Bumpless Transfer Hybrid Non-Fragile Finite-Time Control for Markovian Jump Systems and its Application

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***Abstract*—In this article, the issue of bumpless transfer hybrid non-fragile finite-time H_∞ control for Markovian jump systems (MJSs) is investigated, where the transition rates are partially available. The non-fragile strategy with tolerating both additive and multiplicative perturbations is designed, which greatly relaxes the application scope of the traditional controller. The bumpless transfer control idea is introduced to depict the transient behavior caused by a jumping controller. First, a bumpless transfer constraint condition is provided to restrict the amplitude of the hybrid non-fragile jumping controller, for which the additive and multiplicative perturbations are considered. Then, a bumpless transfer hybrid non-fragile controller is developed to guarantee the solvability of the finite-time H_∞ control issue for MJSs with partially available transition rates. Finally, an electronic circuit system example is applied to illustrate the usefulness of the proposed bumpless transfer hybrid non-fragile control approach.**

***Note to Practitioners*—This article is motivated by the finite-time hybrid non-fragile H_∞ control issue of MJSs with the bump limitation constraint. The bumpless transfer is often encountered in the Markovian jumping control field because it is usually impossible to implement unlimited control signals or bumpy control signals and is a main source of instability, and degradation of performance of MJSs. The traditional bumpless transfer technique because of the constant control gain cannot handle effectively the non-fragile bumpless transfer problem for MJSs. In this paper, the restriction of the constant control gain is relaxed. We develop a bumpless transfer control strategy considering a strong non-fragility of the control gain. In contrast to the existing results, a non-fragile bumpless transfer can tolerate the co-existing additive and multiplicative perturbations with a wider application. This study presents the method for practitioners interested in bumpless transfer controller design.**

***Index Terms*—Bumpless transfer, Markovian jump systems, switched systems, non-fragile control, finite-time control.**

I. Introduction

As a special subclass of switched systems, Markovian jump systems (MJSs) are defined as difference or differential equations and a stochastic function. The cooperation between different equations is realized through a Markovian process [1], [2]. MJSs are adopted to depict complex physical systems subject to sudden variations in structure and parameters. The applicability of MJSs is extended to various fields such as aerospace engineering, robot control, and mechanical manufacturing [3]–[5]. The related work of MJSs has aroused heated discussions among scholars, and many excellent results have been obtained [6]–[8]. For an uncertain nonlinear MJS with Lévy noises, the stability problem is solved by an adaptive sliding mode control method in a mean-square sense [9]. In the conclusions of the above research, the full availability of transition rates is an additional prerequisite. Due to the high cost and difficulty of obtaining transition rates, the transition rates are usually unavailable. On account of the unavailability of transition rates, there is a growing interest in studying MJSs with partially available transition rates [10]–[12]. Based on a decoupling technique, the sufficient conditions ensuring the H_∞ performance of MJSs with multiplicative noise and partially available transition rates are developed [13]. In [14], the condition of mean-square stability is established for singular semi-MJSs with generally unavailable transition rates. Nevertheless, when the transition rates are partially available, how to analyze the system dynamics of MJSs within a finite time interval is a serious challenge, which partially motivates the present study.

In the study of MJSs, the main concern is the traditional stability during an infinite time region. In practice, the dynamic behavior of physical systems deserves more attention within a finite time interval [15]. Finite-time control is used to deal with the dynamic behavior problem of systems within finite time intervals, which is highly effective in enhancing the transient performance of MJSs. Under the influence of a designed controller, the system states are constrained to a preset boundary within a specified time if the initial state starts from a given threshold, which is called the finite-time control method [16], [17]. In recent years, plenty of results about finite-time control of MJSs have been reported [18]. For MJSs with actuator failures, an event-triggered control method is developed to ensure the finite-time boundedness and H_∞ performance under the time-varying transition probabilities [19]. Based on a dwell time method, the asynchronous finite-time control problem is addressed for MJSs subject to actuator and sensor failures [20]. However, the above results ignore the

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influence of the jumping characteristic of a controller on finite-time stability.

In general, the control synthesis problem of MJSs is solved by designing a jumping controller. The controllers can produce different control signals, and result in control bumps during jumping instants [21], [22]. The control bump may destroy the transient performance and even distort the Lyapunov stability of systems [23], [24]. The bumpless transfer scheme provides a potential solution to the control bump issue. Under an average dwell time switching scheme, the bumpless transfer control strategy is developed for switched systems to guarantee the continuity of the control input while ensuring the asymptotic stability of systems [25]. In [26], a control gain interpolation technique is proposed to handle the bumpless transfer issue for positive switched systems, and the L_1 gain is maintained. For switched descriptor systems, by quantifying the state jumping, a switching control strategy that satisfies bumpless transfer performance both input and state is discussed [27].

The above bumpless transfer control strategies are based on the precondition that the controller can be implemented accurately. The designed control strategy should have a certain robustness to system parameters. On account of calculation errors, actuator failures, and control channel interference, the designed bumpless transfer controller should be able to tolerate the uncertainty or parameter perturbation [28], [29]. It is necessary to consider non-fragile bumpless transfer controllers that can withstand uncertainty in gain parameters. As the external environment becomes more complex, the traditional non-fragile controllers that tolerate a single gain perturbation no longer meet the practical engineering [30]. Although the bumpless transfer constraint for the non-fragile controller is anticipated, the developed approaches are still difficult to apply due to the non-fragile behavior. This motivates our study.

In this paper, we present a bumpless transfer hybrid non-fragile control strategy for MJSs to reduce the large control bumps while ensuring the finite-time boundedness and H_∞ index. The significant contributions are summarized as follows:

(i) In the existing work [28], [29], the non-fragile controller is formed by a single feature which is either additive or multiplicative. Although additive and multiplicative perturbations are two fundamental cases, it is not explicitly stated which one might be preferred since the controller is usually affected by the distinct hybrid features. We developed a finite-time hybrid non-fragile H_∞ controller, which can tolerate the co-existing additive and multiplicative perturbations.

(ii) Unlike the traditional bumpless transfer control approaches, the constant control gain without any perturbation is restrained [3], [31]. The additive and multiplicative perturbations lead to big variations of control inputs, which directly affect the establishment of the bumpless transfer controller. The presented bumpless transfer control strategy considering a strong non-fragility of the control gain is developed, which not only reduces control bumps produced by the jumping controller but also allows for application to more intricate scenarios.

(iii) Based on the stochastic multiple Lyapunov function approach, the solvability condition achieving the control objectives is given to guarantee the bumpless transfer hybrid non-fragile finite-time H_∞ performance for MJSs.

II. Problem formulation and preliminaries

Consider a type of MJS described as

$$\begin{aligned}\dot{x}(t) =&A_{m(t)}x(t) + B_{m(t)}u(t) + D_{m(t)}\omega(t),\\ z(t) =&J_{m(t)}x(t) + F_{m(t)}\omega(t),\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^{\mathfrak{n}}$, $u(t) \in \mathbb{R}^{\mathfrak{g}}$, $z(t) \in \mathbb{R}^{\mathfrak{m}}$, and $\omega(t) \in \mathbb{R}^{\mathfrak{q}}$ denote the system state, the control input, the system output, and the disturbance, respectively. Assume that $\omega(t)$ satisfies

$$\int_0^{T_b} \omega^T(s)\omega(s)ds < \varpi, \qquad \varpi \geq 0. \tag{2}$$

$A_{m(t)}$, $B_{m(t)}$, $D_{m(t)}$, $F_{m(t)}$, and $J_{m(t)}$ are available constant matrices. $\{m(t), t \geq 0\}$ is a right continuous Markov process taking values in a set of positive integers $\mathbf{M} = \{1, 2, \ldots, M\}$ with the following transition probability

$$\begin{aligned}&Pr\left\{m(t+\Delta h) = f|m(t) = g\right\}\\ &=\begin{cases}\vartheta_{gf}\Delta h + o(\Delta h), & g \neq f,\\ 1 + \vartheta_{gg}\Delta h + o(\Delta h), & g = f,\end{cases}\end{aligned} \tag{3}$$

in which $\Delta h > 0$ satisfies $\lim\limits_{\Delta h \to 0} \frac{o(\Delta h)}{\Delta h} = 0$, and $\vartheta_{gf} \geq 0$, $g \neq f$ represents the transition rate from system mode g to system mode f with $\sum\limits_{g=1, g\neq f}^{M} \vartheta_{gf} = -\vartheta_{gg}$ for $g, f \in \mathbf{M}$. For more general application scenarios, we consider generalized transition rates where ϑ_{gf} is partially available. For instance, the matrix $\mathcal{M}$ may be described as

$$\mathcal{M} = \begin{bmatrix} \vartheta_{11} & ? & \cdots & ? \\ ? & ? & \cdots & \vartheta_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \vartheta_{M1} & ? & \cdots & \vartheta_{MM} \end{bmatrix}, \tag{4}$$

where '?' denotes that the transition rate is unavailable. Define $\mathbf{M} = \mathbf{M}_g^a \cup \mathbf{M}_g^{ua}$, $\forall g \in \mathbf{M}$, and

$$\begin{aligned}\mathbf{M}_g^a &\triangleq \{f : \vartheta_{gf} \ \text{ is available, for } \ f \in \mathbf{M}\},\\ \mathbf{M}_g^{ua} &\triangleq \{f : \vartheta_{gf} \ \text{ is unavailable, for } \ f \in \mathbf{M}\}.\end{aligned} \tag{5}$$

When $\mathbf{M}_g^a \neq \emptyset$, $\mathbf{M}_g^a$ can be expressed as

$$\mathbf{M}_g^a = \{\mathfrak{f}_1^g, \mathfrak{f}_2^g, \ldots, \mathfrak{f}_p^g\}, 1 \leq p \leq M, \tag{6}$$

in which $\mathfrak{f}_p^g$ denotes the pth transition rate of the set $\mathbf{M}_g^a$ in the gth row of the matrix $\mathcal{M}$.

The hybrid non-fragile controller is developed as

$$u(t) = (K_{m(t)} + \Delta K_{m(t)})x(t) \tag{7}$$

with

$$\Delta K_{m(t)} = \sigma_{m(t)}\Delta K_{1m(t)} + \big(1 - \sigma_{m(t)}\big)\Delta K_{2m(t)}K_{m(t)}, \tag{8}$$

where $K_{m(t)}$ is a nominal control gain, which will be designed later. $\Delta K_{m(t)}$ is the hybrid parameter perturbations of the controller (7). $\sigma_{m(t)}$ with $0 \leq \sigma_{m(t)} \leq 1$ is the weight coefficient satisfying the transition probability (3) of Markov process $m(t)$. $\Delta K_{1m(t)}$ and $\Delta K_{2m(t)}K_{m(t)}$ are the additive and multiplicative perturbations of the form

$$\begin{aligned}\Delta K_{1m(t)} &= G_{1m(t)}\daleth_{1m(t)}(t)Q_{1m(t)},\\ \Delta K_{2m(t)} &= G_{2m(t)}\daleth_{2m(t)}(t)Q_{2m(t)},\end{aligned} \tag{9}$$

where $G_{1m(t)}$, $G_{2m(t)}$, $Q_{1m(t)}$, and $Q_{2m(t)}$ are known matrices, $\daleth_{1m(t)}(t)$ and $\daleth_{2m(t)}(t)$ are unknown matrix functions satisfying $\|\daleth_{1m(t)}(t)\|^2 \leq 1$ and $\|\daleth_{2m(t)}(t)\|^2 \leq 1$.

Remark 1: Owing to the inherent imprecisions and uncertainties, the developed controller should be able to tolerate the perturbation and its own parameter uncertainty. It is worth noting that the perturbation and parameter uncertainty are mainly categorized as multiplicative case and additive case. We develop a hybrid non-fragile controller, where the perturbations that appeared in the controller gains are modeled as uncertain gains. The proposed hybrid non-fragile control strategy is more general, which can describe the more general controller gain variation. In contrast to a signal additive or multiplicative case [28], [29], the designed hybrid non-fragile controller can tolerate the co-existing additive and multiplicative perturbations.

Remark 2: The proposed hybrid non-fragile controller (7) consists of the additive and multiplicative perturbation terms. $\Delta K_{1m(t)}$ is the additive perturbation term which often depicts the small perturbations and uncertainties. $\Delta K_{2m(t)}K_{m(t)}$ is the multiplicative perturbation term that describes the readjustment controller gains. The weight coefficient $\sigma_{m(t)}$ is introduced to establish the relationship between the additive perturbation $\Delta K_{1m(t)}$ and multiplicative perturbation $\Delta K_{2m(t)}K_{m(t)}$. The free weight coefficient can adjust the proportion between the additive and multiplicative perturbations so that the proposed hybrid non-fragile controller tolerates the co-existing additive and multiplicative perturbations. As two special cases, when $\sigma_{m(t)} = 1$, the controller (7) is simplified to an additive non-fragile controller. When $\sigma_{m(t)} = 0$, controller (7) is reduced to a multiplicative non-fragile controller. The classical non-fragile controller is shown to be a special case of the proposed hybrid non-fragile controller.

Remark 3: There is a significant difference between the availability of transition rates and the measurability of jumping signal [32], [33]. Although the transition rates are unavailable, the jumping signal $m(t)$ can be detected by the sensors which is applied to the design of the bumpless transfer hybrid non-fragile controller. In most studies of MJSs, the measurability of the jumping signal has been well recognized as a reasonable and acceptable assumption [32], [33].

Substituting (7)–(9) into MJS (1), the closed-loop system is obtained by

$$\begin{aligned}\dot{x}(t) =& \big[A_{m(t)} + K_{m(t)} + \sigma_{m(t)}B_{m(t)}G_{1m(t)}\daleth_{1m(t)}(t)\\ &\times Q_{1m(t)} + \big(1-\sigma_{m(t)}\big)B_{m(t)}G_{2m(t)}\daleth_{2m(t)}(t)\\ &\times Q_{2m(t)}K_{m(t)}\big]\,x(t) + D_{m(t)}\omega(t), \\ z(t) =& J_{m(t)}x(t) + F_{m(t)}\omega(t),\end{aligned} \tag{10}$$

For simplicity of subsequent expression, define $A_{m(t)} \triangleq A_g$, $B_{m(t)} \triangleq B_g$, $D_{m(t)} \triangleq D_g$, $F_{m(t)} \triangleq F_g$, $J_{m(t)} \triangleq J_g$, $K_{m(t)} \triangleq K_g$, $\sigma_{m(t)} \triangleq \sigma_g$, $G_{1m(t)} \triangleq G_{1g}$, $G_{2m(t)} \triangleq G_{2g}$, $Q_{1m(t)} \triangleq Q_{1g}$, $Q_{2m(t)} \triangleq Q_{2g}$, $\daleth_{1m(t)}(t) \triangleq \daleth_{1g}(t)$, and $\daleth_{2m(t)}(t) \triangleq \daleth_{2g}(t)$, when $m(t) = g$, $g \in \mathbf{M}$.

Due to $(K_g + \Delta K_g)x(t) \neq (K_f + \Delta K_f)x(t)$, $\forall g, f \in \mathbf{M}$, hybrid non-fragile controller (7) is discontinuous at the jumping points. We require that designed hybrid non-fragile controller (7) satisfies the bumpless transfer level

$$\|u^{\star}(t) - u(t)\|^2 < \upsilon\|x(t)\|^2 \tag{11}$$

to make the controller jumping as smooth as possible, in which the scalar $\upsilon > 0$ is called a bumpless transfer level, and $u^{\star}(t) \triangleq K^{\star}x(t)$ is regarded as a reference control input.

Remark 4: The bumpless transfer control aims to limit the amplitude between any two consecutive jumping controllers, not to find a reference control input $u^{\star}(t)$. The idea of introducing $u^{\star}(t)$ is to adjust the jumping controller (7) such that the bumpless transfer level (11) is satisfied. The main task of $u^{\star}(t)$ is not to stabilize the system but to provide a standard for limiting the distance value between $u(t)$ and $u^{\star}(t)$. Therefore, we do not care about the actual value of $u^{\star}(t)$. Moreover, when the real value of $u^{\star}(t)$ is also an important reference, we can take $K^{\star}$ in advance according to the actual demand, then solve the control gain $K_{m(t)}$.

Remark 5: Compared with the existing work [3], [31], the bumpless transfer hybrid non-fragile controller is more general because the multiple perturbations are considered. The additive and multiplicative perturbations directly affect the bumpless transfer constraint, which brings about technical difficulties for the design of the bumpless transfer controller. The bumpless transfer control aims to minimize control bumps between $(K_g + \Delta K_g)x(t)$ and $K^{\star}x(t)$, $\forall g \in \mathbf{M}$.

Definition 1: [34] For given positive scalar T_b, MJS (1) is said to be finite-time bounded with respect to $(\varepsilon_1, \varepsilon_2, S_g, T_b, \varpi)$, if there exist scalars $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ with $\varepsilon_1 < \varepsilon_2$, and matrix $S_g > 0$, $g \in \mathbf{M}$ such that

$$\mathbb{E}\left\{x_0^T S_g x_0\right\} \leq \varepsilon_1 \Rightarrow \mathbb{E}\left\{x^T(t) S_g x(t)\right\} < \varepsilon_2, \forall t \in [0, T_b]. \tag{12}$$

Remark 6: The traditional stability reflects the dynamic behavior of the system during an infinite time region. In practice, the dynamic behavior deserves more attention within a finite time interval [16]. An MJS that is not finite-time stable if its state surpasses a physical threshold during the finite-time interval may be stochastic stable. A finite-time stable MJS could not be stochastic stable.

Bumpless transfer hybrid non-fragile finite-time H_∞ control issue: For MJS (1), find a hybrid non-fragile controller such that

(i) Hybrid non-fragile controller (7) satisfies bumpless transfer level (11);
(ii) MJS (1) is finite-time bounded with respect to $(\varepsilon_1, \varepsilon_2, S_g, T_b, \varpi)$;
(iii) Under zero initial condition, MJS (1) satisfies

$$\int_0^{T_b} z^T(s)z(s)ds \leq \gamma^2 \int_0^{T_b} \omega^T(s)\omega(s)ds. \tag{13}$$

III. Main results

In the section, a hybrid non-fragile controller is designed subject to additive and multiplicative perturbations. Some feasible conditions are developed.

Theorem 1: Given positive scalars T_b, α, ε_1, ε_2, ϖ, σ_g, $\hbar_{\ell g}$, and positive definite matrix S_g, the issue of bumpless transfer hybrid non-fragile finite-time H_∞ control for MJS (1) is solvable, if there exist symmetric matrix R_g, matrices $K^\star$, K_g, positive definite matrices P_g, $\hat{P}_g$, and positive scalars $\breve{\gamma}$, $\hbar_{2g}$, $\hbar_{5g}$, υ_g for $\forall g, f \in \mathbf{M}$, $\ell \in \{1, 3, 4, 6\}$ such that

$$\begin{bmatrix} \Psi_{1g} & P_g D_g & J_g^T & Q_{1g}^T & K_g^T Q_{2g}^T \\ * & -\breve{\gamma}^2 I & F_g^T & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -\hbar_{1g} I & 0 \\ * & * & * & * & -\hbar_{2g} I - \hbar_{3g} I \end{bmatrix} < 0, \tag{14}$$

$$P_f - R_g \le 0, \qquad g \ne f, \qquad f \in \mathbf{M}_g^{ua}, \tag{15}$$

$$P_f - R_g \ge 0, \qquad g = f, \qquad f \in \mathbf{M}_g^{ua}, \tag{16}$$

$$\overline{\rho}\varepsilon_1 + \frac{\varpi \breve{\gamma}^2 \left(1 - e^{-\alpha T_b}\right)}{\alpha} < \underline{\rho}\varepsilon_2 e^{-\alpha T_b}, \tag{17}$$

$$\begin{bmatrix} \Psi_{2g} & K^\star - K_g & 0 & 0 \\ * & \upsilon_g I & Q_{1g}^T & K_g^T Q_{2g}^T \\ * & * & \hbar_{4g} I & 0 \\ * & * & * & \hbar_{5g} I + \hbar_{6g} I \end{bmatrix} > 0, \tag{18}$$

where

$$\begin{aligned} \Psi_{1g} &= A_g^T P_g + P_g A_g + K_g^T B_g^T P_g + P_g B_g K_g - \alpha P_g \\ &\quad + \sum_{f \in \mathbf{M}_g^a} \vartheta_{gf}(P_f - R_g) + \hbar_{1g}\sigma_g^2 P_g B_g G_{1g} G_{1g}^T B_g^T P_g \\ &\quad + \left(\hbar_{2g} + \hbar_{3g}\sigma_g^2\right) P_g B_g G_{2g} G_{2g}^T B_g^T P_g, \\ \Psi_{2g} &= I - \hbar_{4g}\sigma_g^2 G_{1g} G_{1g}^T - \left(\hbar_{5g} + \hbar_{6g}\sigma_g^2\right) G_{2g} G_{2g}^T, \\ \hat{P}_g &= S_g^{-\frac{1}{2}} P_g S_g^{-\frac{1}{2}}, \\ \overline{\rho} &= \max_{g \in \mathbf{M}} \{\rho_{\max}(\hat{P}_g)\}, \\ \underline{\rho} &= \min_{g \in \mathbf{M}} \{\rho_{\min}(\hat{P}_g)\}. \end{aligned}$$

Moreover, the bumpless transfer level with $\upsilon = \max_{g \in \mathbf{M}} \upsilon_g$ and the H_∞ index with $\gamma = \sqrt{e^{\alpha T_b}}\breve{\gamma}$ are ensured.

Proof. Consider the stochastic multiple Lyapunov functions

$$V_{m(t)}(t) = x^T(t) P_{m(t)} x(t). \tag{19}$$

Assume $m(t) = g \in \mathbf{M}$, then $V_{m(t)}(t) = V_g(t)$ and $P_{m(t)} = P_g$. Calculating the weak infinitesimal operator of $V_g(t)$ along the trajectory of system (10) obtains

$$\begin{aligned} &\mathfrak{L}V_g(x(t)) \\ &= x^T(t)\big[A_g^T P_g + P_g A_g + K_g^T B_g^T P_g + P_g B_g K_g \\ &\quad + \sigma_g Q_{1g}^T \daleth_{1g}^T(t) G_{1g}^T B_g^T P_g + \sigma_g P_g B_g G_{1g} \daleth_{1g}(t) Q_{1g} \\ &\quad + (1 - \sigma_g) K_g^T Q_{2g}^T \daleth_{2g}^T(t) G_{2g}^T B_g^T P_g + \sum_{f=1}^{M} \vartheta_{gf} P_f \\ &\quad + (1 - \sigma_g) P_g B_g G_{2g} \daleth_{2g}(t) Q_{2g} K_g\big] x(t) \\ &\quad + \omega^T(t) D_g^T P_g x(t) + x^T(t) P_g D_g \omega(t). \end{aligned} \tag{20}$$

For any symmetric matrix R_g, $g \in \mathbf{M}$, the equation $\sum_{f=1}^{M} \vartheta_{gf} R_g = 0$ always holds. We have

$$\begin{aligned} &\mathfrak{L}V_g(x(t)) \\ &= x^T(t)\big[A_g^T P_g + P_g A_g + K_g^T B_g^T P_g + P_g B_g K_g \\ &\quad + \sum_{f \in \mathbf{M}_g^a} \vartheta_{gf}(P_f - R_g) + \sum_{f \in \mathbf{M}_g^{ua}} \vartheta_{gf}(P_f - R_g) \\ &\quad + K_g^T Q_{2g}^T \daleth_{2g}^T(t) G_{2g}^T B_g^T P_g - \sigma_g K_g^T Q_{2g}^T \daleth_{2g}^T(t) G_{2g}^T B_g^T P_g \\ &\quad + P_g B_g G_{2g} \daleth_{2g}(t) Q_{2g} K_g - \sigma_g P_g B_g G_{2g} \daleth_{2g}(t) Q_{2g} K_g \\ &\quad + \sigma_g Q_{1g}^T \daleth_{1g}^T(t) G_{1g}^T B_g^T P_g + \sigma_g P_g B_g G_{1g} \daleth_{1g}(t) Q_{1g}\big] x(t) \\ &\quad + \omega^T(t) D_g^T P_g x(t) + x^T(t) P_g D_g \omega(t). \end{aligned} \tag{21}$$

According to (21), we conclude

$$\begin{aligned} &\mathfrak{L}V_g(x(t)) + z^T(t) z(t) - \breve{\gamma}^2 \omega^T(t) \omega(t) \\ &= x^T(t)\big[A_g^T P_g + P_g A_g + K_g^T B_g^T P_g + P_g B_g K_g \\ &\quad + \sum_{f \in \mathbf{M}_g^a} \vartheta_{gf}(P_f - R_g) + \sum_{f \in \mathbf{M}_g^{ua}} \vartheta_{gf}(P_f - R_g) \\ &\quad + K_g^T Q_{2g}^T \daleth_{2g}^T(t) G_{2g}^T B_g^T P_g - \sigma_g K_g^T Q_{2g}^T \daleth_{2g}^T(t) G_{2g}^T B_g^T P_g \\ &\quad + P_g B_g G_{2g} \daleth_{2g}(t) Q_{2g} K_g - \sigma_g P_g B_g G_{2g} \daleth_{2g}(t) Q_{2g} K_g \\ &\quad + \sigma_g Q_{1g}^T \daleth_{1g}^T(t) G_{1g}^T B_g^T P_g + \sigma_g P_g B_g G_{1g} \daleth_{1g}(t) Q_{1g}\big] x(t) \\ &\quad + x^T(t) \left(P_g D_g + J_g^T F_g\right) \omega(t) + \omega^T(t) \left(D_g^T P_g \right. \\ &\quad \left. + F_g^T J_g\right) x(t) + \omega^T(t) \left(F_g^T F_g - \breve{\gamma}^2\right) \omega(t). \end{aligned} \tag{22}$$

Denote

$$\begin{aligned} \mathfrak{A}_g(t) &= \begin{bmatrix} \mho_{1g}(t) + J_g^T J_g & P_g D_g + J_g^T F_g \\ * & -\breve{\gamma}^2 I + F_g^T F_g \end{bmatrix} \\ &= \begin{bmatrix} \mho_{1g}(t) & P_g D_g \\ * & -\breve{\gamma}^2 I \end{bmatrix} + \begin{bmatrix} J_g^T J_g & J_g^T F_g \\ * & F_g^T F_g \end{bmatrix}, \end{aligned} \tag{23}$$

where

$$\begin{aligned} &\mho_{1g}(t) \\ &= A_g^T P_g + P_g A_g + K_g^T B_g^T P_g + P_g B_g K_g \\ &\quad + \sum_{f \in \mathbf{M}_g^a} \vartheta_{gf}(P_f - R_g) + \sum_{f \in \mathbf{M}_g^{ua}} \vartheta_{gf}(P_f - R_g) \\ &\quad + \sigma_g Q_{1g}^T \daleth_{1g}^T(t) G_{1g}^T B_g^T P_g + \sigma_g P_g B_g G_{1g} \daleth_{1g}(t) Q_{1g} \\ &\quad + K_g^T Q_{2g}^T \daleth_{2g}^T(t) G_{2g}^T B_g^T P_g - \sigma_g K_g^T Q_{2g}^T \daleth_{2g}^T(t) G_{2g}^T B_g^T P_g \\ &\quad + P_g B_g G_{2g} \daleth_{2g}(t) Q_{2g} K_g - \sigma_g P_g B_g G_{2g} \daleth_{2g}(t) Q_{2g} K_g. \end{aligned}$$

From (23), one gets

$$\mathfrak{B}_g(t) = \begin{bmatrix} \mho_{1g}(t) & P_g D_g & J_g^T \\ * & -\breve{\gamma}^2 I & F_g^T \\ * & * & -I \end{bmatrix}. \tag{24}$$

There exist scalars $\hbar_{1g} > 0$, $\hbar_{2g} > 0$, and $\hbar_{3g} > 0$ such that

$$\begin{aligned} &\sigma_g P_g B_g G_{1g} \daleth_{1g}(t) Q_{1g} + \sigma_g Q_{1g}^T \daleth_{1g}^T(t) G_{1g}^T B_g^T P_g \\ &\le \hbar_{1g}\sigma_g^2 P_g B_g G_{1g} G_{1g}^T B_g^T P_g + \hbar_{1g}^{-1} Q_{1g}^T Q_{1g}, \end{aligned} \tag{25}$$

$$\begin{aligned} &P_g B_g G_{2g} \daleth_{2g}(t) Q_{2g} K_g + K_g^T Q_{2g}^T \daleth_{2g}^T(t) G_{2g}^T B^T P_g \\ &\le \hbar_{2g} P_g B_g G_{2g} G_{2g}^T B_g^T P_g + \hbar_{2g}^{-1} K_g^T Q_{2g}^T Q_{2g} K_g, \end{aligned} \tag{26}$$

and

$$-\sigma_g P_g B_g G_{2g} \daleth_{2g}(t) Q_{2g} K_g - \sigma_g K_g^T Q_{2g}^T \daleth_{2g}^T(t) G_{2g}^T B_g^T P_g$$

$$\leq \hbar_{3g}\sigma_g^2 P_g B_g G_{2g} G_{2g}^T B_g^T P_g + \hbar_{3g}^{-1} K_g^T Q_{2g}^T Q_{2g} K_g. \tag{27}$$

Substituting (25)–(27) into (24) leads to

$$\mathfrak{C}_g(t) = \begin{bmatrix} \mho_{2g}(t) & P_g D_g & J_g^T \\ * & -\breve{\gamma}^2 I & F_g^T \\ * & * & -I \end{bmatrix}, \tag{28}$$

in which

$$\begin{aligned} \mho_{2g}(t) = {} & A_g^T P_g + P_g A_g + K_g^T B_g^T P_g + P_g B_g K_g \\ & + \sum_{f \in \mathbf{M}_g^a} \vartheta_{gf}(P_f - R_g) + \sum_{f \in \mathbf{M}_g^{ua}} \vartheta_{gf}(P_f - R_g) \\ & + \hbar_{1g}\sigma_g^2 P_g B_g G_{1g} G_{1g}^T B_g^T P_g + \hbar_{1g}^{-1} Q_{1g}^T Q_{1g} \\ & + \left(\hbar_{2g} + \hbar_{3g}\sigma_g^2\right) P_g B_g G_{2g} G_{2g}^T B_g^T P_g \\ & + \left(\hbar_{2g}^{-1} + \hbar_{3g}^{-1}\right) K_g^T Q_{2g}^T Q_{2g} K_g. \end{aligned}$$

Applying Schur complement, we know that the inequality $\mathfrak{C}_g(t) < 0$ is guaranteed by condition (14), which implies that $\mathfrak{A}_g(t) < 0$. In view of conditions (14)–(16), we obtain

$$\mathfrak{L}V_g(x(t)) \leq \alpha V_g(x(t)) - z^T(t)z(t) + \breve{\gamma}^2 \omega^T(t)\omega(t). \tag{29}$$

Multiplying both sides of (29) by $e^{-\alpha t}$ yields

$$\mathfrak{L}\left[V_g(x(t))e^{-\alpha t}\right] \leq \left[-z^T(t)z(t) + \breve{\gamma}^2\omega^T(t)\omega(t)\right] e^{-\alpha t}. \tag{30}$$

Moreover, we have

$$\mathfrak{L}V_g(x(t)) \leq e^{-\alpha t}\breve{\gamma}^2 \omega^T(t)\omega(t). \tag{31}$$

Letting $m(t_0) = g_0 \in \mathbf{M}$ yields

$$\mathbb{E}\left\{e^{-\alpha T_b} V_g(x(t)) - V_{g_0}(x_0)\right\} \leq \int_0^{T_b} \breve{\gamma}^2 \varpi e^{-\alpha s} ds. \tag{32}$$

From (32), we can infer that

$$\mathbb{E}\{V_g(x(t))\} \leq e^{\alpha T_b}\left[V_{g_0}(x_0) + \frac{\varpi\breve{\gamma}^2\left(1 - e^{-\alpha T_b}\right)}{\alpha}\right]. \tag{33}$$

Defining $P_g \triangleq S_g^{\frac{1}{2}} \hat{P}_g S_g^{\frac{1}{2}}$, one has

$$\mathbb{E}\{V_g(x(t))\} \leq e^{\alpha T_b}\left[\overline{\rho}\varepsilon_1 + \frac{\varpi\breve{\gamma}^2\left(1 - e^{-\alpha T_b}\right)}{\alpha}\right]. \tag{34}$$

Noting that

$$x^T(t)P_g x(t) \geq \underline{\rho}\mathbb{E}\left\{x^T(t)S_g x(t)\right\}, \tag{35}$$

one obtains

$$\mathbb{E}\left\{x^T(t)S_g x(t)\right\} < \frac{e^{\alpha T_b}\left[\overline{\rho}\varepsilon_1 + \frac{\varpi\breve{\gamma}^2\left(1 - e^{-\alpha T_b}\right)}{\alpha}\right]}{\underline{\rho}} < \varepsilon_2. \tag{36}$$

Clearly, MJS (1) is the finite-time bounded with respect to $(\varepsilon_1, \varepsilon_2, S_g, T_b, \varpi)$.

Applying Dynkin's formula to (30), we derive

$$\begin{aligned} & \mathbb{E}\left\{e^{\alpha t} V_g(x(t)) - V_{g_0}(x_0)\right\} \\ & \leq \mathbb{E}\left\{\int_0^t e^{\alpha s}\left[-z^T(s)z(s) + \breve{\gamma}^2\omega^T(s)\omega(s)\right] ds\right\}. \end{aligned} \tag{37}$$

Based on (37), we get

$$\begin{aligned} & \mathbb{E}\left\{e^{\alpha T_b} V_g(x(t))\right\} \\ & \leq \mathbb{E}\left\{\int_0^{T_b} e^{\alpha s}\left[-z^T(s)z(s) + \breve{\gamma}^2\omega^T(s)\omega(s)\right] ds\right\}, \end{aligned} \tag{38}$$

which means

$$\int_0^{T_b} z^T(s)z(s)ds \leq e^{\alpha T_b}\breve{\gamma}^2 \int_0^{T_b} \omega^T(s)\omega(s)ds. \tag{39}$$

Hence, inequality (13) holds with $\gamma = \sqrt{e^{\alpha T_b}}\breve{\gamma}$.

It can be derived by condition (18) that

$$\begin{bmatrix} I + \mho_{3g} & K^\star - K_g \\ * & \upsilon_g I + \mho_{4g} \end{bmatrix} > 0, \tag{40}$$

where

$$\begin{aligned} \mho_{3g} = & -\hbar_{4g}\sigma_g^2 G_{1g} G_{1g}^T - \left(\hbar_{5g} + \hbar_{6g}\sigma_g^2\right) G_{2g} G_{2g}^T, \\ \mho_{4g} = & -\hbar_{4g}^{-1} Q_{1g}^T Q_{1g} - \left(\hbar_{5g}^{-1} + \hbar_{6g}^{-1}\right) K_g^T Q_{2g}^T Q_{2g} K_g. \end{aligned}$$

There exist positive scalars $\hbar_{4g}$, $\hbar_{5g}$, and $\hbar_{6g}$ such that

$$\begin{aligned} & \begin{bmatrix} \mho_{3g} & 0 \\ 0 & \mho_{4g} \end{bmatrix} \\ & \leq \begin{bmatrix} 0 & -\sigma_g G_{1g} \daleth_{1g}(t) Q_{1g} - (1-\sigma_g) G_{2g} \daleth_{2g}(t) Q_{2g} K_g \\ * & 0 \end{bmatrix}. \end{aligned} \tag{41}$$

Therefore

$$\begin{bmatrix} I & K^\star - K_g - \Delta K_g \\ * & \upsilon_g I \end{bmatrix} > 0. \tag{42}$$

From (42), one has

$$\upsilon_g I > [K^\star - (K_g + \Delta K_g)]^T [K^\star - (K_g + \Delta K_g)], \tag{43}$$

which is equivalent to

$$\begin{aligned} & \upsilon_g x^T(t)x(t) \\ & > x^T(t)[K^\star - (K_g + \Delta K_g)]^T [K^\star - (K_g + \Delta K_g)]x(t). \end{aligned} \tag{44}$$

When the gth hybrid non-fragile controller is activated, (44) guarantees

$$\upsilon_g \|x(t)\|^2 > \|K^\star x(t) - (K_g + \Delta K_g)x(t)\|^2. \tag{45}$$

Obviously, the bumpless transfer level (11) with $\upsilon = \max_{g \in \mathbf{M}} \upsilon_g$ is satisfied for hybrid non-fragile controller (7). This completes the proof. □

Since the terms $K_g^T B_g^T P_g$, $\hbar_{1g}\sigma_g P_g B_g G_{1g} G_{1g}^T B_g^T P_g$, $P_g B_g K_g$, and $\hbar_{2g}(1-\sigma_g) P_g B_g G_{2g} G_{2g}^T B_g^T P_g$ appeared in condition (14) result in the synthesis condition for hybrid non-fragile controller (7) being non-convex. To find the convex solution condition, we give the following result.

Theorem 2: Given positive scalars T_b, α, ε_1, ε_2, ϖ, σ_g, $\hbar_{\ell g}$, and positive definite matrix S_g, the issue of bumpless transfer hybrid non-fragile finite-time H_∞ control for MJS (1) is solvable, if there exist positive scalars $\breve{\gamma}$, $\hbar_{2g}$, $\hbar_{5g}$, $\aleph$, υ_g, matrices $K^\star$, Y_g, symmetric matrix $\mathcal{R}_g$, and positive definite matrix X_g, for $\forall g, f \in \mathbf{M}$, $\ell \in \{1,3,4,6\}$ such that

$$\begin{bmatrix} \Psi_{3g} & D_g & X_g J_g^T & X_g Q_{1g}^T & Y_g^T Q_{2g}^T & \Psi_{4g} \\ * & -\breve{\gamma}^2 I & F_g^T & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 \\ * & * & * & -\hbar_{1g} I & 0 & 0 \\ * & * & * & * & -\mathfrak{W}_g & 0 \\ * & * & * & * & * & -\Psi_{5g} \end{bmatrix} < 0, \quad g \in \mathbf{M}_g^a, \tag{46}$$

$$\begin{bmatrix} \Psi_{6g} & D_g & X_g J_g^T & X_g Q_{1g}^T & Y_g^T Q_{2g}^T & \Psi_{7g} \\ * & -\breve{\gamma}^2 I & F_g^T & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 \\ * & * & * & -\hbar_{1g} I & 0 & 0 \\ * & * & * & * & -\mathfrak{W}_g & 0 \\ * & * & * & * & * & -\Psi_{8g} \end{bmatrix} < 0, \quad g \in \mathbf{M}_g^{ua}, \tag{47}$$

$$\begin{bmatrix} -\mathcal{R}_g & X_g \\ * & -X_f \end{bmatrix} < 0, \quad g \neq f, \quad f \in \mathbf{M}_g^{ua}, \tag{48}$$

$$X_f - \mathcal{R}_g > 0, \quad g = f, \quad f \in \mathbf{M}_g^{ua}, \tag{49}$$

$$\begin{bmatrix} -\varepsilon_2 e^{-\alpha T_b} + \frac{\varpi \breve{\gamma}^2 \left(1 - e^{-\alpha T_b}\right)}{\alpha} & \sqrt{\varepsilon_1} \\ * & -\aleph \end{bmatrix} < 0, \tag{50}$$

$$\aleph S_g^{-1} < X_g < S_g^{-1}, \tag{51}$$

$$\begin{bmatrix} \Psi_{2g} & K^\star & Y_g & 0 & 0 & 0 \\ * & \upsilon_g I & I & I & Q_{1g}^T & 0 \\ * & * & X_g & 0 & 0 & 0 \\ * & * & * & X_g & 0 & Y_g^T Q_{2g}^T \\ * & * & * & * & \hbar_{4g} I & 0 \\ * & * & * & * & * & \hbar_{5g} I + \hbar_{6g} I \end{bmatrix} > 0, \tag{52}$$

where

$$\begin{aligned} \Psi_{3g} &= X_g A_g^T + A_g X_g + Y_g^T B_g^T + B_g Y_g - \alpha X_g \\ &\quad + \hbar_{1g} \sigma_g^2 B_g G_{1g} G_{1g}^T B_g^T + \left(\hbar_{2g} + \hbar_{3g} \sigma_g^2\right) B_g G_{2g} G_{2g}^T B_g^T \\ &\quad - \sum_{f \in \mathbf{M}_g^a} \vartheta_{gf} \mathcal{R}_g + \vartheta_{gg} X_g, \\ \Psi_{4g} &= \Big[\sqrt{\vartheta_{g\mathfrak{f}_1^g}} X_g \;\; \sqrt{\vartheta_{g\mathfrak{f}_2^g}} X_g \;\; \cdots \;\; \sqrt{\vartheta_{g\mathfrak{f}_{\mathfrak{g}-1}^g}} X_g \\ &\quad\quad \sqrt{\vartheta_{g\mathfrak{f}_{\mathfrak{g}+1}^g}} X_g \;\; \cdots \;\; \sqrt{\vartheta_{g\mathfrak{f}_p^g}} X_g \Big], \\ \Psi_{5g} &= \operatorname{diag}\left\{ X_{\mathfrak{f}_1^g}, X_{\mathfrak{f}_2^g}, \ldots, X_{\mathfrak{f}_{\mathfrak{g}-1}^g}, X_{\mathfrak{f}_{\mathfrak{g}+1}^g}, \ldots, X_{\mathfrak{f}_p^g} \right\}, \\ \mathfrak{W}_g &= \hbar_{2g} I + \hbar_{3g} I, \\ \Psi_{6g} &= X_g A_g^T + A_g X_g + Y_g^T B_g^T + B_g Y_g - \alpha X_g \\ &\quad + \hbar_{1g} \sigma_g^2 B_g G_{1g} G_{1g}^T B_g^T + \left(\hbar_{2g} + \hbar_{3g} \sigma_g^2\right) B_g G_{2g} G_{2g}^T B_g^T \\ &\quad - \sum_{f \in \mathbf{M}_g^a} \vartheta_{gf} \mathcal{R}_g, \\ \Psi_{7g} &= \Big[\sqrt{\vartheta_{g\mathfrak{f}_1^g}} X_g \;\; \sqrt{\vartheta_{g\mathfrak{f}_2^g}} X_g \;\; \cdots \;\; \sqrt{\vartheta_{g\mathfrak{f}_p^g}} X_g \Big], \\ \Psi_{8g} &= \operatorname{diag}\left\{ X_{\mathfrak{f}_1^g}, X_{\mathfrak{f}_2^g}, \ldots, X_{\mathfrak{f}_p^g} \right\} \end{aligned}$$

with $\mathfrak{f}_1^g, \mathfrak{f}_2^g, \ldots, \mathfrak{f}_p^g$ described as in (6) and $\mathfrak{f}_\mathfrak{g}^g = g$. The bumpless transfer level with $\upsilon = \max_{g \in \mathbf{M}} \upsilon_g$ and the H_∞ index with $\gamma = \sqrt{e^{\alpha T_b}} \breve{\gamma}$ are achieved. Moreover, controller gain in the hybrid non-fragile controller (7) is given by $K_g = Y_g X_g^{-1}$.

Proof. Define $P_g \triangleq X_g^{-1}$ and $K_g \triangleq Y_g X_g^{-1}$. Performing a transformation to condition (14) by $\operatorname{diag}\{X_g, I, I, I, I\}$, one gets

$$\begin{bmatrix} \Phi_{1g} & D_g & X_g J_g^T & X_g Q_{1g}^T & Y_g^T Q_{2g}^T \\ * & -\breve{\gamma}^2 I & F_g^T & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -\hbar_{1g} I & 0 \\ * & * & * & * & -\hbar_{2g} I - \hbar_{3g} I \end{bmatrix} < 0, \tag{53}$$

in which

$$\begin{aligned} \Phi_{1g} &= X_g A_g^T + A_g X_g + Y_g^T B_g^T + B_g Y_g - \alpha X_g \\ &\quad + \sum_{f \in \mathbf{M}_g^a} \vartheta_{gf} \left(X_g X_f^{-1} X_g - X_g R_g X_g \right) + \hbar_{1g} \sigma_g^2 B_g \\ &\quad \times G_{1g} G_{1g}^T B_g^T + \left(\hbar_{2g} + \hbar_{3g} \sigma_g^2\right) B_g G_{2g} G_{2g}^T B_g^T. \end{aligned}$$

Let $X_g R_g X_g = \mathcal{R}_g$. Due to $\vartheta_{gg} < 0$, $\forall g \in \mathbf{M}$, (53) is dealt with by the two different cases.

Case 1. When $g \in \mathbf{M}_g^a$, (53) is rewritten as

$$\begin{bmatrix} \breve{\Omega}_{1g} & D_g & X_g J_g^T & X_g Q_{1g}^T & Y_g^T Q_{2g}^T \\ * & -\breve{\gamma}^2 I & F_g^T & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -\hbar_{1g} I & 0 \\ * & * & * & * & -\hbar_{2g} I - \hbar_{3g} I \end{bmatrix} < 0, \tag{54}$$

where

$$\begin{aligned} \breve{\Omega}_{1g} &= X_g A_g^T + A_g X_g + Y_g^T B_g^T + B_g Y_g + (\vartheta_{gg} - \alpha) X_g \\ &\quad + \sum_{f \in \mathbf{M}_g^a, f \neq g} \vartheta_{gf} X_g X_f^{-1} X_g - \sum_{f \in \mathbf{M}_g^a} \vartheta_{gf} \mathcal{R}_g + \hbar_{1g} \sigma_g^2 \\ &\quad \times B_g G_{1g} G_{1g}^T B_g^T + \left(\hbar_{2g} + \hbar_{3g} \sigma_g^2\right) B_g G_{2g} G_{2g}^T B_g^T. \end{aligned}$$

One can see that condition (46) is guaranteed by (54).

Case 2. When $g \in \mathbf{M}_g^{ua}$, (53) is rewritten as

$$\begin{bmatrix} \hat{\Omega}_{1g} & D_g & X_g J_g^T & X_g Q_{1g}^T & Y_g^T Q_{2g}^T \\ * & -\breve{\gamma}^2 I & F_g^T & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -\hbar_{1g} I & 0 \\ * & * & * & * & -\hbar_{2g} I - \hbar_{3g} I \end{bmatrix} < 0, \tag{55}$$

where

$$\begin{aligned} \hat{\Omega}_{1g} &= X_g A_g^T + A_g X_g + Y_g^T B_g^T + B_g Y_g - \alpha X_g \\ &\quad + \sum_{f \in \mathbf{M}_g^a, f \neq g} \vartheta_{gf} X_g X_f^{-1} X_g - \sum_{f \in \mathbf{M}_g^a} \vartheta_{gf} \mathcal{R}_g + \hbar_{1g} \sigma_g^2 \\ &\quad \times B_g G_{1g} G_{1g}^T B_g^T + \left(\hbar_{2g} + \hbar_{3g} \sigma_g^2\right) B_g G_{2g} G_{2g}^T B_g^T. \end{aligned}$$

One can see that condition (81) is guaranteed by (55). Pre-and post-multiplying condition (15) by X_g, we obtain

$$X_g X_f^{-1} X_g - \mathcal{R}_g < 0, \quad g \neq f, \quad f \in \mathbf{M}_g^{ua}, \tag{56}$$

which derives condition (48). Performing a transformation to condition (16) by X_g, we know that condition (16) is guaranteed by

$$X_f - \mathcal{R}_f > 0, \quad g = f, \quad f \in \mathbf{M}_g^{ua}. \tag{57}$$

From conditions (50) and (51), we get

$$\aleph^{-1}\varepsilon_1 < \varepsilon_2 e^{-\alpha T_b} + \frac{\varpi\breve{\gamma}^2\left(1-e^{-\alpha T_b}\right)}{\alpha}, \tag{58}$$

$$\aleph I < S_g^{\frac{1}{2}} X_g S_g^{\frac{1}{2}} < I. \tag{59}$$

Together with $\mathfrak{S}_g = S_g^{\frac{1}{2}} X_g S_g^{\frac{1}{2}}$, one has

$$\aleph < \rho(\mathfrak{S}_g) < 1, \forall g \in \mathbf{M}, \tag{60}$$

$$\underline{\rho} < \aleph^{-1}, \tag{61}$$

$$1 < \overline{\rho}, \tag{62}$$

which leads to

$$\overline{\rho}\varepsilon_1 + \frac{\varpi\breve{\gamma}^2\left(1-e^{-\alpha T_b}\right)}{\alpha} < \underline{\rho}\varepsilon_2 e^{-\alpha T_b}. \tag{63}$$

Based on condition (52), we derive

$$\begin{aligned}&\begin{bmatrix} \Psi_{2g} & K^\star & Y_g & 0 \\ * & \upsilon_g I - \Phi_{3g} & I & I \\ * & * & X_g & 0 \\ * & * & * & X_g - \Phi_{4g} \end{bmatrix} \\ &= \begin{bmatrix} I & K^\star & Y_g & 0 \\ * & \upsilon_g I & I & I \\ * & * & X_g & 0 \\ * & * & * & X_g \end{bmatrix} - \begin{bmatrix} \Phi_{2g} & 0 & 0 & 0 \\ * & \Phi_{3g} & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & \Phi_{4g} \end{bmatrix} \\ &> 0,\end{aligned} \tag{64}$$

where

$$\Phi_{2g} = \hbar_{4g}\sigma_g^2 G_{1g}G_{1g}^T + \left(\hbar_{5g} + \hbar_{6g}\sigma_g^2\right)G_{2g}G_{2g}^T,$$
$$\Phi_{3g} = \hbar_{4g}^{-1} Q_{1g}^T Q_{1g}, \quad \Phi_{4g} = \left(\hbar_{5g}^{-1} + \hbar_{6g}^{-1}\right) Y_g^T Q_{2g}^T Q_{2g} Y_g.$$

There exist positive scalars $\hbar_{4g}$, $\hbar_{5g}$, and $\hbar_{6g}$ such that

$$\begin{aligned}&\begin{bmatrix} 0 & \sigma_g G_{1g}\daleth_{1g}(t)Q_{1g} & 0 & -(1-\sigma_g)G_{2g}\daleth_{2g}(t)Q_{2g}Y_g \\ * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} \\ &\leq \begin{bmatrix} \Phi_{2g} & 0 & 0 & 0 \\ * & \Phi_{3g} & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & \Phi_{4g} \end{bmatrix}.\end{aligned} \tag{65}$$

According to (64) and (65), we obtain

$$\begin{bmatrix} I & \Phi_{5g} & Y_g & (1-\sigma_g)G_{2g}\daleth_{2g}(t)Q_{2g}Y_g \\ * & \upsilon_g I & I & I \\ * & * & X_g & 0 \\ * & * & * & X_g \end{bmatrix} \geq 0, \tag{66}$$

where $\Phi_{5g} = K^\star - \sigma_g G_{1g}\daleth_{1g}(t)Q_{1g}$. From (66), one gets

$$\begin{bmatrix} I - Y_g X_g^{-1} Y_g^T - \Phi_{6g} & K^\star - Y_g X_g^{-1} - \Phi_{7g} \\ * & \upsilon_g I - 2X_g^{-1} \end{bmatrix} \geq 0, \tag{67}$$

where

$$\Phi_{6g} = (1-\sigma_g)^2 G_{2g}\daleth_{2g}(t)Q_{2g}Y_g X_g^{-1} Y_g^T Q_{2g}^T \daleth_{2g}^T(t) G_{2g}^T,$$
$$\Phi_{7g} = \sigma_g G_{1g}\daleth_{1g}(t)Q_{1g} + (1-\sigma_g)G_{2g}\daleth_{2g}(t)Q_{2g}Y_g X_g^{-1}.$$

Notice that

$$\begin{bmatrix} Y_g X_g^{-1} Y_g^T + \Phi_{6g} & 0 \\ 0 & 2X_g^{-1} \end{bmatrix} \geq 0. \tag{68}$$

Condition (67) shows

$$\begin{bmatrix} I & K^\star - K_g - \Delta K_g \\ * & \upsilon_g I \end{bmatrix} \geq 0, \tag{69}$$

in which

$$\Delta K_g = \sigma_g G_{1g}\daleth_{1g}(t)Q_{1g} + (1-\sigma_g)G_{2g}\daleth_{2g}(t)Q_{2g}K_g.$$

Bumpless transfer level (11) holds. This completes the proof. □

Remark 7: The parameters ε_1, ε_2, υ_g and g influence the conservatism and computational burden for solving conditions (46)–(52). Too small υ_g may lead to the insolubility of condition (52), and too large υ_g usually brings about big control bumps. A smaller g can lead to less computational burden. In practice, we should choose the parameters ε_1, ε_2, υ_g, and g to realize a trade-off between the actual demand and allowed computational complexity.

IV. Extension

In most existing results, the control problem of MJSs is widely studied based on a prerequisite of synchronization between the controller and the controlled system. However, due to the packet dropout and stochastic perturbation, such kind of ideal synchronization may be difficult to be satisfied in practical applications, which may bring asynchronous phenomena between the modes of controller and system. Recently, the importance of asynchronous control for MJSs has already begun receiving more attention [35]–[38].

In order to handle such an asynchronization case, the hidden Markov model framework in [35] is established to characterize the asynchronous phenomenon between the controller and the controlled system. The passivity asynchronous control issue of delayed singular Markov jump systems is investigated in [36]. In [37], when the asynchronous phenomena occur, the dissipative control is investigated for Markovian fuzzy jump systems by designing a kind of special asynchronous controller. For 2-D MJSs, the issue of asynchronous fault detection is addressed [38]. How to extend the proposed bumpless transfer hybrid non-fragile control strategy to an asynchronous control version for MJSs? It is a significant problem that deserves further study.

Motivated by the above discussions, in this section, we extend the synchronization results developed in Section III to an asynchronous control version. The hybrid non-fragile asynchronous controller is developed as

$$u(t) = (K_{j(t)} + \Delta K_{j(t)})x(t) \tag{70}$$

with

$$\Delta K_{j(t)} = \sigma_{j(t)}\Delta K_{1j(t)} + \left(1-\sigma_{j(t)}\right)\Delta K_{2j(t)}K_{j(t)}, \tag{71}$$

where $K_{j(t)}$ is a nominal control gain, which will be designed later. $\Delta K_{j(t)}$ is the hybrid parameter perturbations of the controller (70). $j(t)$ is a hidden Markov progress, which takes values in another positive integer set $\mathbf{J} = \{1, 2, \ldots, \mathbb{J}\}$, and obeys a conditional probability matrix $\mathcal{J} = [\eta_{gl}]$ with the probability transition as

$$Pr\{j(t) = l | m(t) = g\} = \eta_{gl}. \tag{72}$$

For all $g \in \mathbf{M}$, $l \in \mathbf{J}$, the conditional probability η_{gl} belongs to the interval $[0,1]$ and $\sum_{l=1}^{\mathbb{J}} \eta_{gl} = 1$. $\sigma_{j(t)}$ with $0 \le \sigma_{j(t)} \le 1$ is the weight coefficient satisfying the transition probability (72) of hidden Markov process $j(t)$. $\Delta K_{1j(t)}$ and $\Delta K_{2j(t)} K_{j(t)}$ is with the form in (9).

We rewrite the closed-loop system as

$$\begin{aligned} \dot{x}(t) =& [A_{m(t)} + (1-\sigma_{j(t)}) B_{m(t)} G_{2j(t)} \daleth_{2j(t)}(t) Q_{2j(t)} K_{j(t)} \\ &+ B_{m(t)} K_{j(t)} + \sigma_{j(t)} B_{m(t)} G_{1j(t)} \daleth_{1j(t)}(t) Q_{1j(t)}] x(t) \\ &+ D_{m(t)} \omega(t), \qquad (73) \\ z(t) =& J_{m(t)} x(t) + F_{m(t)} \omega(t). \end{aligned}$$

Define $K_{j(t)} \triangleq K_l$, $\sigma_{j(t)} \triangleq \sigma_l$, $G_{1j(t)} \triangleq G_{1l}$, $G_{2j(t)} \triangleq G_{2l}$, $Q_{1j(t)} \triangleq Q_{1l}$, $Q_{2j(t)} \triangleq Q_{2l}$, $\daleth_{1j(t)}(t) \triangleq \daleth_{1l}(t)$, and $\daleth_{2j(t)}(t) \triangleq \daleth_{2l}(t)$, when $j(t) = l$, $l \in \mathbf{J}$. The main corollaries about the bumpless transfer hybrid non-fragile finite-time asynchronous control issue for MJSs are obtained.

Corollary 1: Given positive scalars T_b, α, ε_1, ε_2, ϖ, $\hbar_{1gl}$, $\hbar_{2gl}$, $\hbar_{3gl}$, $\hbar_{4l}$, $\hbar_{5l}$, $\hbar_{6l}$, and positive definite matrix S_g, the issue of bumpless transfer hybrid non-fragile finite-time H_∞ control for MJS (1) is solvable, if there exist symmetric matrix R_g, matrices $K^\star$, K_l, positive definite matrices P_g, $\hat{P}_g$, and positive scalars $\breve{\gamma}$, σ_l, υ_l for $\forall g \in \mathbf{M}$, $\forall l \in \mathbf{J}$ such that

$$\begin{bmatrix} \Psi_{1gl} & P_g D_g & J_g^T \\ * & -\breve{\gamma}^2 I & F_g^T \\ * & * & -I \end{bmatrix} < 0, \tag{74}$$

$$\begin{bmatrix} \Psi_{2l} & K^\star - K_l & 0 & 0 \\ * & \upsilon_l I & Q_{1l}^T & K_l^T Q_{2l}^T \\ * & * & \hbar_{4l} I & 0 \\ * & * & * & \hbar_{5l} I + \hbar_{6l} I \end{bmatrix} > 0, \tag{75}$$

and (15)–(17) hold, where

$$\begin{aligned} \Psi_{1gl} =& A_g^T P_g + P_g A_g - \alpha P_g + \sum_{l=1}^{\mathbb{J}} \eta_{gl} \big[K_l^T B_g^T P_g + (\hbar_{2gl}^{-1} \\ &+ \hbar_{3gl}^{-1}) K_l^T Q_{2l}^T Q_{2l} K_l + \hbar_{1gl} \sigma_l^2 P_g B_g G_{1l} G_{1l}^T B_g^T P_g \\ &+ (\hbar_{2gl} + \hbar_{3gl} \sigma_l^2) P_g B_g G_{2l} G_{2l}^T B_g^T P_g + \hbar_{1gl}^{-1} Q_{1l}^T Q_{1l} \\ &+ P_g B_g K_l \big] + \sum_{f \in \mathbf{M}_g^a} \vartheta_{gf} (P_f - R_g), \\ \Psi_{2l} =& I - \hbar_{4l} \sigma_l^2 G_{1l} G_{1l}^T - (\hbar_{5l} + \hbar_{6l} \sigma_l^2) G_{2l} G_{2l}^T. \end{aligned}$$

Moreover, the bumpless transfer level with $\upsilon = \max_{l \in \mathbf{J}} \upsilon_l$ and the H_∞ index with $\gamma = \sqrt{e^{\alpha T_b}} \breve{\gamma}$ are ensured.

Proof. Choose the stochastic multiple Lyapunov functions as (19). Notice that $\eta_{gl} \in [0,1]$ and $\sum_{l=1}^{\mathbb{J}} \eta_{gl} = 1$. We get

$$\begin{aligned} &\mathfrak{L} V_g(x(t)) + z^T(t) z(t) - \breve{\gamma}^2 \omega^T(t) \omega(t) \\ =& x^T(t) \Bigg\{ \sum_{f \in \mathbf{M}_g^a} \vartheta_{gf} (P_f - R_g) + \sum_{f \in \mathbf{M}_g^{ua}} \vartheta_{gf} (P_f - R_g) \\ &+ A_g^T P_g + P_g A_g + \sum_{l=1}^{\mathbb{J}} \eta_{gl} \big[K_l^T B_g^T P_g + P_g B_g K_l \\ &+ K_l^T Q_{2l}^T \daleth_{2l}^T(t) G_{2l}^T B_g^T P_g - \sigma_l K_l^T Q_{2l}^T \daleth_{2l}^T(t) G_{2l}^T B_g^T P_g \\ &+ P_g B_g G_{2l} \daleth_{2l}(t) Q_{2l} K_l - \sigma_l P_g B_g G_{2l} \daleth_{2l}(t) Q_{2l} K_l \qquad (76) \\ &+ \sigma_l Q_{1l}^T \daleth_{1l}^T(t) G_{1l}^T B_g^T P_g + \sigma_l P_g B_g G_{1l} \daleth_{1l}(t) Q_{1l} \big] \Big\} x(t) \\ &+ x^T(t) (P_g D_g + J_g^T F_g) \omega(t) + \omega^T(t) (D_g^T P_g \\ &+ F_g^T J_g) x(t) + \omega^T(t) (F_g^T F_g - \breve{\gamma}^2) \omega(t), \end{aligned}$$

with $\sum_{f=1}^{M} \vartheta_{gf} R_g = 0$ for any symmetric matrix R_g, $g \in \mathbf{M}$. By using the proof method similar to (23)–(39) as in *Theorem 1*, we confirm that MJS (1) is the finite-time bounded and satisfies the inequality (13).

It can be derived by condition (75) that

$$\begin{bmatrix} I + \mho_{3l} & K^\star - K_l \\ * & \upsilon_l I + \mho_{4l} \end{bmatrix} > 0, \tag{77}$$

where

$$\begin{aligned} \mho_{3l} =& -\hbar_{4l} \sigma_l^2 G_{1l} G_{1l}^T - (\hbar_{5l} + \hbar_{6l} \sigma_l^2) G_{2l} G_{2l}^T, \\ \mho_{4l} =& -\hbar_{4l}^{-1} Q_{1l}^T Q_{1l} - (\hbar_{5l}^{-1} + \hbar_{6l}^{-1}) K_l^T Q_{2l}^T Q_{2l} K_l. \end{aligned}$$

There exist positive scalars $\hbar_{4l}$, $\hbar_{5l}$, and $\hbar_{6l}$ such that

$$\begin{aligned} &\begin{bmatrix} \mho_{3l} & 0 \\ 0 & \mho_{4l} \end{bmatrix} \\ \le& \begin{bmatrix} 0 & -\sigma_l G_{1l} \daleth_{1l}(t) Q_{1l} - (1-\sigma_l) G_{2l} \daleth_{2l}(t) Q_{2l} K_l \\ * & 0 \end{bmatrix}. \end{aligned} \tag{78}$$

Therefore

$$\begin{bmatrix} I & K^\star - K_l - \Delta K_l \\ * & \upsilon_l I \end{bmatrix} > 0. \tag{79}$$

From (79), one has

$$\upsilon_l I > [K^\star - (K_l + \Delta K_l)]^T [K^\star - (K_l + \Delta K_l)], \tag{80}$$

which is equivalent to

$$\upsilon_l \|x(t)\|^2 > \|K^\star x(t) - (K_l + \Delta K_l) x(t)\|^2. \tag{81}$$

Obviously, the bumpless transfer level (11) with $\upsilon = \max_{l \in \mathbf{J}} \upsilon_l$ is satisfied for hybrid non-fragile asynchronous controller (70). This completes the proof. □

Corollary 2: Given positive scalars T_b, α, ε_1, ε_2, ϖ, $\hbar_{1gl}$, $\hbar_{2gl}$, $\hbar_{3gl}$, $\hbar_{4l}$, $\hbar_{5l}$, $\hbar_{6l}$, and positive definite matrix S_g, the issue of bumpless transfer hybrid non-fragile finite-time H_∞ control for MJS (1) is solvable, if there exist positive scalars $\breve{\gamma}$, σ_l, υ_l, $\aleph$, matrices $K^\star$, Y_l, $\mathfrak{K}$, symmetric matrices $\mathcal{R}_g$, and positive definite matrix X_g, for $\forall g \in \mathbf{M}$, $\forall l \in \mathbf{J}$ such that

$$\begin{bmatrix} -\Upsilon_1 & \Upsilon_{2gl} & 0 & \mathfrak{K}^T J_g^T & \mathfrak{K}^T & 0 & 0 & 0 \\ * & -\Upsilon_{3gl} & D_g & 0 & 0 & \Upsilon_{4gl} & \Upsilon_{6gl} & \Upsilon_{8gl} \\ * & * & -\breve{\gamma}^2 I & F_g^T & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & -\xi X_g & 0 & 0 & 0 \\ * & * & * & * & * & -\Upsilon_{5gl} & 0 & 0 \\ * & * & * & * & * & * & -\Upsilon_{7gl} & 0 \\ * & * & * & * & * & * & * & -\Upsilon_{9gl} \end{bmatrix}$$

$$< 0, \quad g \in \mathbf{M}_g^a, \tag{82}$$

$$\begin{bmatrix} -\Upsilon_1 & \Upsilon_{2gl} & 0 & \mathfrak{K}^T J_g^T & \mathfrak{K}^T & 0 & 0 & 0 \\ * & -\Upsilon_{10gl} & D_g & 0 & 0 & \Pi_{3gl} & \Upsilon_{6gl} & \Upsilon_{11gl} \\ * & * & -\breve{\gamma}^2 I & F_g^T & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & -\xi X_g & 0 & 0 & 0 \\ * & * & * & * & * & -\Upsilon_{5gl} & 0 & 0 \\ * & * & * & * & * & * & -\Upsilon_{7gl} & 0 \\ * & * & * & * & * & * & * & -\Upsilon_{12gl} \end{bmatrix}$$

$$< 0, \quad g \in \mathbf{M}_g^{ua}, \tag{83}$$

$$\begin{bmatrix} \Psi_{2l} & K^\star & Y_l & 0 & 0 & 0 \\ * & \upsilon_l I & I & I & Q_{1l}^T & 0 \\ * & * & \mathfrak{K} & 0 & 0 & 0 \\ * & * & * & \mathfrak{K} & 0 & Y_l^T Q_{2l}^T \\ * & * & * & * & \hbar_{4l} I & 0 \\ * & * & * & * & * & \hbar_{5l} I + \hbar_{6l} I \end{bmatrix} > 0, \tag{84}$$

and (48)–(51) hold, where

$$\Upsilon_1 = \mathfrak{K} + \mathfrak{K}^T, \ \ \Upsilon_{2gl} = \mathfrak{K}^T A_g^T + \sum_{l=1}^{\mathbb{J}} \eta_{gl} Y_l^T B_g^T + X_g,$$

$$\begin{aligned} \Upsilon_{3gl} &= \sum_{f \in \mathbf{M}_g^a} \vartheta_{gf} \mathcal{R}_g - \sum_{l=1}^{\mathbb{J}} \eta_{gl} \big[\hbar_{1gl} \sigma_l^2 B_g G_{1l} G_{1l}^T B_g \\ &\quad + \big(\hbar_{2gl} + \hbar_{3gl} \sigma_l^2\big) B_g G_{2l} G_{2l}^T B_g \big] + \big(\alpha + \xi^{-1} \\ &\quad - \vartheta_{gg}\big) X_g, \\ \Upsilon_{4gl} &= \big[\sqrt{\eta_{g1}} X_g Q_{11}^T \ \ \sqrt{\eta_{g2}} X_g Q_{12}^T \ \ \ldots \ \ \sqrt{\eta_{g\mathbb{J}}} X_g Q_{1\mathbb{J}}^T \big], \\ \Upsilon_{5gl} &= \mathrm{diag}\big\{ \hbar_{1g1} I, \hbar_{1g2} I, \ldots, \hbar_{1g\mathbb{J}} I \big\}, \\ \Upsilon_{6gl} &= \big[\sqrt{\eta_{g1}} (\hbar_{2g1} + \hbar_{3g1}) X_g \mathfrak{K}^{-1^T} Y_1^T Q_{21}^T \ \ \sqrt{\eta_{g2}} (\hbar_{2g2} \\ &\quad + \hbar_{3g2}) X_g \mathfrak{K}^{-1^T} Y_2^T Q_{22}^T \ \ \ldots \ \ \sqrt{\eta_{g\mathbb{J}}} (\hbar_{2g\mathbb{J}} \\ &\quad + \hbar_{3g\mathbb{J}}) X_g \mathfrak{K}^{-1^T} Y_{\mathbb{J}}^T Q_{2\mathbb{J}}^T \big], \\ \Upsilon_{7gl} &= \mathrm{diag}\big\{ (\hbar_{2g1} + \hbar_{3g1}) I, (\hbar_{2g2} + \hbar_{3g2}) I, \ldots, \\ &\qquad (\hbar_{2g\mathbb{J}} + \hbar_{3g\mathbb{J}}) I \big\}, \\ \Upsilon_{8g} &= \Big[\sqrt{\vartheta_{g\mathfrak{f}_1^g}} X_g \ \ \sqrt{\vartheta_{g\mathfrak{f}_2^g}} X_g \ \ \cdots \ \ \sqrt{\vartheta_{g\mathfrak{f}_{\mathfrak{g}-1}^g}} X_g \\ &\quad \sqrt{\vartheta_{g\mathfrak{f}_{\mathfrak{g}+1}^g}} X_g \ \ \cdots \ \ \sqrt{\vartheta_{g\mathfrak{f}_p^g}} X_g \ \Big], \\ \Upsilon_{9g} &= \mathrm{diag} \Big\{ X_{\mathfrak{f}_1^g}, X_{\mathfrak{f}_2^g}, \ldots, X_{\mathfrak{f}_{\mathfrak{g}-1}^g}, X_{\mathfrak{f}_{\mathfrak{g}+1}^g}, \ldots, X_{\mathfrak{f}_p^g} \Big\}, \\ \Upsilon_{10gl} &= \sum_{f \in \mathbf{M}_g^a} \vartheta_{gf} \mathcal{R}_g - \sum_{l=1}^{\mathbb{J}} \eta_{gl} \big[\hbar_{1gl} \sigma_l^2 B_g G_{1l} G_{1l}^T B_g \\ &\quad + \big(\hbar_{2gl} + \hbar_{3gl} \sigma_l^2\big) B_g G_{2l} G_{2l}^T B_g \big] + \big(\alpha + \xi^{-1}\big) X_g, \\ \Upsilon_{11g} &= \Big[\sqrt{\vartheta_{g\mathfrak{f}_1^g}} X_g \ \ \sqrt{\vartheta_{g\mathfrak{f}_2^g}} X_g \ \ \cdots \ \ \sqrt{\vartheta_{g\mathfrak{f}_p^g}} X_g \ \Big], \\ \Upsilon_{12g} &= \mathrm{diag} \Big\{ X_{\mathfrak{f}_1^g}, X_{\mathfrak{f}_2^g}, \ldots, X_{\mathfrak{f}_p^g} \Big\} \end{aligned}$$

with $\mathfrak{f}_1^g, \mathfrak{f}_2^g, \ldots, \mathfrak{f}_p^g$ described as in (6) and $\mathfrak{f}_{\mathfrak{g}}^g = g$. The bumpless transfer level with $\upsilon = \max_{l \in \mathbf{J}} \upsilon_l$ and the H_∞ index with $\gamma = \sqrt{e^{\alpha T_b}} \breve{\gamma}$ are achieved. Moreover, controller gain in the hybrid non-fragile asynchronous controller (70) is given by $K_l = Y_l \mathfrak{K}^{-1}$.

Proof. Conditions (82) and (83) are deduced as

$$\begin{bmatrix} -\Upsilon_1 & \Upsilon_{2gl} & 0 & \mathfrak{K}^T J_g^T & \mathfrak{K}^T \\ * & \Phi_{1gl} & D_g & 0 & 0 \\ * & * & -\breve{\gamma}^2 I & F_g^T & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\xi X_g \end{bmatrix} < 0, \tag{85}$$

where

$$\begin{aligned} \Phi_{1gl} &= \sum_{f \in \mathbf{M}_g^a} \vartheta_{gf} \Big(X_g X_f^{-1} X_g - \mathcal{R}_g \Big) - (\xi^{-1} + \alpha) X_g \\ &\quad + \sum_{l=1}^{\mathbb{J}} \eta_{gl} \big[\big(\hbar_{2gl} + \hbar_{3gl} \sigma_l^2\big) B_g G_{2l} G_{2l}^T B_g^T \\ &\quad + (\hbar_{2gl}^{-1} + \hbar_{3gl}^{-1}) X_g \mathfrak{K}^{-1^T} Y_l^T Q_{2l}^T Q_{2l} Y_l \mathfrak{K}^{-1} X_g \\ &\quad + \hbar_{1gl}^{-1} X_g Q_{1l}^T Q_{1l} X_g + \hbar_{1gl} \sigma_l^2 B_g G_{1l} G_{1l}^T B_g^T \big]. \end{aligned}$$

Define $Y_l \triangleq K_l \mathfrak{K}$, $\hat{\mathfrak{L}}_{gl}^{-1} \triangleq \mathfrak{L}_{gl}$, and

$$\mathfrak{U}_{gl} \triangleq \begin{bmatrix} A_g + \sum_{l=1}^{\mathbb{J}} \eta_{gl} B_g K_l & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ J_g & 0 & 0 & I & 0 \\ I & 0 & 0 & 0 & I \end{bmatrix}. \tag{86}$$

Pre- and post-multiplying (85) by $\mathfrak{U}_{gl}$ and $\mathfrak{U}_{gl}^T$ gets

$$\begin{bmatrix} \Phi_{2gl} & D_g & X_g J_g^T \\ * & -\breve{\gamma}^2 I & F_g^T \\ * & * & -I \end{bmatrix} < 0, \tag{87}$$

in which

$$\begin{aligned} \Phi_{2gl} &= \sum_{f \in \mathbf{M}_g^a} \vartheta_{gf} \Big(X_g X_f^{-1} X_g - \mathcal{R}_g \Big) + \sum_{l=1}^{\mathbb{J}} \eta_{gl} \big[X_g K_l^T B_g^T \\ &\quad + B_g K_l X_g + \big(\hbar_{2gl} + \hbar_{3gl} \sigma_l^2\big) B_g G_{2l} G_{2l}^T B_g^T \\ &\quad + (\hbar_{2gl}^{-1} + \hbar_{3gl}^{-1}) X_g \mathfrak{K}^{-1^T} Y_l^T Q_{2l}^T Q_{2l} Y_l \mathfrak{K}^{-1} X_g \\ &\quad + \hbar_{1gl}^{-1} X_g Q_{1l}^T Q_{1l} X_g + \hbar_{1gl} \sigma_l^2 B_g G_{1l} G_{1l}^T B_g^T \big] \\ &\quad + X_g A_g^T + A_g X_g - \alpha X_g. \end{aligned}$$

Let $X_g = P_g^{-1}$ and $\mathcal{R}_g = X_g R_g X_g$. Performing a transformation to (87) by $\mathrm{diag}\{X_g^{-1}, I, I\}$, one can see that condition (74) is guaranteed.

Based on condition (84), we derive

$$\begin{aligned} &\begin{bmatrix} \Psi_{2l} & K^\star & Y_l & 0 \\ * & \upsilon_l I - \Phi_{4l} & I & I \\ * & * & \mathfrak{K} & 0 \\ * & * & * & \mathfrak{K} - \Phi_{5l} \end{bmatrix} \\ &= \begin{bmatrix} I & K^\star & Y_l & 0 \\ * & \upsilon_l I & I & I \\ * & * & \mathfrak{K} & 0 \\ * & * & * & \mathfrak{K} \end{bmatrix} - \begin{bmatrix} \Phi_{3l} & 0 & 0 & 0 \\ * & \Phi_{4l} & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & \Phi_{5l} \end{bmatrix} \\ &> 0, \end{aligned} \tag{88}$$

where

$$\begin{aligned} \Phi_{3l} &= \hbar_{4l} \sigma_l^2 G_{1l} G_{1l}^T + \big(\hbar_{5l} + \hbar_{6l} \sigma_l^2\big) G_{2l} G_{2l}^T, \\ \Phi_{4l} &= \hbar_{4l}^{-1} Q_{1l}^T Q_{1l}, \quad \Phi_{5l} = \big(\hbar_{5l}^{-1} + \hbar_{6l}^{-1}\big) Y_l^T Q_{2l}^T Q_{2l} Y_l. \end{aligned}$$

There exist positive scalars $\hbar_{4l}$, $\hbar_{5l}$, and $\hbar_{6l}$ such that

$$\begin{bmatrix} I & \Phi_{6l}(t) & Y_l & (1-\sigma_l)G_{2l}\daleth_{2l}(t)Q_{2l}Y_l \\ * & \upsilon_l I & I & I \\ * & * & \mathfrak{K} & 0 \\ * & * & * & \mathfrak{K} \end{bmatrix} \geq 0, \quad (89)$$

where $\Phi_{6l}(t) = K^\star - \sigma_l G_{1l}\daleth_{1l}(t)Q_{1l}$. From (89), one gets

$$\begin{bmatrix} I - Y_l\mathfrak{K}^{-1}Y_l^T - \Phi_{7l}(t) & K^\star - Y_l\mathfrak{K}^{-1} - \Phi_{8l}(t) \\ * & \upsilon_l I - 2\mathfrak{K}^{-1} \end{bmatrix} \geq 0, \quad (90)$$

in which

$$\Phi_{7l}(t) = (1-\sigma_l)^2 G_{2l}\daleth_{2l}(t)Q_{2l}Y_l\mathfrak{K}^{-1}Y_l^T Q_{2l}^T\daleth_{2l}^T(t)G_{2l}^T,$$
$$\Phi_{8l}(t) = \sigma_l G_{1l}\daleth_{1l}(t)Q_{1l} + (1-\sigma_l)G_{2l}\daleth_{2l}(t)Q_{2l}Y_l\mathfrak{K}^{-1}.$$

Notice that

$$\begin{bmatrix} Y_l\mathfrak{K}^{-1}Y_l^T + \Phi_{7l}(t) & 0 \\ 0 & 2\mathfrak{K}^{-1} \end{bmatrix} \geq 0, \quad (91)$$

which shows

$$\begin{bmatrix} I & K^\star - K_l - \Delta K_l \\ * & \upsilon_l I \end{bmatrix} \geq 0, \quad (92)$$

where

$$\Delta K_l = \sigma_l G_{1l}\daleth_{1l}(t)Q_{1l} + (1-\sigma_l)G_{2l}\daleth_{2l}(t)Q_{2l}K_l.$$

Bumpless transfer level (11) holds. This completes the proof. □

Remark 8: When the bumpless transfer hybrid non-fragile finite-time asynchronous control issue is considered for MJSs, the existence of the multiplicative perturbation $\Delta K_{2j(t)}K_{j(t)}$ prevents the production of the linear matrix inequality solving condition from its original version to an asynchronous version. In the future, the non-convex term may allow by use an efficient computational tools of solution. One can solve (82)–(84) by resorting to the existing methods [39]. First, take $\mathfrak{K}$ in advance according to the allowed computational complexity. $X_g, g \in \mathbf{M}$ can be obtained from (48)–(51). Then, $Y_l, l \in \mathbf{J}$ can be obtained from the conditions in *Corollary 2*.

V. An Example

The proposed bumpless transfer finite-time hybrid non-fragile H_∞ control strategy is applied to an electronic circuit model [40] which is shown in Fig. 1. Our control goal is to suppress large power source increments generated at jumping instants while ensuring the finite-time H_∞ performance of the electronic circuit. The electronic circuit consists of a collection of inductors L_g, a collection of capacitors C_g, a resistor R, and a power source increment $U(t)$, $g = 1, 2, 3$. Different inductors and capacitors are changed by jumping. $i_R(t)$ and $i_L(t)$ represent the current increment through the resistor R and the inductor L_g. $U_C(t)$ stands for the voltage increment across the capacitors C_g. Adopting the Kirchhoff current and voltage laws gives rise to

$$\begin{aligned} C_g\dot{U}_C(t) &= -\frac{1}{R}U_C(t) + i_L(t), \\ L_g\dot{i}_L(t) &= -U_C(t) + U(t), \end{aligned} \quad (93)$$

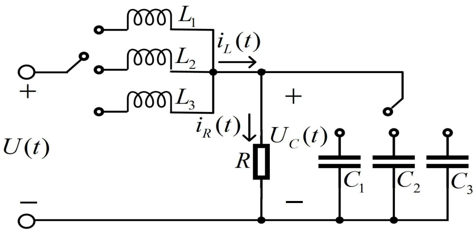

Fig. 1. Electronic circuit.

where $R = 2\ \Omega$, $L_1 = 2$ mH, $L_2 = 4$ mH, $L_3 = 5$ mH, $C_1 = 20$ mF, $C_2 = 10$ mF, and $C_3 = 40$ mF.

Define the system state $x(t) \triangleq \begin{bmatrix} U_C(t) \\ i_L(t) \end{bmatrix}$ and the control input $u(t) \triangleq U(t)$. For $g = 1, 2, 3$, matrices A_g, B_g, D_g, J_g, F_g, G_{1g}, Q_{1g}, G_{2g}, and Q_{2g} are parametrized and expressed by

$$A_1 = \begin{bmatrix} -25 & 50 \\ -500 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 500 \end{bmatrix}, D_1 = \begin{bmatrix} 4 \\ 2 \end{bmatrix},$$
$$J_1 = [\,-2 \ \ -7\,], \ \ F_1 = 0.1, \ \ G_{11} = 0.35,$$
$$Q_{11} = [\,2 \ \ 3\,], \ \ G_{21} = 0.21, \ \ Q_{21} = 5,$$
$$A_2 = \begin{bmatrix} -50 & 100 \\ -250 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 250 \end{bmatrix}, D_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix},$$
$$J_2 = [\,-1 \ \ -5\,], \ \ F_2 = 0.2, \ \ G_{12} = 0.42,$$
$$Q_{12} = [\,1 \ \ 1\,], \ \ G_{22} = 0.52, \ \ Q_{22} = 3,$$
$$A_3 = \begin{bmatrix} -12.5 & 25 \\ -200 & 0 \end{bmatrix}, B_3 = \begin{bmatrix} 0 \\ 200 \end{bmatrix}, D_3 = \begin{bmatrix} 2 \\ 5 \end{bmatrix},$$
$$J_3 = [\,-3 \ \ -2\,], \ \ F_3 = 0.3, \ \ G_{13} = 0.33,$$
$$Q_{13} = [\,1 \ \ 3\,], \ \ G_{23} = 0.23, \ \ Q_{23} = 4.$$

Choose the parameters $x_0 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\varepsilon_1 = 2.2$, $\varepsilon_2 = 4$, $T_b = 3$ s, $S_1 = S_2 = S_3 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix}$, $\alpha = 0.01$, $\omega(t) = \frac{0.0001}{1+t}$, $\sigma_1 = 0.0117$, $\sigma_2 = 0.0297$, $\sigma_3 = 0.0347$, $\daleth_{11}(t) = \daleth_{12}(t) = \daleth_{13}(t) = \cos(\pi t)$, $\daleth_{21}(t) = \daleth_{22}(t) = \daleth_{23}(t) = \sin(\pi t)$, $\hbar_{11} = 6.3$, $\hbar_{12} = 5.3$, $\hbar_{13} = 6.1$, $\hbar_{21} = 0.0028$, $\hbar_{22} = 0.0063$, $\hbar_{23} = 0.0196$, $\hbar_{31} = 2.4$, $\hbar_{32} = 3.4$, $\hbar_{33} = 1.4$, $\hbar_{41} = 10.9$, $\hbar_{42} = 10.5$, $\hbar_{43} = 10.3$, $\hbar_{51} = 2.4003$, $\hbar_{52} = 0.6295$, $\hbar_{53} = 0.8103$, $\hbar_{61} = 21$, $\hbar_{62} = 25$, $\hbar_{63} = 23$, $\mathcal{M} = [\vartheta_{gf}] = \begin{bmatrix} -2 & ? & ? \\ ? & -5 & ? \\ ? & 1 & ? \end{bmatrix}$.

By adopting two different control strategies, we show a comparative study. The proposed bumpless transfer hybrid non-fragile finite-time H_∞ control strategy is represented as Method (A). The finite-time non-fragile H_∞ control strategy in [41] is represented as Method (B). By solving Theorem 2, we obtain

$$K_1 = [\,0.7423 \ \ -0.1596\,], \quad K_2 = [\,0.5360 \ \ -0.3572\,],$$
$$K_3 = [\,0.7788 \ \ -0.1462\,], \quad K^\star = [\,0.7378 \ \ -0.1747\,],$$
$$\gamma = 3.0706, \qquad \varrho = 15.1935.$$

From [41], we get

$$\tilde{K}_1 = [\,0.7637 \ \ 0.1204\,], \quad \tilde{K}_2 = [\,0.5312 \ \ -0.3944\,],$$

$$\tilde{K}_3 = [\, 0.7761 \quad -0.1727 \,], \quad \tilde{\gamma} = 3.0801.$$

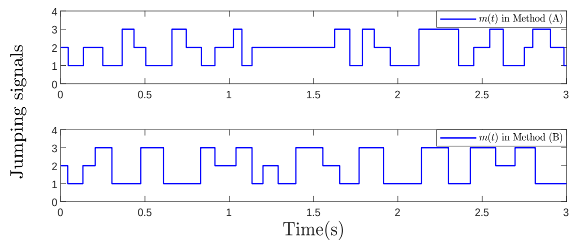

Fig. 2. System mode of MJS.

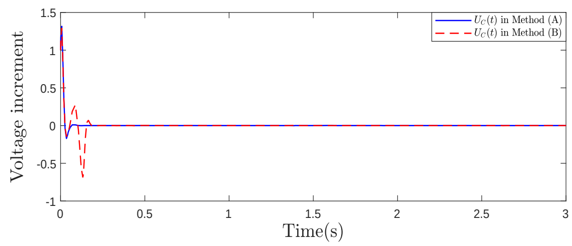

Fig. 3. Response of the voltage increment $U_C(t)$.

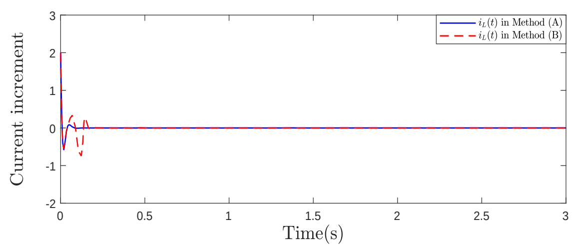

Fig. 4. Response of the current increment $i_L(t)$.

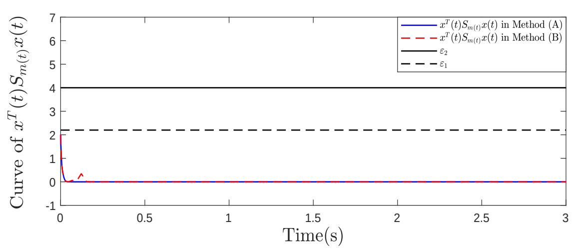

Fig. 5. Curve of $x^T(t)S_{m(t)}x(t)$.

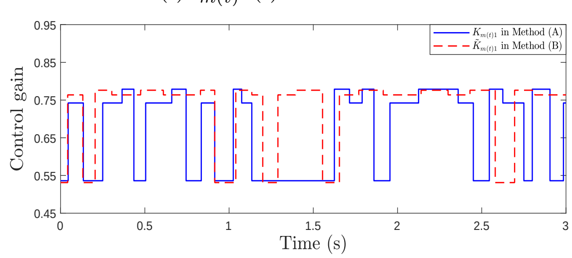

Fig. 6. First component of control gain.

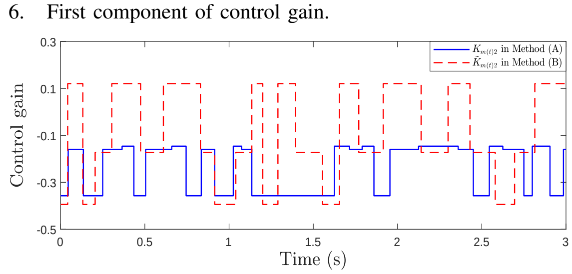

Fig. 7. Second component of control gain.

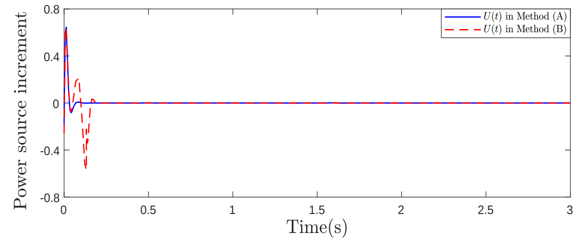

Fig. 8. Power source increment $U(t)$.

The different jumping signals are given in Fig. 2. Responses of the voltage increment $U_C(t)$ and the current increment $i_L(t)$ are given in Figs. 3 and 4. Fig. 5 depicts the evolution of $x^T(t)S_{m(t)}x(t)$. Under the time interval $[0, 3]$, $x^T(t)S_{m(t)}x(t) \leq 3$ is guaranteed by the bumpless transfer hybrid non-fragile controller (7). Figs. 6 and 7 give the variations of control gain, which indicates that the amplitude of control gain varies less in method (A). Fig. 8 is the variation of power source increment $U(t)$. The amplitude of the power source increment $U(t)$ in Method (A) is significantly better than one in Method (B), although the proposed bumpless transfer hybrid non-fragile control strategy largely increases the computational complexity and the control design difficulty. Therefore, the proposed bumpless transfer finite-time hybrid non-fragile H_∞ control technique effectively suppresses the occurrence of large power source increments.

VI. Conclusions

For MJSs, the issue of bumpless transfer hybrid non-fragile finite-time H_∞ control has been dealt with. A non-fragile strategy has been designed to tolerate the additive and multiplicative perturbations. To depict the transient behavior caused by a jumping controller, a bumpless transfer control idea has been given. First, a bumpless transfer constraint condition has been provided to limit the amplitude of the hybrid non-fragile controller. Second, a bumpless transfer hybrid non-fragile controller has been developed to guarantee the solvability of the finite-time H_∞ control issue for MJSs with partially available transition rates. Finally, a practical example has been given to show the usefulness of the proposed bumpless transfer hybrid non-fragile control strategy.

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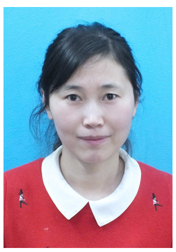

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