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Cross-Diffusion Triggered Multiphysics Wave Instabilities

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Coupled Thermo-Hydro-Mechano-Chemical (THMC) patterns are ubiquitous in nature yet their origin is not yet fully understood. We propose a generic framework for pattern formation in terms of quasi-solitary wave instabilities that are triggered by cross-scale THMC-feedbacks considering a general topology of saturated porous media [1]. We identify the important aspect of cross-diffusion terms and present a linear stability analysis of the governing partial differential equations (pde's). Multiple transient wave instabilities are found as solutions of the coupled THMC pde's and in the standing wave limit (infinite time scale) these waves form the solitary wave patterns frozen into the geosystems at various scales.

Cross diffusion in a complex system is defined by the phenomenon that a gradient of one generalised thermodynamic force drives a generalised thermodynamic flux of another kind. Thermodynamic forces and fluxes in a THMC-system are defined as follows. Thermodynamic forces are the gradients of the THMC-system. The flux (T) represents Fourier's law where thermal conductivity represents its characteristic diffusivity. The flux (H) describes Darcy's law, where the diffusivity depends on the intrinsic permeability of the porous structure and the viscosity of saturating fluid. The flux (M) represents the incremental change in the solid-phase overstress adopting a Representative Elementary Volume (REV) formalism. The fluid phase within the REV, as an immediate environment surrounding the solid matrix, synchronously feels the pressure change, and vice versa. The flux (C) is Fick's law, where chemical reaction and transport processes occur predominantly at/around the solid-fluid interfacial areas.

In order to express the THMC feedback we write the governing reaction diffusion equations as coupled HM equations with generalized source terms depending on temperature, concentration, fluid pressure and solid overstress and further consider the cross-diffusion terms as a generic framework:

$$\frac{Dp_H}{Dt}=D_H\frac{\partial^2 p_H}{\partial z^2}+h_1\frac{\partial^2 \bar{p}_M}{\partial z^2}+f(T,C,p_H,\bar{p}_M)$$

$$\frac{D\bar{p}_M}{Dt}=D_M\frac{\partial^2 \bar{p}_M}{\partial z^2}-h_2\frac{\partial^2 p_H}{\partial z^2}+g(T,C,p_H,\bar{p}_M)$$

where $h_1>0$, $h_2>0$, $h_1+h_2>0$ are the cross-diffusion coefficients [2] triggering wave instabilities from solid-fluid interaction at the microscale. The capital D../Dt denotes the material derivative. In the case that $h_1=h_2=0$ the classical conservation laws are recovered, and no stationary waves are obtained. Propagating waves recorded in laboratory experiments and possible field applications are interpreted with this new approach.

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