doing" for both dialogic teaching and dynamic visualizations.

From Knowing to Doing: Dialogic Teaching Performance

Increased Use of Lower-order APT Talk Moves

A comparison of Ms. Lau's pre- and post-intervention lessons showed that she demonstrated progress in using structured lower-order APT talk moves, and she acknowledged this improvement during the interview. These findings are consistent with observations made by Chen, Chan, et al. (2020), who noted a significant change in the use of the "say more" talk move after a year-long PD intervention. This change may be attributable to the fact that it is relatively easy for teachers to prompt students to provide specific examples. However, Chen, Chan, et al. (2020) found no significant change in the use of the "revoice" talk move, which differs from the results observed in Ms. Lau's case. This inconsistency may be due to the revised definition of "revoice" in this study. First, Ms. Lau was not informed of this change during the workshop sessions to avoid overwhelming her with excessive teaching rules that could lead to confusion, as emphasized by Nolan et al. (2015). Second, as highlighted by Erath et al. (2021), the use of the "revoice" talk move in various scenarios places responsibility for speaking on students, enabling them to verify their understanding. However, in this study, when the content was merely "repeated," the teacher assumed the role of dialogue facilitator. Consequently, the "revoice" talk move functioned as an evaluative approach to assess the students' statements (Erath et al., 2021).

Overall, the comparison between the pre- and post-intervention lessons suggests that Ms. Lau shifted her teaching approach from authoritative to dialogic. In the pre-intervention lesson, as shown in excerpt 1, Ms. Lau engaged in an open chain of interactions (I-R-P-R-P-R…) without final evaluation, following the model described by Aguiar et al. (2010). She prompted (P) various students to respond (R) to real-life examples of rectangles after initiating (I) the question. This interaction mainly involved basic factual questions with low cognitive engagement (Mehan & Cazden, 2015). There was also minimal attention given to student contributions, as indicated by the lack of evaluation and feedback (Scott et al., 2006).

Conversely, in the post-intervention lesson, Ms. Lau demonstrated more dialogic practices. She valued students' voices and ideas, acknowledging their contributions to the discussion (Aguiar et al., 2010; Alexander, 2018; Bossér & Lindahl, 2021). For instance, in excerpt 3, Ms. Lau engaged in dialogue with individual students S5

(Turn 2) and S6 (Turn 4), who had differing views on the topic, while also focusing on the understanding of the entire class (Turn 6). Despite dialogic teaching being a new pedagogical approach for PMTs (Hauk et al., 2023), Ms. Lau utilized lower-order APT talk moves, such as "say more" (e.g., Turn 1) and "revoice" (e.g., Turn 3), to encourage students to elaborate and clarify their ideas. Implementing these APT talk moves helped structure her classroom discussions and held students accountable for mathematical norms (Michaels et al., 2008; Resnick et al., 2010). Her acknowledgment, solicitation, and elaboration were also in line with responsive teaching (Bishop, 2021; Jaber, 2021). Moreover, Ms. Lau's instructional decisions were informed by teacher noticing based on her observations of students' responses (van Es et al., 2017). For example, though no APT talk moves were integrated into excerpt 2 when Ms. Lau used GeoGebra-supported dynamic visualizations, she noticed students' reactions to her demonstrations and subsequently encouraged the whole class to respond to guiding questions (Turns 1 and 3).

Self-awareness of a Higher-order Talk Move

An interesting finding of this study is the difference between perception and implementation of the higher-order APT talk move, "agree/disagree." This talk move involves the teacher encouraging a student to assess another student's reasoning by agreeing or disagreeing (Table 1). Although some students interviewed recalled Ms. Lau using this move occasionally in both lessons, there was a discrepancy between her understanding of its definition and its actual application. As a result, its use was infrequent (decreasing from 2.94% to 0%). During the interview, Ms. Lau explained,

> The way I understand [it] is a bit different. For me, "agree/disagree" means that I ask the whole class whether they agree or disagree with the student's answer. However, it involves asking another student instead of the whole class.

Ms. Lau's self-awareness of the APT talk moves demonstrated that she applied her knowledge through conscious learning and was capable of self-reflection on her classroom practice (Boström & Lassen, 2006; Chapman, 2015). As she stated,

> I had this idea [of using higher-order APT talk moves] before the training sessions, after the [workshop] training, the voice in my mind [telling me to use more higher-order APT talk moves] was louder.

Ms. Lau gained some experience in classroom-based research, and her research topic addressed mathematics teacher classroom behavior. According to her university supervisor, Ms. Lau "was quite interested in this area [dialogic teaching] and involved in a research project on classroom interaction, which laid the foundation for her before she participated in this PD project." Such motivation and prior knowledge allowed her to be aware of her use of APT talk moves and recognize where she could improve teacher-student interactions in her mathematics lessons. However, this finding should be interpreted with caution, as the coding tests conducted in this study did not include the identification of the "agree/disagree" talk move. Therefore, the assessment of Ms. Lau's declarative knowledge regarding this talk move was not examined in a formative manner. Although the interview revealed Ms. Lau's self-awareness of this talk move, it remained unclear how well she understood it from a declarative point of view. Another potential explanation is that the terminology used for the "agree/disagree" talk move may be unclear when implemented in teaching practice, despite being emphasized during the workshops. The practice of teachers requesting choral responses to indicate agreement or disagreement aligns with classroom cultures in East Asian countries influenced by collectivism (Leung, 2001), as seen in previous observations in Chinese mathematics classrooms (Xu & Clarke, 2013, 2019). Thus, this finding calls for a re-evaluation of the appropriateness of talk moves within the APT framework, particularly when considering the integral role of culture.

Video-based PD Supported by CDA

The present study adopted a video-based PD approach supported by the digital platform CDA. Video-based PD has gained popularity, especially during the COVID-19 pandemic, due to its simplicity and feasibility (Chan & Yau, 2021). The findings of this study align with previous research in this area that has demonstrated the positive impact of technology-assisted PD programs on teachers' implementation of dialogic teaching (Chen & Chan, 2022; Jacobs et al., 2022). Unlike traditional face-to-face PD programs, CDA serves as a mediator that overcomes constraints of time and geography, enabling effective and improved PD activities, including collaborative reflections (Borko et al., 2014). In line with the results, CDA provided a unique advantage in enabling the visualization of dialogic teaching through bubble plots; as Ms. Lau commented,

> When I received the first feedback on the bubble plot [of my use of APT talk moves], I felt the need to design more in the second lesson. So, when I prepared for my second class, I referred back to my notes from the workshops and then reflected on where I could add those talk moves to my in-class talk.

The use of CDA during Ms. Lau's teaching practicum encouraged her to reflect on how to better integrate APT talk moves into her subsequent teaching practices. The findings also indicate that digital technologies such as CDA have the potential to provide a hybrid approach capable of uncovering the "black box" of PD programs, thereby meeting teachers' demands for 21st-century skills development (Tondeur et al., 2017; Urbani et al., 2017).

Teacher Presentation and Student Self-creation: Integrating Dynamic Visualizations

Pedagogical Decisions and Student Errors

Before her teaching practicum, Ms. Lau received training in procedural knowledge and hands-on activities using GeoGebra-scaffolded dynamic visualizations through two workshops. The design of the training sessions aimed to address both pedagogical and technological barriers, at least to some extent, in using GeoGebra-scaffolded dynamic visualizations in mathematics teaching, in line with the work of Angeli and Valanides (2009).

During her teaching practice, Ms. Lau used dynamic visualizations twice (Figs. 3 and 4) and she adopted two different pedagogical decisions, i.e., *sherpa at work* in the pre-intervention lesson and *discuss the screen* in the post-intervention lesson, as summarized by Bozkurt and Ruthven (2018) in their work of whole-class lesson segment types relevant to technology use. During her pre-intervention lesson, Ms. Lau invited a student to manipulate the focal dynamic visualization to change the angle of the parallelogram to a rectangle. Ms. Lau adopted the sherpa-at-work decision that the didactical configuration reflects the student presentation to the whole class using the GeoGebra-supported dynamic visualization, and the student acted as the sherpa to carry out the actions the teacher requested (Bozkurt & Ruthven, 2018). Unexpectedly, the sherpa student mistakenly manipulated the dynamic visualization, which wasted a lot of time and disrupted the flow of the lesson as Ms. Lau recalled during the interview. In the post-intervention lesson, Ms. Lau decided to present the dynamic visualization about the ratio of two segments in a hexagon herself. This discuss-the-screen decision involves the scenario that the teacher creates a whole-class discussion about what happens on the dynamic visualizations and intends to enhance the collective instrumental genesis (Bozkurt & Ruthven, 2018). The difference in Ms. Lau's pedagogical decisions may be attributed to how teachers react to student errors (Benecke & Kaiser, 2023; Brodie, 2014; Ingram et al., 2015). The discuss-the-screen decision, to some extent, reflects that Ms. Lau intended to avoid potential student errors in the post-intervention lesson. Just as Ingram et al. (2015) pointed out, errors in classroom interactions continue to be predominantly treated as something to avoid, despite the recommendations that they should be used as opportunities for learning rather than being neglected. Likewise, the contextual factors, including the unforeseen disruption of the prepared lesson plan and Ms. Lau's perception of the ineffectiveness of the sherpa-at-work decision, may bring attention to the PD program that professional development should equip teachers, including PMTs, with strategies to deal with errors constructively at different stages of instruction (Benecke & Kaiser, 2023) and highlight the role of professional learning communities in learning to identify, interpret, and engage with learner errors (Brodie, 2014).

Affordances and Constraints of Dynamic Visualizations

The two GeoGebra-supported dynamic visualizations used by Ms. Lau in both pre- and post-intervention lessons demonstrate their affordances to be manipulated step-by-step and show sequential changes (Wu et al., 2015). In the pre-intervention lesson, for example, the sherpa student was able to constantly move Point D in a parallelogram to form a right angle and, subsequently, a rectangle. Ms. Lau found dynamic visualizations to be more effective than using two separate static graphs of a parallelogram and rectangle, as noted in the interview. In the post-intervention lesson, Ms. Lau dragged the slider to adjust the ratio of segments, continuously helping students observe the enlarging and reducing hexagons. These sequential manipulations are supported by previous research showing that dynamic visualizations, characterized by adjusting the parameters or starting an animation, are more effective than static ones (Kohen et al., 2022; Rolfes et al., 2020). Moreover, Ms. Lau's students found both dynamic visualizations to be vivid, with one student reporting that "it was more interesting, and it could attract my attention and eliminate boredom immediately." The dynamic visualizations also helped consolidate their understanding of geometric figures, eliminating the need to imagine graphic transformation processes in their minds. This aligns with Hsu and Hsu's (2025) work, which suggests that dynamic visualizations can reduce students' extraneous cognitive load when exploring geometric properties compared to static visualizations.

Ms. Lau recognized the benefits of GeoGebra-supported dynamic visualizations for visualizing geometry knowledge and expressed a positive attitude towards future use of them. The results of pre- and post-TPACK surveys confirmed improvements in her procedural knowledge of technology integration since the beginning of her teaching practicum. Consistent with Benning et al. (2023) and Thurm et al. (2024), the GeoGebra-supported dynamic visualization embedded in the PD program impacted teachers' pedagogical beliefs, affective attitudes, knowledge, and skills. However, there are various constraints worth considering for the use of dynamic visualizations in PD initiatives. As reported by Bennison and Goos (2010) for Australian teachers, Ms. Lau also acknowledged that creating GeoGebra-supported dynamic visualizations was time-intensive and added to her workload beyond regular teaching. This was particularly challenging since she had no prior training in this area, except for attending two workshops and receiving informal assistance from the research team during the PD program. Ms. Lau emphasized the need for more support, such as ongoing training and technology resources, to overcome these obstacles, as highlighted by Benning et al. (2023). One possible solution is to incorporate iterative workshops during PD programs, as suggested by Borko et al. (2014), to ensure that PMTs receive sufficient scaffolding, actively participate, and contribute substantially to integrating dynamic visualizations in their teaching instructions.

Additionally, while Thurm et al. (2024) noted that Chinese facilitators and teachers might prefer ready-to-use PD materials and seek more technical training, the two dynamic visualization workshops in the current PD program incorporated not only how to use GeoGebra and existing online resources but also the pedagogical implications of dynamic visualizations for teaching and learning. The first author served as the primary facilitator and encouraged participating PMTs to self-design dynamic visualizations aligned with teaching goals and to invite their students for co-construction. Further discussion of these findings is presented in the following section.

Design Strategies and Student Self-directed Creation

During her pre-intervention lesson, Ms. Lau downloaded an existing dynamic visualization from the GeoGebra official website without modifying it. However, during her post-intervention lesson, she also downloaded an online resource but modified it based on the learning objectives. These methods reflect the concepts of the macro-level interaction framework summarized by Sedig and Liang (2008), which is divided into four levels of interaction, i.e., access, annotation, construction, and combination.

Ms. Lau's approach in her pre-intervention lesson aligned with the first level of interaction, namely *access* (Sedig & Liang, 2008), which involves using accessible online resources to design dynamic visualizations. As Ms. Lau stated, "I searched the website and downloaded a dynamic visualization if I needed to use it, because I didn't know how to make one." However, her modifications to the resource in her post-intervention lesson reflected a higher level of interaction, namely *annotation* (Sedig & Liang, 2008). As she explained, "I just kept the picture and deleted the ratio of similar triangles because that was not the current focus, and I didn't want to confuse them [the students]." While Ms. Lau achieved the first two levels of dynamic visualization creation, it is important that PMTs progress further and become competent in designing dynamic visualizations from scratch and adopting higher levels of interaction to better manipulate dynamic visualizations. This corresponds to the third and fourth levels of the framework, namely *construction*, and *combination*, respectively (Sedig & Liang, 2008).

An unforeseen discovery was made that not only did Ms. Lau use dynamic visualizations as the sole user, but her students also became creators of such visualizations themselves. Ms. Lau intentionally left the two dynamic visualizations on the classroom computers and encouraged students to explore and interact with them during class breaks. The interviews with the students revealed that many of them were curious and attracted to these dynamic visualizations, and actively engaged with them by moving points and dragging sliders. Additionally, some of the students were motivated to search for online videos about GeoGebra-scaffolded dynamic visualizations. They reported that these activities helped them consolidate their understanding of mathematical concepts, such as the ratio of two segments. One student even attempted to create her own dynamic visualization by following online instructions and did not find it difficult. As she explained,

> I found it easy to replicate the parallelogram example using the dynamic visualization software with free online instructional videos.

It is in alignment with the initiatives of the PD program grounded on the national mathematics curriculum standards that teachers are encouraged to guide students to use digital resources such as the GeoGebra-supported dynamic visualizations to strengthen self-directed learning (Ministry of Education, 2022). As the students had no prior experience with virtual manipulatives, engaging in self-exploration not only allowed them to visualize the properties of geometric figures like parallelograms and similar triangles but also helped to reinforce their comprehension of mathematical concepts. In a similar vein, the participating teachers were all motivated by the facilitator to empower their students to become self-creators during the dynamic visualization workshops. The transfer of knowledge from teachers to students served as partial evidence in support of the effectiveness of the PD program. The PD program was also influenced by the curriculum standards for mathematics teacher education, which advocates for the integration of varied training approaches—including technology—to support and enhance teaching and learning practices (Ministry of Education, 2022).

Implications and Limitations

The findings of this study suggest that the proposed video-based PD approach has the potential to expand our understanding of how dialogic teaching and GeoGebra-scaffolded dynamic visualizations can be integrated simultaneously into PD programs for PMTs during their teaching practicum. This PD approach can contribute to bridging the knowing-doing gap in delivering discourse-rich mathematics teaching (Planas & Schütte, 2018; Sfard, 2012) and effectively implementing dynamic visualizations (Zhang et al., 2025). This study also illustrates the affordances of digital platforms, i.e., CDA, in designing and implementing hybrid PD programs.

This study has several limitations that should be acknowledged. First, in terms of dialogic teaching, this study only observed changes in three lower-order talk moves in the APT framework. Changes in higher-order talk moves, such as "agree/disagree," were not evident during the teaching practicum. It is also worth noting that Ms. Lau only had two opportunities to practice teaching at a secondary school. Therefore, it is recommended that future research include more iterative cycles in PD programs and provide PMTs with additional opportunities to apply structured talk moves. Second, this study focused solely on Ms. Lau as a single case study to explore changes in her dialogic teaching and dynamic visualizations throughout the PD program. Data from various sources, such as lesson recordings, surveys, tests, and semi-structured interviews with different stakeholders, were collected to ensure the richness of the data and the credibility of the results. Nevertheless, it is suggested that larger-scale research be conducted to further examine the effectiveness of this PD approach, as highlighted by Erath et al. (2021).

Conclusion

This study aimed to enhance PMTs' dialogic teaching skills and integration of GeoGebra-scaffolded dynamic visualizations into their teaching practicum in a video-based PD program. One PMT was selected as a representative case from the ongoing PD program. Using the QUAL-quan approach, the results showed that during the six-month PD program, the PD approach effectively improved Ms. Lau's declarative knowledge and real teaching practice of lower-order talk moves (i.e., say more, revoice, and press for reasoning) in two mathematics lessons. Her self-awareness and self-reflection on dialogic teaching informed future practices, helping her bridge the knowing-doing gap. Furthermore, the results indicated that Ms. Lau was able to flexibly integrate GeoGebra-scaffolded dynamic visualizations into her teaching with different pedagogical decisions, reflecting how she reacted to student errors and the affordances and constraints of dynamic visualizations. Although opportunities for PMTs to participate in teaching practice may be limited, the effectiveness of the constructivist video-based PD approach relies on the integration of both dialogic teaching and dynamic visualizations. This effectiveness was confirmed by the rich and credible data collected from multiple stakeholders for data triangulation. The findings of this study have some theoretical and practical implications for preservice teachers and teacher educators. The PD approach can serve as pioneering work in simultaneously promoting PMTs' dialogic teaching and GeoGebra-scaffolded dynamic visualizations. Additionally, teacher educators should recognize the potential of integrating digital platforms such as CDA to design hybrid PD programs to enhance PMTs' self-reflection and teaching practices.

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Data Availability The data that support the findings of this study are available from the corresponding author upon reasonable request.

Declarations

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