Journal of Physics Communications

PAPER

An attractor dynamics in a non-Hermitian two-level system

C Li[2,3], P Wang[4], L Jin[1,*] and Z Song[1,*]

[1] School of Physics, Nankai University, Tianjin 300071, People's Republic of China
[2] Department of Physics, The University of Hong Kong, Hong Kong, People's Republic of China
[3] HKU-UCAS Joint Institute of Theoretical and Computational Physics at Hong Kong, People's Republic of China
[4] Graduate School of China Academy of Engineering Physics, Beijing 100193, People's Republic of China
[*] Authors to whom any correspondence should be addressed.

E-mail: jinliang@nankai.edu.cn **and** songtc@nankai.edu.cn

Keywords: Non-Hermitian, Attractor, Nonlinear, Dynamics

OPEN ACCESS

RECEIVED
16 August 2021

REVISED
20 September 2021

ACCEPTED FOR PUBLICATION
1 October 2021

PUBLISHED
14 October 2021

Abstract

Exceptional points in non-Hermitian systems possess fascinating properties. We present an exactly solvable attractor dynamics for the first time from the two-level time-dependent non-Hermitian Hamiltonian. This allows the evolution from a pure or mixed initial state to the coalescence state by varying the imaginary parameter along a specific diabatic passage. In contrast to a chaotic attractor that is ultrasensitive to initial conditions, the designed attractor is insensitive to initial conditions. The attractor-like behavior is applicable to several adiabatic processes.

1. Introduction

Non-Hermiticity is described by imaginary external parameters, which can be imaginary potentials [1–4] or nonreciprocal couplings [5]. Non-Hermiticity leads to considerably unusual features, even in simple systems. These include $\mathcal{PT}$ phase transition [6–10], unidirectional and anomalous transport [11–15], asymmetric reflectionless [16], and loss induced large nonlinearity [17, 18]. An exceptional point (EP) is an exclusive critical point in non-Hermitian systems at which pairs of eigenstates coalesce and exotic features occur, such as invisible defects [19–21], coherent absorption [22] and self sustained emission [23–27], loss-induced revival of lasing [28], laser mode selection [29, 30], and $\mathcal{PT}$ chaos [31]. In contrast to degenerate eigenstates, the coalesced state is immune to tunneling between coalescence eigenstates, stabilizing the target quantum state [32].

EPs possess fascinating properties, for example, the states in a two-level system switch when circling an EP after one circle; moreover, the geometric phase accumulated is circling direction dependent [33–35]. When dynamically encircling an EP non-adiabatically, the energy transfer is nonreciprocal [36–38]. The dissipative dynamics in a non-Hermitian system has been discussed through the master equation approach [39–41]. These observations indicate the possibility of exploring other EP-related dynamical effects. It is interesting to investigate how a quantum state evolves when a system tends toward an EP.

In this work, we propose an exactly solvable time-dependent two-level system. The system can behave as an attractor when it tends toward or crosses an EP. An attractor can be a point, curve, or surface in the phase space of the system to which orbits are attracted. Typical observations are an infinite number of unstable orbits embedded in a chaotic attractor. The chaotic dynamics is sensitive to initial conditions; points on two arbitrarily close trajectories may exhibit distinct dynamics. Efforts have been made to obtain improved performance and multiple uses [42]. A driven nonlinear dissipative system may exhibit chaotic behavior [43], which is extensively investigated [44–46]. Recently, periodic or chaotic dynamics have been demonstrated for both the optical and mechanical modes caused by optomechanical coupling-induced nonlinearity [47]. Here, an attractor dynamics is presented for the first time from the two-level non-Hermitian Hamiltonian. The time evolution is studied when the system approaches its EP. We show that both pure and mixed states evolve to the coalescence state by varying the imaginary parameter along a specific diabatic passage. A chaotic attractor is ultrasensitive to the initial conditions; therefore, we propose an attractor in a time-dependent non-Hermitian system without nonlinearity. The attractor is a limit circle, where the dynamics is insensitive to initial conditions, the evolution

of any state finally converges to one fixed orbit. Numerical simulation shows that the attractor dynamics is applicable to several adiabatic processes.

2. Materials & methods

In a Hermitian two-level system, the transition dynamics is governed by the Landau–Zener formula [48–51], which gives the probability of a diabatic transition between two energy states. In a non-Hermitian system, the EP is different from that in degenerate states, because two eigenstates coalesce into one. Before the construction of a general theory for the dynamics of the time-dependent system, we first present an exactly solvable time-dependent passage.

Any quantum state, being either pure or mixed, can be depicted by a density matrix $\rho = \sum_{i,j} p_{ij} |i\rangle \langle j|$, where $\{|i\rangle\}$ denotes a complete orthonormal set, $\langle i|j\rangle = \delta_{ij}$. For an arbitrary Hamiltonian, either Hermitian or non-Hermitian, the time evolution of the density matrix obeys the equation

$$i\frac{\partial}{\partial t}\rho = [H_+, \rho] + \{H_-, \rho\} \tag{1}$$

where we denote $H_\pm = (H \pm H^\dagger)/2$. Here, the square brackets denote the commutator, and the curly brackets denote the anticommutator. In principle, the dynamics of a mixed state can be obtained from the solution of the equation. However, exact analytical solutions are rare, particularly for a time-dependent non-Hermitian system with $H(t) \neq H^\dagger(t)$.

We consider a simple two-level non-Hermitian system consisting of two coupled cavities, A and B. Energy is constantly exchanged in space or time. The Hamiltonian is

$$H_{\mathrm{AB}} = \begin{pmatrix} i\gamma(t) & \kappa(t) \\ \kappa(t) & -i\gamma(t) \end{pmatrix}, \tag{2}$$

where $\kappa(t)$ is the strength of the coupling and $\gamma(t)$ is the gain or loss of each cavity. The diagonal terms describe the Markovian dissipation (gain) caused by the interaction between the two-level system and the environment baths [52, 53], corresponding characterization in the master equation is the Lindblad form superoperator [54]. All parameters are dependent of time t. The time-varying quantities $\gamma(t)$ and $\kappa(t)$ satisfy

$$\begin{aligned} \gamma(t) = \gamma_n(t) &= \frac{1 - \omega_n^2(t)}{2}, \\ \kappa(t) = \kappa_n(t) &= \frac{1 + \omega_n^2(t)}{2}, \end{aligned} \tag{3}$$

where $\omega_n^2(t) = 2n + 1 - t^2, n = 0, 1, 2, \cdots$. Here we would like to point out that one can always choose arbitrary form of $\gamma(t)$ and $\kappa(t)$ for theoretical study, but only specific forms of them can lead to the analytical exact solutions, which provide a clarification to explain why such model can have the chaotic dynamics-like behavior, also some nonlinear effect. The quantum-optical analogies in optical systems provide versatile platforms for observing quantum phenomena. The equations of motion in the coupled mode theory [55] and the Maxwell's equations under paraxial approximation that describing spatial propagation of light are similar as the temporal Schr ödinger equations [56]. No scheme has been proposed to realize a system for which both $\kappa(t)$ and $\gamma(t)$ can change with time in specific forms In practice, the two-level non-Hamiltonian system imitates

$$H = \begin{pmatrix} i\gamma(t) & \kappa \\ \kappa & -i\gamma(t) \end{pmatrix}, \tag{4}$$

which requires that κ is independent of t; this system has been realized to study the special features of EPs [57], such as the emergence of multiple EPs in coupled acoustic cavity resonators [58] and the properties associated with encirclement of an EP [38]. This system can be also realized using optical waveguides or a series of varying dichroic birefringent plates [59]. According to equation (1), we can do a unitary transformation

$$i\frac{\partial}{\partial t}R\rho = [RH_{\mathrm{AB},+}R^\dagger, R\rho] + \{RH_{\mathrm{AB},-}R^\dagger, R\rho\} \tag{5}$$

for the density matrix ρ of H_{AB}, where a unitary matrix

$$R = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}, \tag{6}$$

changes $H_{\rm AB}$ into

$$\mathcal{H}_n(t) = RH_{\rm AB}R^{\dagger} = \begin{pmatrix} 0 & 1 \\ \omega_n^2(t) & 0 \end{pmatrix}, \tag{7}$$

Note that $\omega_n^2(t) = 2n + 1 - t^2, (n = 0, 1, 2, \ldots)$, and $\omega_n(t)$ can be real or imaginary. For real ω_n, the system has balanced gain and loss. $H_n(t)$ becomes a Jordan-block at $t_c = \pm\sqrt{2n+1}$ $(\omega_n^2(t_c) = 0)$ and two eigenvectors coalesce to $(1,0)^T$. This is the EP of the two level system. Equation (5) has an exact solution. For the matrix $\mathcal{H}_n(t)$, the density matrix can be expressed as

$$\rho_n = R\rho = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}, \tag{8}$$

with the elements p_{ij} satisfying the coupled differential equations

$$\begin{cases} i\dot{p}_{11} = p_{21} - p_{12}, \\ i\dot{p}_{12} = p_{22} - \omega_n^2 p_{11}, \\ i\dot{p}_{21} = \omega_n^2 p_{11} - p_{22}, \\ i\dot{p}_{22} = \omega_n^2(p_{12} - p_{21}). \end{cases} \tag{9}$$

A special solution of the above equations is

$$\begin{aligned} &p_{11} = [x_n(t)]^2,\ p_{22} = [y_n(t)]^2 = [\dot{x}_n(t)]^2, \\ &p_{12} = (p_{21})^* = i\dot{x}_n(t)x_n(t) \end{aligned} \tag{10}$$

with

$$\begin{aligned} x_n(t) &= (2^n n!\sqrt{\pi})^{-1/2} e^{t^2/2} \mathrm{h}_n(t) \\ &= (-1)^n (2^n n!\sqrt{\pi})^{-1/2} e^{t^2/2} \frac{d^n}{dt^n} e^{-t^2}, \end{aligned} \tag{11}$$

where $h_n(t)$ is the Hermite polynomial. The aforementioned solution remains valid for the original Hamiltonian $H_{\rm AB}$ under an inverse transformation of equation (6). The density operator ρ_n is the density matrix of the pure state $(x_n, y_n)^T$, which is the solution of a quantum harmonic oscillator $\ddot{x} + \omega_n^2(t)x = 0$. The eigen functions behave distinctly in three regions. In the center region, the eigen function is a standing wave inside and an evanescent wave outside. This solution governs the dynamics of the system when it varies through the passage $\omega_n^2(t)$. It also provides the exact evolved wave function for a series of initial wave functions as the system varies along a fixed passage. Thus, for a given $\omega_n^2(t)$, the initial and final states are two points, namely $[x_n(t_1), y_n(t_1)]^T$ and $[x_n(t_2), y_n(t_2)]^T$, on the curve $\{x_n(t), y_n(t)\}$, where t_1 and t_2 are the initial and final instants, respectively.

3. Results

3.1. Fixed point of evolution

From the above calculations, $[x_n(t), y_n(t)]$ is not the instantaneous eigenstate of $H_n(t)$. Thus, the time evolution dynamics is a diabatic process and not a adiabatic process. To characterize the process, we employ the normalized Bloch vector $(\sigma_x, \sigma_y, \sigma_z)$ which is defined as

$$\begin{aligned} \sigma_x &= (x_n^* y_n + x_n y_n^*)/\Omega, \\ \sigma_y &= -i(x_n^* y_n - x_n y_n^*)/\Omega, \\ \sigma_z &= (x_n^2 - |y_n|^2)/\Omega, \end{aligned} \tag{12}$$

with $\Omega = 2(|x_n|^2 + |y_n|^2)$. It directly indicates that $\sigma_x = 0$ and $\sigma_y^2 + \sigma_z^2 = 1$, which means that the trajectory of the time evolution for any n is the same fixed longitude line on the Bloch sphere. This line is an asymptotic attractor for any nontrivial initial state. The dynamics is insensitive to the initial states; both pure and mixed states evolve to this orbit.

The solution demonstrates that $[x_n(t), y_n(t)]^T \to [0,0]^T$ as $t \to \pm\infty$. This indicates that state probability decays in the time evolution, similar to complete absorption, which occurs in certain non-Hermitian systems at spectral singularity [24, 25, 27, 60, 61]. Inversely, the time-reversal process is similar to laser emission. However, the rate of the probability increase differs, because the probability gain rate is a function of time square. We demonstrate the features of the dynamics from two limit cases of $n = 0$ and $n \gg 1$. We show that in both cases, the dynamics is diabatic process and the state $[1,0]^T$ can be dynamically prepared. Preparation efficiency is n dependent.

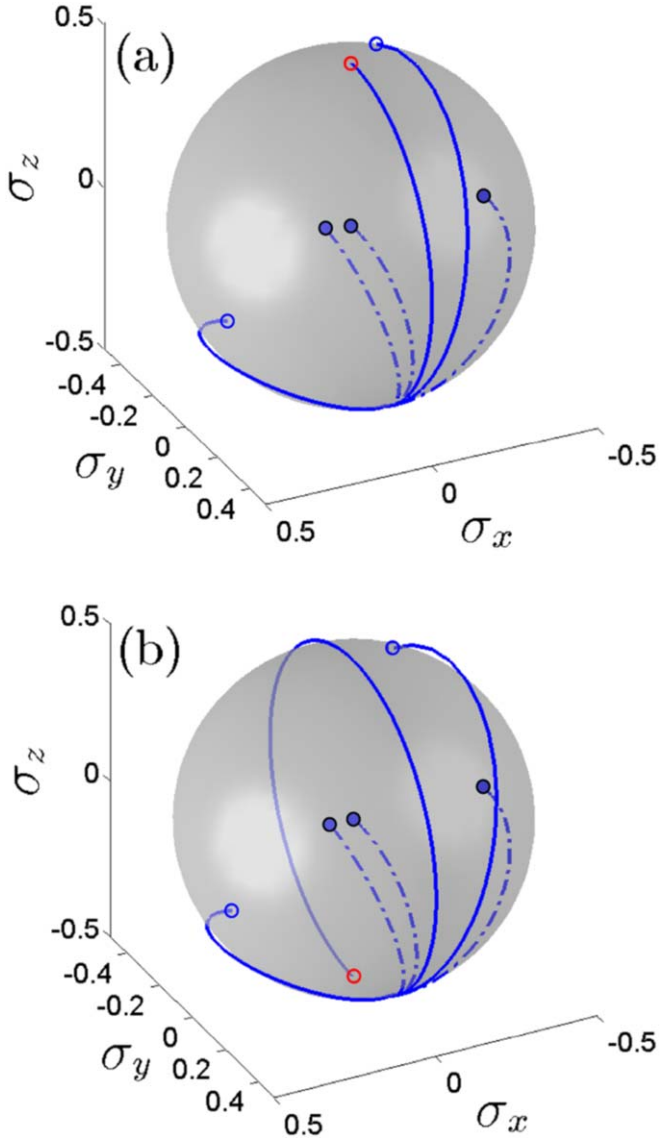

Figure 1. Effective Bloch dynamics of the non-Hermitian two-level system (7) with (a) $n = 0$ and (b) $n = 1$ from different initial states (including pure and mixed states, which are identified by solid and dash lines, respectively). The red points in the two graphs indicate the fixed point (1, 0) and (0, 1), respectively.

When $n = 0$, we have

$$\begin{cases} x_0(t) = e^{-t^2/2}, \\ y_0(t) = -ite^{-t^2/2}. \end{cases} \tag{13}$$

At time $t = t_c = \pm 1$, we have $\omega_0^2(t_c) = 0$ and

$$[x_0(t_c),\, y_0(t_c)]^T = e^{-1/2}[1,\, \mp i]^T, \tag{14}$$

which leads to the EP of the matrix $H_0(t)$. The matrix $H_0(t)$ is defective and reduces to a 2×2 Jordan block, its instantaneous coalesced eigenstate is $[1,0]^T$. The evolved state $[x_0(t_c),\, y_0(t_c)]^T$ is not the instantaneous coalesced eigenstate. However, at the instant $t = 0$, we have the evolved state

$$[x_0(0),\, y_0(0)]^T = [1,0]^T, \tag{15}$$

which is the coalescing state. The matrix $H_0(0) = \sigma_x$ is Hermitian with eigenstates $2^{-1/2}[1,\, \pm 1]^T$. If the initial state is $[x_0(0),\, y_0(0)]^T$ at $t = 0$, the final state at $t \to \infty$ approaches a zero vector. On the other hand, there is an initial state

$$|\phi(t_0)\rangle = (1 + t_0^2)^{-1/2}[i,\, t_0]^T, \tag{16}$$

at $t_0 \to -\infty$ can evolve to state $[1,0]^T$ at $t = 0$, but under infinite amplitude, the normalized state is

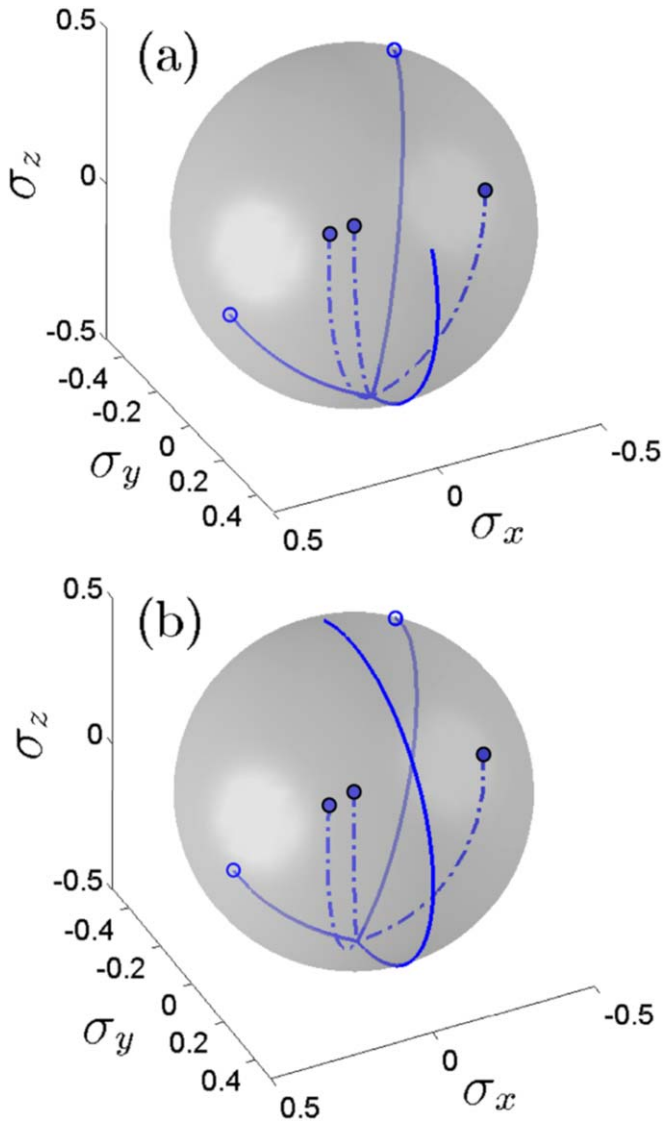

Figure 2. Effective Bloch dynamics of the non-Hermitian two-level system: (a)(23) and (b)(24) from different initial states (including pure and mixed states, which are identified by solid and dash lines, respectively).

$$|\phi(0)\rangle / ||\phi(0)|| = [1,0]^T. \tag{17}$$

Moreover, this feature can be observed at any initial state. Because for an arbitrary state

$$|\phi\rangle = [\cos\theta,\ e^{-i\phi}\sin\theta]^T,$$

the Dirac inner product

$$\langle\phi|\phi(t_0)\rangle = (1 + t_0^2)^{-1/2}(i\cos\theta + t_0 e^{i\phi}\sin\theta), \tag{18}$$

is always nonzero when $t_0 \to -\infty$. This indicates that any state has the component of $|\phi(t_0)\rangle$. This is crucial for the application of an attractor, i.e., any unknown state evolves to $[1,0]^T$. Thus, robust state preparation through dynamical evolution is possible.

When $n \gg 1$, the range of oscillating regions can be estimated from the leftmost or rightmost maximum of the amplitude $x_n(t)$, where we have

$$\frac{d}{dt}x_n(t_b) = 0,\ x_n(t_b) \approx \pm x_{n-1}(t_b), \tag{19}$$

From the recursion identities

$$\begin{cases} \dot{x}_n = \sqrt{\frac{n}{2}}\, x_{n-1} - \sqrt{\frac{n+1}{2}}\, x_{n+1}, \\ t x_n = \sqrt{\frac{n}{2}}\, x_{n-1} + \sqrt{\frac{n+1}{2}}\, x_{n+1}, \end{cases} \tag{20}$$

we have

$$\begin{cases} 0 \approx x_{n-1} - x_{n+1}, \\ \pm t_{\rm b} x_{n-1}(t_{\rm b}) \approx \sqrt{\frac{n}{2}}\,(x_{n-1} + x_{n+1}), \end{cases} \tag{21}$$

which leads to $t_{\rm b} \approx \pm\sqrt{2n}$. We note that at instance $t_{\rm b}$, the solution is the state $[1,0]^T$, i.e., $x_n(t_{\rm b})$ reaches the maximum, whereas $y_n(t_{\rm b})$ as the velocity of x_n vanishes.

When we apply the solution to the dynamics of the state, similar things occur when $n = 0$. If the initial state is $[1,0]^T$ at $t = t_{\rm b}$, the final state at $t \to \infty$ approaches a zero vector. The inverse process occurs if we take an initial state $[x_n(t_0),\ y_n(t_0)]$ at $t_0 \to -\infty$. It can evolve to state $[1,0]^T$ at $t = -t_{\rm b}$, but under infinite amplitude. Compared with the $n = 0$ case, this diabatic process is faster. Figure 1 plots the expression of $x_n(t)$ from the normalized Bloch vector defined in equation (12) for different n values to demonstrate this point, which can be seen as a time evolution process on the Bloch sphere for different initial points. Comparing with the general time evolution process on the Bloch sphere, the main difference is all different initial points evolve to the same final point, which is the behavior of attractor dynamics. Similarly, we note that $\omega_n^2(t_{\rm b}) = 1$, whereas $\omega_n^2(t_{\rm c}) = 0$ with $t_{\rm c} = \sqrt{2n+1}$.

We conclude that a series of diabatic passages can dynamically prepare the coalescing state $[1,0]^T$ from an unknown (arbitrary/any) initial state, including the mixed state. The duration time monotonously depends on n. That is, larger n leads to a faster process.

To demonstrate our conclusions, numerical simulations are performed for the evolutions of pure and mixed states. For a small time increment Δt, the Schrödinger equation for the density matrix becomes

$$\rho(t + \Delta t) \approx \rho(t) - i([H_+(t),\ \rho(t)] - i\{H_-(t),\ \rho(t)\})\Delta t, \tag{22}$$

which is employed to compute the time evolution of the density matrix numerically. A normalized state is depicted by a Bloch vector $\mathbf{a}$, which is defined as $\rho = \frac{1}{2}(I + \mathbf{a}\cdot\boldsymbol{\sigma})$, I is unitary matrix and $\boldsymbol{\sigma}$ is the Pauli matrix. Therefore, the trajectory of $\mathbf{a}$ in the Bloch sphere can describe the time evolution of a state.

We depict the dynamics of the initial states for $n = 0$ and $n = 1$ in figure. 1. All initial states coverage to the analytical solution. The final state is $[1,0]^T$ for $n = 0$ and $[0,1]^T$ for $n = 1$. These imply that when $n \in (0,\ 1)$, there is a possibility that we can dynamically prepare any pure state from an unknown (arbitrary/any) initial state. The numerical results display this tendency.

The dynamics of a $\mathcal{PT}$ dimer always has two fixed points, except at the $\mathcal{PT}$ transition point, where the two fixed points coincide [40]. In exact $\mathcal{PT}$ phase, the orbits are closed circles, which transform nonorthogonal pair states to orthogonal states using complex birefringent material [59]. In broken $\mathcal{PT}$ phase, the orbits start from the fixed source point to the fixed sink point. In this letter, the attractor has different dynamics. Only one fixed point exists, and the pure states on the sphere surface and the mixed states inside the sphere both evolve to the fixed point.

3.2. Adiabatic process

We have proved that the specific forms of $\gamma(t)$ and $\kappa(t)$ in $H_{\rm AB}$ can lead to the dynamics with similar phenomena for complete absorption and laser emission, i.e., implication of an attractor. The natural question is whether an adiabatic passage can accomplish the same task. For example, we consider two passages with κ real constant 1 and (i)

$$\gamma(t) = t, \tag{23}$$

(ii)

$$\gamma(t) = 1 - t^2. \tag{24}$$

The choice of the particular functional forms in above two equations can be arbitrary. Here we just choose a linear time-dependent form and a quadratic time-dependent form as two typical simple examples. Unlike Hermitian systems, no theory has been established to describe the process. No information on tunneling between the two coalescing levels is available. Nevertheless, the diabatic solution implies the possibility of amplification. In this situation, numerical simulation is preferred to follow a quasi-adiabatic passage. Although the position of the EP is different in the two cases, both can pass through (or reach) the EP during the time evolution from a $-t$ satisfied $\kappa^2 - \gamma^2(-t) \ll 0$ to $t = 0$. According to the simulation, the dynamics of these two

cases are the same as we found before. This implies that when the model approaches the EP during the time evolution from the broken area ($\kappa^2 - \gamma^2(t) \ll 0$ at the beginning of the evolution), a fixed point always exists in this dynamical process. The simulation result is shown in figure. 2. Although the speed of the evolution and the position of the fixed point have changed, the asymptotic line, similar to the line in figure. 1, remains even in the adiabatic process with different forms of $\kappa(t)$ and $\gamma(t)$.

4. Discussion and conclusion

We propose a time-dependent non-Hermitian two-level system, in which the dynamics has an exactly solvable passage. By varying the imaginary parameter along a specific diabatic passage, the two-level system behaves as an attractor that is insensitive to initial conditions. Comparing with the earlier works involving non-Hermitian attractors dynamics in a non-Hermitian one-dimensional chain [62] or a non-Hermitian kicked top [63], attractor-like behavior is found for the first time in a non-Hermitian two-level system. Our results prove that arbitrary pure and mixed states can evolve to the coalescence state. Moreover, the numerical results present two other indications. (i) Except for the coalescing state, we can dynamically prepare several target states from an unknown (arbitrary/any) initial state. (ii) The same phenomenon exists even in adiabatic passages with different parameters of the cavity. Finally, the dynamic properties of our model may have applications on the state control in the time evolution-like process likes the arbitrary control over pairs of polarization states in [59] or dynamic encirclement of an EP [38], based on the possibility that any pure state can be dynamically prepared from an unknown (arbitrary/any) initial state.

Acknowledgments

We acknowledge the support from National Nature Science Foundation of China (Grant No. 11 874 225, 11 975 128, and 11 605 094).

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

ORCID iDs

C Li https://orcid.org/0000-0003-1178-5520
L Jin https://orcid.org/0000-0003-1898-6317
Z Song https://orcid.org/0000-0002-3315-4589

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