Stability Analysis of Cyclic Switched Linear Systems: An Average Cycle Dwell Time Approach

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Abstract

In this paper, the stability problem of switched linear systems with a class of cyclic switching signals is investigated. Firstly, a new concept of average cycle dwell time (ACDT) is introduced to relax the conservativeness of cycle dwell time that is extensively used in the literature. In addition, the ACDT is further extended to stable cyclic switching sequence dependent average cycle dwell time (S-ACDT) and unstable cyclic switching sequence dependent average cycle dwell time (U-ACDT). Secondly, the stability criteria for cyclic switched linear (or nonlinear) systems with ACDT or both S-ACDT and U-ACDT are derived by resorting to a technique that uses multiple Lyapunov functions. Both cyclic switched linear systems and cyclic switched nonlinear systems which contain all stable subsystems or partly stable subsystems are studied. Finally, a numerical example is given to demonstrate the feasibility of the proposed techniques.

Keywords: Stability; Cyclic switched linear systems; Average cycle dwell time; Stable (or unstable) cyclic switching sequence dependent average cycle dwell time

1. Introduction

Due to the fact that switched systems can provide natural mathematical models for many complex practical systems with switching phenomena, switched systems theory

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has wide applications in daily life such as networked control systems [1] and complex networks (see [2, 3]). Switched systems which consist of a group of subsystems and an arbitrary switching rule governing the switching among them, are a typical type of hybrid systems (see [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]). During the past decades, research results on switched systems mainly focus on stability, controller synthesis and robustness (see [19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32]). Among these issues, stability is a fundamental requirement of running switched systems, and therefore attracts a great deal of attention (see [33, 34]). In particular, stability of a class of switched systems is studied by using the concept of dwell time in [35]. Next, the asymptotic stability analysis and state-feedback control design for a class of discrete-time-switched piecewise-affine systems are investigated in [36], where the dwell-time, smooth approximation technique and multiple Lyapunov functions are utilized. However, the obtained dwell time has some conservativeness, for example, a switched system may be stabilized by a switching law which has a smaller dwell time than the given value. Furthermore, to reduce this conservativeness, a large number of results based on the average dwell time method have appeared (see [37, 38]). In particular, a standard H_∞ filtering problem for a class of discrete-time two-dimensional (2-D) switched systems is considered in [39] to design a full-order filter, where the extended average dwell time technique is introduced under the restricted switching signal. Subsequently, a quasi-synchronization problem for a class of discrete-time Lur'e-type switched systems with parameter mismatches and transmission channel noises is studied in [40] to find the synchronization criteria, where the average dwell time constraints combined with the persistent dwell-time are considered simultaneously in [40] to relax the limitation of dwell time requirements and to improve the flexibility of the persistent dwell-time switching signal design. Nevertheless, the average dwell time method requires all subsystems share a common average dwell time, which is also conservative to some extent. To overcome this issue, the concept of mode-dependent average dwell time is introduced in [4], where the mode-dependent average dwell time is also extensively used to study stability issues of switched systems, and each subsystem has its own average dwell time (see [41, 5, 42]). However, most of the above results with constrained switching laws are obtained based on arbitrary switching rules and are somewhat conservative.

As a typical class of switched systems, a cyclic switched system uses a cyclic switching rule to govern the switching among its subsystems. The cyclic switching rule here means that the order that each subsystem is activated in the cyclic switched system is cyclic, but the activation time of same subsystem in different cycles can be different. So far, cyclic switched systems have attracted many research interests in different field-s such as switched flow networks [43] and automotive transmission [44]. In addition, theoretical results of cyclic switched systems have been obtained (see [45, 46, 47]). In the above mentioned results, stability analysis of cyclic switched systems is also a fundamental issue. Due to the characteristics of cyclic switched systems, paper [47] proposed the concept of cycle dwell time in cyclic switched nonlinear systems and obtained finite time stable results. Here, the cycle dwell time method refers to the residence time T_i of a cyclic switching rule in any i^{th} $(i = 1, 2, \cdots)$ cycle is not less than a given scalar T^*. Similar to dwell time method [35], the cycle dwell time method also has some same conservativeness. That is, the mechanism of cycle dwell time is similar to the idea (i.e., the dwell time requires that the dwell time of a switching signal in each mode is not less than a threshold) of dwell time, and a cyclic switched system may be stabilized by an actual cyclic switching law which has a smaller cycle dwell time than the given positive constant. Up to date, the theoretical research of switched systems with a class of cyclic switching signals is still relatively few from the perspective of stability. In addition, it is worth mentioning that how to find a more effective way to solve the stability problem of cyclic switched linear systems remain an open problem. To the best of our knowledge, there is still much room for improvements.

In this paper, the stability problem for a class of cyclic switched linear system-s is investigated with average cycle dwell time (ACDT), stable cyclic switching sequence dependent average cycle dwell time (S-ACDT) and unstable cyclic switching sequence dependent average cycle dwell time (U-ACDT). The contributions of the paper are summarized as follows: First, the ACDT, S-ACDT and U-ACDT concepts are proposed for the unique characteristics of cyclic switched systems. The proposed ACDT, S-ACDT and U-ACDT methods are more flexible than the cycle dwell time based method since ACDT or both S-ACDT and U-ACDT switching laws may contain cyclic switching signals that occasionally have consecutive cyclic discontinuities separated by less than a threshold. Second, based on the multiple Lyapunov function method, the stability conditions of a class of cyclic switched nonlinear systems with two different cases: 1) all subsystems are stable, 2) only part of subsystems are stable are established based on the ACDT method and both S-ACDT and U-ACDT methods, respectively. Third, based on the aforementioned cyclic switched nonlinear systems, the stability criteria of cyclic switched linear systems are also given in terms of linear matrix inequalities, which can be checked numerically.

The remainder of the paper is organized as follows. In Section 2, related descriptions and properties of cyclic switched linear systems are given. Section 3 gives the main results of this paper. An example is given in Section 4 to show the effectiveness of the obtained results. Section 5 gives the conclusions of the paper.

Notation $\mathbb{R}$ and $\mathbb{Z}^+$ denote the sets of real and positive integer numbers, respectively. $\mathbb{R}^n$ denotes the set of n dimensional real numbers, and $\mathbb{R}^{m\times n}$ denotes the set of $m\times n$ dimensional real matrices. $\|\cdot\|$ denotes the Euclidean norm on $\mathbb{R}^n$. A function $\gamma:\mathbb{R}\geq 0\to\mathbb{R}\geq 0$ is of class $\mathscr{K}_\infty$, i.e., $\gamma\in\mathscr{K}_\infty$ if: 1) it is continuous, 2) zero at zero, 3) strictly increasing and 4) γ grows unbounded as its argument grows unbounded.

2. Problem formulation and preliminaries

Consider the following cyclic switched linear system:

$$\begin{cases}\dot{x}(t)=A_{\sigma(t)}x(t),\\ x(t_0)=x_0,\end{cases}\tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector and $x_0\in\mathbb{R}^n$ is the initial state. $\sigma:\mathbb{R}\to\mathscr{Q}=\{1,2,\cdots,m\}$ represents the discrete cyclic switching signal taking values in the finite set $\mathscr{Q}$ and m is the total number of subsystems. For any $\sigma(t)=i\in\mathscr{Q}$, $A_i\in\mathbb{R}^{n\times n}$. Suppose that the switching happens at the time $t_1,t_2,\cdots,t_k,t_{k+1}\cdots$. When $t\in[t_k,t_{k+1})$ $(\forall k=0,1,\cdots)$, we say the subsystem $\sigma(t_k)\in\mathscr{Q}$ is activated. In addition, we assume that all subsystems can be activated.

Now, let us first introduce some concepts and properties, which will be used later.

Definition 1.[47, 45] A switching signal $\sigma(t)$ of system (1) is cyclic if $\sigma(t_k)=\sigma(t_{k+m})$

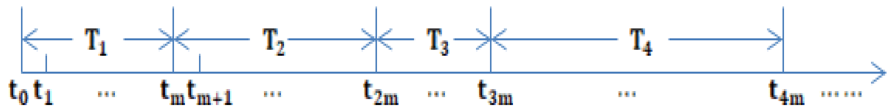

Figure 1: Diagram of the cyclic period time.

and $\sigma(t_{k+i}) \neq \sigma(t_{k+j})$, $\forall k \in \{0,1,\cdots\}, i,j \in \{0,1,\cdots,m-1\}$ and $j \neq i$.

Definition 2. Given a cyclic switching signal $\sigma(t)$ of system (1), an ordered cyclic switching sequence is defined as $\Xi = \{\sigma(t_{(N-1)m}), \sigma(t_{(N-1)m+1}), \cdots, \sigma(t_{(N-1)m+m-1})\}$, where $\sigma(t_{(N-1)m+i}) \neq \sigma(t_{(N-1)m+j}), \forall N \in \{1,2,\cdots\}, i,j \in \{0,1,\cdots,m-1\}$ and $j \neq i$.

Definition 3. A stable (or unstable) cyclic switching sequence of a given cyclic switching sequence Ξ is a subsequence of Ξ that all the stable (or unstable) subsystems are activated in the cyclic switching sequence Ξ.

Remark 1. Definitions 1, 2 and 3 show that the switching signal of system (1) is a cyclic switching signal and the switching sequence of system (1) under the cyclic switching signal is a cyclic form. That is, each (stable (unstable)) subsystem within the (stable (unstable)) cyclic switching sequence is not repeated in the same cycle and each (stable (unstable)) subsystem within the (stable (unstable)) cyclic switching sequence is repeated in different cycles. For example, if system (1) has 4 subsystems, the cyclic switching sequence Ξ of system (1) can be $\{1,2,3,4\}$ or $\{2,4,3,1\}$. That is, the activation order of each subsystem of system (1) under cyclic switching sequence $\{1,2,3,4\}$ is $1,2,3,4,1,2,3,4,1,2,\cdots$, and the activation order of each subsystem of system (1) under cyclic switching sequence $\{2,4,3,1\}$ is $2,4,3,1,2,4,3,1,2,4,\cdots$. In addition, suppose subsystems 1, 2 are stable and subsystems 3, 4 are unstable. For a given cyclic switching sequence $\Xi = \{2,4,1,3\}$, then the stable and unstable cyclic switching sequence are $\{2,1\}$ and $\{4,3\}$, respectively.

Definition 4.[47] The k^{th} cyclic period time of the cyclic switching sequence Ξ defined in Definition 2 is $T_k := t_{km} - t_{(k-1)m}$ ($\forall k = 1,2,\cdots$), where Figure 1 gives an illustration of the cyclic period time.

Remark 2. Definition 4 shows that the cyclic period time of the cyclic switching sequence Ξ in each cycle can be different, and the residence time of each subsystem within the cyclic switching sequence Ξ in different cycles can also be different.

Definition 5. [48] The equilibrium $x = 0$ of system (1) is globally uniformly exponentially stable under a cyclic switching signal $\sigma(t)$ if for any initial condition $x(t_0) \in \mathbb{R}^n$, there exist constants $K > 0$ and $\gamma > 0$ such that the solution of system (1) under $\sigma(t)$ satisfies

$$\|x(t)\| \leq K\|x(t_0)\|e^{-\gamma(t-t_0)},\ t \geq t_0.$$

Next, the definitions of average cycle dwell time (ACDT), stable cyclic switching sequence dependent average cycle dwell time (S-ACDT) and unstable cyclic switching sequence dependent average cycle dwell time (U-ACDT) will be given for the first time, which will be used to obtain the main results of the paper.

Definition 6. Consider a cyclic switching signal $\sigma(t)$ of system (1). Let $N_\sigma(t_2,t_1)$ denote the number of switching cycles that the cyclic switching sequence has been completed in the time interval $[t_1,t_2]$ with $t_2 \geq t_1 \geq 0$. Then T_c is called the ACDT if there exists a positive constant N_0 such that

$$N_\sigma(t_2,t_1) \leq N_0 + \frac{t_2 - t_1}{T_c},\ \forall t_2 \geq t_1 \geq 0,$$

where N_0 is called the chatter bound of the cyclic switching sequence.

Remark 3. Definition 6 shows that if there exists a positive constant T_c such that a cyclic switching signal has the ACDT characteristic, the ACDT method between any two consecutive cyclic switching is no smaller than a positive constant T_c for all cyclic switching sequences.

Definition 7. Consider a cyclic switching signal $\sigma(t)$ of system (1). Let $N_\sigma^-(t_2,t_1)$ denote the number of switching cycles that the stable cyclic switching sequence has been completed in the time interval $[t_1,t_2]$ with $t_2 \geq t_1 \geq 0$, and $T_\sigma^-(t_2,t_1)$ denote the total activated time of all subsystems in the stable cyclic switching sequence over the time interval $[t_1,t_2]$. Then T_c^- is called the S-ACDT if there exists a positive constant N_0^- such that

$$N_\sigma^-(t_2,t_1) \leq N_0^- + \frac{T_\sigma^-(t_2,t_1)}{T_c^-},\ \forall t_2 > t_1 > 0,$$

where N_0^- is called the chatter bound of the stable cyclic switching sequence.

Definition 8. Consider a cyclic switching signal $\sigma(t)$ of system (1). Let $N_\sigma^+(t_2,t_1)$ denote the number of switching cycles that the unstable cyclic switching sequence has been completed in the time interval $[t_1,t_2]$ with $t_2 \geq t_1 \geq 0$, and $T_\sigma^+(t_2,t_1)$ denote the total activated time of all subsystems in the unstable cyclic switching sequence over the time interval $[t_1,t_2]$. Then T_c^+ is called the U-ACDT if there exists a positive constant N_0^+ such that

$$N_\sigma^+(t_2,t_1) \geq N_0^+ + \frac{T_\sigma^+(t_2,t_1)}{T_c^+}, \forall t_2 > t_1 > 0,$$

where N_0^+ is called the chatter bound of the unstable cyclic switching sequence.

Remark 4. Definition 7 (or Definition 8) shows that if there exists a positive constant T_c^- (T_c^+) such that a cyclic switching signal has the S-ACDT (or U-ACDT) property, the average time among the intervals associated with the stable cyclic switching sequence (or unstable cyclic switching sequence) is larger than (or less than) a positive constant T_c^- (T_c^+). In fact, Definition 7 and Definition 8 can ensure that the running time of the stable cyclic switching sequence and the unstable cyclic switching sequence of system (1) is long enough and short enough under S-ACDT and U-ACDT, respectively. In addition, the S-ACDT and U-ACDT methods are extensions of the ACDT method and allow stable cyclic switching sequence and unstable cyclic switching sequence in system (1) to have their own ACDT.

In the next section, we will use these concepts to study the stability of cyclic switched linear systems.

3. Main results

In this section, in order to obtain the main stability results of cyclic switched linear systems, we first give the general lemmas of cyclic switched nonlinear systems with ACDT (or both S-ACDT and U-ACDT) cyclic switching schemes, and then give the main theorems of cyclic switched linear systems with ACDT (or both S-ACDT and U-ACDT) cyclic switching schemes.

Case 1: Suppose that each subsystem of cyclic switched linear (or nonlinear) systems is stable.

Firstly, we will present the stability conditions of cyclic switched nonlinear systems with ACDT cyclic switching in Case 1.

Lemma 1. Consider the cyclic switched nonlinear system

$$\begin{cases} \dot{x}(t) = f_{\sigma(t)}(x(t)), \\ x(t_0) = x_0, \end{cases} \tag{2}$$

and for each subsystem $i \in \mathscr{Q}$, let $\alpha_i > 0$ and $\beta_i > 1$ be given scalars. Suppose that there exist continuous differentiable functions $V_i(x(t)) : \mathbb{R}^n \to \mathbb{R}{\geq 0}$ $(i \in \mathscr{Q})$, and class $\mathscr{K}_\infty$ functions γ_{1i} $(i \in \mathscr{Q})$ and γ_{2i} $(i \in \mathscr{Q})$ such that

$$\gamma_{1i}(\|x(t)\|) \leq V_i(x(t)) \leq \gamma_{2i}(\|x(t)\|), \tag{3}$$

$$\dot{V}_i(x(t)) \leq -\alpha_i V_i(x(t)), \tag{4}$$

and $\forall \sigma(t_k) = i \in \mathscr{Q}$ and $\forall \sigma(t_k^-) = j \in \mathscr{Q}$ with $j \neq i$, the following inequality holds

$$V_i(x(t_k)) \leq \beta_i V_j(x(t_k^-)). \tag{5}$$

Then system (2) is globally uniformly exponentially stable for any cyclic switching signal with ACDT T_c satisfying

$$T_c \geq \frac{\sum_{i=1}^{m} \ln \beta_i}{\alpha - \gamma^*} \ (\alpha > \gamma^* > 0), \tag{6}$$

where $\alpha = \min\{\alpha_1, \alpha_2, \ldots, \alpha_m\}$, γ^* is the global uniform exponential convergence rate of system (2), and for any $\sigma(t) = i \in \mathscr{Q}$, each nonlinear function f_i satisfies the locally Lipschitz continuous condition with $f_i(0) = 0$. In addition, the other notations are the same as those of system (1).

Proof: Without loss of generality, let $t_0 = 0$ and $\forall t \in [t_k^{N+1}, t_{k+1}^{N+1}) = [t_{Nm+k}, t_{Nm+k+1})$, where the right superscript $N+1 \in \{1, 2, \cdots\}$ of t_k^{N+1} denotes the number of $(N+1)^{th}$ cycles of the cyclic switching sequence Ξ, the right subscript $k \in \{0, 1, \cdots, m-1\}$ of t_k^{N+1} denotes the $(k+1)^{th}$ subsystem in the $(N+1)^{th}$ cyclic period. Then, according to conditions $(4)-(5)$, we have

$$
\begin{aligned}
& V_{\sigma(t)}(x(t)) \\
& \leq \beta_{\sigma(t_k^{N+1})} \exp\{-\alpha_{\sigma(t_k^{N+1})}(t-t_k^{N+1})\} V_{\sigma((t_k^{N+1})^-)}(x((t_k^{N+1})^-)) \\
& = \beta_{\sigma(t_k)} \exp\{-\alpha_{\sigma(t_k)}(t-t_k^{N+1})\} V_{\sigma(t_k^-)}(x((t_k^{N+1})^-)) \\
& \leq \cdots \\
& \leq \prod_{i=1}^{k} \beta_{\sigma(t_i)} \exp\{-\alpha_{\sigma(t_k)}(t-t_k^{N+1}) - \alpha_{\sigma(t_{k-1})}(t_k^{N+1} - t_{k-1}^{N+1}) \\
& \quad - \cdots - \alpha_{\sigma(t_0)}(t_1^{N+1} - t_0^{N+1})\} V_{\sigma(t_0)}(x(t_0^{N+1})) \\
& \leq \cdots \\
& \leq \prod_{i=1}^{k} \beta_{\sigma(t_i)} (\beta_{\sigma(t_{m-1})} \beta_{\sigma(t_{m-2})} \ldots \beta_{\sigma(t_0)})^N \exp\{-\alpha_{\sigma(t_k)}(t- \\
& \quad t_k^{N+1}) - \alpha_{\sigma(t_{k-1})} \triangle t_k^{N+1} - \cdots - \alpha_{\sigma(t_0)} \triangle t_1^{N+1} - \\
& \quad \alpha_{\sigma(t_{m-1})} \triangle t_m^N - \cdots - \alpha_{\sigma(t_0)} \triangle t_1^1\} V_{\sigma(0)}(x(0)),
\end{aligned}
$$

where $\triangle t_j^i = t_j^i - t_{j-1}^i, i \in \{1,2,\ldots,N+1\}$, $j \in \{1,2,\ldots,m\}$, and t_j^i is the j^{th} moment of the i^{th} cycle.

From $\alpha = \min\{\alpha_1, \alpha_2, \ldots, \alpha_m\} = \min\{\alpha_{\sigma(t_0)}, \alpha_{\sigma(t_1)}, \ldots, \alpha_{\sigma(t_{m-1})}\}$ and $\beta_{\sigma(t_{m-1})} \beta_{\sigma(t_{m-2})} \ldots \beta_{\sigma(t_0)} = \beta_m \beta_{m-1} \ldots \beta_1$, we have

$$
\begin{aligned}
& V_{\sigma(t)}(x(t)) \\
& \leq \prod_{i=1}^{k} \beta_{\sigma(t_i)} (\beta_m \beta_{m-1} \ldots \beta_1)^N \exp\{-\alpha(t - t_k^{N+1}) \\
& \quad - \alpha \triangle t_k^{N+1} - \cdots - \alpha \triangle t_1^1\} V_{\sigma(0)}(x(0)).
\end{aligned}
$$

Therefore, when $t \in [t_0^{N+1}, t_1^{N+1}) = [t_{Nm}, t_{Nm+1})$ and under the ACDT switching, we have

$$
\begin{aligned}
& V_{\sigma(t)}(x(t)) \\
& \leq (\beta_m \beta_{m-1} \ldots \beta_1)^N \exp\{-\alpha t\} V_{\sigma(0)}(x(0)) \\
& \leq (\beta_m \beta_{m-1} \ldots \beta_1)^{N_0 + \frac{t}{T_c}} \exp\{-\alpha t\} V_{\sigma(0)}(x(0)) \\
& = \exp\{N_0 \sum_{i=1}^{m} \ln \beta_i\} \exp\{(\frac{\sum_{i=1}^{m} \ln \beta_i}{T_c} - \alpha) t\} V_{\sigma(0)}(x(0)).
\end{aligned}
$$

If there exists a positive constant γ^* $(\alpha > \gamma^* > 0)$, and T_c satisfying $T_c \geq \frac{\sum_{i=1}^{m} \ln \beta_i}{\alpha - \gamma^*}$, we have

$$V_{\sigma(t)}(x(t)) \leq \exp\{N_0 \sum_{i=1}^{m} \ln \beta_i\} \exp\{-\gamma^* t\} V_{\sigma(0)}(x(0)).$$

Thus, $V_{\sigma(t)}(x(t))$ converges to zero with convergence rate γ^* as $t \to \infty$. Then, global uniform exponential stability can be deduced with the aid of (3). ■

Remark 5. Lemma 1 shows the global uniform exponential stability conditions of cyclic switched nonlinear system (2) with all stable subsystems under the methods of ACDT (T_c) and multiple Lyapunov functions. Compared with the average dwell time (τ_a) method proposed in paper [37] for system (2) with arbitrary switching signals (note that system (2) in paper [37] obtains the global uniform asymptotical stability conditions under any switching signal with average dwell time $\tau_a \geq \frac{\ln \mu}{\lambda}$, where parameters $\lambda > 0$ and $\mu > 1$ are the same for all subsystems), we obtain the global uniform exponential stability conditions of system (2) under the cyclic switching signal with ACDT $T_c \geq \frac{\sum_{i=1}^{m} \ln \beta_i}{\alpha - \gamma^*}$ $(\min\{\alpha_1, \alpha_2, \ldots, \alpha_m\} = \alpha > \gamma^* > 0, \forall i \in \mathcal{Q}, \alpha_i > 0, \beta_i > 1)$. In addition, compared with the ACDT method proposed for cyclic switching signals (see automotive transmission [44] and switched flow networks [43] with cyclic switching in practical life), average dwell time method [37] is proposed for arbitrary switching signals and is somewhat ideality and uncommon in practice. Obviously, the stability conditions obtained by the ACDT switching law are the further extension of the stability conditions designed by the average dwell time switching law. In fact, the switched system with cyclic switching signal has infinite switchings, but the switching sequence forms a cycle and the cycle is repeated (see [45]). That is, since the cyclic switched system has infinite switching times, the stability conditions obtained by the average dwell time switching law have some conservative and computational burdens, where the average dwell time is designed for all switching subsystems over the time interval. For example, for a given cyclic switching sequence $\Xi = \{1, 2, 3, 4\}$ with 10 cyclic periods, the ACDT switching law and average dwell time switching law are designed for 10 switching cycles and 40 subsystems, respectively. Obviously, the ACDT (T_c) method designed for the number of switching cycles of the cyclic switching sequence is a further extension of the average dwell time (τ_a) method designed for the number of discontinuous switching of the subsystem. Finally, compared with cycle dwell time method [47] (i.e., for any $k = 1, 2, \cdots$, the cycle dwell time method refers to the cyclic period time T_k of a cyclic switching rule in any k^{th} cycle is not less than a given positive constant T^*) proposed for cyclic switched systems, the ACDT method requires all switching cycles of the cyclic switching sequence Ξ to have a common ACDT, and the actual ACDT method may contain a cyclic switching signal that occasionally has a consecutive cyclic period time separated by less than a given positive constant T^*.

Secondly, based on Lemma 1, we will present the stability conditions of cyclic switched linear systems with ACDT cyclic switching in Case 1.

Theorem 1. Consider cyclic switched linear system (1) with any initial state $x(t_0) \in \mathbb{R}^n$. For each subsystem $i \in \mathscr{Q}$, let $\alpha_i > 0$ and $\beta_i > 1$ be given scalars. Suppose that there exist matrices $P_i > 0$ $(i \in \mathscr{Q})$ such that

$$A_i^T P_i + P_i A_i \leq -\alpha_i P_i, \tag{7}$$

and $\forall \sigma(t_k) = i \in \mathscr{Q}$ and $\forall \sigma(t_k^-) = j \in \mathscr{Q}$ with $j \neq i$, the following inequality holds

$$P_i \leq \beta_i P_j. \tag{8}$$

Then system (1) is globally uniformly exponentially stable for any cyclic switching signal with ACDT T_c satisfying (6) in Lemma 1, where the other notations are the same as those of Lemma 1.

Proof: For each subsystem $i \in \mathscr{Q}$, consider the following Lyapunov function candidate:

$$V_i(x(t)) = x^T(t) P_i x(t), \quad i \in \mathscr{Q} \tag{9}$$

where for each $i \in \mathscr{Q}$, P_i is a positive definite matrix satisfying (7) and (8).

Then, from (1), (4) , (5) and (9), we have

$$
\begin{aligned}
&\dot{V}_i(x(t)) + \alpha_i V_i(x(t)) \\
&= \dot{x}^T(t) P_i x(t) + x^T(t) P_i \dot{x}(t) + \alpha_i x^T(t) P_i x(t) \\
&= x^T(A_i^T P_i + P_i A_i) x(t) + \alpha_i x^T(t) P_i x(t) \\
&= x^T(A_i^T P_i + P_i A_i + \alpha_i P_i) x(t), \\
&V_i(x(t)) - \beta_i V_j(x(t)) \\
&= x^T(t) P_i x(t) - \beta_i x^T(t) P_j x(t) \\
&= x^T(t)(P_i - \beta_i P_j) x(t).
\end{aligned}
$$

Thus, if (7)-(8) hold, system (1) is globally uniformly exponentially stable for cyclic switching signal with ACDT T_c satisfying condition (6) by Lemma 1. ■

Remark 6. Theorem 1 gives the global uniform exponential stability criteria of cyclic switched linear system (1) with all stable submodes under linear matrix inequalities and cyclic switching signal with ACDT $T_c \geq \frac{\sum_{i=1}^{m} \ln \beta_i}{\alpha - \gamma^*}$ ($\min\{\alpha_1, \alpha_2, \ldots, \alpha_m\} = \alpha > \gamma^* > 0, \alpha_i > 0, \beta_i > 1, i \in \mathscr{Q}$).

Case 2: Suppose that there are stable (at least one stable subsystem) and unstable subsystems in the cyclic switched linear (or nonlinear) system. Without loss of generality, suppose that the first r ($1 \leq r < m, r \in \mathbb{Z}^+$) subsystems inside the cyclic switching sequence are stable and the last $m - r$ subsystems inside the cyclic switching sequence are unstable.

Firstly, we will present the stability conditions of cyclic switched nonlinear systems with S-ACDT and U-ACDT switching schemes in Case 2.

Lemma 2. Consider cyclic switched nonlinear system (2) with any initial state $x(t_0) \in \mathbb{R}^n$. For each subsystem $i \in \mathscr{Q}$, let α_i and β_i be given scalars. Suppose that there exist continuous differentiable functions $V_i(x(t)):\ \mathbb{R}^n \to \mathbb{R}{\geq 0}$ ($i \in \mathscr{Q}$), and class $\mathscr{K}_\infty$ functions γ_{1i} ($i \in \mathscr{Q}$) and γ_{2i} ($i \in \mathscr{Q}$) such that

$$\gamma_{1i}(\|x(t)\|) \leq V_i(x(t)) \leq \gamma_{2i}(\|x(t)\|), \tag{10}$$

$$\dot{V}_i(x(t)) \leq \alpha_i V_i(x(t)), \tag{11}$$

and $\forall\sigma(t_k)=i\in\mathscr{Q}$ and $\forall\sigma(t_k^-)=j\in\mathscr{Q}$ with $j\neq i$, the following inequality holds

$$V_i(x(t_k))\leq\beta_i V_j(x(t_k^-)). \tag{12}$$

Then system (2) is globally uniformly exponentially stable for any cyclic switching signal with S-ACDT T_c^- and U-ACDT T_c^+ satisfying

$$\begin{cases} T_c^-\geq\frac{\sum_{i=1}^{r}\ln\beta_i}{\alpha^--\gamma^*} \quad (\alpha^->\gamma^*>0),\\ \frac{\sum_{i=r+1}^{m}\ln\beta_i}{-\alpha^+-\gamma^*}\geq T_c^+>0 \quad (\alpha^+>0,\gamma^*>0),\end{cases} \tag{13}$$

where $\beta_i>1$ $(1\leq i\leq r, i\in\mathbb{Z}^+)$ and $0<\beta_i<1$ $(r+1\leq i\leq m, i\in\mathbb{Z}^+)$; $0>-\alpha^-=\max\{\alpha_1,\alpha_2,\ldots,\alpha_r\}$ $(\alpha_i<0, 1\leq i\leq r, i\in\mathbb{Z}^+)$ and $0<\alpha^+=\max\{\alpha_{r+1},\alpha_{r+2},\ldots,\alpha_m\}$ $(\alpha_i>0, r+1\leq i\leq m, i\in\mathbb{Z}^+)$; T_j^- and T_j^+ correspond to the total running time of all stable and unstable subsystems in the j^{th} $(j\in\mathbb{Z}^+)$ cycle, respectively. The other notations are the same as those of Lemma 1.

Proof: Suppose $t_0=0$, let $\forall t=t_m^N\in[t_{m-1}^N,t_m^N]=[t_{Nm-1},t_{Nm}]$. Then, according to conditions $(11)-(12)$ and Lemma 1, we have

$$\begin{aligned}&V_{\sigma(t)}(x(t))\\&\leq(\beta_{\sigma(t_{m-1})}\beta_{\sigma(t_{m-2})}\ldots\beta_{\sigma(t_0)})^N\exp\{\alpha_{\sigma(t_{m-1})}\triangle t_m^N+\cdots\\&\quad+\alpha_{\sigma(t_0)}\triangle t_1^N\}\cdot\exp\{\alpha_{\sigma(t_{m-1})}\triangle t_m^{N-1}+\cdots+\alpha_{\sigma(t_0)}\triangle t_1^{N-1}\}\cdot\\&\quad\cdots\exp\{\alpha_{\sigma(t_{m-1})}\triangle t_m^1+\cdots+\alpha_{\sigma(t_0)}\triangle t_1^1\}V_{\sigma(0)}(x(0)).\end{aligned}$$

Furthermore, without loss of generality, we consider $\alpha_{\sigma(t_0)}=\alpha_1,\alpha_{\sigma(t_1)}=\alpha_2,$ $\ldots,\alpha_{\sigma(t_{m-1})}=\alpha_m$ and $\beta_{\sigma(t_{m-1})}\beta_{\sigma(t_{m-2})}\ldots\beta_{\sigma(t_0)}=\beta_m\beta_{m-1}\ldots\beta_1$. That is, we have

$$\begin{aligned}&V_{\sigma(t)}(x(t))\\&\leq(\beta_m\beta_{m-1}\ldots\beta_1)^N\exp\{\sum_{j=1}^{N}(\alpha_1\triangle t_1^j+\cdots+\alpha_m\triangle t_m^j)\}V_{\sigma(0)}(x(0)).\end{aligned}$$

From $-\alpha^-=\max\{\alpha_1,\alpha_2,\ldots,\alpha_r\}$ and $\alpha^+=\max\{\alpha_{r+1},\alpha_{r+2},\ldots,\alpha_m\}$, we have

$$\begin{aligned}&V_{\sigma(t)}(x(t))\\&\leq(\beta_m\beta_{m-1}\ldots\beta_1)^N\exp\{\sum_{j=1}^{N}(\alpha^+T_j^+-\alpha^-T_j^-)\}V_{\sigma(0)}(x(0)).\end{aligned}$$

Therefore, under the S-ACDT T_c^- and the U-ACDT T_c^+, we have

$$
\begin{aligned}
&V_{\sigma(t)}(x(t)) \\
&\leq (\beta_r\beta_{r-1}\dots\beta_1)^{N_0^-+\frac{\sum_{j=1}^N T_j^-}{T_c^-}} \cdot (\beta_m\beta_{m-1}\dots\beta_{r+1})^{N_0^++\frac{\sum_{j=1}^N T_j^+}{T_c^+}} \\
&\quad \cdot \exp\{\sum_{j=1}^N (\alpha^+T_j^+ - \alpha^-T_j^-)\}V_{\sigma(0)}(x(0)) \\
&= \exp\{(N_0^- + \frac{\sum_{j=1}^N T_j^-}{T_c^-})(\sum_{i=1}^r \ln\beta_i)\}\exp\{(N_0^+ + \frac{\sum_{j=1}^N T_j^+}{T_c^+}) \\
&\quad \cdot (\sum_{i=r+1}^m \ln\beta_i)\}\exp\{\sum_{j=1}^N (\alpha^+T_j^+ - \alpha^-T_j^-)\}V_{\sigma(0)}(x(0)) \\
&= \exp\{N_0^-(\sum_{i=1}^r \ln\beta_i)\}\cdot\exp\{N_0^+(\sum_{i=r+1}^m \ln\beta_i)\}\cdot\exp\{\frac{(\sum_{i=1}^r \ln\beta_i)(\sum_{j=1}^N T_j^-)}{T_c^-}\} \\
&\quad \cdot \exp\{\frac{(\sum_{i=r+1}^m \ln\beta_i)(\sum_{j=1}^N T_j^+)}{T_c^+}\}\cdot\exp\{\sum_{j=1}^N(\alpha^+T_j^+ - \alpha^-T_j^-)\}V_{\sigma(0)}(x(0)) \\
&= \exp\{N_0^-(\sum_{i=1}^r \ln\beta_i) + N_0^+(\sum_{i=r+1}^m \ln\beta_i)\}\prod_{j=1}^N \exp\{(\alpha^+ + \frac{\sum_{i=r+1}^m \ln\beta_i}{T_c^+})T_j^+ \\
&\quad + (\frac{\sum_{i=1}^r \ln\beta_i}{T_c^-} - \alpha^-)T_j^-\}V_{\sigma(0)}(x(0)).
\end{aligned}
$$

If there exists a positive constant γ^* ($\alpha^- > \gamma^* > 0$), and both T_c^- and T_c^+ satisfying (13), we have

$$
\begin{aligned}
&V_{\sigma(t)}(x(t)) \\
&\leq \exp\{N_0^-(\sum_{i=1}^r \ln\beta_i) + N_0^+(\sum_{i=r+1}^m \ln\beta_i)\}\cdot\exp\{-\gamma^* t\}V_{\sigma(0)}(x(0)).
\end{aligned}
$$

Thus, $V_{\sigma(t)}(x(t))$ converges to zero with convergence rate γ^* as $t\to\infty$. Then, global uniform exponential stability can be deduced with the aid of (10). ■

Remark 7. Lemma 2 shows the global uniform exponential stability conditions of cyclic switched nonlinear system (2) with partly unstable subsystems under the methods of S-ACDT, U-ACDT and multiple Lyapunov functions. Obviously, according to condition (13), Lemma 2 with Case 2 is a further extension of Lemma 1 with Case 1.

Secondly, based on Lemma 2, we will present the stability conditions of cyclic switched linear systems with S-ACDT and U-ACDT switching schemes in Case 2.

Theorem 2. Consider cyclic switched linear system (1) with any initial state $x(t_0) \in \mathbb{R}^n$. For each subsystem $i \in \mathcal{Q}$, let α_i and β_i be given scalars. Suppose that there exist matrices $P_i > 0$ $(i \in \mathcal{Q})$ such that

$$A_i^T P_i + P_i A_i \leq \alpha_i P_i, \tag{14}$$

and $\forall \sigma(t_k) = i \in \mathcal{Q}$ and $\forall \sigma(t_k^-) = j \in \mathcal{Q}$ with $j \neq i$, the following inequality holds

$$P_i \leq \beta_i P_j. \tag{15}$$

Then system (1) is globally uniformly exponentially stable for any cyclic switching signal with S-ACDT T_c^- and U-ACDT T_c^+ satisfying (13) in Lemma 2, where the other notations are the same as those of Lemma 2.

Proof: For each subsystem $i \in \mathcal{Q}$, consider the following Lyapunov function candidate:

$$V_i(x(t)) = x^T(t) P_i x(t), \quad i \in \mathcal{Q} \tag{16}$$

where for each $i \in \mathcal{Q}$, P_i is a positive definite matrix satisfying (14) and (15).

Then, from (1), (11) , (12) and (16), we have

$$\begin{aligned}
&\dot{V}_i(x(t)) - \alpha_i V_i(x(t)) \\
&= \dot{x}^T(t) P_i x(t) + x^T(t) P_i \dot{x}(t) - \alpha_i x^T(t) P_i x(t) \\
&= x^T (A_i^T P_i + P_i A_i) x(t) - \alpha_i x^T(t) P_i x(t) \\
&= x^T (A_i^T P_i + P_i A_i - \alpha_i P_i) x(t), \\
&V_i(x(t)) - \beta_i V_j(x(t)) \\
&= x^T(t) P_i x(t) - \beta_i x^T(t) P_j x(t) \\
&= x^T(t)(P_i - \beta_i P_j) x(t).
\end{aligned}$$

Thus, if (14)-(15) hold, system (1) is globally uniformly exponentially stable for cyclic switching signal with S-ACDT T_c^- and U-ACDT T_c^+ satisfying condition (13) by Lemma 2. ■

Remark 8. Theorem 2 gives the global uniform exponential stability criteria of cyclic switched linear system (1) with partly stable and partly unstable submodes under linear matrix inequalities and cyclic switching signal with S-ACDT T_c^- ($T_c^- \geq \frac{\sum_{i=1}^{r} \ln \beta_i}{\alpha^- - \gamma^*}, \alpha^- > \gamma^* > 0, 0 > -\alpha^- = \max\{\alpha_1, \alpha_2, \ldots, \alpha_r\}, \alpha_i < 0, \beta_i > 1, 1 \leq i \leq r, i \in \mathbb{Z}^+$) and U-ACDT T_c^+ ($\frac{\sum_{i=r+1}^{m} \ln \beta_i}{-\alpha^+ - \gamma^*} \geq T_c^+ > 0, \gamma^* > 0, 0 < \alpha^+ = \max\{\alpha_{r+1}, \alpha_{r+2}, \ldots, \alpha_m\}, \alpha_i > 0, 0 < \beta_i < 1, r+1 \leq i \leq m, i \in \mathbb{Z}^+$). Obviously, Theorem 2 is a further extension of Theorem 1.

4. Numerical Example

In this section, a numerical example is presented to demonstrate the validity of the obtained results.

Example 1. Consider a cyclic switched linear system consisting of four subsystems with

$$A_1 = \begin{bmatrix} 2.5 & -1 \\ 1.5 & 1.3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.1 & -3 \\ 1.7 & 0.17 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -1 & -0.2 \\ 2 & -1.3 \end{bmatrix}, \quad A_4 = \begin{bmatrix} -2.4 & 0.2 \\ -2 & -2 \end{bmatrix}.$$

Our purpose here is to find a cyclic switching signal with S-ACDT and U-ACDT such that the system is stable. Furthermore, to illustrate the advantages of the proposed S-ACDT and U-ACDT based switching, the results of cycle dwell time [47] based switching are also obtained and compared with S-ACDT and U-ACDT switching.

First, one can easily see that the subsystems A_1 and A_2 are unstable, A_3 and A_4 are stable, and the calculation results ($T_c^- \geq 1.22964$ and $0.32850 \geq T_c^+ > 0$) of S-ACDT and U-ACDT switching schemes are obtained by setting the parameters ($\alpha_1 = 0.8, \beta_1 = 0.8, \alpha_2 = 0.3, \beta_2 = 0.9, \alpha_3 = -1.2, \beta_3 = 1.8, \alpha_4 = -1.5$ and $\beta_4 = 1.9$) appropriately. Then, by applying the parameters obtained, we can obtain the state responses and cyclic switching signal with S-ACDT, U-ACDT and cycle dwell time of the cyclic switched linear system, as shown in Figures 2-3. Next, as can be seen from Figure 2 and Figure 3, a cyclic switching sequence with the S-ACDT and U-ACDT switching and the cycle dwell time switching is generated, i.e., the switching signal of the cyclic switched linear system evolves in the following order: subsystem 1 $\rightarrow$ subsystem 2 $\rightarrow$ subsystem 3 $\rightarrow$ subsystem 4 $\rightarrow$ subsystem 1$\rightarrow$ subsystem 2$\rightarrow \cdots$, which means

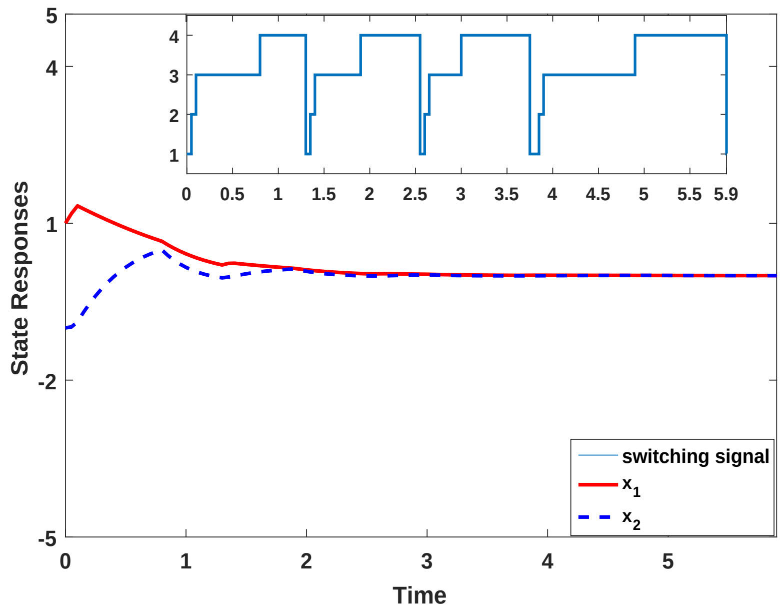

Figure 2: State responses under S-ACDT and U-ACDT.

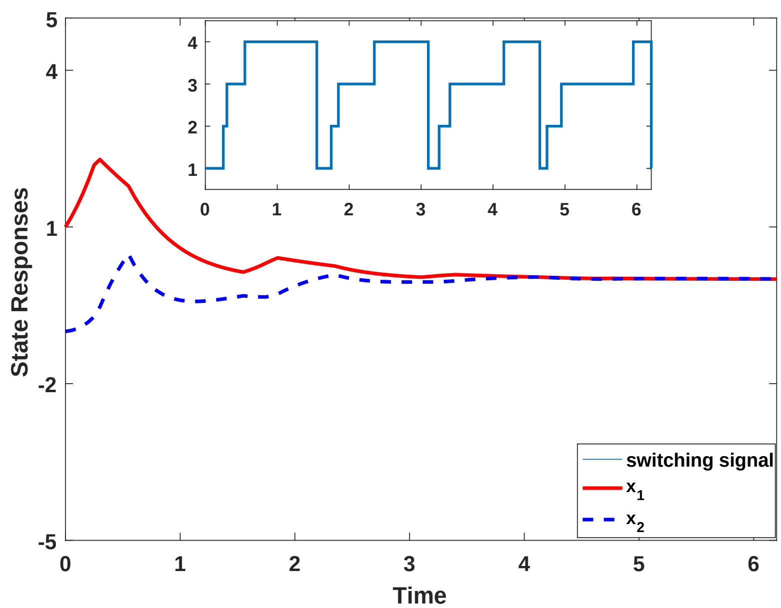

Figure 3: State responses under cycle dwell time [47].

the cyclic switching sequence stays in $\Xi = \{1,2,3,4\}$, where $1,2,3,4$ in the ordinate axis of Figure 2 or Figure 3 represent the subsystem 1, subsystem 2, subsystem 3 and subsystem 4, respectively. In addition, due to the existence of unstable and stable subsystems in the cyclic switched linear system, the curves of state $x_1(t)$ and state $x_2(t)$ of the system show divergence and convergence phenomena when each subsystem is activated in turn.

Second, from Figure 2, the cyclic switched linear system globally exponentially converges to the equilibrium point 0 under the cyclic switching sequence $\Xi = \{1,2,3,4\}$ with convergence rate $\gamma^* = 0.2$ provided the S-ACDT T_c^- and U-ACDT T_c^+ satisfying $T_c^- \geq 1.22964$ and $0.32850 \geq T_c^+ > 0$. Furthermore, one can see from Figure 2 that the state curves of the system under S-ACDT and U-ACDT switching schemes are more smooth. However, Figure 3 shows that although the state responses of the system are ultimately stable under cycle dwell time switching, the state curves of the system will fluctuate sharply. That is, the activation time of the unstable subsystem in Figure 3 under the cycle dwell time switching is longer than that of the unstable subsystem in Figure 2 under S-ACDT and U-ACDT switching schemes.

Finally, as can be seen from Figure 2, the cyclic period time T_k ($T_1^- = 1.2 < 1.22964, T_1^+ = 0.1; T_2^- = 1.15 < 1.22964, T_2^+ = 0.1; T_3^- = 1.1 < 1.22964, T_3^+ = 0.1; T_4^- = 2, T_4^+ = 0.15; k = 1,2,3,4$) can be different in different cycle periods. Obviously, the proposed S-ACDT T_c^- may allows the actual dwell time of all stable subsystems in different cycle periods to be less than a constant 1.22964. However, for Figure 3, according to the definition of cycle dwell time in [47], the dwell time of all stable subsystems in different cycle periods must not be less than T_c^- ($T_c^- \geq 1.22964$). Therefore, the proposed S-ACDT and U-ACDT switching schemes are more flexible and feasible in numerical example than the cycle dwell time switching scheme.

5. Conclusions

The stability problem of cyclic switched linear systems with ACDT (or both S-ACDT and U-ACDT) switching schemes has been studied. Firstly, the ACDT, S-ACDT and U-ACDT concepts have been introduced for the first time. Next, stability conditions for cyclic switched linear (or nonlinear) systems with all stable or partly unstable subsystems have been obtained. Finally, an example has been used to verified the effectiveness of the proposed S-ACDT and U-ACDT schemes. In the future, the problems of stabilization of cyclic switched control systems and stability of cyclic switched time-delay systems deserve attention.

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