Dynamic Resource Allocation on Multi-Category Two-Sided Platforms

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Abstract

Platform businesses are typically resource-intensive and must scale up their business quickly in the early stage to compete successfully against fast-emerging rivals. We study a critical question faced by such firms in the novel context of multi-category two-sided platforms: how to optimally make investment decisions across two sides, multiple categories, and different time periods to achieve fast and sustainable growth. We first develop a two-category two-period theoretical model and propose optimal resource allocation strategies that account for heterogeneous within-category direct and indirect network effects and cross-category interdependence. We find that the proposed strategy shares the spirit of the allocation rules for multi-product non-platforms firms and single-product platform firms, yet it does not amount to a simple combination of the existing rules. Interestingly, the business model that platforms adopt crucially determines the optimal strategy. Platforms that charge by user should adopt a "reinforcing" rule for both within- and cross-category allocations by allocating more resources toward the stronger growth driver. Platforms that charge by transaction should also adopt the "reinforcing" rule for within-category allocation, but follow a "compensatory" rule for cross-category and intertemporal allocations by allocating more resources toward the weaker growth driver. We use data from the daily deals industry to empirically identify the network effects, propose alternative allocation strategies stemming from our theoretical findings, and use simulations to show the benefits of these strategies. For instance, we show that re-allocating 10% of the average observed investment from Fitness to Beauty can increase profits by up to 15.5% for some cities.

Keywords: dynamic resource allocation; two-sided platform; business model; network effects; daily deals

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1 Introduction

Two-sided platforms have become ubiquitous in such diverse industries as online retail, software, mobile apps, video games, and many others. Successful examples of platform businesses include such well-known enterprises as Amazon and eBay, as well as more recently established companies, such as Airbnb and Uber. Given that these platforms are typically highly resource-intensive, they must scale up their business quickly in the early stage to compete successfully against fast-emerging rivals. Consequently, one of the most important decisions such companies need to make in the early stage is how to allocate available scarce resources across multiple priorities. A poorly made resource allocation decision at the early stage can lead to quick company demise.

Solving the resource allocation problem becomes substantially more challenging when a two-sided platform features multiple categories. While some platforms concentrate on providing a marketplace in a single product category, numerous others feature multiple categories of goods and services (e.g., daily deals platforms such as Groupon and Living-Social; home services platforms such as TaskRabbit and Thumbtack). Besides the direct and indirect network effects associated with traditional single-product platforms, there is an additional cross-category effect arising on multi-category platforms, whereby the numbers of buyers and sellers in one category can influence the numbers of buyers and sellers in other categories. Furthermore, categories can differ in the sizes of the three types of effects, requiring the platforms to properly account for heterogeneity across categories.

Our paper studies the critical question faced by multi-category two-sided platform firms: how to optimally make investment decisions to achieve fast and sustainable growth. Despite the importance of such decisions, the understanding of this problem in the novel context of multi-category platforms is scant both in the academic literature and in practice. Extant literature has documented the optimal investment strategy for traditional multi-product non-platform firms (e.g., Fischer et al. 2011) and single-product platform firms (e.g., Armstrong 2006, Wely 2010, Hagiu 2014, Sridhar et al. 2011). The former suggests that non-platform firms should invest more in stronger products with higher growth potential; heuristics that are commonly used in industry practice (e.g., "percentage-of-

sales" heuristics) share a similar logic. The latter suggests that platforms should invest more (subsidize or reduce prices) in stronger side with weaker indirect network effects and more independence in growth to optimize the platform as a whole. However, our context differs from both of these settings: unlike the classic multi-product non-platform context, we consider two sides within each product (category), and account for the interdependence between the two sides; unlike the single-product two-sided platform context, we account for additional cross-category effects among multiple categories. We examine whether and how the allocation rules for traditional multi-product non-platform firms and single-product platform firms can be applied to multi-category platforms.

Another layer of complexity is that the investment decisions may also depend on the type of business model employed by a multi-category platform. There are two widely adopted business models in practice: the per-transaction model, in which platforms charge a fee per transaction as commission, and the per-user model, in which platforms charge a fee per user as commission on one side or both sides. A daily deals platform such as Groupon or LivingSocial is an example of the first type, where the revenue of the platform depends on the total transactions occurred on the platform. A shopping mall is an example of the latter as it charges a fixed rent per merchant, which does not explicitly vary with the consumer traffic or transaction volume.

In this paper, we focus on three key strategic decisions in investment allocation: On which side should a platform invest more, the buyer side or seller side? In which category should a platform invest more? When a platform should invest more, earlier or later periods? This is a challenging problem as the three questions are intertwined. We answer these questions for the two types of business models. We first introduce a stylized theoretical framework of two-sided platforms with two categories and propose optimal investment strategies that account for the heterogeneity in within-category direct and indirect network effects and cross-category interdependence. Importantly, we show how and why the investment strategy differs for the two business models and for within-category and cross-category allocations. We find that under the per-user model, the platform should adopt a "reinforcing" rule for both within- and cross-category allocations:

it should invest more in the side/category that is the stronger driver of platform growth. Under the per-transaction model, however, while also adopting the "reinforcing" rule for within-category allocation, it should follow a "compensatory" rule for dynamic and cross-category allocations by investing less in the category that is the stronger driver of platform growth.

Why would the rules differ for the per-transaction and the per-user models and for within-category and cross-category allocations? We posit that this difference arises from the relationship between sides and categories in the two business models. In the per-user model, the platform charges by the number of users on each side for each category; the size of one side (category) does not directly influence how much the platform charge the other side (category). In this case, the within-category and cross-category relationships are the same, so the platform should adopt the same rule for both allocation decisions. In the per-transaction model, the relationship across categories is the same as in the per-user model: the size of one category does not directly influence how much the platform can charge another category. However, the relationship within a category is different: the platform charges by the number of transactions which is jointly determined by both sides within a category. In this case, the within-category and cross-category relationships are different, so the platform should adopt different rules for within- and cross-category allocation decisions.

In addition to the theoretical derivation, we empirically demonstrate the relative advantage of our proposed investment strategy in the context of the daily deals industry, which adopts the per-transaction business model. We use data on Groupon deals offerings, sales and investment across four major categories to estimate category-specific network effects. We substitute the empirical estimates into our theoretical framework to obtain model recommendations and construct alternative investment schemes detailing which side, which category, and when to invest more. We simulate alternative platform growth paths and compare the performance of these investment schemes with the observed allocation strategies. The results suggest that merely re-allocating the resources based on our recommendations could have improved Groupon's profitability. For instance, re-

allocating 10% of the average observed investment from consumer side to merchant side in the Entertainment category can lead to up to 16.5% increase in profits for some cities.

Our research is the first to show that the optimal allocation strategy for multi-category two-sided platforms crucially depends on the business model (per-transaction versus per-user) that the platform adopts and differs for within-category and cross-category allocations. We contribute to the broader literature on investment allocation among multiple products or projects (e.g., Bish and Wang 2004, Klingebiel and Rammer 2014). The unique feature of our contribution is that we consider both the cross-side network effect of platform and the cross-product spillover effect of different categories. Therefore, it is related to the extensive literature on the cross-side network effects of two-sided platforms (e.g., Chen and Xie 2007, Dubé, Hitsch and Chintagunta 2010, Rochet and Tirole 2003, Armstrong 2006, Caillaud and Jullien 2003, Kaiser and Wright 2006, Hagiu 2009, Weyl 2010, Hagiu 2014, Jin and Rysman 2015). It also contributes to the literature on how interdependence across categories can affect optimal category management. In particular, the notion of purchase complementarity (sales in one category can be affected by pricing and promotions in another category) has received much attention in the retail setting (e.g., Fader and Lodish 1990, Mulhern and Leone 1991, Walters 1991, Raju 1992, Narasimhan et al. 1996, Manchanda et al. 1999).

Our research is also related to the literature on the daily deal industry. Prior research studied group buying as a selling mechanism (Hu et al. 2013, Wu et al. 2014) and as advertising (Edelman et al. 2016), explored strategic implications of displaying daily deal sales (Subramanian and Rao 2016) and revenue sharing (Bhardwaj and Sajeesh 2017), and examined operational advantages of threshold discounts (Marinesi et al. 2017). Using similar panel datasets on deal supply and sales, Cao et al. (2018) tested empirical effects of discounts, Kitchens et al. (2018) examined geographic competition among local firms, and Li et al. (2018) showed how local market characteristics affect platform growth. In this research, we use both deal data and investment data from the daily deal industry to study the optimal resource allocation problem in the broader context of multi-category two-sided platforms.

2 Theoretical Model

2.1 Model Setup

We consider a multi-category two-sided platform. We construct a general framework of how such a platform grows in response to investments on both sides (e.g., Naik, Prasad, and Sethi 2008, Simon and Arndt 1980) that parsimoniously captures all key interdependencies, including own dynamic effects (i.e., direct network effect), cross-side effects (i.e., indirect network effect), and cross-category effects. For better tractability, we focus on solving the optimal dynamic resource allocation problem for the case of two categories and two periods. Let n_t denote the number of merchants and s_t denote the number of consumers in the focal category. Let $\tilde{\cdot}$ denote variables from the other category. Let u_t and v_t denote the investment that the platform allocates to the merchant side and the consumer side, respectively. Following the standard marketing response function used in two-sided platform literature (e.g., Sridhar et al. 2011), we model how the two sides of the platform grow over time in the following form:[1]

$$
(1) \qquad n_t = \beta_u u_t + \overbrace{\lambda_n n_{t-1}}^{\text{own dynamic}} + \overbrace{\theta_n s_{t-1}}^{\text{cross-side}} + \overbrace{\gamma_{nn}\tilde{n}_{t-1} + \gamma_{ns}\tilde{s}_{t-1}}^{\text{cross-category}}
$$

$$
s_t = \beta_v v_t + \underbrace{\lambda_s s_{t-1}}_{\text{own dynamic}} + \underbrace{\theta_s n_{t-1}}_{\text{cross-side}} + \underbrace{\gamma_{ss}\tilde{s}_{t-1} + \gamma_{sn}\tilde{n}_{t-1}}_{\text{cross-category}}
$$

In particular, the sizes of the two sides of the platform, n_t and s_t, depend on the current-period investment and the three types of interdependencies from the past.[2] First, investment on the merchant side (e.g., sales force) and the consumer side (e.g., advertising

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[1]The linearly additive functional form can be derived from the dynamic marketing response functions, $N_t = U_t^{\beta_u} N_{t-1}^{\lambda_n} S_{t-1}^{\theta_n} \tilde{N}_{t-1}^{\gamma_{nn}} \tilde{S}_{t-1}^{\gamma_{ns}}$ and $S_t = V_t^{\beta_v} S_{t-1}^{\lambda_s} N_{t-1}^{\theta_s} \tilde{S}_{t-1}^{\gamma_{ss}} \tilde{N}_{t-1}^{\gamma_{sn}}$, which closely follow that in Sridhar et al. (2011). Sridhar et al. (2011) study the optimal investment strategies for single-category two-sided platforms. We build on their work and introduce the impact of other categories to the system. Setting $\beta_u < 1$ and $\beta_v < 1$ allows the marketing response function to exhibit diminishing marginal return to investment. Taking log on both sides of this marketing response function and denoting $x_t \equiv \ln X_t$ would yield the linearly additive functional form in our setup. We show in the online appendix that the main results and the propositions continue to hold when we do not take log of the variables. We use the specification with log form as the main specification because it yields closed-form solutions while the specification without log form can only be solved numerically.

[2]For simplicity, we assume that merchants in the same category are homogenous. This assumption does not change the main insights of the model. We relax this assumption and control for merchant heterogeneity in the empirical analysis.

spending) enhance platform attractiveness and increase the size of the corresponding side. We allow for diminishing marginal return to investment by restricting the coefficient on investment $\beta_u < 1$ and $\beta_v < 1$. Second, within each category, the size of the focal side in the last period can also have an impact, which is captured by the own dynamic effect (λ_n and λ_s) or the direct network effect in the two-sided platform literature; the size of the other side in the last period can also have an impact, which is captured by the cross-side effect (θ_n and θ_s) or the indirect network effect in the two-sided platform literature. Third, across categories, the sizes of the two sides in the other category can impact the size of the focal category, captured by the cross-category effects (γ_{nn}, γ_{ns}, γ_{ss} and γ_{sn}). These effects are unique to the multi-category platform context and allow the focal category to grow by leveraging the growth from other categories. Similarly, we set up the constraints for the other category and allow the two categories to have different effect sizes:

(2)
$$\tilde{n}_t = \beta_u \tilde{u}_t + \tilde{\lambda}_n \tilde{n}_{t-1} + \tilde{\theta}_n \tilde{s}_{t-1} + \tilde{\gamma}_{nn} n_{t-1} + \tilde{\gamma}_{ns} s_{t-1}$$

$$\tilde{s}_t = \beta_v \tilde{v}_t + \tilde{\lambda}_s \tilde{s}_{t-1} + \tilde{\theta}_s \tilde{n}_{t-1} + \tilde{\gamma}_{ss} s_{t-1} + \tilde{\gamma}_{sn} n_{t-1}$$

The firm solves for the optimal investment strategy in each side, each category and each period. We consider two types of firm objective functions, each corresponding to a business model commonly adopted by platform businesses in practice. The first type of objective function corresponds to a **per-user model** in which platforms charge a fixed fee per user as commission on at least one side. For example, some e-commerce platform charges listing fees on the seller side and membership fees on the buyer side. The platform maximizes the following objective function subject to the conditions specified in Equation 1 and Equation 2:

$$\max \quad \sum_{t=1,2} \left(\sqrt{n_t} + \sqrt{s_t} - u_t - v_t \right) + \left(\sqrt{\tilde{n}_t} + \sqrt{\tilde{s}_t} - \tilde{u}_t - \tilde{v}_t \right)$$

where we take square root of the number of users to keep the concavity of the problem.[3]

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[3]An alternative approach to keep concavity is to have a linear benefit and a quadratic cost of in-

Intuitively, platforms may charge a lower fee per user as the userbase becomes larger. We allow for asymmetric fees on the two sides as model extensions in Section 2.4.

The second objective function corresponds to a **per-transaction model** in which platforms charge a fixed fee per transaction as commission. A typical example is the daily deal industry: Groupon charges a fraction of the revenue generated per transaction as commission. The number of transactions depends on the sizes of both the consumer side and the merchant side. To operationalize the relationship, we adjust the definition of n_t and s_t and let n_t represent the number of merchants and s_t represent the sales *per merchant*.[4] The number of transactions equals the number of merchants times the sales per merchant, so the platform maximizes the following objective function subject to the conditions specified in Equation 1 and Equation 2:

$$\max \sum_{t=1,2} (n_t s_t - u_t - v_t) + (\tilde{n}_t \tilde{s}_t - \tilde{u}_t - \tilde{v}_t)$$

For both types of objective functions, the trade-offs for the firm's investment decision are related to how investment in a particular side-category-period can affect the platform growth according to Equation 1 and Equation 2. For instance, increasing the merchant-side investment in the current period can lead to a larger number of merchants in the focal category through β_u, which can further affect the future size of the merchant side through the own dynamic effect λ_n, the future size of the consumer side through the cross-side effect θ_s, and the future size of the other category through the cross-category effects $\tilde{\gamma}_{nn}$ and $\tilde{\gamma}_{sn}$. As these effects may differ across categories and sides, the platform must properly account for them when optimally choosing which category, which side, and when to allocate resources to maximize total profits.

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vestment (i.e., $n_t + s_t - u_t^2 - v_t^2$). We derive the optimal investment solutions under this alternative model specification and find that the main results and propositions continue to hold. We present the results from a square root benefit of investment and a linear cost of investment so that the cost function is consistent with that of the per-transaction model. Studies in the extant literature have used both approaches. For instance, Deal (1979) and Fruchter and Kalish (1997) use a linear term for advertising effectiveness and a quadratic term for advertising costs, whereas Erickson (1985) and Chintagunta and Vilcassim (1992) use the square root and the linear function, respectively.

[4]Despite having different definitions for s_t, the per-transaction and the per-user models have the same conditions for $n_t, s_t, \tilde{n}_t, \tilde{s}_t$ as specified in Equation 1 and Equation 2 (the values of the coefficients can be different). We use the same notation for consistency.

2.2 Optimal Investment Solution

In solving the optimization problem, we assume the platform starts with no merchants and no consumers, $n_0 = s_0 = \tilde{n}_0 = \tilde{s}_0 = 0$. The optimal allocations are functions of the parameters in the system.[5]

For the per-transaction model, the optimal investment solution for period 1 is

$$
\begin{aligned}
u_1^* &= \frac{1}{\beta_u \beta_v} \left(1 - \lambda_s - \theta_n \frac{\beta_v}{\beta_u} - \tilde{\gamma}_{ss} - \tilde{\gamma}_{ns} \frac{\beta_v}{\beta_u} \right) \\
v_1^* &= \frac{1}{\beta_u \beta_v} \left(1 - \lambda_n - \theta_s \frac{\beta_u}{\beta_v} - \tilde{\gamma}_{nn} - \tilde{\gamma}_{sn} \frac{\beta_u}{\beta_v} \right)
\end{aligned}
$$

(3)

The solution indicates that the platform can invest less in period 1 if the own dynamic effects, cross-side effects, or cross-category effects are strong. The reason is that these effects indicate the ability of the category to grow organically without further investment, either by relying on the existing sizes of the two sides or the existing size of the other category. More investment is required if these effects are not sufficiently strong.

The optimal investment solution for period 2 is

$$
\begin{aligned}
u_2^* &= \frac{1}{\beta_u \beta_v} - \left(\lambda_n u_1^* + \theta_n \frac{\beta_v}{\beta_u} v_1^* + \gamma_{nn} \tilde{u}_1^* + \gamma_{ns} \frac{\beta_v}{\beta_u} \tilde{v}_1^* \right) \\
v_2^* &= \frac{1}{\beta_u \beta_v} - \left(\lambda_s v_1^* + \theta_s \frac{\beta_u}{\beta_v} u_1^* + \gamma_{ss} \tilde{v}_1^* + \gamma_{sn} \frac{\beta_u}{\beta_v} \tilde{u}_1^* \right)
\end{aligned}
$$

(4)

The solution indicates that the platform can invest less in period 2 if period 1 investment is large on either side of the focal or the other category, moderated by the effect sizes. If the effects are weak or the period 1 investments are small, more investment is required in period 2.

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[5]Details of model derivation are provided in the appendix.

Similarly, we can derive the optimal investment solutions for the per-user model:

$$u_1^* = \frac{\beta_u}{4\left(1 - \lambda_n - \theta_s \frac{\beta_u}{\beta_v} - \tilde{\gamma}_{nn} - \tilde{\gamma}_{sn} \frac{\beta_u}{\beta_v}\right)^2}$$

(5)
$$v_1^* = \frac{\beta_v}{4\left(1 - \lambda_s - \theta_n \frac{\beta_v}{\beta_u} - \tilde{\gamma}_{ss} - \tilde{\gamma}_{ns} \frac{\beta_v}{\beta_u}\right)^2}$$

$$u_2^* = \frac{\beta_u}{4} - \left(\lambda_n u_1^* + \theta_n \frac{\beta_v}{\beta_u} v_1^* + \gamma_{nn} \tilde{u}_1^* + \gamma_{ns} \frac{\beta_v}{\beta_u} \tilde{v}_1^*\right)$$

$$v_2^* = \frac{\beta_v}{4} - \left(\lambda_s v_1^* + \theta_s \frac{\beta_u}{\beta_v} u_1^* + \gamma_{ss} \tilde{v}_1^* + \gamma_{sn} \frac{\beta_u}{\beta_v} \tilde{u}_1^*\right)$$

The closed-form solutions of period 1 and 2 investment illustrate the novelty of our setting with multi-category platforms, compared with single-category platforms. The key difference between a multi-category platform and a single-category platform is the existence of cross-category effects. Note that letting $\gamma_{nn} = \gamma_{ns} = \gamma_{ss} = \gamma_{sn} = 0$ and $\tilde{\gamma}_{nn} = \tilde{\gamma}_{ns} = \tilde{\gamma}_{ss} = \tilde{\gamma}_{sn} = 0$ in the optimal investment solutions yields the investment decision when platforms ignore the cross-category effects. Comparing the investment decisions with and without cross-category effects, we show in the appendix that ignoring the cross-category effects would lead to over-spending in period 1 and erroneous resource allocation in period 2. This is because a multi-category platform can leverage the categories' ability to grow by relying on each other and save resources in early periods. Depending on the strength of effects within and across categories, the platform should then invest more or less in later periods.

2.3   Investment Rules Across Sides, Categories, and Periods

In this subsection, we derive how the sizes of the own dynamic effects, cross-side effects, and cross-category effects determine how much the platform should investment across sides, categories, and time periods. In particular, we first take derivatives of the optimal investment with respect to each effect to examine the effect-specific impact on investment. We then take the total derivative and compute the overall impact of effects on investment.

Interestingly, we find that the optimal investment rules crucially depend on the business model that the platform adopts.

Per-Transaction Model We start with examining **which side** should receive more resources, or the within-category allocation rules across two sides, by comparing the merchant-side investment $(u_1^* + u_2^*)$ with the consumer-side investment $(v_1^* + v_2^*)$.[6] A positive (negative) derivative of $(v_1^* + v_2^*) - (u_1^* + u_2^*)$ with respect to each effect indicates that a stronger effect size leads to relatively more investment on the consumer (merchant) side.

Proposition 1: Holding everything else the same, a stronger own dynamic effect (λ_n, λ_s) on one side increases the optimal relative investment to that side.

If the consumer (merchant) side has a stronger own dynamic effect, the platform should invest more on the consumer (merchant) side. A stronger own dynamic effect implies the greater ability of one side to grow on its own. The result suggests that the platform should invest more on that side to leverage the higher growth potential.

Proposition 2: Holding everything else the same, a stronger cross-side effect (θ_n, θ_s) on one side decreases the optimal relative investment to that side.

If one side values more the number of participants on the other side and relies more on the other side to grow, the platform should invest less on the focal and more on the other side. This is consistent with the key insights from the economic literature on two-sided platform pricing: platforms should charge a higher price or offer less subsidy (invest less in our setting) on the side that depends more on the presence of the other side (e.g., Armstrong 2006, Wely 2010, and Hagiu 2014).

Combining the insights from Propositions 1 and 2, we find that the within-category optimal investment strategy for the per-transaction model follows a "reinforcing" rule: the platform should invest *more* on the side that is the stronger driver of platform growth. When the focal side has a stronger own dynamic effect or is less reliant on the other

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[6]The same results apply if we replace the variables of the focal category (u^*, v^*, λ, θ) with the variables of the other category ($\tilde{u}^*$, $\tilde{v}^*$, $\tilde{\lambda}$, $\tilde{\theta}$).

side, it has stronger ability to grow on its own. The platform should invest more on this side. Such "reinforcing" rule is in the spirit of the recommended investment strategy for multi-product non-platform firms in the literature. For instance, Fischer et al. (2011) study multi-product non-platform firms and find that the optimal investment should be larger for stronger products with higher growth potential. Heuristics that are commonly used in industry practice (e.g., "percentage-of-sales" heuristics) also share similar logic.

We then examine **which category** should receive more resources, or the cross-category allocation rules, by comparing the focal category's investment $(v_1^* + v_2^* + u_1^* + u_2^*)$ with the other category's investment $(\tilde{v}_1^* + \tilde{v}_2^* + \tilde{u}_1^* + \tilde{u}_2^*)$. A positive (negative) derivative of $(v_1^* + v_2^* + u_1^* + u_2^*) - (\tilde{v}_1^* + \tilde{v}_2^* + \tilde{u}_1^* + \tilde{u}_2^*)$ with respect to each effect indicates that a stronger effect size leads to relatively more investment on the focal (other) category.

Proposition 3: Holding everything else the same, a stronger own dynamic effect (λ_n, λ_s) or a stronger cross-side effect (θ_n, θ_s) of a category decreases the optimal relative investment to that category.

Stronger own dynamic and cross-side effects in comparison to the other category implies that the focal category is "stronger" in terms of its ability to grow endogenously by leveraging the sizes of both sides within the category. The optimal allocation is to invest more in the other category, which is "weaker" in its ability to grow endogenously.

Proposition 4: Holding everything else the same, a stronger cross-category effect (γ_{nn}, γ_{ns}, γ_{ss}, γ_{sn}) of a category increases the optimal relative investment to that category.

When the cross-category effects are stronger, the expansion of the focal category relies more on the other category. The solution suggests that the platform should allocate more resources to this focal category in order to boost overall platform growth and less to the other category which is more independent.

Combining the insights from Propositions 3 and 4, we find that the cross-category investment rule differs from the within-category investment rule (rooted in Propositions 1 and 2) and follows a "compensatory" rule: the platform should invest *less* in the category that is the stronger driver of platform growth. When the focal category has stronger own

dynamic effects and cross-side effects, it has a stronger ability to leverage its own past size to grow and requires less investment. When the focal category has larger cross-category effects, it is more reliant on the size of the other category to grow and requires more investment.

We further examine **when** to allocate more, or the cross-period allocation rules, by comparing the early period allocation $(u_1^* + v_1^*)$ with the later period allocation $(u_2^* + v_2^*)$. A positive (negative) derivative of $(v_1^* + u_1^*) - (v_2^* + u_2^*)$ with respect to each effect indicates that a stronger effect size leads to relatively more investment in earlier (later) periods.

Proposition 5: Holding everything else the same, a stronger own dynamic effect (λ_n, λ_s), a stronger cross-side effect (θ_n, θ_s), or a larger cross-category effect ($\tilde{\gamma}_{nn}$, $\tilde{\gamma}_{ns}$, $\tilde{\gamma}_{ss}$, $\tilde{\gamma}_{sn}$) decreases the early-period investment (relative to the later-period investment) in that category.

These effects indicate how fast the platform can grow based on the past performance or the other category. Proposition 5 suggests that the cross-time investment rule follows the "compensatory" logic similar to the one guiding the cross-category investment rule. When the focal category has stronger own dynamic and cross-side effects, its own growth momentum calls for less investment in its early period (relative to its later period). When the other category can better leverage the focal category's growth, less investment is required for the focal category in the early period (relative to its later period).

The allocation rules for the per-transaction model (Propositions 1 to 5) are summarized in Table 1(a).

Per-User Model   Similarly, we derive which side, which category, and when to invest more for the per-user model and summarize the propositions for the per-user model in Table 1(b). The results suggest that the platform should adopt the "reinforcing" rule for both within- and cross-category allocations.

Model Comparison   Interestingly, we find that the optimal allocation rules differ by business model. When the platform adopts a per-transaction model, the within- and cross-category allocations have opposite rules: the platform should use the "reinforcing"

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rule for within-category allocation and the "compensatory" rule for cross-category allocation. When the platform adopts a per-user model, it should use the same "reinforcing" rule for both within- and cross-category allocations as well as for cross-time allocation.

As the two business models share the same marketing response functions (Equations 1 and 2) and only differ in the composition of the revenue function, we posit that how the platform charge the users is the key driver of the allocation rule difference. In the per-user model, the revenue generating process within a category and across categories are similar, as reflected by the additive relationship between the two sides and two categories in the revenue function $\sqrt{n_t} + \sqrt{s_t} + \sqrt{\tilde{n}_t} + \sqrt{\tilde{s}_t}$. The size of the other side (category) does not directly affect how much the platform charge the focal side (category). In this case, the investment decision follows similar rules for within-category and cross-category allocations. In the per-transaction model, however, the within-category and cross-category relationships are different: within a category, the transaction volume, on which the platform charges, crucially depends on both sides of the platform, as reflected by the multiplicative relationship between the two sides $n_t * s_t$; across categories, the size of one category does not directly influence how much the platform can charge another category, as reflected by the additive relationship between the two categories $n_t * s_t + \tilde{n}_t * \tilde{s}_t$. Therefore, the optimal investment rule is different for within- and cross-category allocations in this case.

2.4   Model Extensions

We consider several extensions to the model by allowing for asymmetry across sides and categories in the platform's revenue generating ability, which alters the objective functions of the platforms. We find that the optimal allocation rules are robust and remain qualitatively the same; the asymmetry only amplifies or dampens the propositions. We provide the details of the derivation in the online appendix and summarize the results in this subsection.

Category Asymmetry   Suppose the platform charges different commissions across

Table 1: Proposition Summary

(a) Per-Transaction Model

	When ... stronger	invest ... focal		
Which side	λ_n, λ_s	more	Prop 1	"reinforcing"
	θ_n, θ_s	less	Prop 2	"reinforcing"
Which category	λ_n, λ_s or θ_n, θ_s	less	Prop 3	"compensatory"
	$\gamma_{nn}, \gamma_{ns}, \gamma_{ss}, \gamma_{sn}$	more	Prop 4	"compensatory"
When	λ_n, λ_s or θ_n, θ_s	less earlier	Prop 5	"compensatory"
	or $\tilde{\gamma}_{nn}, \tilde{\gamma}_{ns}, \tilde{\gamma}_{ss}, \tilde{\gamma}_{sn}$			

(b) Per-User Model

	When ... stronger	invest ... focal		
Which side	λ_n, λ_s	more	Prop 1	"reinforcing"
	θ_n, θ_s	less	Prop 2	"reinforcing"
Which category	λ_n, λ_s or θ_n, θ_s	more	oppo. Prop 3	"reinforcing"
	$\gamma_{nn}, \gamma_{ns}, \gamma_{ss}, \gamma_{sn}$	less	oppo. Prop 4	"reinforcing"
When	λ_n, λ_s or θ_n, θ_s	more earlier	oppo. Prop 5	"reinforcing"
	or $\tilde{\gamma}_{nn}, \tilde{\gamma}_{ns}, \tilde{\gamma}_{ss}, \tilde{\gamma}_{sn}$			

categories. Without loss of generality, let the category-specific commissions be $\{a, 1\}$ for categories 1 and 2. The optimal investment problem of the per-transaction model is

$$\max \quad \sum_{t=1,2} (an_t s_t - u_t - v_t) + (\tilde{n}_t \tilde{s}_t - \tilde{u}_t - \tilde{v}_t)$$

The optimal investment problem of the per-user model is

$$\max \quad \sum_{t=1,2} (a\sqrt{n_t} + a\sqrt{s_t} - u_t - v_t) + \left(\sqrt{\tilde{n}_t} + \sqrt{\tilde{s}_t} - \tilde{u}_t - \tilde{v}_t \right)$$

We examine how changes in a affect the optimal investment (e.g., $\frac{\partial u_1^*}{\partial a}$) and whether it amplifies or dampens the propositions (e.g., $\frac{\partial^2 (u_1^* + u_2^*) - (v_1^* + v_2^*)}{\partial \lambda_n \partial a}$).

For the per-transaction model, first, we find that a higher commission of a category a decreases the relative investment to that category. This is consistent with the "compensatory" rule in terms of which category to invest in the per-transaction model. Second, we find that the propositions still qualitatively hold, while the magnitudes of the effects change with a. In particular, a higher commission of the focal category a will dampen an effect when the driver of the effect is the focal category (e.g., $\lambda_n, \theta_n, \tilde{\gamma}_{nn}, \tilde{\gamma}_{ns}$); a higher

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commission of the focal category a will have no effect when the driver of the effect is the other category (e.g., γ_{nn}, γ_{ns}). This is again consistent with the "compensatory" rule.

For the per-user model, first, we find that a higher commission of a category a increases the relative investment to that category. This is consistent with the "reinforcing" rule in terms of which category to invest in the per-user model. Second, the propositions still qualitatively hold, while the magnitudes of the effects change with a. In particular, a higher commission of the focal category a will amplify an effect when the driver of the effect is the focal category (e.g., λ_n, θ_n, $\tilde{\gamma}_{nn}$, $\tilde{\gamma}_{ns}$); a higher commission of the focal category a will have no effect when the driver of the effect is the other category (e.g., γ_{nn}, γ_{ns}). This is again consistent with the "reinforcing" rule.

Side Asymmetry   Suppose the platform charges different commissions across sides. Without loss of generality, let the side-specific commissions be $\{1, \alpha\}$ on the merchant and consumer sides, respectively. The optimal investment problem of the per-user model is

$$\max \quad \sum_{t=1,2} \left(\sqrt{n_t} + \alpha\sqrt{s_t} - u_t - v_t \right) + \left(\sqrt{\tilde{n}_t} + \alpha\sqrt{\tilde{s}_t} - \tilde{u}_t - \tilde{v}_t \right)$$

We do not examine side asymmetry in the per-transaction model as the commission is not charged by side in the per-transaction model.

We examine how changes in α affect the optimal investment (e.g., $\frac{\partial u_1^*}{\partial \alpha}$) and whether it amplifies or dampens the propositions (e.g., $\frac{\partial^2 \left(u_1^* + u_2^* \right) - \left(v_1^* + v_2^* \right)}{\partial \lambda_n \partial \alpha}$). First, we find that a larger fee on the consumer side increases the relative investment to the consumer side. This is consistent with the "reinforcing" rule in terms of which side to invest for the per-user model. Second, the propositions still qualitatively hold, while the magnitudes of the effects change with α. In particular, a larger fee on the consumer side α will amplify an effect when the driver of the effect is the consumer side (e.g., θ_n, γ_{ns}, $\tilde{\gamma}_{ns}$); a larger fee on the consumer side α will have no effect when the driver of the effect is the merchant side (e.g., λ_n, γ_{nn}, $\tilde{\gamma}_{nn}$). This is again consistent with the "reinforcing" rule.

In sum, we find that introducing asymmetry across categories and sides does not

qualitatively change the optimal allocation rules.

2.5   Overall Model Recommendations

The propositions in Table 1 describe how each effect separately affects the optimal investment under each business model, holding everything else the same. In practice, multiple effects jointly determine in which side, in which category, and when to invest. Furthermore, categories can differ in multiple effects, each of which may have a separate, sometimes even opposite, impact on the optimal investment. To derive an overall model recommendation regarding the joint impact of multiple effects, we utilize the rule of total derivatives. For instance, consider the decision of which category to invest in more. Following the rule of total derivatives, the difference between the optimal investment levels of the two categories can be expressed as

$$
\begin{aligned}
\triangle I =& \frac{\partial I}{\partial \lambda_n}\triangle \lambda_n + \frac{\partial I}{\partial \theta_n}\triangle \theta_n + \frac{\partial I}{\partial \lambda_s}\triangle \lambda_s + \frac{\partial I}{\partial \theta_s}\triangle \theta_s \\
&+ \frac{\partial I}{\partial \gamma_{nn}}\triangle \gamma_{nn} + \frac{\partial I}{\partial \gamma_{ns}}\triangle \gamma_{ns} + \frac{\partial I}{\partial \gamma_{ss}}\triangle \gamma_{ss} + \frac{\partial I}{\partial \gamma_{sn}}\triangle \gamma_{sn} \\
&+ \frac{\partial I}{\partial \tilde{\gamma}_{nn}}\triangle \tilde{\gamma}_{nn} + \frac{\partial I}{\partial \tilde{\gamma}_{ns}}\triangle \tilde{\gamma}_{ns} + \frac{\partial I}{\partial \tilde{\gamma}_{ss}}\triangle \tilde{\gamma}_{ss} + \frac{\partial I}{\partial \tilde{\gamma}_{sn}}\triangle \tilde{\gamma}_{sn}
\end{aligned}
$$

(6)

where $I = u_1^* + v_1^* + u_2^* + v_2^*$ is the total optimal investment level of one category. The difference between the total optimal investment levels of the two categories, $\triangle I$, can be derived from the partial derivatives of the optimal investment with respect to each effect (e.g., $\frac{\partial I}{\partial \lambda_n}$), multiplied by the difference in the effects (e.g., $\Delta \lambda_n$), and summed up over all the effects. The partial derivatives can be directly derived from Equations 3, 4 (for the per-transaction model), or 5 (for the per-user model). When ΔI is larger, more investment is required for the focal category. We can use the same logic to derive the joint impact of multiple effects regarding which side and when to invest.

3   Empirical Model

The theoretical framework can be used to generate practical recommendations for platform firms regarding which side, which category, and when to invest more by substituting the empirically estimated effects into the equations of total derivatives. In this section, we describe an application of this method using daily deals industry as an example. This industry context is appropriate as a daily deals website represents a typical two-sided multi-category platform which adopts a per-transaction model. Participating merchants can offer deals in different categories on one side and consumers can buy deals on the other side. The platform charges a fraction of the revenue from each transaction. We use data on Groupon, the leading player in the daily deals industry, to empirically show the validity and benefits of our proposed strategy relative to the common industry practice in allocating marketing resources.

The empirical validation involves two steps. In the first step, we use the data on observed investment, deal offerings and sales on Groupon to estimate the sizes of category-specific own dynamic effects, cross-side effects and cross-category effects. In the second step, given the estimated effects, we construct alternative investment schemes based on our model recommendations and simulate alternative platform growth paths. We compare the performance of these investment schemes with the observed scheme to demonstrate how Groupon's profitability could potentially be improved with our approach.

3.1   Data Description

We obtained data about Groupon deal offerings and sales in all the major U.S. markets between March 2010 and January 2012.[7] To control for the competition effect, we also obtained data about the presence of other daily deal platforms in each market. We focus on the four largest categories of Groupon: Beauty, Restaurants, Fitness, and Entertainment, accounting for 17.6%, 14.1%, 7.2%, and 16.7% of the deals, respectively.

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[7]The data is provided by a company that specializes in aggregating deal offerings from most of the daily deals websites.

We observe the basic characteristics of each deal listed in these categories during the data period, including deal starting date, ending date, price, and discount level, as well as the deal sales. In this business context during our sample period, all deals are local. Our data includes deals from 99 Metropolitan Statistical Areas (MSA). Groupon entered different markets at different times. In our analysis below, we aggregate the deal information at the category-market-week level, where "week" is the number of weeks since the platform entered the market. We use historical instead of calendar week because the former is more relevant for characterizing platform growth in this localized context. For variables in our empirical model, we construct their category-market-week level values and present the summary statistics in Table 2. Although there is significant variation in terms of average deal price and sales by category, the average duration and discount level across categories are highly concentrated and comparable. This is because, according to various media coverage of merchants' experiences, Groupon typically uses standard terms of a 50% discount rate and follows previous deals of comparable merchants and products when determining deal characteristics (Sunil 2012). Therefore, there is no significant variation in discount rate and duration across deals. We also plot the median number of deals, per deal sales, and total deal sales (equals number of deals times per deal sales) in a category-market-week over time in Figure 1. The x-axis is the number of weeks since the platform entered the market. As the platform grew in a market, the number of deals increased over time, especially for Beauty and Entertainment. The per deal sales decreased over time. Restaurant has the largest per deals sales. The increase in number of deals dominated the decrease in per deal sales so that the total deal sales increased over time.

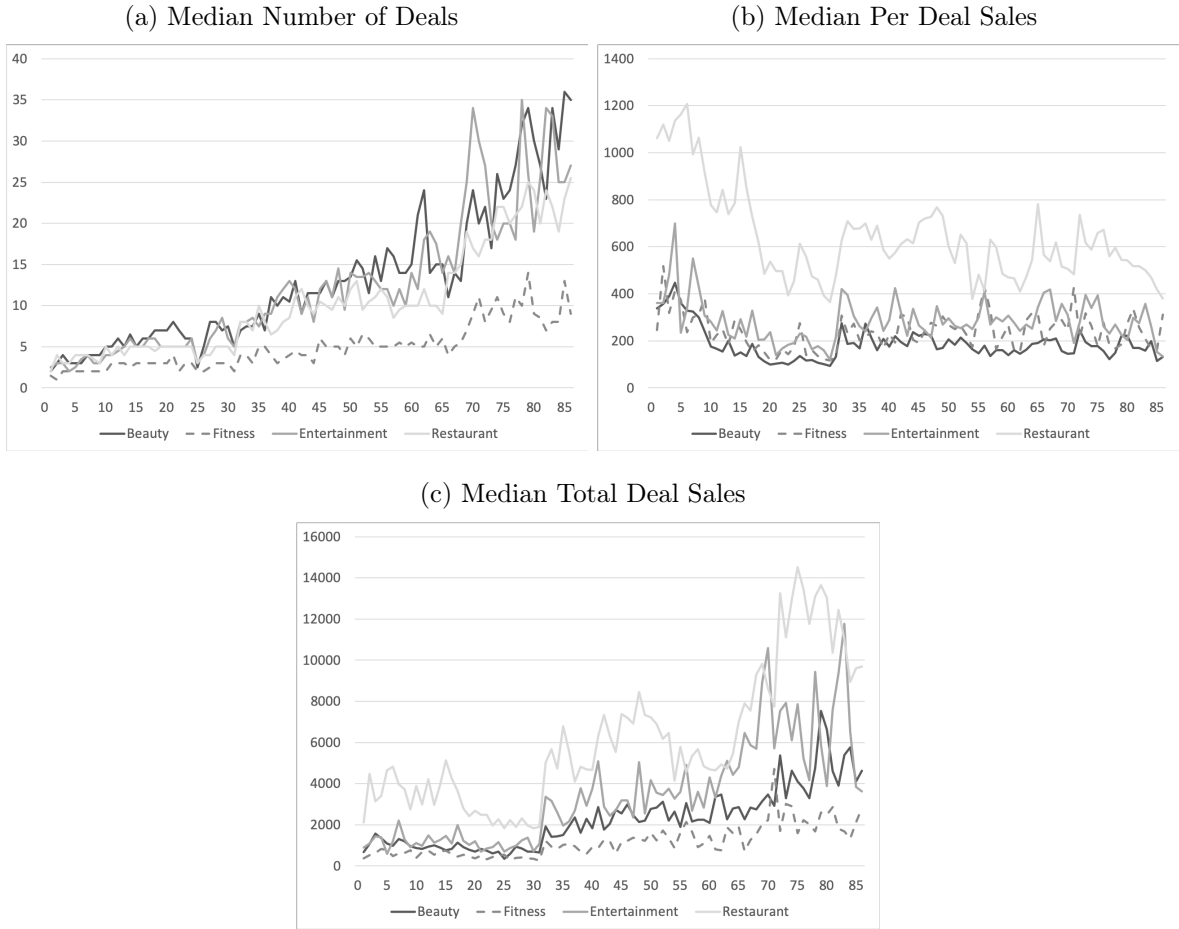Figure 1: Data Patterns: Number of Deals and Per Deal Sales

(a) Median Number of Deals

(b) Median Per Deal Sales

(c) Median Total Deal Sales

Table 2: Summary Statistics

	Beauty		Fitness		Entertainment		Restaurant	
	Mean	(Std.)	Mean	(Std.)	Mean	(Std.)	Mean	(Std.)
# of deals (weekly)	11.15	(10.21)	4.64	(4.02)	10.46	(10.18)	8.25	(7.70)
Per deal sales	261.15	(359.71)	360.53	(623.16)	435.28	(666.48)	790.36	(678.73)
% returning merchant	46.98	(32.36)	36.47	(35.77)	40.56	(32.36)	30.03	(29.80)
Past experience	1.06	(1.21)	0.65	(1.00)	1.03	(1.75)	0.59	(1.26)
Duration	2.90	(1.09)	2.63	(1.14)	2.89	(1.14)	2.43	(1.00)
Price	99.96	(71.89)	44.28	(37.09)	55.20	(53.09)	21.03	(28.30)
Discount	59.62	(6.61)	63.87	(9.04)	52.58	(3.97)	51.51	(2.08)

For our analysis, we also obtained information on Groupon's resource allocation on the consumer and merchant sides. We obtained data on Groupon's aggregate marketing expense and sales force at the quarter level from Groupon's SEC filings and news releases

and linearly interpolate within each quarter to obtain the monthly values.[8] We use the marketing expense as a proxy for Groupon's consumer-side investment and the size of the sales force as a proxy for Groupon's merchant-side investment.

Groupon only publishes aggregate information on advertising and sales force, while our analysis requires data on market- and category-specific investments. On the merchant side, to impute the market- and category-specific investment from the aggregate investment, we assume that the actual allocation of sales force per market per category in a period is a function of the linear and quadratic terms of the observed number of deals and the fraction of new merchants (i.e., who have not offered deals before) per category per market in that period.[9] We empirically estimate the function parameters and use the function to impute the market-category-week level investment (more details in the appendix). This approach can flexibly capture potential learning effect or economy of scale in terms of merchant acquisition. It also allows for different costs of acquiring new versus returning merchants. The underlying assumption is that the way the investment systematically changes with the number of merchants (e.g., how quickly learning happens) and the fraction of new merchants (e.g., how much less costly it is to acquire returning merchants) is the same across categories and markets. As the sales force typically follows a standardized procedure when approaching merchants, this assumption can be a reasonable representation of industry practice. Note that this assumption is agnostic about Groupon's investment rules in practice and does not preclude any strategic investment behavior. To ensure that the empirical results are robust to this assumption, we conduct robustness checks by assuming that 1) Groupon followed a "reinforcing" rule in practice so that the category-market level investment in a period is positively proportional to the observed category-market level number of deals in the *last* period; 2) Groupon followed a "compensatory" rule in practice so that the category-market level investment in a pe-

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[8]See https://www.sec.gov/Archives/edgar/data/1490281/000144530512000922/groupon10-k.htm and http://investor.groupon.com/news-releases/news-release-details/groupon-announces-first-quarter-2012-results.

[9]Note that there is a few months' time lag between when the merchant signs the contract with Groupon and the actual release of the deal. When operationalizing this assumption, we use the current number of deals and fraction of new deals per category per market and the one-period lagged aggregate investment to reflect this industry practice.

riod is negatively proportional to the observed category-market level number of deals in the *last* period. As shown in the online appendix, the estimation results are robust to different assumptions regarding the observed Groupon investment rule.

On the consumer side, Groupon's marketing expense focuses primarily on online marketing.[10] Given the nature of Groupon's marketing expense, we proxy market-specific investment as proportional to the number of website visits in that market because the marketing expense per click is relatively constant in practice. Groupon would incur more costs in markets where there are more consumer clicks and website visits. We utilize the Comscore consumers' website browsing records data during our sample period (March 2010 – January 2012) to construct the measures. For each website visit, we observe the machine ID, starting and ending time stamps, website visited, last website visited before the focal visit, and household information such as zip code. We aggregate over the website visits of Groupon in a market-period and assume that investment in a particular market-period is proportional to the observed website visits in that market-period. Note that we assume the marketing expense in a given market-period is the same across categories because Groupon appears not conducting category-specific advertising in practice. First, Groupon's display ads usually feature only the overall brand instead of a particular category of deals.[11] Second, once consumers click on the ad, they are routed to the main website instead of a category-specific page. We include more details on how the consumer-side investment is calculated in the appendix.

3.2   Deal Supply and Demand

We focus on estimating the sizes of the category-specific own dynamic effects, cross-side effects and cross-category effects through deal supply and demand. These effects are allowed to vary across categories, as categories might be inherently different. For example, consumers might be more likely to share word-of-mouth about deals in some

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[10]See http://investor.groupon.com/static-files/2951404e-7171-47b7-9492-36cec02b1d50, also http://adage.com/article/digital/groupon-marketing-spending-works/230777/.

[11]For a sample display ad, see http://ppcblog.com/how-does-groupon-win-new-markets/.

categories than deals in other categories. It implies that the own dynamic effect on the consumer side could differ by category. Similarly, the cross-side effect on sales could be stronger for some categories than for others due to the consumption pattern and the substitutability between deals within a category.

Our empirical specification closely follows the theoretical model setup. The platform's deal offerings N_{jmt} and per deal sales S_{jmt} in category j in market m at period t are modeled as follows:[12]

$$(7) \quad \ln N_{jmt} = \beta_u \ln u_{jmt}^o + \lambda_{nj} \ln N_{jmt-1} + \theta_{nj} \ln S_{jmt-1} + \gamma_{nnj} \ln \tilde{N}_{jmt-1} + \gamma_{nsj} \ln \tilde{S}_{jmt-1}$$
$$+ \alpha_W W_{jmt} + \alpha_Z Z_{jm} + \alpha_D D_{mt} + \alpha_T T_t + \epsilon_{jmt}^N$$

$$(8) \quad \ln S_{jmt} = \beta_v \ln v_{mt}^o + \lambda_{sj} \ln S_{jmt-1} + \theta_{sj} \ln N_{jmt-1} + \gamma_{ssj} \ln \tilde{S}_{jmt-1} + \gamma_{snj} \ln \tilde{N}_{jmt-1}$$
$$+ \beta_X X_{jmt} + \beta_Z Z_{jm} + \beta_D D_{mt} + \beta_T T_t + \epsilon_{jmt}^S$$

where u_{jmt}^o and v_{mt}^o represent the observed platform investment in category j in market m at week t to the merchant side and the consumer side, respectively.

Equation 7 describes the determinants of the number of deals in category j of the week in the market. The first term λ_{nj} stands for the own dynamic effect, or how the number of deal offerings in the preceding period affects the deal supply of the current period. More merchants using the platform in the past may lead to more merchants participating in the future. The effect reflects the ability of the merchant side to grow on its own. The second term θ_{nj} captures the cross-side effect, or how per deal sales affect the supply of deals in this category. The effect reflects the ability of the merchant side to grow with the consumer side. The third and fourth terms γ_{nnj} and γ_{nsj} represent the cross-category effects from number of deals and per deal sales in other categories. More merchants or more consumers using the platform in other categories can indicate higher platform attractiveness in general and impact the focal category. The traffic generated by other categories can be helpful for the focal category as well. To link the two-category theory model and the four-category empirical model, we operationalize the

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[12]We use log transformation as the variables are highly skewed.

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number of deals in other categories as the sum of deal supply in the other three categories, $\ln \tilde{N}_{jmt-1} = \ln \left(\sum_{-j} N_{-jmt-1} \right)$. We operationalize the per deal sales in other categories as the average per deal sales in the other three categories, $\ln \tilde{S}_{jmt-1} = \ln \left(\frac{\sum_{-j} S_{-jmt-1}}{3} \right)$.

Equation 8 describes the per deal sales of the category. It is affected by the per deal sales in the preceding period, which is captured by the own dynamic effect λ_{sj}. The idea is that past purchases in the category may trigger word-of-mouth sharing among consumers, which then may help sales in the current period. The per deal sales can also be affected by the number of deals available to consumers in the market, which is captured by the cross-side effect θ_{sj}. On the one hand, a larger number of available deals may attract more consumers and increase deal sales, which is consistent with a positive indirect network effect in the two-sided platform literature. On the other hand, more deals may lead to more competition among merchants and decrease *per deal* sales. The cross-side effect in our model is the net effect of the two forces. Finally, the per deal sales and the number of deals in other categories can also have an impact, which is captured by the cross-category effects γ_{ssj} and γ_{snj}.

We also control for category and market fixed effects Z_{jm}, market structure D_{mt}, and the time effect T_t on both sides. On the deal supply side, we further control for merchants' experience with the platform in the market, W_{jmt}. On the deal demand side, we further control for the average deal characteristics of the category X_{jmt}. The variables are as follows:

X_{jmt}: average deal characteristics, including duration, price, and discount;

Z_{jm}: category and market fixed effects;

W_{jmt}: merchants' past experience with the platform, including the percentage of deals offered by returning merchants who have offered deals in this category before period t and the average number of times that the merchants in this category have used Groupon before period t;

D_{mt}: market structure, including total number of incumbents, presences of the second, third, fourth, and fifth entrants;

T_t: time trend and seasonality, including number of weeks since the platform first

entered the market, its squared term, year dummy, and month dummy.

3.3   Estimation Method

We allow the error terms of the two equations to be correlated by running a three-stage least square (3SLS). In particular, the lagged dependent variables $\ln N_{jmt-1}$, $\ln S_{jmt-1}$, $\ln \tilde{N}_{jmt-1}$ and $\ln \tilde{S}_{jmt-1}$ can be endogenous if the error terms in the two equations are serially correlated. We control for category- and market-specific fixed effects and use the following exogenous variables as instruments to treat the endogeneity issue of the lagged terms: (a) the two-period lagged terms; (b) the deal characteristics X_{jmt}; (c) the merchant experience terms W_{jmt}; (d) the average deal characteristics and merchant experience terms of other categories; and (e) the market conditions Z_{jm} and D_{mt}.[13] Exclusion restrictions come from the variables of merchants' experience W_{jmt} and the deal characteristics X_{jmt}. On the one hand, merchants' past experience affects their decisions to offer deals again. However, it does not directly affect consumers' deal purchase. W_{jmt} only affects deal supply and not deal sales. On the other hand, deal characteristics X_{jmt} such as discount and duration only affect deal sales and do not directly affect deal supply. The partial F-statistics for instruments X_{jmt} and W_{jmt} are 585.18 and 299.95, respectively, suggesting that the instruments satisfy the relevance condition. To test for serial correlation, we follow the approach proposed by Godfrey (1994) as it can be applied to dynamic models with endogenous variables. We obtain the fitted residuals from each equation and use the lagged residuals as new regressors to re-estimate the model. The estimated coefficients on the lagged residuals are insignificant, suggesting that serial correlation of the error terms is not present after treating the endogeneity issue of the lagged terms.

Another potential endogeneity issue is that we impute the merchant-side investment

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[13]The two-period lagged terms are valid instruments if serial correlation goes down to zero in two periods. We test the correlation between the error terms in period $t-2$ and t and find that the correlations are not significantly different from zero (-0.0070 and 0.0019 for the merchant and the consumer side equations respectively), suggesting that serial correlation is not a serious concern after treating the endogeneity issue.

u_{jmt}^o as a function of the number of deals and the fraction of new merchants in the focal category, so u_{jmt}^o can be potentially endogenous to unobserved shocks in the current period. We treat the potential endogeneity issue of u_{jmt}^o by using the fraction of returning merchants in *other* categories in the current period as instruments for u_{jmt}^o when running the main 3SLS regression. The fraction of returning merchants in other categories is a valid instrument because it correlates with u_{jmt}^o: it affects the investment to other categories and in turn affects the investment to the focal category u_{jmt}^o (given that investments across all categories are subject to a single total investment). However, it does not directly affect the dependent variable N_{jmt}.

We treat the deal characteristics X_{jmt} as exogenous based on how deal characteristics are determined in practice. Merchants are usually approached by Groupon with the standard 50% discount rate and a 50/50 revenue split (Sunil 2012). Their negotiation power is highly limited: they may be able to negotiate their revenue split by leveraging their local market awareness or product differentiation, but they were rarely able to negotiate the discount rates (Agrawal 2011). This is also reflected in the highly concentrated deal discount rates and durations across categories and markets in the data as shown in Table 2. Li et al. (2018) treat the deal characteristics in the same way given similar industry settings.

3.4   Estimation Results

Table 3 presents the estimation results. As expected, we find that different categories exhibit different magnitudes of the own dynamic, cross-side, and cross-category effects. First, from the merchant-side number of deals regression, we find a positive and significant own dynamic effect. More merchants choosing a particular platform results in more merchants for the same category in the future. Such effect is the strongest for Restaurant and the weakest for Fitness.

Second, we find a positive and significant cross-side effect on the merchant side for Entertainment; larger per deal sales encourage merchants to offer deals. The effect is

negative for Beauty and Restaurant, which is not entirely surprising given the industry context. Given high discount rates (typically 50%) and the 50/50 revenue split to the platform, merchants often collect only 25% or less of the face value. After accounting for the cost of serving customers, the merchant may even lose money in deal sales. Survey results in Dholakia (2011) show that only 55.5% of the deals are profitable. Edelman et al (2016) show that merchants use daily deals platforms mainly as advertising tools. Consistent with our finding that Beauty and Restaurant categories has negative cross-side effect, Gupta et al. (2012) show that merchants in the restaurant category "lose money from daily deals on average" and that high-priced, high-margin categories (e.g., salon and spa treatments and high-end cosmetic treatments) "had the worst discounts and up-spending and among the worst redemption rates and planned future visits by new customers," which may discourage merchants from offering deals.

Third, the cross-category effect on the merchant side has mixed results. It is positive for Beauty and negative for Entertainment. It suggests that some categories can benefit from more deals or higher per deal sales in other categories, while other categories can be hurt by them.

From the consumer-side per deal sales regression, we find a positive and significant own dynamic effect and a negative cross-side effect, while the magnitudes differ across categories. The positive own dynamic effect suggests that higher sales today lead to higher sales tomorrow. This can be interpreted as the ability of a category to grow based on its own demand. We find that Entertainment exhibits the highest own dynamic effect. The negative cross-side effect is the net effect of two forces. On the one hand, more deals on the merchant side attract more consumers, which is the classic indirect network effect in the two-sided platform literature. On the other hand, more deals means stronger competition. Overall, the net effect is that more deals in the same category lead to lower sales per deal on average. This negative effect is the strongest in Beauty and relatively weak in Fitness and Entertainment.

Finally, we find a mostly insignificant cross-category effect from the other categories' per deal sales. For Entertainment, more deal sales in other categories can hurt focal

category sales. The cross-category effect from the other categories' number of deals is positive for Beauty and negative for Fitness and Entertainment. It suggests that, besides bringing more traffic, more deals offered in other categories today can also lead to stronger competition among deals and lower per deal sales in some focal categories tomorrow.

In terms of the rest of the parameters, the estimated coefficients on the investment are positive and significant for both the consumer and the merchant sides. The effects of deal characteristics on deal sales are mostly as expected. Higher deal price leads to lower deal sales, and larger discount rate leads to higher deal sales. The percentage of returning merchants or past experience with the platform has a positive impact on deal offerings. Finally, the presence of entrants in the local market has a negative impact on the deal supply of the focal platform.

4   Simulation

Given the empirical estimates of the effects, we can use our theoretical framework for the per-transaction model to generate overall model recommendations regarding category, side, and timing for Groupon's investment decision. As discussed in Section 2.5, we substitute the empirical estimates of the effects into Equation 6 and use the partial derivatives with respect to each effect as "weights" of each single effect to jointly evaluate the overall prediction of multiple effects. We account for the fact that categories are asymmetric in their prices when deriving the partial derivatives, as illustrated in the model extensions. The results are summarized in Table 4 (the differences in the investment levels $\triangle I$ and the effect sizes such as $\Delta \lambda_n$ are measured relative to zero).

In terms of which side to invest in, our model recommends that Entertainment, relative to other categories, should receive a higher proportion of investment on the merchant side than the consumer side. In terms of which category to invest in, our model recommends that the platform should invest more in the Beauty and the Restaurant categories and invest less in Fitness. Finally, in terms of when to invest, our model recommends that the platform should invest earlier in Beauty and later in Entertainment.
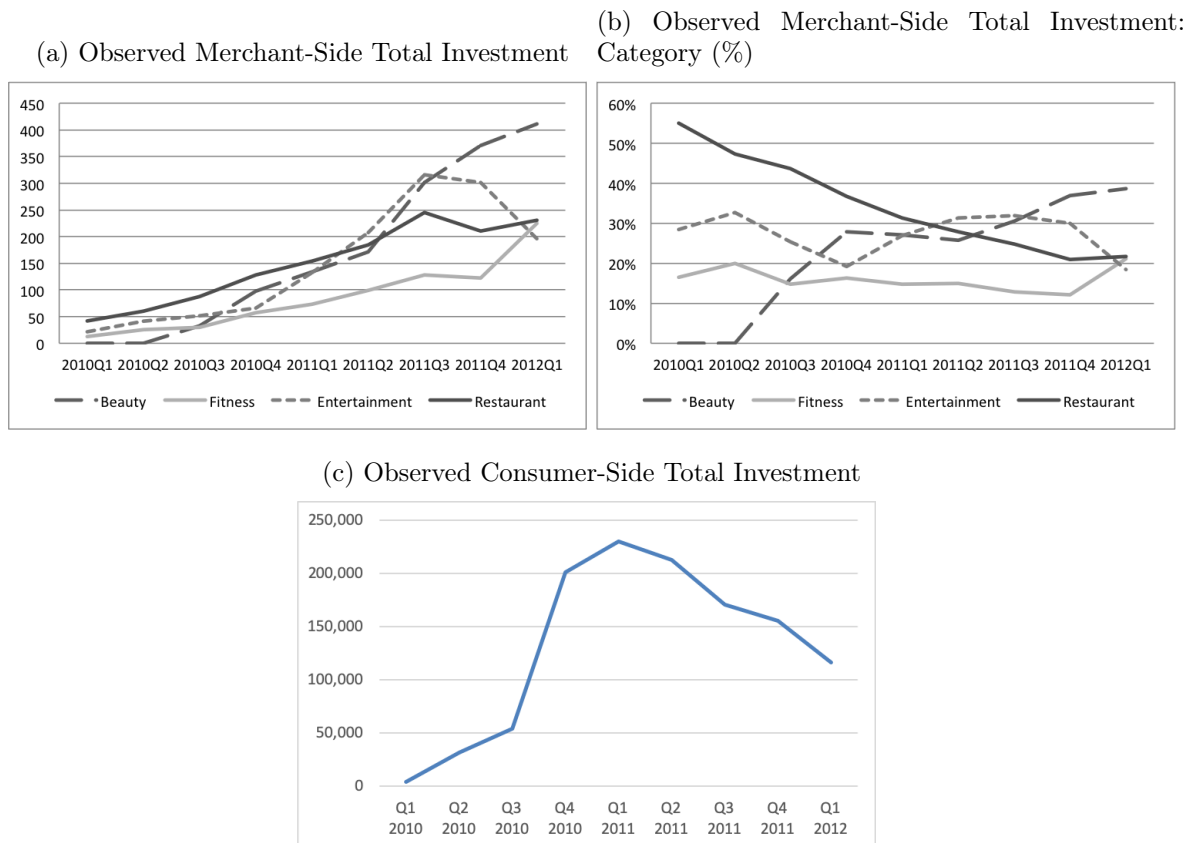
Table 3: Estimation Results

Merchant-side Regression		Est.	Std.	Consumer-side Regression		Est.	Std.
Investment β_u		0.479***	(0.0713)	Investment β_v		0.0532***	(0.0173)
Own dynamic λ_{nj}	$j=1$	0.288***	(0.0683)	Own dynamic λ_{sj}	$j=1$	0.0840	(0.0729)
	$j=2$	0.238**	(0.101)		$j=2$	0.0744	(0.0634)
	$j=3$	0.374***	(0.0547)		$j=3$	0.348***	(0.0691)
	$j=4$	0.339***	(0.0519)		$j=4$	-0.0364	(0.0828)
Cross-side θ_{nj}	$j=1$	-0.232***	(0.0460)	Cross-side θ_{sj}	$j=1$	-0.466***	(0.107)
	$j=2$	-0.0407	(0.0394)		$j=2$	0.105	(0.160)
	$j=3$	0.141***	(0.0445)		$j=3$	-0.119	(0.0874)
	$j=4$	-0.0970*	(0.0533)		$j=4$	-0.223***	(0.0806)
Cross-category γ_{nnj}	$j=1$	0.0665	(0.0762)	Cross-category γ_{ssj}	$j=1$	-0.0135	(0.0715)
	$j=2$	0.0589	(0.0740)		$j=2$	0.111	(0.0713)
	$j=3$	-0.147***	(0.0538)		$j=3$	-0.188**	(0.0756)
	$j=4$	-0.0731*	(0.0410)		$j=4$	0.0633	(0.0942)
Cross-category γ_{nsj}	$j=1$	0.128***	(0.0448)	Cross-category γ_{snj}	$j=1$	0.215*	(0.118)
	$j=2$	-0.0447	(0.0435)		$j=2$	-0.258**	(0.114)
	$j=3$	-0.148***	(0.0503)		$j=3$	-0.164*	(0.0862)
	$j=4$	0.164***	(0.0592)		$j=4$	-0.0214	(0.0648)
Return		0.811***	(0.0793)	Discount		0.605***	(0.0791)
Past experience		0.0694***	(0.00491)	Price		-0.356***	(0.0160)
				Duration		-0.105***	(0.0321)
Time: linear		-0.00495	(0.00452)	Time: linear		-0.00280	(0.00692)
Time: quadratic		-5.40e-05***	(1.86e-05)	Time: quadratic		3.01e-07	(2.38e-05)
No. incumbents		-0.00797	(0.133)	No. incumbents		0.275	(0.210)
Second entrant		-0.143***	(0.0394)	Second entrant		-0.0180	(0.0620)
Third entrant		-0.0297	(0.0275)	Third entrant		-0.0415	(0.0434)
Fourth entrant		-0.0253	(0.0235)	Fourth entrant		0.105***	(0.0371)
Fifth entrant		-0.0587***	(0.0222)	Fifth entrant		0.0251	(0.0349)
Market FEs		YES		Market FEs		YES	
Category FEs		YES		Category FEs		YES	
Month year FEs		YES		Month year FEs		YES	
N.		9,663		N.		9,663	
R-squared		0.611		R-squared		0.534	

Notes: $j = 1, 2, 3, 4$ represent Beauty, Fitness, Entertainment, Restaurant categories, respectively. Standard errors are in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Table 4: Model Recommendations

	△I		
	Which side	**Which category**	**When**
Definition of I	$I = (v_1^* + v_2^*) - (u_1^* + u_2^*)$	$I = u_1^* + u_2^* + v_1^* + v_2^*$	$I = (v_1^* + u_1^*) - (v_2^* + u_2^*)$
Beauty	8.62	6.93	0.94
Fitness	0.031	0.96	-2.04
Entertainment	-0.085	1.76	-4.43
Restaurant	19.7	16.9	-0.90
Recommendation:	If larger, invest more consumer side.	If larger, invest more in this category.	If larger, invest earlier.

Figure 2: Observed Investment Patterns

(a) Observed Merchant-Side Total Investment

(b) Observed Merchant-Side Total Investment: Category (%)

(c) Observed Consumer-Side Total Investment

The observed investment patterns of Groupon do not always follow our recommendations. Figure 2 plots Groupon's observed investment patterns on the merchant and the consumer sides.[14] For instance, Groupon invests the most in Restaurant, which is consistent with our model recommendation. However, its investment in Entertainment is relatively stable over time as shown in Figure 2(b), while our model suggests that it should invest later in Entertainment.

To illustrate the benefits of our proposed investment strategies, we conduct simulation analysis by varying the investment allocation according to our recommendations while keeping the total investment fixed. The results suggest that merely re-allocating investments based on our recommendations can improve Groupon's profitability. We do not

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[14]The investment on the merchant side increased over time for all categories; the relative investment across categories varied over time. The investment on the consumer side peaked during the middle of our sample period when Groupon went IPO in 2011. Marketing expense dropped after the second quarter of 2011 because "Groupon's vitality is offsetting the need to spend money" (see Groupon's IPO roadshow presentation, available at http://www.businessinsider.com/groupon-ipo-roadshow-slides-2011-10#welcome-to-the-presentation-1, or news press http://adage.com/article/digital/groupon-marketing-spending-works/230777/).

solve for a full optimization problem of Groupon's dynamic investment allocation for the following reasons. First, the goal of the simulation is to validate the theoretical model predictions and provide suggestive evidence that the proposed strategy can improve firm profitability. Second, we do not observe the budget constraints faced by Groupon over time during our sample period. Without such information, setting up a realistic optimization problem is infeasible. Instead, we re-allocate observed investment in our simulation while keeping the total investment fixed. For instance, the total investment is fixed per market per period in the first and the second simulations; the total investment is fixed per market in the third simulation while we delay the investment in one category. These re-allocation changes are feasible to implement in practice without changing the total amount of available investment.

The simulation proceeds as follows. We first obtain the estimated coefficients on investment (β_u and β_v), the estimated effect sizes (e.g., λ_{nj}), and the fitted values of other control variables (e.g., category and market fixed effects, market structure, and time fixed effects) $\hat{\Omega}_{jmt}^N$ and $\hat{\Omega}_{jmt}^S$ in Equations 7 and 8:

$$
\begin{aligned}
(9) \qquad \ln \hat{N}_{jmt} = {} & \beta_u \ln u_{jmt}^o + \beta_u \ln U_{jmt} + \lambda_{nj} \ln N_{jmt-1} + \theta_{nj} \ln S_{jmt-1} \\
& + \gamma_{nnj} \ln \tilde{N}_{jmt-1} + \gamma_{nsj} \ln \tilde{S}_{jmt-1} + \hat{\Omega}_{jmt}^N
\end{aligned}
$$

$$
\begin{aligned}
(10) \qquad \ln \hat{S}_{jmt} = {} & \beta_v \ln v_{jmt}^o + \beta_v \ln V_{jmt} + \lambda_{sj} \ln S_{jmt-1} + \theta_{sj} \ln N_{jmt-1} \\
& + \gamma_{ssj} \ln \tilde{S}_{jmt-1} + \gamma_{snj} \ln \tilde{N}_{jmt-1} + \hat{\Omega}_{jmt}^S
\end{aligned}
$$

We keep these values fixed throughout the simulation while varying the investments. Given these values, the paths of number of deals and per deal sales $\{N_{jmt}, S_{jmt}\}_t$ can be generated by initializing the system with $\{\ln N_{jm0}, \ln S_{jm0}\}$ and substituting the sequence of investments $\{u_{jmt}, v_{jmt}\}_t$ into Equations 9 and 10. Let superscript o denote the values associated with observed investment. Let $\{N_{jmt}^o, S_{jmt}^o\}$ denote the simulated paths from the observed investment $\{u_{jmt}^o, v_{jmt}^o\}_t$ and let $\{N_{jmt}, S_{jmt}\}_t$ denote the simulated paths from the alternative investment $\{u_{jmt}, v_{jmt}\}_t$. We initialize the simulated path using the observed $\{\ln N_{jm0}^o, \ln S_{jm0}^o\}$ for $t = 0$ and substitute in alternative investment strategies

to obtain alternative paths of $\{N_{jmt}, S_{jmt}\}_t$. We account for category-specific prices P_j and compare the simulated profits $\pi = \sum_t \sum_m \sum_j N_{jmt} * S_{jmt} * P_j - u_{jmt} - v_{jmt}$ under alternative investment strategies to the simulated profits under the observed strategy $\pi^o = \sum_t \sum_m \sum_j N^o_{jmt} * S^o_{jmt} * P_j - u^o_{jmt} - v^o_{jmt}$. Throughout the simulations, we allow the consumer-side investment to be category-specific rather than being the same across categories in the observed investment. Category-specific investment can be achieved by highlighting specific categories rather than promoting all categories in the marketing communications. Also, this illustrates a more general two-sided platform context, in which a multi-category platform can choose whether to engage in category-specific promotions.

The first simulation tests our recommendation regarding which category the platform should invest more in. Our model recommends that the platform should invest more in Beauty and Restaurant and invest less in Fitness. As Groupon seems to have invested the most in Restaurant, we focus on Beauty and Fitness. In particular, we keep the total investment of the platform fixed while re-allocating some of the resources from Fitness to Beauty as follows:

$$u_{jmt} = u^o_{jmt} + k^u, \; v_{jmt} = v^o_{jmt} + k^v, \qquad \text{for } j = \text{Beauty}$$

$$u_{jmt} = u^o_{jmt} - k^u, \; v_{jmt} = v^o_{jmt} - k^v, \qquad \text{for } j = \text{Fitness}$$

where k^u and k^v represent the amount of re-allocated investment on the merchant side and the consumer side, respectively. We let these amounts take the values from 0% to 10% of the average observed investment level within each market and run multiple simulations to show the robustness of our findings.[15] Figure 3(a) plots the profit change $(\frac{\pi - \pi^o}{\pi^o})$ with respect to the amount of re-allocation (k^u and k^v). We find that the profit increases when more resources are allocated from Fitness to Beauty (i.e., k^u and k^v are larger). The profit decreases when the resource allocation is in the opposite direction (i.e., k^u and k^v are smaller). In particular, re-allocating 10% of the average observed

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[15]The average investment level varies across markets. We let k^u and k^v take the values from 0% to 10% of the average observed investment level within each market. For instance, if the average category-side-period level investment on the merchant side is 10 for market m, we let k^v take the values from 0 to 1 for market m.

investment from Fitness to Beauty can lead to up to 15.5% increase in profits for some cities. The results suggest that implementing our model recommendation can improve firm profitability.

The second simulation tests our recommendation regarding which side the platform should invest more in. The observed Groupon investment was lower on the merchant side and higher on the consumer side during our sample period.[16] Our model recommends that the platform should invest more on the merchant side in Entertainment. We keep the total investment in Entertainment fixed while re-allocating some of the resources from the consumer side to the merchant side as follows:[17]

$$u_{jmt} = u_{jmt}^o + k^u, \, v_{jmt} = v_{jmt}^o - k^v, \qquad \text{for } j = \text{Entertainment}$$

As shown in Figure 3(b), re-allocating resources from consumer side to merchant side increases profits while re-allocating in the opposite direction decreases profits. Re-allocating 10% of the average observed investment from consumer side to merchant side in Entertainment can lead to up to 16.5% increase in profits for some cities.

The third simulation tests our recommendation regarding when to invest more, earlier or later. Our model recommends that the platform should invest earlier in Beauty and invest later in Entertainment. We focus on the intertemporal re-allocation of Entertainment as it is more feasible to shift investment later rather than earlier in practice. We keep the total investment in Entertainment fixed while re-allocating some of the resources from the first half of the periods to the second half of the periods as follows:

$$u_{jmt} = u_{jmt}^o - k^u, \, v_{jmt} = v_{jmt}^o - k^v \text{ if } t < T/2, \qquad \text{for } j = \text{Entertainment}$$

$$u_{jmt} = u_{jmt}^o + k^u, \, v_{jmt} = v_{jmt}^o + k^v \text{ if } t > T/2, \qquad \text{for } j = \text{Entertainment}$$

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[16]See Groupon's "Marketing Expense" and "Sales, General and Administrative" in `http://investor.groupon.com/static-files/2951404e-7171-47b7-9492-36cec02b1d50`. "Sales, General and Administrative" is generally lower than "Marketing Expense" while the sales force expenditure is a subset of "Sales, General and Administrative".

[17]The underlying assumption is that X% of the consumer-side investment is equivalent in cost to X% of the merchant-side investment for Groupon.

Figure 3: Simulation Results

(a) Which Category to Invest   (b) Which Side to Invest   (c) When to Invest
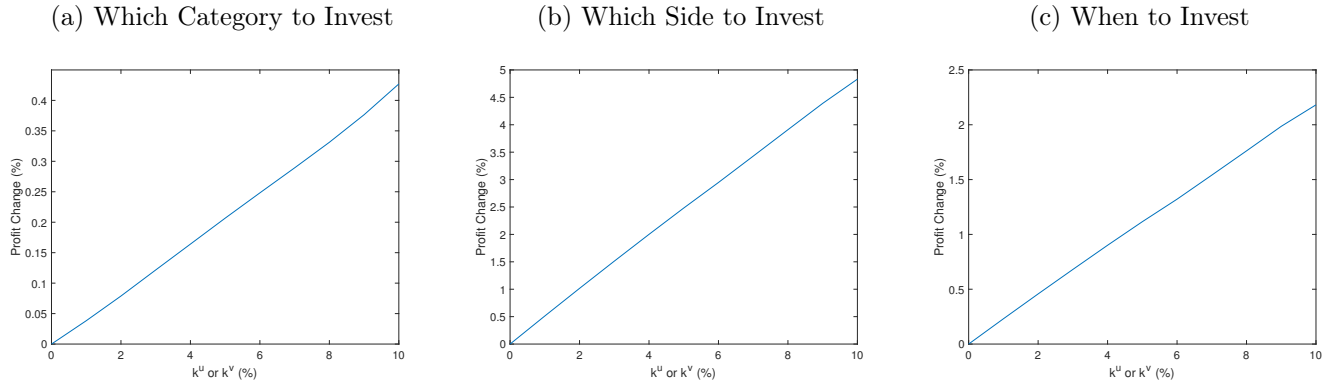
Notes: The percentage in the x-axis indicates the re-allocated amount as a percentage of the average observed investment within each market.

As shown in Figure 3(c), re-allocating resources from earlier periods to later periods increases profits. Re-allocating 10% of the average observed investment from earlier periods to later periods in Entertainment can lead to up to 10.8% increase in profits for some cities.

Overall, the simulation results provide suggestive evidence that merely re-allocating the resources based on our recommendations can improve Groupon's profitability. The proposed combination of within-category allocation rule of "reinforcing" and the cross-category allocation rule of "compensatory" for the per-transaction model benefits the firm by leveraging the heterogeneous own dynamic, cross-side, and cross-category effects across categories.

5   Conclusion

This research focuses on an important challenge that multi-category two-sided platform businesses face: how should scarce resources or limited investment be dynamically allocated across two sides, multiple categories, and different time periods to achieve long-term sustainable growth? We first used a two-category two-period theory model to highlight how the optimal investment strategy crucially depends on which business model, per-transaction versus per-user, the platform adopts. We also illustrate the importance of ac-

counting for the heterogeneity of direct and indirect network effects within each category as well as the inter-dependence across categories in determining the optimal allocation solution. We find that platforms that adopt a per-user model and charge by the number of users should use the same "reinforcing" rule for both within-category and cross-category allocations. Platforms that adopt a per-transaction model and charge by transaction should use the "reinforcing" rule for within-category allocation and the "compensatory" rule for cross-category and cross-period allocations.

We believe that these findings can improve the quality of decisions related to allocating scarce resources in growing platform businesses, which is particularly important for new start-ups. Practitioners frequently use heuristic methods when determining such allocations. For example, in determining marketing spend allocation, the most commonly used rules are "percentage-of-sales," "objective-and-task," and "affordability" methods (Bigné 1995). These rules "usually yield results that are rather far from the optimal profit-maximizing budget" (Fischer et al. 2011). One of the reasons why firms commonly adopt these types of rules is that investment decisions are made separately for different products across different markets. However, for the firms operating two-sided multi-category platforms, not only are the two sides of the platform inter-related through indirect network effects, but also the categories are intrinsically different in their relative dependency on each other. The optimal investment allocation has to take these interactive effects into consideration. Using the daily deals industry data and simulations, we demonstrate how our proposed strategy may outperform the observed investment heuristics in practice.

The insights from this research apply not only to our empirical setting (the daily deals industry) but are also highly relevant for other platform economies that are capital investment-intensive and struggling in their growth. Our research offers a systematic examination of which category, which side, and when to invest in a multi-category platform context. We also highlight the importance of accounting for the business model when choosing which allocation rules to follow.

An important assumption of our study is that, while the resource input can affect the evolution on either side and either category of the platform, it does not change the

within-category and cross-category network effects discussed above, which we treat as inherent category characteristics. Future research can relax this assumption and explore the cases when investment can influence platform dynamics through both direct and indirect mechanisms. In addition, one may consider the scenario under which the category characteristics may evolve over time. In either case, the investment allocation problem will become even more challenging.

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APPENDIX

A. Derivation of the Two-Period Theoretical Model

Let subscript 1 and 2 denote periods 1 and 2. λ_n and λ_s represent own dynamic effects. θ_n and θ_s represent cross-side effects. γ_{nn}, γ_{ns}, γ_{ss}, and γ_{sn} represent cross-category effects. u_t represents merchant-side allocation and v_t represents consumer-side allocation. The allocations affect the consumer and the merchant sides through coefficients β^u and β^v.

Assume the platform starts with no merchants and no consumers, $n_0 = s_0 = \tilde{n}_0 = \tilde{s}_0 = 0$. The marketing response functions are

$$n_1 = \beta_u u_1$$

$$s_1 = \beta_v v_1$$

$$\tilde{n}_1 = \beta_u \tilde{u}_1$$

$$\tilde{s}_1 = \beta_v \tilde{v}_1$$

$$n_2 = \beta_u u_2 + \lambda_n n_1 + \theta_n s_1 + \gamma_{nn} \tilde{n}_1 + \gamma_{ns} \tilde{s}_1$$

$$s_2 = \beta_v v_2 + \lambda_s s_1 + \theta_s n_1 + \gamma_{ss} \tilde{s}_1 + \gamma_{sn} \tilde{n}_1$$

$$\tilde{n}_2 = \beta_u \tilde{u}_2 + \tilde{\lambda}_n \tilde{n}_1 + \tilde{\theta}_n \tilde{s}_1 + \tilde{\gamma}_{nn} n_1 + \tilde{\gamma}_{ns} s_1$$

$$\tilde{s}_2 = \beta_v \tilde{v}_2 + \tilde{\lambda}_s \tilde{s}_1 + \tilde{\theta}_s \tilde{n}_1 + \tilde{\gamma}_{ss} s_1 + \tilde{\gamma}_{sn} n_1$$

Per-Transaction Model   The optimal investment problem is

$$\max \quad (n_1 s_1 - u_1 - v_1) + (\tilde{n}_1 \tilde{s}_1 - \tilde{u}_1 - \tilde{v}_1) + (n_2 s_2 - u_2 - v_2) + (\tilde{n}_2 \tilde{s}_2 - \tilde{u}_2 - \tilde{v}_2)$$

Substituting the conditions into the objective function yields

$$\max \quad \pi \equiv [\beta_u u_1 \beta_v v_1 - u_1 - v_1] + [\beta_u \tilde{u}_1 \beta_v \tilde{v}_1 - \tilde{u}_1 - \tilde{v}_1]$$
$$+ [\beta_u u_2 + \lambda_n \beta_u u_1 + \theta_n \beta_v v_1 + \gamma_{nn} \beta_u \tilde{u}_1 + \gamma_{ns} \beta_v \tilde{v}_1]$$
$$* [\beta_v v_2 + \lambda_s \beta_v v_1 + \theta_s \beta_u u_1 + \gamma_{ss} \beta_v \tilde{v}_1 + \gamma_{sn} \beta_u \tilde{u}_1] - u_2 - v_2$$
$$+ \left[\beta_u \tilde{u}_2 + \tilde{\lambda}_n \beta_u \tilde{u}_1 + \tilde{\theta}_n \beta_v \tilde{v}_1 + \tilde{\gamma}_{nn} \beta_u u_1 + \tilde{\gamma}_{ns} \beta_v v_1\right]$$
$$* \left[\beta_v \tilde{v}_2 + \tilde{\lambda}_s \beta_v \tilde{v}_1 + \tilde{\theta}_s \beta_u \tilde{u}_1 + \tilde{\gamma}_{ss} \beta_v v_1 + \tilde{\gamma}_{sn} \beta_u u_1\right] - \tilde{u}_2 - \tilde{v}_2$$

The new objective function is a function of β^u, β^v, the eight allocations (2 category * 2 periods * 2 sides), and the three types of effects. The first-order conditions with respect

to u_1, v_1, u_2, and v_2 are

$$\frac{\partial \pi}{\partial u_1} = \beta_u \beta_v v_1 - 1$$

$$+ \lambda_n \beta_u \left(\beta_v v_2 + \lambda_s \beta_v v_1 + \theta_s \beta_u u_1 + \gamma_{ss} \beta_v \tilde{v}_1 + \gamma_{sn} \beta_u \tilde{u}_1 \right)$$

$$+ \theta_s \beta_u \left(\beta_u u_2 + \lambda_n \beta_u u_1 + \theta_n \beta_v v_1 + \gamma_{nn} \beta_u \tilde{u}_1 + \gamma_{ns} \beta_v \tilde{v}_1 \right)$$

$$+ \tilde{\gamma}_{nn} \beta_u \left(\beta_v \tilde{v}_2 + \tilde{\lambda}_s \beta_v \tilde{v}_1 + \tilde{\theta}_s \beta_u \tilde{u}_1 + \tilde{\gamma}_{ss} \beta_v v_1 + \tilde{\gamma}_{sn} \beta_u u_1 \right)$$

$$+ \tilde{\gamma}_{sn} \beta_u \left(\beta_u \tilde{u}_2 + \tilde{\lambda}_n \beta_u \tilde{u}_1 + \tilde{\theta}_n \beta_v \tilde{v}_1 + \tilde{\gamma}_{nn} \beta_u u_1 + \tilde{\gamma}_{ns} \beta_v v_1 \right) = 0$$

$$\frac{\partial \pi}{\partial v_1} = \beta_u \beta_v u_1 - 1$$

$$+ \lambda_s \beta_v \left(\beta_u u_2 + \lambda_n \beta_u u_1 + \theta_n \beta_v v_1 + \gamma_{nn} \beta_u \tilde{u}_1 + \gamma_{ns} \beta_v \tilde{v}_1 \right)$$

$$+ \theta_n \beta_v \left(\beta_v v_2 + \lambda_s \beta_v v_1 + \theta_s \beta_u u_1 + \gamma_{ss} \beta_v \tilde{v}_1 + \gamma_{sn} \beta_u \tilde{u}_1 \right)$$

$$+ \tilde{\gamma}_{ss} \beta_v \left(\beta_u \tilde{u}_2 + \tilde{\lambda}_n \beta_u \tilde{u}_1 + \tilde{\theta}_n \beta_v \tilde{v}_1 + \tilde{\gamma}_{nn} \beta_u u_1 + \tilde{\gamma}_{ns} \beta_v v_1 \right)$$

$$+ \tilde{\gamma}_{ns} \beta_v \left(\beta_v \tilde{v}_2 + \tilde{\lambda}_s \beta_v \tilde{v}_1 + \tilde{\theta}_s \beta_u \tilde{u}_1 + \tilde{\gamma}_{ss} \beta_v v_1 + \tilde{\gamma}_{sn} \beta_u u_1 \right) = 0$$

$$\frac{\partial \pi}{\partial u_2} = \beta_u \left(\beta_v v_2 + \lambda_s \beta_v v_1 + \theta_s \beta_u u_1 + \gamma_{ss} \beta_v \tilde{v}_1 + \gamma_{sn} \beta_u \tilde{u}_1 \right) - 1 = 0$$

$$\frac{\partial \pi}{\partial v_2} = \beta_v \left(\beta_u u_2 + \lambda_n \beta_u u_1 + \theta_n \beta_v v_1 + \gamma_{nn} \beta_u \tilde{u}_1 + \gamma_{ns} \beta_v \tilde{v}_1 \right) - 1 = 0$$

The first-order conditions with respect to the investment in the other category, $\tilde{u}_1, \tilde{v}_1, \tilde{u}_2, \tilde{v}_2$, can be similarly derived. The eight first-order conditions with respect to the eight investments $(u_1, v_1, u_2, v_2, \tilde{u}_1, \tilde{v}_1, \tilde{u}_2, \tilde{v}_2)$ form a system of eight equations with eight unknowns. Solving the system of equations yields the optimal investment solutions as in Equations 3 and 4.

Per-User Model   The optimal investment problem is

$$\max \quad \left(\sqrt{n_1} + \sqrt{s_1} - u_1 - v_1 \right) + \left(\sqrt{\tilde{n}_1} + \sqrt{\tilde{s}_1} - \tilde{u}_1 - \tilde{v}_1 \right)$$

$$+ \left(\sqrt{n_2} + \sqrt{s_2} - u_2 - v_2 \right) + \left(\sqrt{\tilde{n}_2} + \sqrt{\tilde{s}_2} - \tilde{u}_2 - \tilde{v}_2 \right)$$

Substituting the conditions into the objective function yields

$$\max \quad \pi \equiv \sqrt{\beta_u u_1} + \sqrt{\beta_v v_1} - u_1 - v_1 + \sqrt{\beta_u \tilde{u}_1} + \sqrt{\beta_v \tilde{v}_1} - \tilde{u}_1 - \tilde{v}_1$$

$$+ \sqrt{\beta_u u_2 + \lambda_n \beta_u u_1 + \theta_n \beta_v v_1 + \gamma_{nn} \beta_u \tilde{u}_1 + \gamma_{ns} \beta_v \tilde{v}_1}$$

$$+ \sqrt{\beta_v v_2 + \lambda_s \beta_v v_1 + \theta_s \beta_u u_1 + \gamma_{ss} \beta_v \tilde{v}_1 + \gamma_{sn} \beta_u \tilde{u}_1} - u_2 - v_2$$

$$+ \sqrt{\beta_u \tilde{u}_2 + \tilde{\lambda}_n \beta_u \tilde{u}_1 + \tilde{\theta}_n \beta_v \tilde{v}_1 + \tilde{\gamma}_{nn} \beta_u u_1 + \tilde{\gamma}_{ns} \beta_v v_1}$$

$$+ \sqrt{\beta_v \tilde{v}_2 + \tilde{\lambda}_s \beta_v \tilde{v}_1 + \tilde{\theta}_s \beta_u \tilde{u}_1 + \tilde{\gamma}_{ss} \beta_v v_1 + \tilde{\gamma}_{sn} \beta_u u_1} - \tilde{u}_2 - \tilde{v}_2$$

The new objective function is a function of β^u, β^v, the eight allocations (2 category * 2 periods * 2 sides), and the three types of effects. The first-order conditions with respect to u_1, v_1, u_2, and v_2 are

$$\frac{\partial \pi}{\partial u_1} = \frac{\beta_u}{2\sqrt{\beta_u u_1}} - 1 + \frac{\lambda_n \beta_u}{2\sqrt{n_2}} + \frac{\theta_s \beta_u}{2\sqrt{s_2}} + \frac{\tilde{\gamma}_{nn} \beta_u}{2\sqrt{\tilde{n}_2}} + \frac{\tilde{\gamma}_{sn} \beta_u}{2\sqrt{\tilde{s}_2}} = 0$$

$$\frac{\partial \pi}{\partial v_1} = \frac{\beta_v}{2\sqrt{\beta_v v_1}} - 1 + \frac{\lambda_s \beta_v}{2\sqrt{s_2}} + \frac{\theta_n \beta_v}{2\sqrt{n_2}} + \frac{\tilde{\gamma}_{ss} \beta_v}{2\sqrt{\tilde{s}_2}} + \frac{\tilde{\gamma}_{ns} \beta_v}{2\sqrt{\tilde{n}_2}} = 0$$

$$\frac{\partial \pi}{\partial u_2} = \frac{\beta_u}{2\sqrt{n_2}} - 1 = \frac{\beta_u}{2\sqrt{\beta_u u_2 + \lambda_n \beta_u u_1 + \theta_n \beta_v v_1 + \gamma_{nn} \beta_u \tilde{u}_1 + \gamma_{ns} \beta_v \tilde{v}_1}} - 1 = 0$$

$$\frac{\partial \pi}{\partial v_2} = \frac{\beta_v}{2\sqrt{s_2}} - 1 = \frac{\beta_v}{2\sqrt{\beta_v v_2 + \lambda_s \beta_v v_1 + \theta_s \beta_u u_1 + \gamma_{ss} \beta_v \tilde{v}_1 + \gamma_{sn} \beta_u \tilde{u}_1}} - 1 = 0$$

The first-order conditions with respect to the investment in the other category, $\tilde{u}_1, \tilde{v}_1, \tilde{u}_2, \tilde{v}_2$, can be similarly derived. The eight first-order conditions with respect to the eight investments $(u_1, v_1, u_2, v_2, \tilde{u}_1, \tilde{v}_1, \tilde{u}_2, \tilde{v}_2)$ form a system of eight equations with eight unknowns. Solving the system of equations yields the optimal investment solutions in Equation 5.

B. Multi- versus Single-Category: Ignoring Cross-Category Effects

We examine how multi-category platforms differ from single-category platforms. Extant literature on two-side platforms have documented the optimal investment strategy for single-category platforms: platforms should charge a higher price or offer less subsidy (invest less in our setting) on the side that depends more on the presence of the other

side (e.g., Armstrong 2006, Wely 2010, and Hagiu 2014). The findings illustrates how own dynamic and cross-side effects influence optimal investment. We show that multi-category platforms should further account for the presence of cross-category effects (i.e., γ in our model); failing to account for the interdependency across categories would lead to misallocation of resources.

To show the uniqueness of multi-category setting relatively to the traditional single-category setting, we compare the optimal investments with and without accounting for cross-category effects in this context. We use the per-transaction model as an example. Let subscript 0 denote the values without cross-category effects. When there is no cross-category effects (i.e., $\gamma_{nn} = \gamma_{ns} = \gamma_{ss} = \gamma_{sn} = 0$, $\tilde{\gamma}_{nn} = \tilde{\gamma}_{ns} = \tilde{\gamma}_{ss} = \tilde{\gamma}_{sn} = 0$), the optimal investment solutions in period 1 and 2 are

$$u_{10}^* = \frac{1}{\beta_u \beta_v} \left(1 - \lambda_s - \theta_n \frac{\beta_v}{\beta_u} \right), \quad v_{10}^* = \frac{1}{\beta_u \beta_v} \left(1 - \lambda_n - \theta_s \frac{\beta_u}{\beta_v} \right)$$

$$u_{20}^* = \frac{1}{\beta_u \beta_v} [1 - \lambda_n \left(1 - \lambda_s - \theta_n \frac{\beta_v}{\beta_u} \right) - \theta_n \frac{\beta_v}{\beta_u} \left(1 - \lambda_n - \theta_s \frac{\beta_u}{\beta_v} \right)]$$

$$v_{20}^* = \frac{1}{\beta_u \beta_v} [1 - \lambda_s \left(1 - \lambda_n - \theta_s \frac{\beta_u}{\beta_v} \right) - \theta_s \frac{\beta_u}{\beta_v} \left(1 - \lambda_s - \theta_n \frac{\beta_v}{\beta_u} \right)]$$

These investment solutions are the platform's investment strategies when following the traditional single-category platform investment rule and accounting for only own dynamic and cross-side effects. Comparing the optimal investment in period 1 in this case with the proposed optimal multi-category platform investment rule yields

$$\Delta u_1^* \equiv u_{10}^* - u_1^* = \frac{1}{\beta_u \beta_v} \left(\tilde{\gamma}_{ss} + \tilde{\gamma}_{ns} \frac{\beta_v}{\beta_u} \right)$$

$$\Delta v_1^* \equiv v_{10}^* - v_1^* = \frac{1}{\beta_u \beta_v} \left(\tilde{\gamma}_{nn} + \tilde{\gamma}_{sn} \frac{\beta_u}{\beta_v} \right)$$

We find that ignoring the cross-category effects will result in erroneous investment allocation in period 1. If the cross-category effects γ are positive, ignoring the cross-category effects will result in **over-spending** in period 1. This deviation from the optimal allocation grows larger when the cross-category effects are stronger. Intuitively, positive cross-category effects indicate that one category can grow by leveraging the growth of

other categories. In other words, multi-category platforms are "stronger" than single-category platforms in terms of inherent capability for endogenous growth. Therefore, lower resource allocation would suffice in early periods. Ignoring the cross-category effects would lead to erroneous resource allocation. The stronger the cross-category effects, the greater the over-spending.

Comparing the optimal investment in period 2 with and without cross-category effects yields

$$
\Delta u_2^* \equiv u_{20}^* - u_2^* = \overbrace{\gamma_{nn}\tilde{u}_1^* + \gamma_{ns}\frac{\beta_v}{\beta_u}\tilde{v}_1^*}^{\text{over-spend due to other category}} \overbrace{-\lambda_n\Delta u_1^* - \theta_n\frac{\beta_v}{\beta_u}\Delta v_1^*}^{\text{under-spend due to period 1 bias}}
$$

$$
\Delta v_2^* \equiv v_{20}^* - v_2^* = \underbrace{\gamma_{ss}\tilde{v}_1^* + \gamma_{sn}\frac{\beta_u}{\beta_v}\tilde{u}_1^*}_{\text{over-spend due to other category}} \underbrace{-\lambda_s\Delta v_1^* - \theta_s\frac{\beta_u}{\beta_v}\Delta u_1^*}_{\text{under-spend due to period 1 bias}}
$$

We find that ignoring the cross-category effects will result in resource misallocation in period 2. If all effects are positive, ignoring the cross-category effects will result in **over-spending** (**under-spending**) in period 2 if the cross-category effects γ (the own dynamic effects λ and cross-side effects θ) are relatively stronger; the deviation increases with the sizes of these effects. Intuitively, ignoring the cross-category effects leads to one of two kinds of allocation errors in period 2. On the one hand, similar to period 1, the platform does not realize that the categories are "stronger" in terms of growing by relying on each other, which leads to over-spending again in period 2. This deviation from the optimal investment grows larger when the cross-category effects are stronger. On the other hand, there is under-spending due to the over-spending in period 1. As the platform overspends in period 1, it can leverage the larger sizes of both sides in period 1 given the own dynamic effects and cross-side effects. Therefore, it can spend less in period 2. This deviation is larger when the own dynamic and cross-side effects are stronger.

C. Impute Category and Market Level Investment

Merchant-side Investment We empirically estimate how investment changes with the number of merchants (deals) and the type of merchants. Specifically, we let the investment in a category-market-week u_{jmt} be a function of the linear and quadratic terms of the number of merchants N_{jmt} as well as the fraction of new merchants f_{jmt}^{new}:

$$(11) \qquad\qquad u_{jmt} = \beta_0 + \beta_1 N_{jmt} + \beta_2 N_{jmt}^2 + \beta_3 f_{jmt}^{new}$$

This specification can flexibly capture learning/economy of scale (through the quadratic term of the number of merchants) and potentially lower cost of acquiring returning merchants (through the fraction of new merchants). It also allows the data to determine the relationship rather than making assumptions by ourselves. We empirically estimate the coefficients $\beta_0, \beta_1, \beta_2, \beta_3$ by matching the sum of the monthly investment across categories and markets to the observed total monthly investment u_T^o:

$$(12) \qquad u_T^o = \Sigma_{j,m} u_{jmT} = \Sigma_{jm}(\beta_0 + \beta_1 N_{jmT} + \beta_2 N_{jmT}^2 + \beta_3 f_{jmT}^{new})$$

Here T represents month T. We construct the conditions above at the monthly level because sales force compensation is paid at the monthly level in practice. Overall, there are 23 months and 23 conditions/equations. The coefficients $\beta_0, \beta_1, \beta_2, \beta_3$ are identified from how total investment changes with the number and the type (new versus returning) of deals across markets over time.

The estimated coefficients have expected signs. The coefficient on the linear number of deals β_1 is 0.171 and the coefficient on the quadratic number of deals β_2 is -0.000353, suggesting that there is learning and economy of scale in acquiring merchants. The coefficient on the fraction of new merchant is 35.06, suggesting that new merchant acquisition is more costly. Given the estimated coefficients, we can impute the category-market-week level investment in two ways. The first approach uses Equation 11 ($u_{jmt}^o = \beta_0 + \beta_1 N_{jmt} + \beta_2 N_{jmt}^2 + \beta_3 f_{jmt}^{new}$) to obtain the category-market-week level

investment. The second approach assumes that weeks in the same month have the same investment and imputes the category-market-week level investment using $u^o_{jmt} = u_{jmT} = \beta_0 + \beta_1 N_{jmT} + \beta_2 N^2_{jmT} + \beta_3 f^{new}_{jmT}$, where t represents the weeks in month T. We use both approaches to obtain the imputed investments and run the main regression. We find that the results are very robust to the choice of the approach. We use the second approach in the main text because it produces a larger R-squared of the main regression.

We would like to note that the variables included $(N_{jmt}, N^2_{jmt}, f^{new}_{jmt})$ have a large explanatory power. A parsimonious way to assess the explanatory power of these variables is, instead of estimating the system of Equation 12, regressing the total monthly investment (u^o_T) on the linear and quadratic total monthly level number of merchants (N_T, N^2_T) and the fraction of new merchants (f^{new}_T) and checking the explanatory power. We obtain an R-squared of 0.90, suggesting that including these three variables can well explain the variation in investment over time. Therefore, we do not include additional variables (e.g., category fixed effects or market condition variables) because the explanatory power is already large enough and 23 observations/conditions may not identify more parameters.

Consumer-side Investment Let B_{mt} denote the number of website visits in a market-period. We assume that investment in a particular market-period is proportional to the observed website visits in that market-period. We make this assumption because the marketing expense per click is relatively constant in practice and that the total marketing expense is proportional to the total number of clicks. Given the assumption, we can impute the market-week level investment in two ways. The first approach obtains the market-week level investment from the total monthly level investment using $v^o_{mt} = \frac{B_{mt}}{\Sigma_{t \in T} \Sigma_m B_{mt}} v^o_T$, where $t \in T$ denote the weeks within month T and v^o_T denote the observed total monthly investment. The second approach assumes that weeks in the same month have the same investment and obtains the market-week level investment using $v^o_{mt} = v^o_{mT} = \frac{B_{mT}}{\Sigma_m B_{mT}} v^o_T$. We use both approaches to impute market-week level investments and run the main regression. The results are very robust to the choice of the approach. We use the second approach because it produces a larger R-squared of the main regression.

Online Appendix

A. Robustness: Theoretical Model without Taking Logarithm

As discussed in Footnote 1, the linear additive functional form of the marketing response functions (e.g., $n_t = \beta_u u_t + \lambda_n n_{t-1} + \theta_n s_{t-1} + \gamma_{nn} \tilde{n}_{t-1} + \gamma_{ns} \tilde{s}_{t-1}$) can be derived by taking log of the dynamic marketing response functions (e.g., $N_t = U_t^{\beta_u} N_{t-1}^{\lambda_n} S_{t-1}^{\theta_n} \tilde{N}_{t-1}^{\gamma_{nn}} \tilde{S}_{t-1}^{\gamma_{ns}}$) in Sridhar et al. (2011). In this section, we use numerical solutions to show that the results of the per-transaction and per-user models continue to hold without taking log of the variables.

1) Per-Transaction Model   The objective function, without taking log of the variables, is $N_t S_t - U_t - V_t = e^{n_t} e^{s_t} - e^{u_t} - e^{v_t}$. The optimal investment problem becomes

$$\max \quad \left(e^{n_1} e^{s_1} - e^{u_1} - e^{v_1}\right) + \left(e^{\tilde{n}_1} e^{\tilde{s}_1} - e^{\tilde{u}_1} - e^{\tilde{v}_1}\right) + \left(e^{n_2} e^{s_2} - e^{u_2} - e^{v_2}\right) + \left(e^{\tilde{n}_2} e^{\tilde{s}_2} - e^{\tilde{u}_2} - e^{\tilde{v}_2}\right)$$

The marketing response functions are Equations 1 and 2. The first-order conditions with respect to the focal category investments u_1, v_1, u_2, and v_2 are

$$\frac{\partial \pi}{\partial u_1} = \beta_u e^{\beta_u u_1 + \beta_v v_1} - e^{u_1} + e^{n_2} e^{s_2} \beta_u \left(\lambda_n + \theta_s\right) + e^{\tilde{n}_2} e^{\tilde{s}_2} \beta_u \left(\tilde{\gamma}_{nn} + \tilde{\gamma}_{sn}\right) = 0$$

$$\frac{\partial \pi}{\partial v_1} = \beta_v e^{\beta_u u_1 + \beta_v v_1} - e^{v_1} + e^{n_2} e^{s_2} \beta_v \left(\lambda_s + \theta_n\right) + e^{\tilde{n}_2} e^{\tilde{s}_2} \beta_v \left(\tilde{\gamma}_{ss} + \tilde{\gamma}_{ns}\right) = 0$$

$$\frac{\partial \pi}{\partial u_2} = \beta_u e^{n_2} e^{s_2} - e^{u_2} = 0$$

$$\frac{\partial \pi}{\partial v_2} = \beta_v e^{n_2} e^{s_2} - e^{v_2} = 0$$

Similarly for $\tilde{u}_1, \tilde{v}_1, \tilde{u}_2, \tilde{v}_2$. As the system is highly non-linear, it is infeasible to obtain closed-form solutions. We numerically solve for the optimal investments, which are high-dimensional functions of the effect parameters. To show how the optimal investment changes with the effects (i.e., the propositions) in a trackable way, we first parametrize the model using values similar to the estimated ones in the empirical model.[18] We then vary the effect parameters one at a time and plot how the optimal investment changes with that particular effect in Figure 4a. For instance, to test Proposition 1 regarding which side to invest, we vary λ_n while keep the rest of the parameters fixed. We then plot how $I = (u_1^* + u_2^*) - (v_1^* + v_2^*)$ changes on the y-axis with respect to λ_n on the x-axis in the first plot of Figure 4a. We find that the results are consistent with the original propositions of the per-transaction model, suggesting that the original propositions continue to hold when we do not take log of the variables. Note that this exercise can also show the

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[18]Specifically, we let the focal category parameters take the values of $\lambda_n = \tilde{\lambda}_n = 0.35, \theta_n = \tilde{\theta}_n = -0.2, \lambda_s = \tilde{\lambda}_s = 0.35, \theta_s = \tilde{\theta}_s = -0.2, \gamma_{nn} = \tilde{\gamma}_{nn} = -0.1, \gamma_{ns} = \tilde{\gamma}_{ns} = 0.1, \gamma_{ss} = \tilde{\gamma}_{ss} = -0.1, \gamma_{sn} = \tilde{\gamma}_{sn} = -0.1$. The results are robust if we let the two categories to be asymmetric in the effect sizes and if we adopt alternative parametrization.

Figure 4: Alternative Functional Form: without Taking ln

(a) Per-Transaction Model
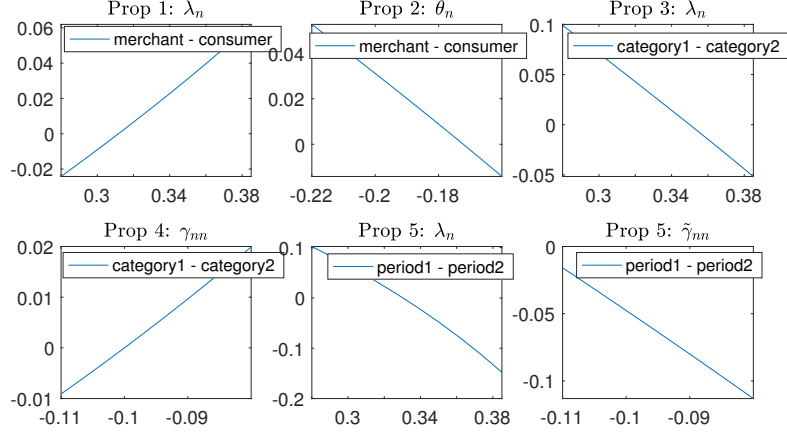
(b) Per-User Model
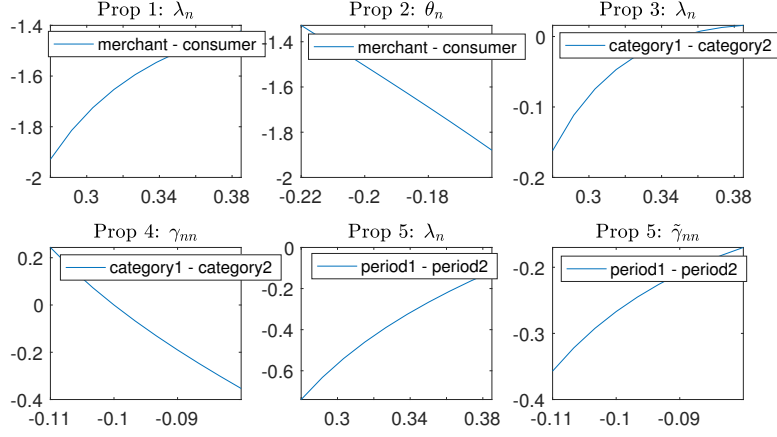
Notes: We plot the optimal investment I on the y-axis and the effect parameter on the x-axis. For Propositions 1 and 2 regarding which side to invest, $I = (u_1^* + u_2^*) - (v_1^* + v_2^*)$. For Propositions 3 and 4 regarding which category to invest, $I = (v_1^* + v_2^* + u_1^* + u_2^*) - (\tilde{v}_1^* + \tilde{v}_2^* + \tilde{u}_1^* + \tilde{u}_2^*)$. For Proposition 5 regarding when to invest, $I = (v_1^* + v_2^* + u_1^* + u_2^*) - (\tilde{v}_1^* + \tilde{v}_2^* + \tilde{u}_1^* + \tilde{u}_2^*)$.

robustness of our findings to alternative parametrization of the model.

2) Per-User Model   The objective function, without taking log of the variables, is the following:

$$\max \quad \sum_{t=1,2} \left(\sqrt{e^{n_t}} + \sqrt{e^{s_t}} - e^{u_t} - e^{v_t} \right) + \left(\sqrt{e^{\tilde{n}_t}} + \sqrt{e^{\tilde{s}_t}} - e^{\tilde{u}_t} - e^{\tilde{v}_t} \right)$$

The marketing response functions are Equations 1 and 2. The first-order conditions

with respect to the focal category investments u_1, v_1, u_2, and v_2 are

$$\frac{\partial \pi}{\partial u_1} = \frac{\beta_u}{2}\sqrt{e^{\beta_u u_1}} - e^{u_1} + \frac{\lambda_n \beta_u}{2}\sqrt{e^{n_2}} + \frac{\theta_s \beta_u}{2}\sqrt{e^{s_2}} + \frac{\tilde{\gamma}_{nn}\beta_u}{2}\sqrt{e^{\tilde{n}_2}} + \frac{\tilde{\gamma}_{sn}\beta_u}{2}\sqrt{e^{\tilde{s}_2}} = 0$$

$$\frac{\partial \pi}{\partial v_1} = \frac{\beta_v}{2}\sqrt{e^{\beta_v v_1}} - e^{v_1} + \frac{\lambda_s \beta_v}{2}\sqrt{e^{s_2}} + \frac{\theta_n \beta_v}{2}\sqrt{e^{n_2}} + \frac{\tilde{\gamma}_{ss}\beta_v}{2}\sqrt{e^{\tilde{s}_2}} + \frac{\tilde{\gamma}_{ns}\beta_v}{2}\sqrt{e^{\tilde{n}_2}} = 0$$

$$\frac{\partial \pi}{\partial u_2} = \frac{\beta_u}{2}\sqrt{e^{n_2}} - e^{u_2} = 0$$

$$\frac{\partial \pi}{\partial v_2} = \frac{\beta_v}{2}\sqrt{e^{s_2}} - e^{v_2} = 0$$

Similar to the per-transaction model, we numerically solve for the optimal investments using the same parametrization and plot how the optimal investment changes with each effect in Figure 4b. We find that the results are consistent with the original propositions of the per-user model. Again, it suggests that the original propositions continue to hold when we do not take log of the variables.

B. Derivation of Model Extensions: Category Asymmetry

Per-Transaction Model   Suppose the category-specific fees are $\{a, 1\}$ for categories 1 and 2. The optimal investment problem is

$$\max \quad (a n_1 s_1 - u_1 - v_1) + (\tilde{n}_1 \tilde{s}_1 - \tilde{u}_1 - \tilde{v}_1) + (a n_2 s_2 - u_2 - v_2) + (\tilde{n}_2 \tilde{s}_2 - \tilde{u}_2 - \tilde{v}_2)$$

The optimal investment solutions are

$$u_1^* = \frac{1}{a\beta_u\beta_v}\left(1 - \lambda_s - \theta_n \frac{\beta_v}{\beta_u} - \tilde{\gamma}_{ss} - \tilde{\gamma}_{ns}\frac{\beta_v}{\beta_u}\right)$$

$$v_1^* = \frac{1}{a\beta_u\beta_v}\left(1 - \lambda_n - \theta_s \frac{\beta_u}{\beta_v} - \tilde{\gamma}_{nn} - \tilde{\gamma}_{sn}\frac{\beta_u}{\beta_v}\right)$$

$$u_2^* = \frac{1}{a\beta_u\beta_v} - \left(\lambda_n u_1^* + \theta_n \frac{\beta_v}{\beta_u}v_1^* + \gamma_{nn}\tilde{u}_1^* + \gamma_{ns}\frac{\beta_v}{\beta_u}\tilde{v}_1^*\right)$$

$$v_2^* = \frac{1}{a\beta_u\beta_v} - \left(\lambda_s v_1^* + \theta_s \frac{\beta_u}{\beta_v}u_1^* + \gamma_{ss}\tilde{v}_1^* + \gamma_{sn}\frac{\beta_u}{\beta_v}\tilde{u}_1^*\right)$$

Similarly for $\tilde{u}_1, \tilde{v}_1, \tilde{u}_2, \tilde{v}_2$. We examine how changes in a affect the optimal investment (e.g., $\frac{\partial u_1^*}{\partial a}$) and whether it amplifies or dampens the propositions (e.g., $\frac{\partial^2 (u_1^* + u_2^*) - (v_1^* + v_2^*)}{\partial \lambda_n \partial a}$).

First, when a is larger, the investments in the focal category $\{u_1^*, v_1^*, u_2^*, v_2^*\}$ decrease

Table 5: Extension: Asymmetric Category-Specific Prices

(a) Per-Transaction Model

		Focal Parameter	Original Proposition	Effect of a	
Which side	Prop 1	λ_n	$\frac{\partial I}{\partial \lambda_n} > 0$	$\frac{\partial^2 I}{\partial \lambda_n \partial a} < 0$	dampen
$I = (u_1^* + u_2^*) - (v_1^* + v_2^*)$	Prop 2	θ_n	$\frac{\partial I}{\partial \theta_n} < 0$	$\frac{\partial^2 I}{\partial \theta_n \partial a} > 0$	dampen
Which category	Prop 3	λ_n	$\frac{\partial I}{\partial \lambda_n} < 0$	$\frac{\partial^2 I}{\partial \lambda_n \partial a} > 0$	dampen
$I = (v_1^* + v_2^* + u_1^* + u_2^*)$		θ_n	$\frac{\partial I}{\partial \theta_n} < 0$	$\frac{\partial^2 I}{\partial \theta_n \partial a} > 0$	dampen
$- (\tilde{v}_1^* + \tilde{v}_2^* + \tilde{u}_1^* + \tilde{u}_2^*)$	Prop 4	γ_{nn}	$\frac{\partial I}{\partial \gamma_{nn}} > 0$	$\frac{\partial^2 I}{\partial \gamma_{nn} \partial a} = 0$	no change
		γ_{ns}	$\frac{\partial I}{\partial \gamma_{ns}} > 0$	$\frac{\partial^2 I}{\partial \gamma_{ns} \partial a} = 0$	no change
When	Prop 5	λ_n	$\frac{\partial I}{\partial \lambda_n} < 0$	$\frac{\partial^2 I}{\partial \lambda_n \partial a} > 0$	dampen
$I = (v_1^* + u_1^*) - (v_2^* + u_2^*)$		θ_n	$\frac{\partial I}{\partial \theta_n} < 0$	$\frac{\partial^2 I}{\partial \theta_n \partial a} > 0$	dampen
		$\tilde{\gamma}_{nn}$	$\frac{\partial I}{\partial \tilde{\gamma}_{nn}} < 0$	$\frac{\partial^2 I}{\partial \tilde{\gamma}_{nn} \partial a} > 0$	dampen
		$\tilde{\gamma}_{ns}$	$\frac{\partial I}{\partial \tilde{\gamma}_{ns}} < 0$	$\frac{\partial^2 I}{\partial \tilde{\gamma}_{ns} \partial a} > 0$	dampen

(b) Per-User Model

		Focal Parameter	Original Proposition	Effect of a	
Which side	Prop 1	λ_n	$\frac{\partial I}{\partial \lambda_n} > 0$	$\frac{\partial^2 I}{\partial \lambda_n \partial a} > 0$	amplify
$I = (u_1^* + u_2^*) - (v_1^* + v_2^*)$	Prop 2	θ_n	$\frac{\partial I}{\partial \theta_n} < 0$	$\frac{\partial^2 I}{\partial \theta_n \partial a} < 0$	amplify
Which category	Prop 3	λ_n	$\frac{\partial I}{\partial \lambda_n} > 0$	$\frac{\partial^2 I}{\partial \lambda_n \partial a} > 0$	amplify
$I = (v_1^* + v_2^* + u_1^* + u_2^*)$		θ_n	$\frac{\partial I}{\partial \theta_n} > 0$	$\frac{\partial^2 I}{\partial \theta_n \partial a} > 0$	amplify
$- (\tilde{v}_1^* + \tilde{v}_2^* + \tilde{u}_1^* + \tilde{u}_2^*)$	Prop 4	γ_{nn}	$\frac{\partial I}{\partial \gamma_{nn}} < 0$	$\frac{\partial^2 I}{\partial \gamma_{nn} \partial a} = 0$	no change
		γ_{ns}	$\frac{\partial I}{\partial \gamma_{ns}} < 0$	$\frac{\partial^2 I}{\partial \gamma_{ns} \partial a} = 0$	no change
When	Prop 5	λ_n	$\frac{\partial I}{\partial \lambda_n} > 0$	$\frac{\partial^2 I}{\partial \lambda_n \partial a} > 0$	amplify
$I = (v_1^* + u_1^*) - (v_2^* + u_2^*)$		θ_n	$\frac{\partial I}{\partial \theta_n} > 0$	$\frac{\partial^2 I}{\partial \theta_n \partial a} > 0$	amplify
		$\tilde{\gamma}_{nn}$	$\frac{\partial I}{\partial \tilde{\gamma}_{nn}} > 0$	$\frac{\partial^2 I}{\partial \tilde{\gamma}_{nn} \partial a} > 0$	amplify
		$\tilde{\gamma}_{ns}$	$\frac{\partial I}{\partial \tilde{\gamma}_{ns}} > 0$	$\frac{\partial^2 I}{\partial \tilde{\gamma}_{ns} \partial a} > 0$	amplify

$(\frac{\partial u_1^*}{\partial a} < 0,\ \frac{\partial v_1^*}{\partial a} < 0,\ \frac{\partial u_2^*}{\partial a} < 0,\ \frac{\partial v_2^*}{\partial a} < 0)$ while the investments in the other category $\{\tilde{u}_1^*, \tilde{v}_1^*\}$ remain the same for period 1 and $\{\tilde{u}_2^*, \tilde{v}_2^*\}$ increase for period 2, so a higher fee of a category decreases the relative investments to that category. This is consistent with the "compensatory" rule in terms of which category to invest for the per-transaction model.

Second, the propositions still qualitatively hold, while the magnitudes of the effects change with a. In particular, as shown in Table 5a, the column of "original proposition" has the same signs as before, indicating that introducing a produces the same qualitative results. The column of "effect of a" suggests that a higher fee of the focal category a will dampen an effect when the driver of the effect is the focal category (e.g., λ_n, θ_n, $\tilde{\gamma}_{nn}$, $\tilde{\gamma}_{ns}$); a higher fee of the focal category a will have no effect when the driver of the effect is the other category (e.g., γ_{nn}, γ_{ns}).

Per-User Model   Suppose the category-specific fees are $\{a, 1\}$ for categories 1 and 2. The optimal investment problem is

$$
\begin{aligned}
\max \quad & (a\sqrt{n_1} + a\sqrt{s_1} - u_1 - v_1) + \left(\sqrt{\tilde{n}_1} + \sqrt{\tilde{s}_1} - \tilde{u}_1 - \tilde{v}_1\right) \\
& + (a\sqrt{n_2} + a\sqrt{s_2} - u_2 - v_2) + \left(\sqrt{\tilde{n}_2} + \sqrt{\tilde{s}_2} - \tilde{u}_2 - \tilde{v}_2\right)
\end{aligned}
$$

The optimal solutions are

$$
\begin{aligned}
u_1^* &= \frac{a^2 \beta_u}{4\left(1 - \lambda_n - \theta_s \frac{\beta_u}{\beta_v} - \tilde{\gamma}_{nn} - \tilde{\gamma}_{sn}\frac{\beta_u}{\beta_v}\right)^2} \\
v_1^* &= \frac{a^2 \beta_v}{4\left(1 - \lambda_s - \theta_n \frac{\beta_v}{\beta_u} - \tilde{\gamma}_{ss} - \tilde{\gamma}_{ns}\frac{\beta_v}{\beta_u}\right)^2} \\
u_2^* &= \frac{a^2 \beta_u}{4} - \left(\lambda_n u_1^* + \theta_n \frac{\beta_v}{\beta_u} v_1^* + \gamma_{nn}\tilde{u}_1^* + \gamma_{ns}\frac{\beta_v}{\beta_u}\tilde{v}_1^*\right) \\
v_2^* &= \frac{a^2 \beta_v}{4} - \left(\lambda_s v_1^* + \theta_s \frac{\beta_u}{\beta_v} u_1^* + \gamma_{ss}\tilde{v}_1^* + \gamma_{sn}\frac{\beta_u}{\beta_v}\tilde{u}_1^*\right)
\end{aligned}
$$

Similarly for $\tilde{u}_1, \tilde{v}_1, \tilde{u}_2, \tilde{v}_2$. We examine how changes in a affect the optimal investment (e.g., $\frac{\partial u_1^*}{\partial a}$) and whether it amplifies or dampens the propositions (e.g., $\frac{\partial^2 \left(u_1^* + u_2^*\right) - \left(v_1^* + v_2^*\right)}{\partial \lambda_n \partial a}$).

First, when a is larger, the investments in the focal category $\{u_1^*, v_1^*, u_2^*, v_2^*\}$ increase ($\frac{\partial u_1^*}{\partial a} > 0$, $\frac{\partial v_1^*}{\partial a} > 0$, $\frac{\partial u_2^*}{\partial a} > 0$, $\frac{\partial v_2^*}{\partial a} > 0$) while the investments in the other category $\{\tilde{u}_1^*, \tilde{v}_1^*\}$ remain the same for period 1 and $\{\tilde{u}_2^*, \tilde{v}_2^*\}$ decrease for period 2, so a higher fee of a category increases the relative investment to that category. This is consistent with the "reinforcing" rule in terms of which category to invest for the per-user model.

Second, the propositions still qualitatively hold, while the magnitudes of the effects change with a. In particular, as shown in Table 5b, the column of "original proposition" has the same signs as before, indicating that introducing a produces the same qualitative results. The column of "effect of a" suggests that a higher fee of the focal category a will amplify an effect when the driver of the effect is the focal category (e.g., λ_n, θ_n, $\tilde{\gamma}_{nn}$, $\tilde{\gamma}_{ns}$); a higher price of the focal category a has no effect when the driver of the effect is the other category (e.g., γ_{nn}, γ_{ns}).

C. Derivation of Model Extensions: Side Asymmetry

We examine side asymmetry for the per-user model as the fee is not charged by side in the per-transaction model. Suppose in the per-user model platforms charge asymmetric fees $\{1, \alpha\}$ on the merchant- and consumer-side users, respectively. The optimal investment problem becomes

$$
\begin{aligned}
\max \quad & (\sqrt{n_1} + \alpha\sqrt{s_1} - u_1 - v_1) + \left(\sqrt{\tilde{n}_1} + \alpha\sqrt{\tilde{s}_1} - \tilde{u}_1 - \tilde{v}_1\right) \\
& + (\sqrt{n_2} + \alpha\sqrt{s_2} - u_2 - v_2) + \left(\sqrt{\tilde{n}_2} + \alpha\sqrt{\tilde{s}_2} - \tilde{u}_2 - \tilde{v}_2\right)
\end{aligned}
$$

The optimal solution becomes

$$
\begin{aligned}
u_1^* &= \frac{\beta_u}{4\left(1 - \lambda_n - \theta_s\frac{\beta_u}{\beta_v} - \tilde{\gamma}_{nn} - \tilde{\gamma}_{sn}\frac{\beta_u}{\beta_v}\right)^2} \\
v_1^* &= \frac{\alpha^2 \cdot \beta_v}{4\left(1 - \lambda_s - \theta_n\frac{\beta_v}{\beta_u} - \tilde{\gamma}_{ss} - \tilde{\gamma}_{ns}\frac{\beta_v}{\beta_u}\right)^2} \\
u_2^* &= \frac{\beta_u}{4} - \left(\lambda_n u_1^* + \theta_n\frac{\beta_v}{\beta_u}v_1^* + \gamma_{nn}\tilde{u}_1^* + \gamma_{ns}\frac{\beta_v}{\beta_u}\tilde{v}_1^*\right) \\
v_2^* &= \frac{\alpha^2 \cdot \beta_v}{4} - \left(\lambda_s v_1^* + \theta_s\frac{\beta_u}{\beta_v}u_1^* + \gamma_{ss}\tilde{v}_1^* + \gamma_{sn}\frac{\beta_u}{\beta_v}\tilde{u}_1^*\right)
\end{aligned}
$$

We examine how changes in α affect the optimal investment (e.g., $\frac{\partial u_1^*}{\partial \alpha}$) and whether it amplifies or dampens the propositions (e.g., $\frac{\partial^2 \left(u_1^* + u_2^*\right) - \left(v_1^* + v_2^*\right)}{\partial \lambda_n \partial \alpha}$).

First, when α is larger, the consumer-side investments v_1^*, v_2^* increase ($\frac{\partial v_1^*}{\partial \alpha} > 0$, $\frac{\partial v_2^*}{\partial \alpha} > 0$) while the merchant-side investment u_1^* remains the same and u_2^* decreases ($\frac{\partial u_2^*}{\partial \alpha} < 0$), so a larger fee on the consumer side increases the relative investment to the consumer side. This is consistent with the "reinforcing" rule in terms of which side to invest for the per-user model.

Second, the propositions still qualitatively hold, while the magnitudes of the effects change with α. In particular, as shown in Table 6, the column of "original proposition" has the same signs as before, indicating that introducing α produces the same qualitative results. The column of "effect of α" suggests that a larger fee on the consumer side α will

Table 6: Extension: Asymmetric Fees in Per-User Model

		Focal Parameter	Original Proposition	Effect of α	
Which side	Prop 1	λ_n	$\frac{\partial I}{\partial \lambda_n} > 0$	$\frac{\partial^2 I}{\partial \lambda_n \partial \alpha} = 0$	no change
$I = (u_1^* + u_2^*) - (v_1^* + v_2^*)$	Prop 2	θ_n	$\frac{\partial I}{\partial \theta_n} < 0$	$\frac{\partial^2 I}{\partial \theta_n \partial \alpha} < 0$	amplify
Which category	Prop 3	λ_n	$\frac{\partial I}{\partial \lambda_n} > 0$	$\frac{\partial^2 I}{\partial \lambda_n \partial \alpha} = 0$	no change
$I = (v_1^* + v_2^* + u_1^* + u_2^*)$		θ_n	$\frac{\partial I}{\partial \theta_n} > 0$	$\frac{\partial^2 I}{\partial \theta_n \partial \alpha} > 0$	amplify
$-(\tilde{v}_1^* + \tilde{v}_2^* + \tilde{u}_1^* + \tilde{u}_2^*)$	Prop 4	γ_{nn}	$\frac{\partial I}{\partial \gamma_{nn}} < 0$	$\frac{\partial^2 I}{\partial \gamma_{nn} \partial \alpha} = 0$	no change
		γ_{ns}	$\frac{\partial I}{\partial \gamma_{ns}} < 0$	$\frac{\partial^2 I}{\partial \gamma_{ns} \partial \alpha} < 0$	amplify
When	Prop 5	λ_n	$\frac{\partial I}{\partial \lambda_n} > 0$	$\frac{\partial^2 I}{\partial \lambda_n \partial \alpha} = 0$	no change
$I = (v_1^* + u_1^*) - (v_2^* + u_2^*)$		θ_n	$\frac{\partial I}{\partial \theta_n} > 0$	$\frac{\partial^2 I}{\partial \theta_n \partial \alpha} > 0$	amplify
		$\tilde{\gamma}_{nn}$	$\frac{\partial I}{\partial \tilde{\gamma}_{nn}} > 0$	$\frac{\partial^2 I}{\partial \tilde{\gamma}_{nn} \partial \alpha} = 0$	no change
		$\tilde{\gamma}_{ns}$	$\frac{\partial I}{\partial \tilde{\gamma}_{ns}} > 0$	$\frac{\partial^2 I}{\partial \tilde{\gamma}_{ns} \partial \alpha} > 0$	amplify

amplify an effect when the driver of the effect is the consumer side (e.g., θ_n, γ_{ns}, $\tilde{\gamma}_{ns}$); a larger fee on the consumer side α will have no effect when the driver of the effect is the merchant side (e.g., λ_n, γ_{nn}, $\tilde{\gamma}_{nn}$).

D. Robustness Check: Vary Assumptions on Observed Investment

The main specification assumes that Groupon's merchant-side investment in a category-market-period is a function of the observed category-market level number of deals and fraction of new merchants in the *current* period. We conduct robustness checks by re-estimating the model under two alternative assumptions: 1) Groupon followed a "reinforcing" rule in practice so that the category-market level investment in a period is positively proportional to the observed category-market level number of deals in the *last* period; 2) Groupon followed a "compensatory" rule in practice so that the category-market level investment in a period is negatively proportional to the observed category-market level number of deals in the *last* period.[19] As shown in Tables 7 and 8, the estimation results are robust to different assumptions on the observed investment.

---

[19]To operationalize "positively proportional" and "negatively proportional," suppose the number of deals in a particular category-market is n_i and the total number of deals is $N \equiv \sum_i n_i$. "Positively proportional" suggests that the fraction of investment allocated to that category-market is $\frac{n_i}{N}$ so that a larger n_i leads to more investment. "Negatively proportional" suggests that the fraction of investment allocated to that category-market is $\frac{N - n_i}{\sum_i (N - n_i)}$ so that a larger n_i leads to less investment.

Table 7: Robustness Checks: Merchant-side Regression Estimation Results

Merchant-side Regression		Main	Reinforcing	Compensatory
Investment (β_u)		0.479***	0.157***	0.163**
		(0.0713)	(0.0191)	(0.0808)
Own dynamic (λ_{nj})	Beauty	0.288***	0.190***	0.307***
		(0.0683)	(0.0720)	(0.0695)
	Fitness	0.238**	0.250**	0.318***
		(0.101)	(0.105)	(0.102)
	Entertainment	0.374***	0.303***	0.396***
		(0.0547)	(0.0586)	(0.0554)
	Restaurant	0.339***	0.314***	0.404***
		(0.0519)	(0.0538)	(0.0516)
Cross-side (θ_{nj})	Beauty	-0.232***	-0.212***	-0.234***
		(0.0460)	(0.0466)	(0.0461)
	Fitness	-0.0407	0.0578	-0.00698
		(0.0394)	(0.0397)	(0.0408)
	Entertainment	0.141***	0.223***	0.216***
		(0.0445)	(0.0442)	(0.0439)
	Restaurant	-0.0970*	-0.178***	-0.187***
		(0.0533)	(0.0523)	(0.0537)
Cross-category (γ_{nnj})	Beauty	0.0665	0.198**	0.144*
		(0.0762)	(0.0771)	(0.0760)
	Fitness	0.0589	-0.0993	-0.0740
		(0.0740)	(0.0717)	(0.0727)
	Entertainment	-0.147***	-0.0730	-0.103*
		(0.0538)	(0.0546)	(0.0541)
	Restaurant	-0.0731*	-0.0254	-0.0680
		(0.0410)	(0.0420)	(0.0416)
Cross-category (γ_{nsj})	Beauty	0.128***	0.117**	0.137***
		(0.0448)	(0.0455)	(0.0454)
	Fitness	-0.0447	-0.0660	-0.0205
		(0.0435)	(0.0443)	(0.0454)
	Entertainment	-0.148***	-0.253***	-0.252***
		(0.0503)	(0.0490)	(0.0488)
	Restaurant	0.164***	0.217***	0.233***
		(0.0592)	(0.0594)	(0.0609)
Return		0.811***	0.307***	0.314***
		(0.0793)	(0.0243)	(0.0241)
Past experience		0.0694***	0.0656***	0.0664***
		(0.00491)	(0.00490)	(0.00487)
Market fixed effects		YES	YES	YES
Category fixed effects		YES	YES	YES
Market structure		YES	YES	YES
Time		YES	YES	YES
N.		9,663	9,663	9,663
R-squared		0.611	0.577	0.581

Notes: Standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table 8: Robustness Checks: Consumer-side Regression Estimation Results

Consumer-side Regression		Main	Reinforcing	Compensatory
Investment (β_v)		0.0532***	0.0474***	0.0423**
		(0.0173)	(0.0170)	(0.0173)
Own dynamic (λ_{sj})	Beauty	0.0840	0.0473	0.0599
		(0.0729)	(0.0720)	(0.0728)
	Fitness	0.0744	0.0791	0.0970
		(0.0634)	(0.0628)	(0.0628)
	Entertainment	0.348***	0.363***	0.363***
		(0.0691)	(0.0684)	(0.0694)
	Restaurant	-0.0364	-0.0234	-0.0773
		(0.0828)	(0.0817)	(0.0813)
Cross-side (θ_{sj})	Beauty	-0.466***	-0.524***	-0.500***
		(0.107)	(0.106)	(0.107)
	Fitness	0.105	-0.0274	0.130
		(0.160)	(0.154)	(0.161)
	Entertainment	-0.119	-0.172**	-0.108
		(0.0874)	(0.0851)	(0.0878)
	Restaurant	-0.223***	-0.246***	-0.224***
		(0.0806)	(0.0797)	(0.0811)
Cross-category (γ_{ssj})	Beauty	-0.0135	0.0244	-0.00298
		(0.0715)	(0.0704)	(0.0718)
	Fitness	0.111	0.117*	0.0835
		(0.0713)	(0.0706)	(0.0706)
	Entertainment	-0.188**	-0.198***	-0.208***
		(0.0756)	(0.0748)	(0.0760)
	Restaurant	0.0633	0.0559	0.107
		(0.0942)	(0.0930)	(0.0926)
Cross-category (γ_{snj})	Beauty	0.215*	0.261**	0.263**
		(0.118)	(0.117)	(0.117)
	Fitness	-0.258**	-0.186*	-0.281**
		(0.114)	(0.110)	(0.114)
	Entertainment	-0.164*	-0.133	-0.168*
		(0.0862)	(0.0848)	(0.0864)
	Restaurant	-0.0214	-0.0197	-0.00879
		(0.0648)	(0.0642)	(0.0649)
Discount		0.605***	0.611***	0.614***
		(0.0791)	(0.0779)	(0.0790)
Price		-0.356***	-0.359***	-0.360***
		(0.0160)	(0.0157)	(0.0159)
Duration		-0.105***	-0.136***	-0.144***
		(0.0321)	(0.0316)	(0.0320)
Market fixed effects		YES	YES	YES
Category fixed effects		YES	YES	YES
Market structure		YES	YES	YES
Time		YES	YES	YES
N.		9,663	9,663	9,663
R-squared		0.534	0.532	0.516

Notes: Standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.