Letter

Helical symmetry breaking and quantum anomaly in massive Dirac fermions

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Helical symmetry of massive Dirac fermions is broken explicitly in the presence of electric and magnetic fields. Here we present two equations for the divergence of helical and axial vector currents following the Jackiw-Johnson approach to the anomaly of the neutral axial vector current. We discover the contribution from the helical symmetry breaking is attributed to the occupancy of the two states at the top of the valence band and the bottom of the conduction band. The explicit symmetry breaking fully cancels the anomalous correction from quantum fluctuation in the band gap. The chiral anomaly can be derived from the helical symmetry breaking. It provides an alternative route to understanding the chiral anomaly from the point of view of the helical symmetry breaking. The pertinent physical consequences in condensed matter are the helical magnetic effect, which means a charge current circulating at the direction of the magnetic field, and the mass dependent positive longitudinal magnetoconductivity as a transport signature. The discovery not only reflects anomalous magnetotransport properties of massive Dirac materials, but also reveals the close relation between the helical symmetry breaking and the physics of chiral anomaly in quantum field theory and high energy physics.

Introduction. The chiral anomaly of massless Dirac fermions is a purely quantum mechanical effect, and is an extraordinarily rich subject in quantum field theory and elementary particle physics [1–5]. It is regarded as a consequence of spontaneous symmetry breaking induced by the quantum fluctuation in the presence of electric and magnetic field. However, the helicity represents the projection of the particle spin at the direction of motion and is also conserved for Dirac fermions. The helical symmetry is broken explicitly in an electric field. In the massless case the helicity and chirality become identical for the positive energy and differ by an opposite sign for the negative energy [6,7]. This raises a question of whether or not the chiral anomaly is closely related to the explicit symmetry breaking of helicity. In recent years, the discovery of Weyl semimetals revived research interests on chiral anomaly for massless Dirac fermions in condensed matter physics [8–14]. A negative longitudinal magnetoresistance was regarded as a significant signature to support the existence of chiral anomaly in gapless Weyl semimetals and Dirac semimetals [15–22]. However, as more and more topological materials with finite band gap [23–27] also exhibit negative magnetoresistance, the mechanism of chiral anomaly is obviously challenged as the chiral symmetry has already been broken explicitly by a finite mass. Very recently, Andreev and Spivak proposed that the helicity imbalance of massive Dirac fermions may also produce negative magnetoresistance [28]. Because of the close relation between the helical and chiral symmetry, it deserves an investigation on the helical symmetry breaking of massive Dirac fermions and its transport signature in the presence of electric and magnetic fields.

Furthermore, it may reveal the deep relevance of the helicity symmetry breaking and the physics of chiral anomaly.

In this Letter, following the Jackiw-Johnson approach to the anomaly of the neutral axial vector current [29] we derive the equations for the divergence of the helical current and axial vector current for massive Dirac fermions in the presence of electric and magnetic fields. We find the discontinuity of helicity at the momentum $q_z = 0$ at the zeroth Landau levels leads to the helical symmetry breaking in the presence of the electric field. The anomalous correction from the quantum fluctuation is exactly canceled by the explicit symmetry breaking in the band gap. The mass term strongly revises the coefficient in the equation for the axial vector current, but keeps the coefficient as constant in the equation of the helical current. The two equations become equivalent in the higher energy and massless case. The identical form of the equations for the helicity and chirality provides deep insight into the role of helical symmetry breaking in the physics of chiral anomaly. Physically, the helical symmetry breaking leads to a charge current circulating along the direction of the magnetic field, termed the helical magnetic effect. The effect gives rise to the mass-dependent negative magnetoresistance in massive Dirac materials.

Helicity in a magnetic field. We start with the massive Dirac fermions in a finite magnetic field B along the z direction,

$$H_0 = \gamma^0(\gamma^i v \Pi_i + m v^2), \tag{1}$$

where $\gamma^0 = \tau_1 \sigma_0$ and $\gamma^i = -i \tau_2 \sigma_i$, with τ and σ being Pauli matrices of orbital and spin degree of freedom. m is the Dirac mass and v is the effective velocity. $\Pi = \hbar \mathbf{q} + e \mathbf{A}$ is the kinematical momentum with the vector potential $\mathbf{A} = (-By, 0, 0)$. Without loss of generality, we assume $eB > 0$. Since the presence of the vector potential $\mathbf{A}$ does not break the translation symmetry along the x and z direction, q_x and
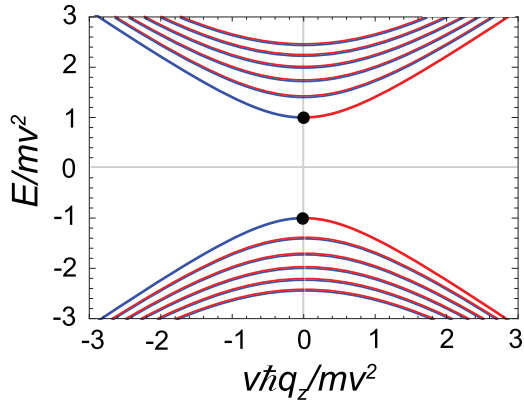
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FIG. 1. Energy dispersion spectrum of the Landau levels with helicity distribution (blue line for right-handed and red for left-handed helicity). The two black dots indicate the discontinuity of helicity in the Landau levels of $n = 0$.

q_z are still good quantum numbers. The operator $\Sigma \cdot \Pi = \tau_0 \sigma \cdot \Pi$ defines the projection of the particle spin at the direction of motion. By taking advantage of the ladder operators $a = (\Pi_x - i\Pi_y)/\sqrt{2eB\hbar}$ and $a^\dagger = (\Pi_x + i\Pi_y)/\sqrt{2eB\hbar}$ [30], we can obtain the eigenvalues and eigenstates for the operator [31],

$$\sigma \cdot \Pi |n, q_x, q_z, \chi_n\rangle = \chi_n \sqrt{\hbar^2 q_z^2 + n\hbar^2 \Omega^2/v^2} |n, q_x, q_z, \chi_n\rangle, \quad (2)$$

where $n = 0, 1, 2, \ldots$ are the indices of the Landau levels and $\Omega \equiv \sqrt{2}v\ell_B^{-1}$ is the cyclotron frequency with the magnetic length $\ell_B = \sqrt{\hbar/eB}$. χ_n stands for the helicity of massive Dirac fermions: $\chi_n = \pm 1$ for $n > 0$ and $\chi_0 = -\text{sgn}(q_z)$ for $n = 0$. The sign change of χ_0 around $q_z = 0$ is a peculiar feature of the Landau level of $n = 0$ (see the black dots in Fig. 1). In the basis of helicity eigenstates, the helicity operator is expressed as

$$\hat{h} = \sum_{n, q_x, q_z, \chi_n} \chi_n \tau_0 |n, q_x, q_z, \chi_n\rangle\langle n, q_x, q_z, \chi_n|. \quad (3)$$

The helicity operator commutes with the Hamiltonian, $[\hat{h}, H_0] = 0$; thus the helical symmetry survives in a finite magnetic field. In the helicity basis, the Hamiltonian is reduced to an effective one-dimensional system $H_0 = \chi_n \sqrt{v^2\hbar^2 q_z^2 + n\hbar^2 \Omega^2} \tau_3 + mv^2 \tau_1$. Thus the energy eigenvalues are $\varepsilon_{n\zeta\chi_n} = \zeta \sqrt{v^2\hbar^2 q_z^2 + m^2 v^4 + n\hbar^2 \Omega^2}$, where $\zeta = +1$ for the conduction band and -1 for the valence band. The corresponding eigenstate for each Landau level is [31]

$$|n, \zeta, \chi_n; q_x, q_z\rangle = \begin{pmatrix} \cos\frac{\phi_{n\zeta\chi_n}}{2} \\ \zeta \sin\frac{\phi_{n\zeta\chi_n}}{2} \end{pmatrix} \otimes |n, q_x, q_z, \chi_n\rangle, \quad (4)$$

where $\cos\phi_{n\zeta\chi_n} = \chi_n \sqrt{n\hbar^2\Omega^2 + v^2\hbar^2 q_z^2}/\varepsilon_{n\zeta\chi_n}$. These eigenstates are orthogonal to each other as $\langle n', \zeta', \chi'_n; q'_x, q'_z | n, \zeta, \chi_n; q_x, q_z\rangle = \delta_{nn'}\delta_{\zeta\zeta'}\delta_{\chi_n\chi'_n}\delta(q'_x - q_x)\delta(q'_z - q_z)$. All the Landau levels with different q_x are degenerated with the degeneracy $n_L = eB/2\pi\hbar$ per unit area in the x-y plane. Besides each Landau level has additional double degeneracy for helicity when $n > 0$.

Continuity equation for helicity. The presence of an electric field breaks the helical symmetry for the massive Dirac fermions. Consider the electric potential $V(\mathbf{r}) = e\mathbf{E} \cdot \mathbf{r}$ for a uniform electric field $\mathbf{E}$. Since the helicity operator is a function of momentum, which does not commute the position operator $\mathbf{r}$, $[\hat{h}, \hat{V}] \neq 0$. To establish the equation of the divergence of helical currents, we follow the Jackiw-Johnson approach to the anomaly of the neutral axial vector current [4,29], and define the gauge-invariant helical currents

$$\hat{j}_h^i(z) = \lim_{\epsilon_\alpha \to 0} \bar{\psi}\left(r_\alpha + \frac{\epsilon_\alpha}{2}\right)\gamma^i v\hat{h}\psi\left(r_\alpha - \frac{\epsilon_\alpha}{2}\right)e^{-i\phi(t,\epsilon_0)}, \quad (5)$$

with $\phi(t, \epsilon_0) = \int_{t-\epsilon_0/2}^{t+\epsilon_0/2} V(r_\alpha)dt/\hbar$. ψ and $\bar{\psi} = \psi^\dagger\gamma^0$ are the Dirac spinors. The local density and current are obtained by taking ϵ to be small. $\hat{\rho}_h = \lim_{\epsilon \to 0} j_h^0(z, \epsilon)$ and $\hat{j}_h^i = \lim_{\epsilon \to 0} j_h^i(z, \epsilon)$. Utilizing the time-dependent Dirac equation, the divergence of helical currents is given by

$$\partial_t \rho_h + \partial_i j_h^i = -\frac{e^2}{2\pi^2\hbar^2}\mathbf{E} \cdot \mathbf{B} - \frac{i}{\hbar}\langle\bar{\psi}\gamma^0[\hat{h}, \hat{V}]\psi\rangle, \quad (6)$$

where ρ_h and j_h^i are the expectation values of helical density and current density at zero temperature. The first term in the right-hand side of Eq. (6) is given by the anomalous correction from the quantum fluctuation

$$\hat{S}(z) = i\hbar^{-1} \lim_{\epsilon_\alpha \to 0}[\hat{j}_h^0(z, \epsilon)\epsilon_3 - \hat{j}_h^3(z, \epsilon)\epsilon_0]\partial_z V(\mathbf{r}) \quad (7)$$

for small, but nonzero ϵ_0 and ϵ_3. The divergence of the helical density as $1/\epsilon_3$ is caused by the infinity of the Fermi sea in the valence bands, which was first encountered in the anomaly of neutral axial vector current [29]. Besides, the second term in the right-hand side of Eq. (6) comes from the explicit helical symmetry breaking. In the basis of the eigenenergy, we find [31]

$$[\hat{h}, \hat{V}]_0 = i2eE_z \sum_{q_x, q_z} \delta(q_z)|0, \zeta, \chi_0; q_x, q_z\rangle\langle 0, \zeta, \chi_0; q_x, q_z| \quad (8)$$

for the Landau levels of $n = 0$. It is noted that the delta function originates from the discontinuity of helicity around $q_z = 0$, i.e., $\partial_{q_z}\text{sgn}(q_z) = 2\delta(q_z)$ (see Fig. 1). The chemical potential determines the occupancy of the two states $|n = 0, \zeta = \pm, \chi_0; q_x, q_z\rangle$ at $q_z = 0$: each state may contribute one $\frac{e^2}{2\pi^2\hbar^2}\mathbf{E} \cdot \mathbf{B}$. Combining with the term of anomalous correction, we obtain the equation for the divergence of the helical density ρ_h and current $\mathbf{j}_h$ in a more compact form as

$$\partial_t \rho_h + \nabla \cdot \mathbf{j}_h = C_h \frac{e^2}{2\pi^2\hbar^2}\mathbf{E} \cdot \mathbf{B}. \quad (9)$$

Here $C_h = \text{sgn}(\mu)$ when the chemical potential is in the conduction or valence band and $C_h = 0$ at the half filling. Thus the explicit symmetry breaking term and the anomalous correction are exactly canceled in the right-hand side of Eq. (6) within the gap. The sign change in the conduction and valence bands is caused by the opposite velocities of fermions with identical helicity at the direction of the magnetic field. For the higher Landau levels of $n > 0$, the diagonal elements of $[\hat{h}, \hat{z}]_n$ always vanish and the off-diagonal elements may contribute to higher order corrections from the electric field (see details in

Ref. [31]). Thus Eq. (9) holds for a finite magnetic field and a weak electric field. It is one of the key results of this Letter.

Chiral anomaly of the massless fermions. The helical symmetry breaking may provide an alternative approach to derive the chiral anomaly for massless Dirac fermions. In the basis of the eigenenergy levels, the chirality operator $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ becomes

$$\gamma^5 = \sum_{n,\zeta,\chi_n,q_x,q_z} \zeta\,\chi_n |n,\zeta,\chi_n;q_x,q_z\rangle\langle n,\zeta,\chi_n;q_x,q_z|. \quad (10)$$

For $m = 0$, γ_5 commutes with the Hamiltonian and is conserved. In the conduction band of $\zeta = +1$, $\gamma^5 P_+ = \hat{h}P_+$, but in the valence band of $\zeta = -1$, $\gamma^5 P_- = -h\hat{P}_-$, where the band projection operator $P_\zeta = \sum_{n,q_x,q_z,\chi_n} |n,\zeta,\chi_n;q_x,q_z\rangle\langle n,\zeta,\chi_n;q_x,q_z|$. Thus the helicity and chirality become identical in the conduction band and opposite in the valence band. Substituting the relation into Eq. (9), one can obtain the continuity equation for chiral anomaly,

$$\partial_t\rho_5 + \nabla\cdot\mathbf{j}_5 = \mathrm{sgn}(|\mu|)\frac{e^2}{2\pi^2\hbar^2}\mathbf{E}\cdot\mathbf{B}, \quad (11)$$

where ρ_5 and $\mathbf{j_5}$ are the chirality density and the corresponding axial vector current, respectively. This provides an alternative approach to derive the chiral anomaly from the point of view of the helical symmetry breaking. Unlike the helicity, the chirality operator is independent of momentum and the presence of an electric field does not break the chiral symmetry.

Pseudoscalar density and continuity equation of chirality for massive Dirac fermions. In the presence of a finite mass m, γ^5 does not commute with the finite mass term, $\psi^\dagger[\gamma^5, H_0]\psi = -2imv^2\mathfrak{n}_P$ with the pseudoscalar density $\mathfrak{n}_P = -\bar{\psi}i\gamma^5\psi$ [34,35]. After including the contribution from the quantum fluctuation, the divergence of the axial vector currents is given by [29,34]

$$\partial_t\rho_5 + \nabla\cdot\mathbf{j}_5 = \frac{e^2}{2\pi^2\hbar^2}\mathbf{E}\cdot\mathbf{B} - \frac{2}{\hbar}mv^2\langle n_P\rangle. \quad (12)$$

In the absence of an electric field, all diagonal elements of n_p vanish in the helicity basis and the off-diagonal elements connect the conduction and valence bands with the same Landau index. The expectation value of pseudoscalar density $\langle\mathfrak{n}_P\rangle$ is equal to zero for a free gas of Dirac fermions. In the presence of electric field, the electric potential may couple the two states of the same momentum and Landau index. Due to the double degeneracy of the states of the Landau levels of $n > 0$, it is found that only the nondegenerated Landau levels of $n = 0$ contribute to the nonzero value of $\langle\mathfrak{n}_P\rangle$. The perturbation approach up to the linear electric field $\mathbf{E}$ gives

$$\langle\mathfrak{n}_P\rangle = (1 - C_5)\frac{e^2}{4\pi^2\hbar mv^2}\mathbf{E}\cdot\mathbf{B}. \quad (13)$$

The coefficient $C_5 = \sqrt{1 - \frac{m^2v^4}{\mu^2}}$ for $|\mu| \geqslant mv^2$ and $C_5 = 0$ otherwise [36]. Then, the equation for the axial vector currents becomes [31]

$$\partial_t\rho_5 + \nabla\cdot\mathbf{j}_5 = C_5\frac{e^2}{2\pi^2\hbar^2}\mathbf{E}\cdot\mathbf{B}. \quad (14)$$
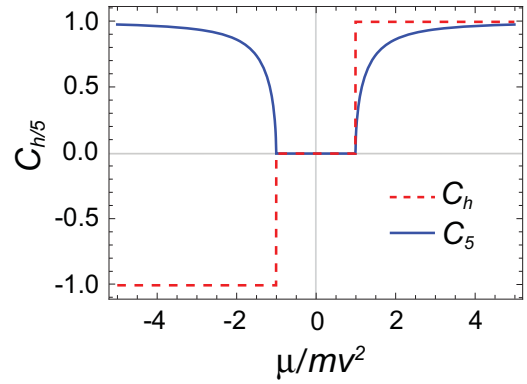

FIG. 2. Comparison of the coefficients $C_{h/5}$ in the equations for the divergence of the helical current and axial vector currents in Eqs. (9) and (14).

Again the symmetry breaking term caused by the mass cancels the anomalous correction $\frac{e^2}{2\pi^2\hbar^2}\mathbf{E}\cdot\mathbf{B}$ from quantum fluctuation out when the valence band is fully filled. Clearly, the chirality is not a good quantum number for a nonzero mass. We cannot define a chirality-dependent potential via the energy levels of free fermions as we do for the helicity, but the chirality density are still closely related to the helicity density. Consider a tiny helical potential μ_h near the chemical potential μ. It is found that $\rho_5 = \sqrt{1 - \frac{m^2v^4}{\mu^2}}\rho_h$. This demonstrates that the two quantities tend to be equal when μ is much larger than the band gap $2mv^2$ or the mass approaches zero. When μ is located near the band bottom, the chirality density approaches zero. Thus the equations for helical current and axial current density are consistent with each other. A straightforward comparison of the coefficients on the right-hand side in Eqs. (9) and (14) is presented in Fig. 2. It illustrates clearly the difference and connection between the two equations for a different chemical potential.

Helical magnetic effect. For a nearly free gas of massive Dirac fermions, the helicity density is equal to zero. If the helicity balance is broken, the chemical potentials for fermions of different helicity deviate from the equilibrium value μ: one increases and another decreases, i.e., $\mu_\pm = \mu \pm \mu_h/2$. If there is no other interaction, the helicity is still conserved. The helicity-dependent current for $\chi = \pm1$ can be calculated independently. The electric current density is given by the difference of the helical currents for two distinct helicities [31],

$$\mathbf{j} = \frac{e^2}{4\pi^2\hbar^2}(|\mu_+| - |\mu_-|)\mathbf{B}. \quad (15)$$

This means that, if two chemical potentials are not equal, a charge current circulates at the direction of the magnetic field. The field dependent current was named the *helical magnetic effect* [37,38]. The effect is equivalent to the chiral magnetic effect when the mass m approaches zero as the helicity and chirality become identical [28,39–41].

One remarkable transport consequence of the helical magnetic effect is the magnetoconductivity in Dirac/Weyl semimetals. In the presence of impurity scattering, the interhelicity scattering process can maintain a nonzero helical charge imbalance near the Fermi energy μ in the background

of the electromagnetic field. The scattering potentials are functions of position and do not commute with the helicity operator. Thus we assume the scattering potentials V_s are not so strong such that the averaged value of $[\hat{h}, V_s]$ is still negligible. With a characteristic relaxation time τ_{h} between different helical electrons, one can introduce a relaxation term in the continuity equation, $\partial_t \rho_{\mathrm{h}} = \mathrm{sgn}(\mu)\frac{e^2}{2\pi^2\hbar^2}\mathbf{E}\cdot\mathbf{B} - \frac{\rho_{\mathrm{h}}}{\tau_{\mathrm{h}}}$. For the equilibrium state $\partial_t \rho_{\mathrm{h}} = 0$, the solution for ρ_{h} is found as $\rho_{\mathrm{h}} = \mathrm{sgn}(\mu)\frac{e^2}{2\pi^2\hbar^2}\mathbf{E}\cdot\mathbf{B}\tau_{\mathrm{h}}$ [42]. When $|\mu_{\mathrm{h}}| \ll |\mu|$, the corresponding helical chemical potential can be found as $\mu_{\mathrm{h}} \approx \frac{2\rho_{\mathrm{h}}}{g(\mu)}$, where $g(\mu)$ is the total density of states at the Fermi energy. Then, the helical magnetic effect leads to a nonzero field-dependent current density as

$$\mathbf{j}_{\mathrm{HME}} = \left(\frac{e}{\pi\hbar}\right)^4 \frac{\tau_{\mathrm{h}}}{4g(\mu)}\mathbf{E}\cdot\mathbf{BB}. \tag{16}$$

Accordingly, the helicity-induced magnetoconductivity is given by $\sigma_{ij}^h = \frac{e^4}{4\pi^4\hbar^4 g(\mu)}\tau_{\mathrm{h}}B_iB_j$. The interhelicity scattering time τ_{h} is determined by the impurity scattering potentials. This equation is valid from the weak magnetic field to the quantum limit regime. In the weak magnetic field, the density of state at the Fermi energy is $g(\mu) = \frac{\mu q_F}{\pi^2 v^2\hbar^2}$, with $q_F = \sqrt{\mu^2 - m^2v^4}/v\hbar$. The matrix element of magnetoconductivity tensor due to the helical magnetic effect reads $\sigma_{ij}^h = \frac{e^2}{4\pi^2\hbar}\frac{e^2v^3}{\mu v\hbar q_F}\tau_{\mathrm{h}}B_iB_j$. The longitudinal magnetoconductivity is consistent with the result for massive and massless cases [12,17,28], while the transverse magnetoresistance gives rise to the planar Hall effect [43]. In the Born approximation, the interhelicity scattering time is found to reach a maximal value at $m = 0$, and decays with the increase of the mass, which results in a mass-dependent magnetoconductance in Dirac materials [31]. In the quantum limit regime, the density of state at Fermi energy μ is found as $g(\mu) = \frac{\mu}{2\pi^2\ell_B^2 v^2\hbar^2 q_F}$, and the corresponding longitudinal magnetoconductivity becomes $\sigma_{zz}^{\mathrm{h}} = \frac{e^3 v}{2\pi^2\hbar^2}\frac{v\hbar q_F}{\mu}\tau_{\mathrm{h}}B$, which is a linear function of the magnetic field once τ_h is a constant in the massless limit. For a moderately strong magnetic field, the density of states will oscillate with the magnetic field, and there are quantum oscillations in the magnetoconductivity.

Discussion and conclusion. The full cancellation of the explicit symmetry breaking and the anomalous correction in the band gap reflects the quantum anomaly in the massive Dirac fermions. The anomalous correction arises by introducing the gauge invariant currents in Eq. (5), and is an electromagnetic

response from the infinite Dirac sea. Here it is worth pointing out that the approach is different from the method based on the variation of charge density of particles in the zeroth Landau levels [15,28], in which all other negative energy levels are neglected and is actually irrelevant to physics of the quantum anomaly. This point can be further clarified in the following example. Consider the nonrelativistic Pauli Hamiltonian for a free electron gas in a magnetic field, $H_P = (\sigma \cdot \Pi)^2/2m = \Pi^2/2m + \frac{e\hbar}{2m}B\sigma_z$. The helicity is conserved: $[\sigma \cdot \Pi, H_P] = 0$. The helical symmetry breaking in an electric field leads to an identical continuity equation for the divergence of helical density and current as in Eq. (9) for $\mu > 0$ [31], which is consistent with the picture of the Landau levels. However, it is unrelated to the physics of quantum anomaly since there are no infinite negative energy states at all. The helical symmetry breaking in this system may also give rise to a negative longitudinal magnetoresistance. Of course it should be noted that the effect disappears if the Zeeman field is absent, i.e., in $H = \Pi^2/2m$.

In summary, we derived the two equations for the divergence of helical current and axial vector current in electric and magnetic fields. We discovered the discontinuity of helicity at the zeroth Landau levels leads to the helical symmetry breaking in the presence of the electric field. The occupancy of the states at the top of the valence band and the bottom of the conduction band contributes one in the unit of $\frac{e^2}{2\pi^2\hbar^2}\mathbf{E}\cdot\mathbf{B}$ in the equation of the divergence of the helical currents. The anomalous corrections from the quantum fluctuation for both helicity and chirality are canceled exactly by the explicit symmetry breaking to guarantee the conservation laws when the chemical potential is located within the energy gap. In the case of higher energy or tiny mass $|\mu| \gg mv^2$, the equations for helicity and chirality become equivalent (only differing by a sign for positive and negative energy). This provides an alternative route to understanding the chiral anomaly from the point of view of the helical symmetry breaking. The two equations may shed some insights into the physics of the chiral anomaly in the quantum field theory, as well as peculiar transport behaviors in condensed matter. For instance, as a peculiar feature of massive Dirac fermions in a magnetic field, the helical magnetic effect can give rise to a mass-dependent positive magnetoconductance.

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