Train Stop Deployment Planning in the Case of Complete Blockage: An Integer Linear Programming Model

Shuguang Zhan[1,2], S. C. Wong[3], Qiyuan Peng[1,2], S. M. Lo[4]

[1] *School of Transportation and Logistics, Southwest Jiaotong University, Chengdu 610031, Sichuan, China*
shuguangzhan@my.swjtu.edu.cn; qiyuan-peng@swjtu.edu.cn

[2] *National United Engineering Laboratory of Integrated and Intelligent Transportation, Southwest Jiaotong University, Chengdu, Sichuan, China*

[3] *Department of Civil Engineering, The University of Hong Kong, Hong Kong*
hhecwsc@hku.hk

[4] *Department of Architecture and Civil Engineering, City University of Hong Kong, Hong Kong*
bcsmli@cityu.edu.hk

Abstract

This paper focuses on train stop deployment planning for high-speed railway lines during a major disruption, when a segment is completely blocked for a relatively long period of time. Due to this disruption, trains approaching the disrupted area must be stopped at appropriate stations based on their types for both safety and operational reasons. Stopped trains should not hinder other trains that can continue to run during the disruption. Train stop deployment planning is applied to handle this problem. A mixed integer linear programming (MILP) model is formulated to minimize the number of canceled trains and the total weighted train deviations. The model is tested on a real-world instance, the Beijing-Shanghai high-speed railway, and related lines are implicitly taken into account because of cross-line trains. The results show that our model can obtain good solutions within a relatively short computation time for disruptions lasting no more than 90 minutes.

Keywords: High-speed railway; Segment blockage; Train stop; Integer linear programming

1 Introduction and Motivation

Disruptions to railway systems inevitably occur due to various external and internal factors, such as bad weather conditions, a faulty switch on a busy track, a broken signaling system or rolling stock, or damaged overhead wires. These disruptions require trains to deviate from their original timetable, which negatively affects passenger service.

Immediately after a disruption occurs, dispatchers must control affected trains and update the original timetable to a disposition timetable based on the latest information. In a serious disruption, in which some track segments are temporary blocked, trains that need use the disrupted area cannot continue their journey. Dispatchers must immediately and appropriately handle trains that are approaching the disrupted area. Two strategies are typically used to address these situations. The first is to short-turn disrupted trains in an appropriate station adjacent to the disrupted area (Nielsen et al. [2012], Louwerse and Huisman [2013], Veelenturf et al. [2016], Ghaemi et al. [2017] and Ghaemi et al. [2018]). This train rescheduling strategy is used in a number of European countries, such as the Netherlands, where railway lines are relatively short and seat reservations are not used. However, in other countries, such as China and Japan, short-turning is not favored for long distance high-speed railway lines. Instead, trains approaching the disrupted area must stop and wait at suitable stations until the end of the disruption; this is called the disrupted train service waiting strategy (Hirai et al. [2009] and Zhan et al. [2015]). Railway systems that use this strategy usually have rather long high-speed railway lines and use seat reservations. Thus, it is often difficult for passengers to find other ways to continue their journey if trains short-turn mid-trip, and it is not convenient for passengers to transfer from one train to another when seat reservations are used. In this study, we focus on long-distance high-speed railway lines with seat reservations, and use the disrupted train service waiting strategy for disrupted train service. Under this strategy, the issue of how to stop disrupted trains at suitable stations to wait for the disruption to end (*train stop deployment planning*, for short)is both critical and complex (Hirai et al. [2009]). Until now, train stop deployment planning during a serious disruption has mainly been handled manually by dispatchers, which is quite challenging. In addition, to the best of our knowledge, few previous papers have investigated this important issue. These considerations motivate our research.

In a situation of serious disruption, in which all tracks in a segment are blocked (also called complete blockage), train rescheduling on a long-distance high-speed railway line with dense traffic is complicated. Trains in both directions (inbound and outbound trains) must be rescheduled efficiently. In a short-turning strategy, only trains on one side of the disrupted line are considered, as train rescheduling on the other side is similar, see Louwerse and Huisman [2013] and Veelenturf et al. [2016] for examples. On a double-track high-speed railway line, inbound (outbound) trains use inbound (outbound) tracks in each segment normally. In a train service waiting strategy, trains in both directions do not interrupt each other if we assume that inbound (outbound) trains are only allowed to use inbound (outbound) sidings in each station. Thus, train services in each direction can be rescheduled independently. That is, we only reschedule train services in one direction; see Zhan et al. [2015]. However, it is likely that trains in both directions will share sidings at each station in a complete blockage, which is helpful for reducing the impact of disruption on train operations. Accordingly, we allow trains in both directions to share sidings at a station in this study. Thus, trains in both directions, including local trains and cross-line trains, must be rescheduled simultaneously.

We mainly focus on train stop deployment planning in a seriously disrupted situation, which is a critical step for train rescheduling. As soon as a complete blockage occurs, railway dispatchers must stop trains approaching the disrupted area. Because various types of trains run on a high-speed railway network, dispatchers have to decide which train should stop at which station, both to ensure safety and improve operational efficiency. For example, lower-speed trains should not hinder higher-speed trains, and local trains that can continue to run should not be blocked by trains that will be stopped for a long time. We discuss train stop deployment planning in more detail in the problem description section. A new mixed integer linear programming (MILP) model is formulated to solve the train stop deployment planning problem at a macroscopic level. A real-world high-speed railway line in China, the Beijing-Shanghai high-speed railway line, and the other lines connected to this line are used to test our model.

This paper's contributions are twofold. First, we formulate a MILP model to handle real-time train stop deployment planning on long high-speed railway lines during a complete blockage, taking local trains and cross-line trains into account. Second, unlike most previous studies, which only considered part of trains in a complete blockage(trains running in one direction, or trains running on one side of the location of the disruption), our model considers trains in both directions simultaneously, because trains in both directions can share station tracks or even main lines during a serious disruption. To this end, our results can be used to reduce the impact of disruptions on train operations.

The remainder of this paper is organized as follows. Section 2 gives an overview of the related literature. In Section 3, we describe the problem in detail. In Section 4, a MILP model is formulated to model train stop deployment planning during disruptions. Section 5 presents the computational results based on a series of disruption scenarios. Conclusions and future research are discussed in Section 6.

2 Literature review

Train rescheduling has recently become an active area for researchers. Numerical research has been conducted in this area; see, for example, recent surveys by Cacchiani et al. [2014], Corman and Meng [2015], and Fang et al. [2015].

Unforeseen incidents caused by internal or external factors can be classified as disturbances or disruptions, according to their degree of influence on the railway system (Cacchiani et al. [2014]). A disturbance is a relatively small perturbation, such as a train being delayed by several minutes due to longer passenger boarding time at a station. However, a disruption caused by a serious accident, such as a locomotive breakdown or blockage on a railway line, may significantly affect the railway system. According to Cacchiani et al. [2014], most previous research on real-time train rescheduling has focused on disturbances. In addition, most research has focused on rescheduling trains at the microscopic level instead of a macroscopic level. At the microscopic level, more details of the railway network and train information need to be considered. For example, a railway line must be divided into block sections instead of sections between two successive stations, which is the approach normally used in macroscopic train rescheduling. For more information about microscopic and macroscopic train rescheduling, we refer to Cacchiani et al. [2014]. D'Ariano et al. [2008] and Corman et al. [2010, 2011a, 2012, 2014] reschedule trains during disturbances at a microscopic level. These studies develop a real-time traffic management system, called ROMA, based on the alternative graph model. However, because ROMA considers railway infrastructure at a microscopic level, solving large real-world instances is time consuming. Another research area for small railway disturbances is railway delay management. The railway delay management model is utilized to determine which connections should be maintained during small delays. Several studies have been conducted to solve this problem at the macroscopic level; see Schöbel [2007, 2009], Schachtebeck and Schöbel [2010], and Dollevoet et al. [2012].

Fewer studies have considered real-time train rescheduling during a disruption (Cacchiani et al. [2014]). Corman et al. [2011b] reschedule trains during a situation in which one line of a segment is unavailable for a time at a microscopic level. They consider a network that is divided into several dispatching areas. In their experiments, they compare the solutions obtained by centralized and distributed dispatching strategies, and find that the latter is more effective. Meng and Zhou [2011] study meet-pass plans for trains on a single-track railway line when a track section is blocked for a relatively short time.

At the macroscopic level of disruption management, Brucker et al. [2002] reschedule trains when one track of a railway segment is unavailable due to construction. They determine the sequence of trains in both directions passing the construction segment to minimize total train lateness. They assume that the sequence of trains in each direction running outside the construction segment is fixed; however, in a complex railway system with trains of various speeds, the sequence of trains running in one direction will not be fixed. Zhan et al. [2016] also focus on train rescheduling given a partial blockage. They introduce a MILP model to determine the order of trains traveling in opposite directions that must share the only available track in the blocked segment. In their model, the order of trains is not fixed in advance, and trains can change their order to reduce the influence of the disruption on train operations.

Albrecht et al. [2013] study train rescheduling at a macroscopic level for a disruption in which track maintenance occurs. They consider both train operations and track maintenance by regarding each track maintenance task as a pseudo train. They apply a problem space search meta-heuristic method to quickly generate disposition timetables to minimize the total delay for trains and maintenance, and test this approach on a single-track rail network in Queensland, Australia.

Louwerse and Huisman [2013] adjust a train timetable to form a disposition timetable for both partial and complete segment blockages. They describe the railway system using an event-activity graph, and formulate an integer programming model to minimize the number of canceled trains and the total train delay while taking train balance into account. Veelenturf et al. [2016] extend the model in Louwerse and Huisman [2013] to consider railway station capacity in each station and explicitly integrate rolling stock circulation. Furthermore, unlike Louwerse and Huisman [2013], they reschedule trains for the entire period of disruption, including the transfer from the original timetable to the disposition timetable at the occurrence of the disruption and from the disposition timetable to the original timetable at the end. As previously mentioned, both Louwerse and Huisman [2013] and Veelenturf et al. [2016] allow trains to short-turn before they arrive at the disrupted area instead of stopping trains to wait for the disruption to end.

Ghaemi et al. [2017, 2018] focus on the train short-turning strategy at the microscopic and macroscopic levels. In their studies, trains approaching the blocked location must short-turn at intermediate stations before the disrupted area. Due to limited station capacity, trains may need to short-turn several stations before the disrupted area. Therefore, their model determines which trains should short-turn at which station and then run as another train in the opposite direction. However, not all of the world's railway systems use the short-turning strategy to deal with disruptions.

For some railway systems, it is preferable to let disrupted trains stop at intermediate stations to wait for the disruption to end. To the best of our knowledge, few papers have investigated in detail how to stop trains during a blockage. Hirai et al. [2009] consider how to stop trains at appropriate stations during a disruption in which the line is blocked by an accident for a long time. This decision-making process is called *train stop deployment planning*. They formulate a petri-net-based integer programming model to solve the train stop deployment planning problem, but because they describe the railway network at a microscopic level, and a large number of parameters are required, the model can become very large. Hence, it cannot solve large real-world problems. Zhan et al. [2015] reschedule trains during a blockage, and partially solve the train stop deployment planning problem. However, they only handle long-distance trains in one direction, and their approach cannot handle local and cross-line trains.

3 Problem description

This paper focuses on high-speed train rescheduling on double-track railway lines given a complete blockage. Because of the complete blockage of a segment, trains cannot pass the disrupted area and must deviate from the original timetable. Railway dispatchers must handle all of the disrupted trains efficiently. The train rescheduling process during a complete disruption can be divided into two stages. The first is the train handling process during the disruption, also called *train stop deployment planning*; the second is train rescheduling after the disruption ends. This paper focuses on the first stage, which is critical for the whole train rescheduling problem. In the following, we describe our main problem based on the simple example shown in Figure 1.

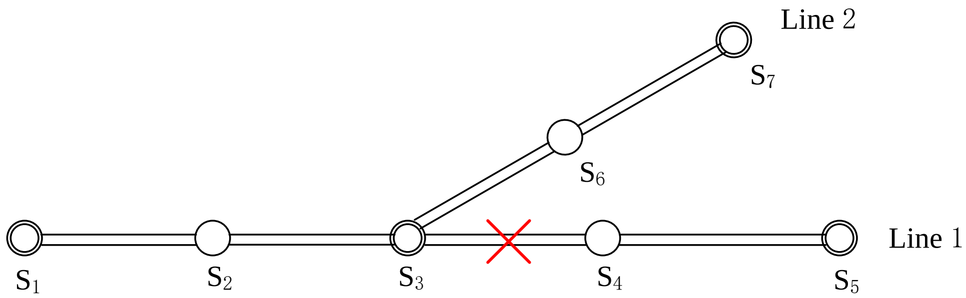

Figure 1: A simple example of a disrupted railway network

In Figure 1, there is a small high-speed railway network consisting of two high-speed railway lines. Line 1 is from station S_1 to S_5. Line 2 consists of stations S_3, S_6, and S_7. Lines 1 and 2 connect with each other at Station S_3. Each circle denotes a station, and the stations denoted by double circles are large stations with shunting yards, which can be used as origins and destinations for some trains. A disruption occurs on line 1 in the segment between stations S_3 and S_4, shown by the red cross. Because of the disruption, no trains on line 1 can pass the disrupted area until the end of the disruption.

Immediately after the disruption occurs, dispatchers must stop any trains approaching the disrupted area at appropriate stations. Because each station has a limited number of available tracks (also called *station capacity*) and because each track can be occupied by at most one train at a certain time, the number of disrupted trains stopped at each station, both inbound and outbound, cannot exceed station capacity. In addition, medium-speed trains should not hinder high-speed trains. Furthermore, some trains run on other high-speed lines that diverge from the disrupted high-speed railway line ahead of the disrupted area, e.g., trains may run from station S_1 to station S_3 on line 1 and then on line 2 to station S_7. Such trains can continue their journeys on line 2 during the disruption. When dispatchers consider stopping disrupted trains, these running trains must not be hindered by the stopped trains, so that delay on one line does not spread to connecting lines. Finally, local trains whose destinations are ahead of the disrupted area, e.g., trains running from station S_1 to station S_3, can continue to run to their destinations during the disruption. Thus, stopping long-distance trains should not hinder these local trains. Specifically, long-distance trains running from station S_1 to station S_5 must stop at stations before the disrupted area to wait for the disruption to end, and cannot hinder local trains that run from station S_1 to station S_3.

In the train stop deployment planning stage, dispatchers must determine which trains should be stopped and which can continue to run, and which station to send trains to, with the goal of minimizing the total impact of the disruption on train operations, and preventing conflicts among trains. We refer to Hirai et al. [2009] for more details about the *train stop deployment planning problem.*

4 Model formulation

4.1 Assumptions

In our model, the following assumptions are made. However, some of them can be altered.

- Each track in a station can be used by trains to dwell or pass. Trains from each track in a segment can enter any station track.
- Trains that have entered the disrupted area when a disruption occurs can continue according to the original timetable. Trains already present in the disrupted area when the disruption occurs should be carefully handled based on detailed information. If they have passed the disrupted area, they can continue their journey; otherwise, they cannot pass. Because we reschedule trains at a macroscopic level, we make this assumption to ensure that these trains have passed the disrupted area at the start of the disruption.
- Train services can only be canceled if they are scheduled to depart from their origin after a disruption occurs. After they have departed, they must continue until they arrive at their destination, possibly with significant delays.

4.2 Basic model

The train stop deployment planning problem is also part of the train rescheduling problem, and it can be formulated using an event-activity network; see, for example, Zhan et al. [2015, 2016] for instance. An event-activity network can be denoted by a directed graph $N = (E, A)$, where E is the set of events, and A is the set of activities. An event $e \in E$ consists of the arrival or departure of a train at a station. Accordingly, the set of events E can be further divided into two subsets: $E^{arr} \subset E$, which contains all of the arrival events, and $E^{dep} \subset E$, which contains all of the departure events. Furthermore, let E_s^{arr} and E_s^{dep} be the subsets of the arrival events and departure events at station s, respectively. An activity $a \in A$ connects two events. Activities can be divided into train activities and headway activities. A train activity, $a \in A_{train}$, can either be a running activity $a \in A_{run}$ between a departure event from a station and an arrival event at the successive station, or a dwell activity $a \in A_{dwell}$ between an arrival event and a departure event of the same train at the same station. A headway activity, $a \in A_{head}$, can either be a segment headway activity $a \in A_{head}^{track}$ that expresses the headway time between two trains running on the same track, or a station headway activity $a \in A_{head}^{station}$ that denotes the minimum headway time between the departure and arrival of two consecutive trains on the same track at a station.

Let T be the set of trains, which can be divided into two subsets T^{up} and T^{down}. T^{up} is the subset that contains all upside trains (inbound trains), while the subset T^{down} contains all downside trains (outbound trains). Let S be the set of stations and Seg the set of segments. For an event $e \in E$, parameter t_e is the train corresponding to event e, and parameter q_e is the scheduled time of event e in the original timetable. In addition, parameters μ_e^+ and μ_e^- are the penalties for a time unit of tardiness and a time unit of earliness for train event $e \in E$. For each train t, parameter γ_t is the penalty for canceling train t.

Stations s_t^o and s_t^d are the origin and destination stations of train t. Assume that a disruption starts at time H_{dis}^{start} and ends at time H_{dis}^{end} in the segment between station s_k and station s_l, $(s_k, s_l) \in Seg$, (see Figure 2).

Delays for arrival events and departure events differ in two ways. First, passengers care more about arrival delays. This means that an arrival delay usually has a higher penalty than a departure delay. Second, trains are allowed to arrive earlier than scheduled, but they cannot depart before the scheduled time. If a train arrives earlier than scheduled, it causes a deviation from the original timetable, and thus earliness should be minimized. We therefore consider the deviation of arrival events and departure events separately.

For each event $e \in E_{train}$, we define a decision variable D_e^+ as the delay (tardiness) of event e. For each arrival event $e \in E^{arr}$, we define a decision variable D_e^- as the earliness of arrival event e. The decision variable x_e is the real time of event e in the disposition timetable. For each train $t \in T$, we introduce a binary variable y_t, which is defined as follows:

$$y_t = \begin{cases} 1 & \text{if train } t \in T \text{ is canceled} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Canceled train services are moved beyond the end of the day, which prevents canceled trains from hindering other trains or each other; see constraint (3). Parameter M_1 equals the time duration of a whole day, i.e., 1,440 min. Using the notations above, we develop the following basic model for train operations. Some additional train operational constraints are added in Subsection 4.3, and constraints on stopping trains are explained in detail in Subsection 4.4.

$$\min \sum_{t \in T} \gamma_t y_t + \sum_{e \in E} \mu_e^+ D_e^+ + \sum_{e \in E^{arr}} \mu_e^- D_e^- \tag{2}$$

subject to

$$2M_1 y_{t_e} - M_1 \le x_e - q_e \le M_1 \qquad \forall e \in E, t_e \in T \tag{3}$$

$$x_e \ge q_e \qquad \forall e \in E^{dep} \tag{4}$$

$$D_e^+ \ge x_e - q_e - M_1 y_{t_e} \qquad \forall e \in E, t_e \in T \tag{5}$$

$$D_e^- \ge q_e - x_e \qquad \forall e \in E^{arr} \tag{6}$$

$$x_e = q_e \qquad \forall e \in E : q_e < H_{dis}^{start} \tag{7}$$

$$y_{t_e} = 0 \qquad \forall t_e \in T, e \in E_{s_{t_e}^o}^{dep} : q_e \le H_{dis}^{start} \tag{8}$$

$$y_t \in \{0, 1\} \qquad \forall t \in T \tag{9}$$

$$x_e, D_e^+, D_e^- \ge 0 \qquad \forall e \in E \tag{10}$$

Objective (2) specifies that, we minimize train cancelations and the total weighted train deviation (tardiness/earliness). Constraint (3) ensures that the events of the canceled trains occur at time $q_e + M_1$, thus moving canceled trains beyond the end of the day. Constraint (4) prevents trains from departing from a station before the scheduled time. Constraint (5) indicates that no delays for events of canceled trains are considered, and the delay for event e of a non-canceled train is at least $x_e - q_e$. Constraint (6) expresses the earliness of the arrival event e. Constraint (7) ensures that trains run as scheduled before the occurrence of the disruption. Constraint (8) indicates that trains departing from their origin before the occurrence of the disruption are not canceled. Constraints (9) and (10) specify the domains of the variables.

4.3 Additional train operational constraints

As mentioned, train stop deployment planning is a critical element in solving the whole train rescheduling problem. To ensure safety and prevent conflicts among trains, the minimum running and dwell time, headway between trains, and overtaking constraint should be respected; these constraints are similar to those in train rescheduling problems. We discuss these three constraints in the following sub-sections.

4.3.1 Single-train precedence constraints

For a single train, the minimum running time in each segment and minimum dwell time in each station that it passes should be respected. The minimum duration of such a train activity a is L_a. If $a \in A_{run}$ is a running activity, L_a indicates the minimum running time, otherwise, L_a is the minimum dwell time. Then, a single-train precedence constraint can be modeled as follows.

$$x_f - x_e \geq L_a \quad \forall a = (e, f) \in A_{train} \tag{11}$$

Constraint (11) indicates that the minimum running time of a train in each segment and the minimum dwell time in each intermediate station should be respected. Here f is the successive event of event e corresponding to the same train, and $a = (e, f)$ is a train activity.

4.3.2 Headway constraints between trains

A railway track is divided into a sequence of blocks by signals, and each block can be occupied by at most one train at a time. The headway time between two consecutive trains on any block cannot be less than the minimum headway time. To fulfill this requirement, we require that the headway for the complete segment be equal to the largest headway between the blocks.

Due to the different speeds of trains, especially during a disruption, it is common for a fast train to overtake a slow train at an intermediate station. To keep track of the order of two events on a track, we first introduce the set of headway activities A_{head}^{track}. An activity $a = (e, f) \in A_{head}^{track}$ corresponds to two events $e \in E$ and $f \in E$, with $t_e \neq t_f$, that are either both arrival events or both departure events in the same direction at the same station. We introduce a binary decision variable λ_a as defined in equation (12).

$$\lambda_a = \begin{cases} 1 & \text{if event } e \text{ takes place before event } f \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

The following two constraints ensure that λ_a takes the correct value, and that the headway times between two consecutive trains in the same direction are respected. Note that we consider various types of trains, and cross-line trains that run through different high-speed lines. Two trains that arrive at the same junction station from different lines in the same direction are not allowed to be too close to each other. This is also the case for trains that depart from the same junction station to different lines in the same direction. We call this the *junction station constraint*. This constraint is summarized by

Constraints (13) and (14).

$$\lambda_a + \lambda_{a'} = 1 \qquad \forall a = (e,f) \in A^{track}_{head} \wedge a' = (f,e) \in A^{track}_{head} \tag{13}$$

$$x_f - x_e + M_2 \times (1 - \lambda_a) \geq L_a \qquad \forall a = (e,f) \in A^{track}_{head} \tag{14}$$

In Constraint (13), "$\wedge$" means that both the conditions before and after it are fulfilled. Constraint (13) ensures that event e takes place before or after event f. Constraint (14) indicates that, if event $e \in E$ takes place after event $f \in E$, then $x_f - x_e < 0$, and thus λ_a must be equal to 0. Furthermore, if event $e \in E$ takes place before event $f \in E$, then $\lambda_a = 1$ due to (13), and hence the minimum headway time L_a between events e and f is respected.

Note that the parameter M_2 is a large positive number. Because we move events $e \in E$ corresponding to canceled trains beyond the time horizon at time $q_e + M_1$, the value of M_2 should be larger than that of M_1. Otherwise, when $\lambda_a = 0$, constraint (14) cannot always be satisfied.

4.3.3 Overtaking constraints

Note that we consider one-way double-track high-speed lines, and each track is used for trains running in one direction. Trains are unable to overtake each other on the same track in a segment, which means that the departure order of two trains from a station must be the same as the arrival order at the successive station. To this end, we introduce order activity pairs.

Between two trains in a segment (s, s'), where s' is the successive station of s, there are two headway activities: $a = (e, f)$ and $a' = (e', f')$, where $e, f \in E^{dep}_s$ and $e', f' \in E^{arr}_{s'}$, $t_e = t_{e'}$ and $t_f = t_{f'}$. Both activities a and a' are headway activities, $a, a' \in A^{track}_{head}$, but a denotes the departure headway while a' denotes the arrival headway of the same train. Then the pair (a, a') is an order activity pair, and B is defined as the set of all order activity pairs. To prohibit overtaking in a segment, we introduce Constraint (15).

$$\lambda_a - \lambda_{a'} = 0 \quad \forall (a, a') \in B \tag{15}$$

By Constraints (13) and (14), if $x_e > x_f$, then $\lambda_a = 0$, otherwise $\lambda_a = 1$. Similarly, if $x_{e'} > x_{f'}$, then $\lambda_{a'} = 0$, otherwise $\lambda_{a'} = 1$. Thus $\lambda_a = \lambda_{a'}$ indicates that trains t_e and t_f arrive at station s' in the same order that they departed from station s. That is, trains t_e and t_f do not overtake each other between stations s and s'.

4.4 Train stop deployment planning during a disruption

As mentioned earlier, train stop deployment planning is a vital stage of the train rescheduling process. This main goal of this stage is to determine how to stop trains approaching the disrupted area to prevent them from hindering trains that can still continue to run during the disruption. Specifically, the stopped trains should not block trains that can continue to run on other related lines during the disruption. In the following, we formulate how to stop disrupted trains.

4.4.1 Definition of directly influenced train services

To handle disrupted train services during the disruption, we must first identify all of the train services that have been disrupted. We assume that there is no way of predicting the occurrence of or end to the disruption. Hence, the train services that are directly influenced by the disruption are those that *fully* or *partly* run during the disruption; see Figure 2.

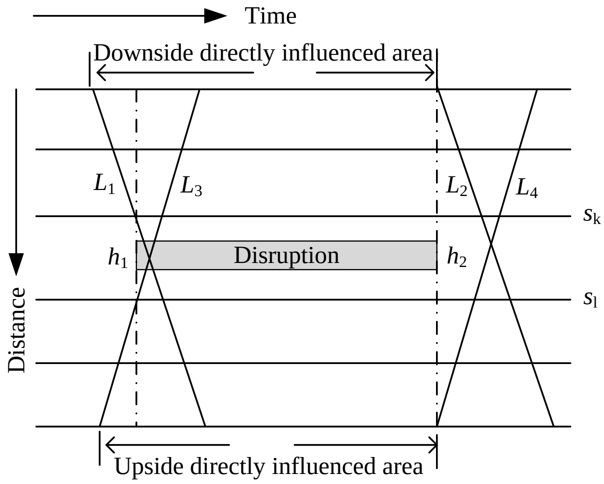

Figure 2: Area that is directly influenced by the disruption

We define T_{dis} as the subset of train services that are influenced directly by the disruption, which can be further divided into a directly influenced downside train service set T_{dis}^{down} and a directly influenced upside train service set T_{dis}^{up}. Assume that a disruption occurs at time h_1 and lasts until time h_2 in the segment between stations s_k and s_l; see Figure 2. In this figure, train services running in the area between lines L_1 and L_2 belong to T_{dis}^{down}, while train services running in the area between lines L_3 and L_4 belong to T_{dis}^{up}. Note that no anticipation of the end of the disruption is allowed. Therefore, all trains that start during the disruption period are regarded as directly influenced trains even if the disruption has already ended by the time of their planned arrival at the disrupted area based on the estimation.

To formulate T_{dis} in detail, we further divide T_{dis}^{down} and T_{dis}^{up} into two subsets each. That is, T_{dis}^{down} is divided into $T_{dis}^{down,short}$ and $T_{dis}^{down,long}$, and T_{dis}^{up} is divided into $T_{dis}^{up,short}$ and $T_{dis}^{up,long}$. Here $T_{dis}^{down,short}$ and $T_{dis}^{up,short}$ include all of the disrupted downside and upside train services that do not need to pass the disrupted area, as they arrived at their destinations before entering the disrupted segment. $T_{dis}^{down,long}$ and $T_{dis}^{up,long}$ include all of the directly influenced downside and upside train services that need to pass the disrupted area. Stations are numbered along the downside direction.

For a downside train service $t \in T^{down}$, event $e \in E_{s_t^o}^{dep}$ is the original departure event of train service t, event $e' \in E_{s_t^d}^{arr}$ is the final arrival of train service t, and event $e'' \in E_{s_k}^{dep}$ is the departure event of train service t at station s_k. Accordingly, the subsets of $T_{dis}^{down,short}$ and $T_{dis}^{down,long}$ can be formulated as follows:

$$T_{dis}^{down,short} = \{t \in T^{down} : e \in E_{s_t^o}^{dep} : q_e \leq h_2 \wedge e' \in E_{s_t^d}^{arr} : q_{e'} \geq h_1\} \quad \forall s_t^o, s_t^d \in S : s_t^o < s_t^d \leq s_k \quad (16)$$

$$T_{dis}^{down,long} = \{t \in T^{down} : e \in E_{s_t^o}^{dep} : q_e \leq h_2 \wedge e'' \in E_{s_k}^{dep} : q_{e''} \geq h_1\} \quad \forall s_t^o, s_t^d \in S : s_t^o \leq s_k < s_t^d \quad (17)$$

In equation (16), $T_{dis}^{down,short}$ includes all of the short-distance trains that depart from their origin before the end of the disruption and arrive at their destination after the occurrence of the disruption; similarly, in equation (17), $T_{dis}^{down,long}$ includes all of the long-distance trains that depart from their origin before the end of the disruption and are scheduled to depart from station s_k after the occurrence of the disruption.

All of the downside train services that are directly influenced by the disruption can be formulated as follows:

$$T_{dis}^{down} = T_{dis}^{down,short} \cup T_{dis}^{down,long} \tag{18}$$

Similarly, for an upside train service $t \in T^{up}$, let $e \in E_{s_t^o}^{dep}$ be the original departure event of train service t, $e' \in E_{s_t^d}^{arr}$ be the final arrival of train service t, and $e'' \in E_{s_l}^{dep}$ be the departure event of train service t at station s_l. Then, the subsets of $T_{dis}^{up,short}$ and $T_{dis}^{up,long}$ can be formulated as follows:

$$T_{dis}^{up,short} = \{t \in T^{up} : e \in E_{s_t^o}^{dep} : q_e \le h_2 \wedge e' \in E_{s_t^d}^{arr} : q_{e'} \ge h_1\} \quad \forall s_t^o, s_t^d \in S : s_t^o > s_t^d \ge s_l \tag{19}$$

$$T_{dis}^{up,long} = \{t \in T^{up} : e \in E_{s_t^0}^{dep} : q_e \le h_2 \wedge e'' \in E_{s_l}^{dep} : q_{e''} \ge h_1\} \quad \forall s_t^o, s_t^d \in S : s_t^o \ge s_l > s_t^d \tag{20}$$

The meaning of equation (19) is similar to that of equation (16), while the meaning of equation (20) is similar to that of equation (17).

Then, all the upside train services that directly influenced by the disruption are formulated as follows:

$$T_{dis}^{up} = T_{dis}^{up,short} \cup T_{dis}^{up,long} \tag{21}$$

Based on the preceding formulations, all of the train services that are directly influenced by the disruption can be depicted by equation (22):

$$T_{dis} = T_{dis}^{down} \cup T_{dis}^{up} \tag{22}$$

4.4.2 Formulation of train stop deployment planning

Each directly influenced train $t \in T_{dis}$ has to take one of the following four actions to avoid entering the disrupted area.

- Train t may be canceled if it is scheduled to depart from its origin after the disruption has occurred.
- Train t may have to stop and wait at an intermediate station until the end of the disruption.
- Train t may arrive at its destination if it is a short-distance train, $t \in T_{dis}^{down,short} \cup T_{dis}^{up,short}$.
- Train t may wait outside station s if no station track is available there if train t is running in segment (s', s) at the time the disruption occurs.

(1) Train stopping and waiting at an intermediate station

To determine whether train t needs to stop at station s to wait for the disruption to be resolved, for each train $t \in T_{dis}$ and each station $s \in S$ ($s \le s_k$ for downside train services and $s \ge s_l$ for upside train services), we define a binary variable $w_{t,s}$ according to equation (23).

$$w_{t,s} = \begin{cases} 1 & \text{if train } t \text{ stops at station } s \text{ to wait until the end of the disruption} \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

For each train $t \in T_{dis}$, if it stops at a certain intermediate station s to wait for the disruption to be resolved, it has to dwell there until the disruption is over before it can depart, because no anticipation of the end of the disruption is allowed, as we do not know the exact time the disruption will really end. However, if we allow trains to depart slightly before the end of the disruption, the delay to the trains may decrease, but the schedule is not robust; see Zhan et al. [2016] for more details. We define $e \in E_s^{dep}$ as the departure event of train t_e from station s. We have the following constraints for upside and downside train $t_e \in T_{dis}$.

$$x_e > H_{dis}^{end} * w_{t_e,s} \quad \begin{cases} \forall e \in E_s^{dep}, s \in S : s \leq s_k & \text{if } t_e \in T_{dis}^{down} \\ \forall e \in E_s^{dep}, s \in S : s \geq s_l & \text{if } t_e \in T_{dis}^{up} \end{cases} \tag{24}$$

We define a station set $\tilde{S} \subset S$ that includes all of the intermediate stations. Because of the first assumption in Section 4.1, a train can uses any track at a station and trains in both directions can share the same track at a station. This can help to model the station capacity constraint as follows. During the disruption, the number of trains that dwell at station s, $s \in \tilde{S}$, ($s \leq s_k$ for downside trains and $s \geq s_l$ for upside trains) waiting for the disruption to end cannot exceed the capacity of station s. Parameter C_s is introduced to denote the capacity of station s. Furthermore, to guarantee that stopped trains do not block other trains that can continue to run, at least one residual track should be available at each intermediate station. Thus the station capacity constraint for trains that dwell at an intermediate station s until the end of disruption is as follows:

$$\sum_{t \in T_{dis}} w_{t,s} \leq C_s - 1 \quad \begin{cases} \forall s \in \tilde{S} : s \leq s_k & \text{if } t \in T_{dis}^{down} \\ \forall s \in \tilde{S} : s \geq s_l & \text{if } t \in T_{dis}^{up} \end{cases} \tag{25}$$

(2) Trains waiting outside a station

It is possible that trains running in a segment when disruption occurs cannot dwell at the successive station because of a lack of station capacity. We assume that such trains wait outside the successive station until the end of the disruption. Note that only the directly influenced trains that are running on the disrupted line when the disruption occurs may need to wait outside the successive station because of a lack of station capacity. Trains that dwell at stations when the disruption occurs can stop there until the end of the disruption, but they are not allowed to stop in the segment. To this end, we define a train service subset $T_{dis}^{run} \subset T_{dis}$ for trains that are running in the segment when the disruption occurs. Train service set T_{dis}^{run} is further divided into downside train service set $T_{dis}^{run,down}$ and upside train service set $T_{dis}^{run,up}$, respectively.

In principle, it is unusual that a train running in a segment when the disruption occurs cannot be accommodated at successive stations due to a lack of station capacity, because the number of trains running simultaneously in a segment is relatively limited due to the headway constraint. Furthermore, a relatively large number of tracks are available at each station for trains when a train is allowed to enter any station track. However, we take this situation into account to ensure that our model can handle it.

Because short-distance and cross-line trains can continue to run during the disruption, we first need to consider whether a long-distance train has to wait outside a station. A binary variable $O_{t,s}$ is introduced to indicate whether train $t \in T_{dis}^{run} \bigcap (T_{dis}^{down,long} \cup T_{dis}^{up,long})$ has to wait outside station s. The definition of $O_{t,s}$ is given in equation (26).

$$O_{t,s} = \begin{cases} 1 & \text{if train } t \text{ waits outside station } s \text{ for the disruption to end} \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

As mentioned previously, if a train t is running in segment (s', s'') when the disruption occurs, where station s'' is the successive station of s' for train t, it may only wait outside station s''. That is, train t is not allowed to wait outside any other station. This is formulated in equation (27).

$$O_{t,s} = 0 \quad \forall t \in T_{dis}^{run}, s \in \tilde{S} : s \neq s'' \tag{27}$$

To check whether a long-distance train has to wait outside a station, we need to find out whether there is a residual station capacity available in the successive stations that are ahead of the disruption location. Specifically, for train $t_f \in T_{dis}^{run,down} \bigcap T_{dis}^{down,long}$, we assume that train t_f is running in segment (s_m, s_n) when the disruption occurs, $s_n \leq s_k$. Here event f is a departure event of train t_f from station s_m, $f \in E_{s_m}^{dep}$. We further define T_{t_f,s_m}^1 as the subset of disrupted downside trains that depart from station s_m before train t_f. We define $A_{t_f,s_m}^1 \subset A_{head}^{track}$ as the subset of headway activities between the departure train t_f at station s_m and any other train $t_e \in T_{dis}^{down}$ that departs from station s_m. Finally, we define T_{t_f,s_m}^2, which contains the trains in set T_{t_f,s_m}^1 that are running on the disrupted line when the disruption occurs. The value of binary variable O_{t_f,s_n} for downside train t_f is specified as constraints (28).

$$\sum_{a=(e,f)\in A_{t_f,s_m}^1} \lambda_a - \sum_{t_e\in T_{t_f,s_m}^2} \sum_{s_n<s\leq s_k} O_{t_e,s} \geq O_{t_f,s_n} \times \sum_{s_n\leq s\leq s_k} (C_s - 1) \quad \forall t_f \in T_{dis}^{run,down} \cap T_{dis}^{down,long} \tag{28}$$

In constraint (28), $\sum_{a=(e,f)\in A_{t_f,s_m}^1} \lambda_a$ counts the total number of downside trains that arrive at station s_n before train t_f; $\sum_{t_e\in T_{t_f,s_m}^2} \sum_{s_n<s\leq s_k} O_{t_e,s}$ is the total number of downside trains in front of train t_f that have to wait outside a station before station s_n, and $\sum_{s_n\leq s\leq s_k}(C_s - 1)$ denotes the total cumulative capacity of stations s_n to s_k that is available. Therefore, $\sum_{a=(e,f)\in A_{t_f,s_m}^1} \lambda_a - \sum_{t_e\in T_{t_f,s_m}^2} \sum_{s_n<s\leq s_k} O_{t_e,s}$ is the total number of trains that can occupy station capacity. Supposing $\sum_{a=(e,f)\in A_{t_f,s_m}^1} \lambda_a - \sum_{t_e\in T_{t_f,s_m}^2} \sum_{s_n<s\leq s_k} O_{t_e,s} \geq \sum_{s_n\leq s\leq s_k}(C_s - 1)$, train t_f has to wait outside station t_n, because no residual capacity of the successive stations is available for train t_f. Note that when $\sum_{a=(e,f)\in A_{t_f,s_m}^1} \lambda_a - \sum_{t_e\in T_{t_f,s_m}^2} \sum_{s_n<s\leq s_k} O_{t_e,s} \geq \sum_{s_n\leq s\leq s_k}(C_s - 1)$, O_{t_f,s_n} can be either 1 or 0, but in combination with constraint (35), the preferred value of O_{t_f,s_n} is 1.

For each downside train $t_e \in T_{dis}^{down}$, $e \in E_s^{dep}$, if it stops outside station s, it must wait there until the end of the disruption. This is denoted as follows:

$$x_e > H_{dis}^{end} \times O_{t_e,s} \quad \forall t_e \in T_{dis}^{run,down}, e \in E_s^{dep}, s \in \tilde{S} : s \leq s_k \tag{29}$$

We assume that train $t_f \in T_{dis}^{run,up}$ is running in segment (s_p, s_r) when the disruption occurs, $s_r \geq s_l$. We define T_{t_f,s_p}^1 as the subset of disrupted upside trains that depart from station s_p before train t_f. We define $A_{t_f,s_p}^1 \subset A_{head}^{track}$ as the subset of headway activities between the departing train t_f at station s_p and any other train $t_e \in T_{dis}^{up}$ departing from station s_p. T_{t_f,s_p}^2 contains the trains in set T_{t_f,s_p}^1 running on the disrupted line at the time that the disruption occurs. The value of binary variable O_{t_f,s_r} for upside train t_f is specified as constraint (30).

$$\sum_{a=(e,f)\in A_{t_f,s_p}^1} \lambda_a - \sum_{t_e\in T_{t_f,s_p}^2} \sum_{s_l\leq s<s_r} O_{t_e,s} \geq O_{t_f,s_r} \times \sum_{s_l\leq s\leq s_r} (C_s - 1) \quad \forall t_f \in T_{dis}^{run,up} \cap T_{dis}^{up,long} \tag{30}$$

The meaning of constraint (30) is similar to that of constraint (28).

For each upside train $t_e \in T_{dis}^{up}$, $e \in E_s^{dep}$, if it stops outside station s to wait for the disruption, it must wait there until the end of the disruption. This is denoted as follows:

$$x_e > H_{dis}^{end} \times O_{t_e,s} \quad \forall t_e \in T_{dis}^{run,up}, e \in E_s^{dep}, s \in \tilde{S} : s \geq s_l \tag{31}$$

(3) Short-distance and cross-line trains arriving at their destinations

As mentioned in the problem description section, train services of multiple types may run on a high-speed line. Some short-distance trains and cross-line trains may still be able to reach their destinations during the disruption. However, if a short-distance train runs on a segment following a long-distance train and the long-distance train has to wait outside the successive station, then this short-distance train is hindered by the long-distance train. We assume that this short-distance train must also wait outside the successive station. To this end, we have to determine which short-distance and long-distance trains are running in which segments when the disruption occurs. The set of downside long-distance trains that are running in segment (s, s'), where s' is the next station of s and $s' \leq s_k$, can be formulated as follows:

$$T_{dis}^{run(s,s'),down,1} = \{t \in T_{dis}^{down,long} | q_e < H_{dis}^{start} \wedge q_f > H_{dis}^{end}\}, e \in E_s^{dep}, f \in E_{s'}^{arr}, t_e = t_f = t \tag{32}$$

Similarly, the set of downside short-distance trains that are running in segment (s, s'), where s' is the next station of s and $s' \leq s_k$, can be formulated as follows:

$$T_{dis}^{run(s,s'),down,2} = \{t \in T_{dis}^{down,short} | q_e < H_{dis}^{start} \wedge q_f > H_{dis}^{end}\}, e \in E_s^{dep}, f \in E_{s'}^{arr}, t_e = t_f = t \tag{33}$$

Any two trains t and t' run in segment (s, s') when the disruption occurs, where train $t \in T_{dis}^{run(s,s'),down,2}$ and $t' \in T_{dis}^{run(s,s'),down,1}$. Event e is the departure event of t from station s, $e \in E_s^{dep}$ and event f is the departure event of t' from s, $f \in E_s^{dep}$. Therefore, activity $a = (e, f)$ is the headway activity of trains t and t' departing from station s. Furthermore, event e' is the departure event of t from station s', $e' \in E_{s'}^{dep}$. Based on this notation, whether short-distance train t is hindered by long-distance train t' can be formulated as follows:

$$x_{e'} > H_{dis}^{end}(o_{t',s'} + \lambda_a - 1) \tag{34}$$

Constraint (34) denotes that if a long-distance train t' departs from station s before a short distance train t ($\lambda_a = 1$), and train t' has to wait outside successive station s' ($o_{t',s'} = 1$), then train t also has to wait outside station s' ($x_{e'} > H_{dis}^{end}$). If not, then train t does not need to wait outside station s' until the disruption ends.

Note that we have handled downside short-distance trains using constraint (34). The same method can also be used to handle upside short-distance trains. To simplify, we have omitted the formulation for upside short distance trains.

(4) Action for directly influenced long-distance train services

For each directly influenced downside long-distance train $t \in T_{dis}^{down,long}$, if it is not canceled, it either stops at a station to wait until the disruption is over or waits outside the successive station s_n (supposing that train $t \in T_{dis}^{run}$ is in the segment just before station s_n when the disruption occurs) until the end of the disruption. Our focus is on long-distance high-speed railways with a seat reservation system, and it is quite difficult within this system for passengers to temporarily transfer from one train to another during their journey. The third assumption in Section 4.1 ensures that a train cannot be canceled mid-trip. The activity for a directly influenced long-distance train is denoted as follows:

$$\sum_{s\in S:s_t^o\le s\le s_k} w_{t,s} + y_t + O_{t,s_n} = 1 \quad \forall t \in T_{dis}^{down,long}, s_n \in \tilde{S} : s_n \le s_k \tag{35}$$

Similarly, for each directly influenced upside train $t \in T_{dis}^{up,long}$, if not canceled, it either stops at a station to wait until the end of the disruption or it waits outside the successive station s_r (if train $t \in T_{dis}^{run}$ is running in the segment before station s_r when the disruption occurs) until the end of the disruption. This is denoted as follows:

$$\sum_{s\in S:s_l\le s\le s_t^o} w_{t,s} + y_t + O_{t,s_r} = 1 \quad \forall t \in T_{dis}^{up,long}, s_r \in \tilde{S} : s_r \ge s_l \tag{36}$$

5 Computational Experiments

To confirm our analysis of the train stop deployment planning problem during a complete blockage, we now present experimental results based on the Chinese high-speed railway network. IBM ILOG CPLEX 12.8 is used as the solver, with CPLEX parameters set to their default values. All of our experiments are run on an Intel(R)Core(TM)i7-7700 Processor CPU @3.60GHz 3.60GHz, 16.0GB RAM desktop.

5.1 Test instance and parameter values

To test the model, a real-world instance from China's high-speed railway is utilized. Here we focus on the Beijing-Shanghai high-speed line, but other high-speed lines related to this line are implicitly considered due to cross-line trains, such as the Wuhan-Shanghai high-speed line and the Jinan-Qingdao high-speed line, which are shown as dotted lines in Figure 3. Interested readers can find more details about China's high-speed railway network in Zhan et al. [2016].

The Beijing-Shanghai high-speed line is one of the longest in the world, with a length of 1,318 km. Furthermore, many high-speed lines in this area are connected to the Beijing-Shanghai high-speed line, and trains can cross between lines. There are in total 23 stations located on the Beijing-Shanghai high-speed line, dividing the whole line into 22 segments. According to the practical timetable applied in 2013, 172 trains in total (86 inbound and 86 outbound trains) operate on this line; 84 of these are long-distance trains that travel the whole line, while the others are local or cross-line trains.

The total number of tracks at each intermediate station is given in Table 1. The minimum train running times in each segment for high-speed and medium-speed trains are shown in Table 2. Note that the minimum running times for inbound and outbound trains in the same segment are the same. In addition, the dwell time for each train in each station is based on the original timetable. The arrival and departure headway times between two consecutive trains at each station in the same direction are set to 3 minutes and 4 minutes, respectively. Note that because we have changed the departure headway time to 4 min, the minimum running time in the segment may be slightly different from that in Zhan et al. [2016]. We have made this modification because we believe that a departure time interval of 4 min is more practical than 2 min. The headway time between a train that departs from a station track and another train that arrives at the same track is set to 3 minutes.

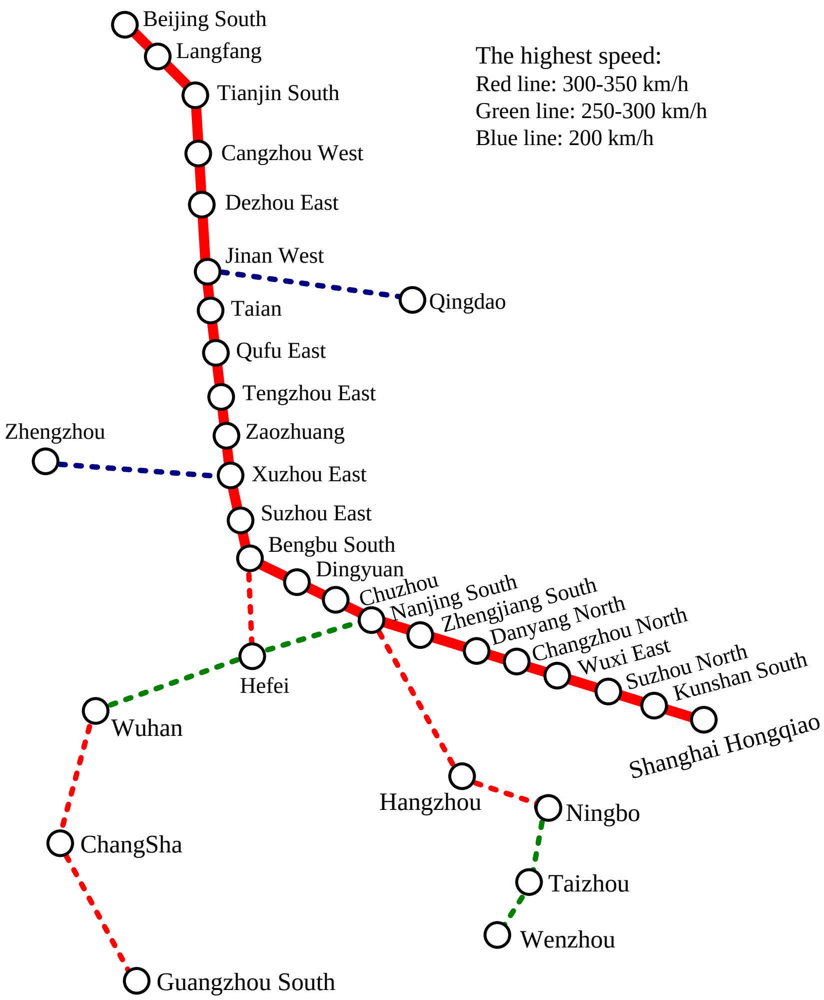

Figure 3: Chinese high-speed railway network connected with the Beijing-Shanghai high-speed line by cross-line trains

In the case study, the penalty values for canceling a high-speed train service and a medium-speed train service (γ_t) are 5000 and 3000, respectively. The penalty values for a one-minute arrival delay (tardiness) for a high speed train and a medium speed train (μ_e^+) are 5 and 3, respectively. Similarly, the penalty values for one minute arrival earliness for a high-speed train and a medium-speed train (μ_e^-) are 2 and 1, respectively. However, the penalty values for a one-minute departure delay (tardiness) for a high-speed train and a medium-speed train (μ_e^+) are 3 and 2, respectively. Because canceling a train service has more serious effects on passenger service, we assign a much larger penalty value to canceling than to delaying a train. In addition, because passengers care more about arrival delay than departure delay, we assign a larger penalty value to an arrival delay. Finally, M_1 is 1,440 minutes, which is the duration of a whole day, and M_2 is 2,880 minutes. For readers' convenience, we show the definitions and values of the parameters in Table 3.

To test our approach, we assume various disruption scenarios based on occurrence time, and the location and duration of the disruptions. Six scenarios are considered in our experiments, as shown in the first column of Table 4. For each disruption scenario, the three numbers in brackets are the occurrence time, the location, and the disruption duration. For example, disruption scenario (10, 9, 60) indicates that the disruption occurs at 10:00 in the morning, is located in segment 9, and lasts 60 min. Based

Table 1: Number of tracks at each intermediate station of the Beijing-Shanghai high-speed railway

NO.	Station	Tracks	NO.	Station	Tracks
1	Langfang	4	12	Bengbu South	11
2	Tianjin South	6	13	Dingyuan	4
3	Cangzhou West	6	14	Chuzhou South	6
4	Dezhou East	7	15	Nanjing South	10
5	Jinan West	17	16	Zhengjiang North	6
6	Taian	6	17	Dangyang North	4
7	Qufu East	6	18	Changzhou North	6
8	Tengzhou East	4	19	Wuxi East	6
9	Zaozhuang West	6	20	Suzhou North	6
10	Xuzhou East	15	21	Kunshan South	12
11	Suzhou East	6			

Table 2: Minimum running time for high-speed and medium-speed trains in each segment

Segment	High-speed (min)	Medium-speed (min)	Segment	High-speed (min)	Medium-speed (min)
1	15	17	12	17	21
2	13	15	13	11	14
3	16	20	14	12	15
4	21	24	15	12	15
5	17	21	16	14	14
6	10	15	17	6	6
7	11	16	18	6	7
8	10	14	19	11	14
9	7	7	20	5	6
10	11	16	21	6	6
11	13	17	22	12	13

The segments are numbered in order from Beijing to Shanghai.

on these assumed disruption scenarios and the parameter setting above, we obtain the results for each scenario.

5.2 Computational results

Using the assumed disruption scenarios and the parameter setting in the previous subsection, we obtain the results for each disruption scenario; see Table 4. The computation time for each scenario is limited to 5 min. Except for the computation time limitation, if a solution with a gap of 0 is obtained, the computation is also stopped. The total number of trains considered in each scenario is around 40. In Tables 4 and 5, the first column is the disruption scenario, the second is the objective value for the model, the third shows the total train deviation, the fourth shows the number of canceled trains, and the last two columns illustrate the computation time and the gap (the difference between the best upper bound and lower bound obtained in the given time).

From Table 4, we can see that CPLEX can solve the train stop deployment planning problem for each disruption scenario within 5 min, with an average gap of around 10%. A disruption of longer duration tends to have a more serious impact on train operations than one occurring at the same time and in the same location but with a shorter duration. In addition, a longer duration disruption is more difficult to solve than a shorter duration one. Some trains may be canceled due to disruption, but the number of canceled trains is not more than one or two in our cases. We can obtain the timetable for each disruption scenario and illustrate the train stop plan. Due to limited space, we only show the timetable for one

Table 3: Values for the parameters used in our case study

Parameter	Definition	Value
λ_a	Departure headway between two trains in the same direction	4
λ_a	Arrival headway between two trains in the same direction	3
λ_a	Headway between two trains depart from and arrive at the same station track	3
γ_t	Penalty for cancelling a high-speed train	5000
γ_t	Penalty for cancelling a medium-speed train	3000
μ_e^+	Penalty for a unit arrival delay of a high-speed train	5
μ_e^+	Penalty for a unit arrival delay of a medium-speed train	3
μ_e^-	Penalty for a unit arrival earliness of a high-speed train	2
μ_e^-	Penalty for a unit arrival earliness of a medium-speed train	1
μ_e^+	Penalty for a unit departure delay of a high-speed train	3
μ_e^+	Penalty for a unit departure delay of a medium-speed train	2
M_1	A large positive number	1440
M_2	A large positive number	2880

Note that parameters with the same form are specified by a specific event e or activity a.

Table 4: Results for various disruption scenarios for all of the trains operating during the disruption

Scenario	Objective	Total deviation (min)	Cancelation	Time(s)	Gap(%)
(10, 9, 60)	34934	7605	1	300	14.09
(10, 9, 90)	90191	20381	2	300	21.99
(12, 14, 60)	32190	8433	0	300	6.18
(12, 14, 90)	83006	21979	0	300	10.87
(15, 17, 60)	22822	5784	0	88	0
(15, 17, 90)	60940	14203	1	300	11.22

disruption scenario (10, 9, 60) (Figure 4).

In Figures 4 and 5, the horizontal axis shows time, while the vertical axis is the station dimension. The stations are numbered starting at 0, from Beijing South Station to Shanghai Hongqiao Station. Red lines in the figure are operation lines for high-speed trains, while the blue lines are for medium-speed trains. Note that we use black lines to specify the deviations in train operation. The disrupted area is shown as a green rectangle.

In Figure 4, we can see that trains approaching the disrupted area are stopped at appropriate stations to wait for the disruption to end. For trains traveling from Beijing toward Shanghai, three trains can stop at Station 8 to wait for the disruption, as in total four tracks are available at this station and one must be reserved for emergency rescue trains or other running trains, as discussed earlier. In the opposite direction, five trains stop at Station 9 to wait for the disruption to end, as the station capacity is six. Five trains also wait at Station 10 for the disruption to end because this station also has six tracks. As Figure 4 shows, our model can stop trains at the appropriate stations in a blocked situation.

In practice, trains that have already started from their origin are the most critical, as trains that have not departed from their origin can simply wait at their origin stations. Therefore, to reduce the complexity of computation, we apply the train stop deployment planning model only for trains that are operating when the disruption occurs. Figure 4 shows that trains that start earlier from their origin tend to be more influenced by the disruption, while trains that arrive at the disrupted area a long time after the end of the disruption are usually less influenced. This also shows that it is reasonable to consider only trains that are urgent instead of all directly influenced trains. Table 5 shows the results for each

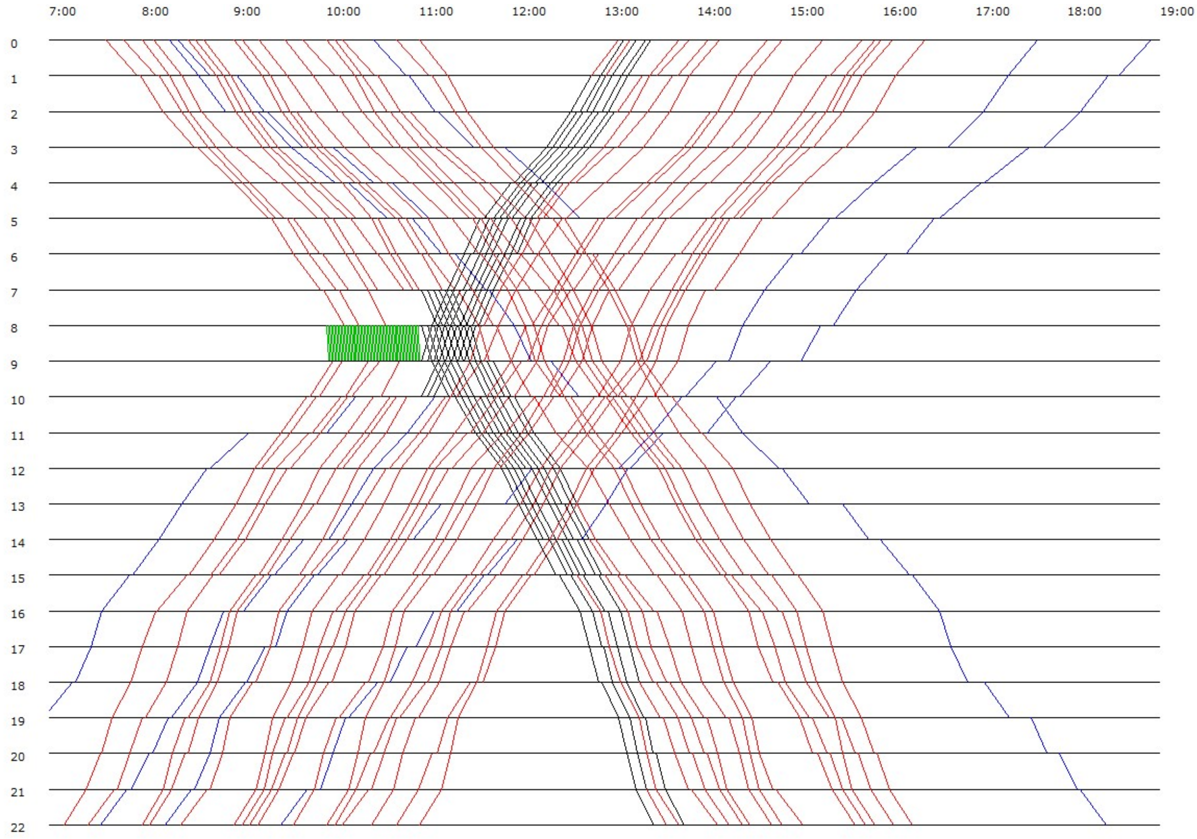

Figure 4: Rescheduled timetable for disruption scenario (10,9,60), including all of the directly influenced trains

scenario. Because the railway dispatchers place much more emphasis on short computation time than on an optimal solution, we limit the computation time to 100 seconds and the relative gap to 5%.

Table 5: Results for various disruption scenarios for trains that departing from their origin before the disruption occurs

Scenarios	Objective	Total deviation (min)	Time(s)	Gap(%)
(10, 9, 60)	28929	7345	73	5
(10, 9, 90)	81579	20665	100	12.75
(12, 14, 60)	31929	8365	35	5
(12, 14, 90)	76519	19871	72	5
(15, 17, 60)	21742	5502	6	0
(15, 17, 90)	49361	12518	39	0
(10, 9, 90)	78289	19891	**3600**	8.14

It is much easier to handle trains that are already operating on the railway lines at the start of the disruption. We can obtain the solutions for all of the assumed disruption scenarios except scenario (10, 9, 90) within 100 seconds, with a gap no larger than 5%. This short computation time is acceptable for railway dispatchers who need to handle urgent disrupted trains. We also test disruption scenario (10, 9, 90) with a relatively long computation time, 3,600 seconds; see the last row of Table 5. The solution for this disruption scenario obtained in 3,600 seconds is only 4% better than that obtained in 100 seconds. Thus, we can say that the results obtained in 100 seconds are good enough for practical use. We can obtain a rescheduled timetable to assist dispatchers in each disruption scenario. Figure 5 shows the disposition timetable for the same scenario (10, 9, 60).

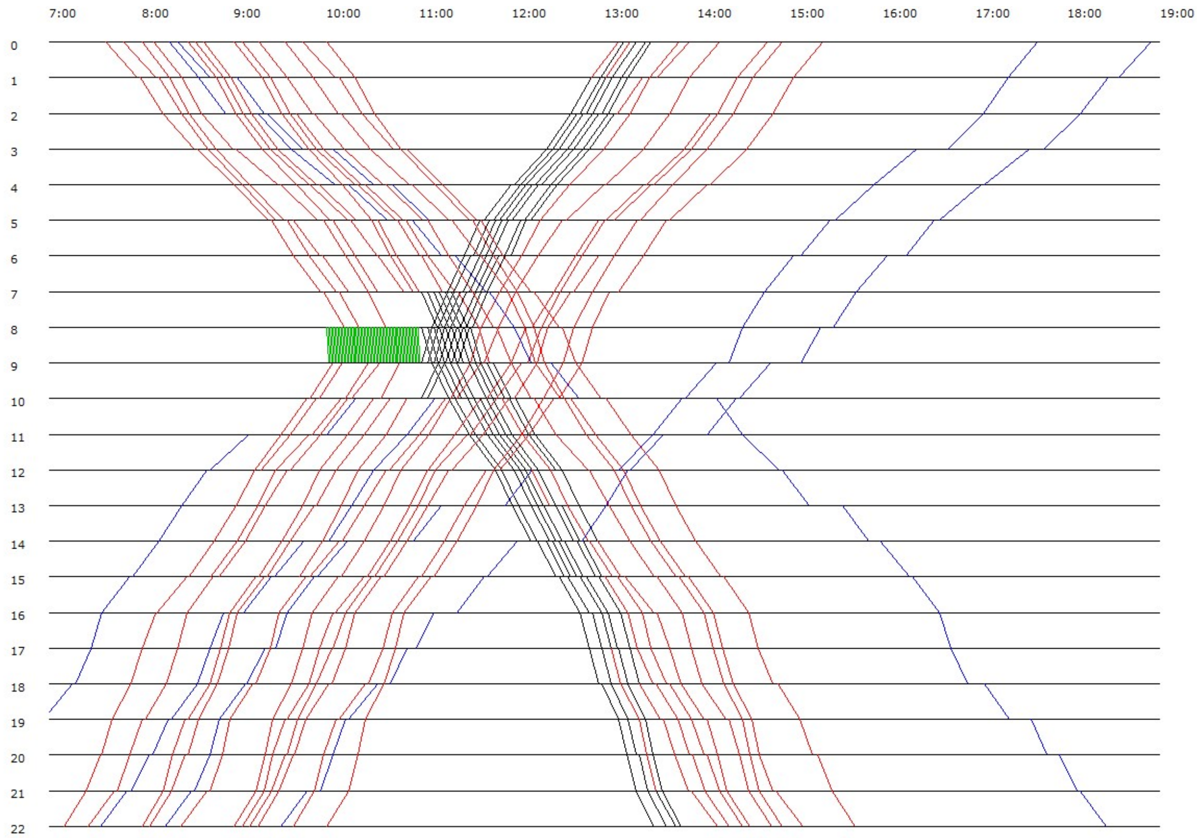

Figure 5: Rescheduled timetable for disruption scenario (10, 9, 60), including trains originally departing before the occurrence of the disruption

In Figure 5, we can see that the train stopping pattern is similar to that in Figure 4. This indicates that we can solve the train stop deployment planning problem in a short computation time by considering fewer trains. Furthermore, all of the trains that deviate from the original timetable (trains with part of their operation lines in black in the figure) in Figure 4 are also rescheduled in Figure 5. Thus, we can see that the solutions obtained within 100 seconds are adequate for practical use.

6 Conclusions

In this paper, we have studied train stop deployment planning in a highly disrupted situation for a long-distance high-speed railway line and lines connected with it by cross-line trains. The railway network is considered at the macroscopic level using an event-activity graph, and a Mixed Integer Linear Programming model is proposed to solve the train stop deployment planning problem, which is a critical part of the whole train rescheduling problem. A real-world instance of the Beijing-Shanghai High-speed Railway and other lines connected with it by cross-line trains is used to test our model.

Because we allow inbound trains and outbound trains to share station tracks, we take trains in both directions into account to minimize the impact of disruptions on train operations. Trains approaching the disrupted area are stopped at appropriate intermediate stations to wait for the disruption to end. Due to limited station capacity, the number of trains waiting for the disruption cannot exceed capacity. In addition, stopping trains should not hinder other running trains. In particular, it is important to prevent the delay of trains on one high-speed railway line from propagating to other connected lines by cross-line trains as much as possible.

From our experiments, we find that a longer disruption not only means a more serious impact on train operations, but also greater model computation time. If we consider all of the directly influenced trains, we can obtain rescheduling solutions for all of the disruption scenarios within 5 min, and the average gap is about 11%, although the maximum gap increases to 21.99%. We consider about 40 trains simultaneously to further reduce computation time, and show that we can solve the train stop deployment planning problem for only trains that are operating on the railway line when the disruption occurs instead of all of the directly influenced trains. This is reasonable from a practical point of view, as running trains require more urgent attention. In addition, we find that considering only trains that are operating at the time the disruption occurs does not greatly influence the solutions found in our experiments. We can therefore obtain a rescheduled timetable for each disruption scenario within 100 seconds, which is acceptable for real-time applications. The model developed in this research can help railway dispatchers to handle trains approaching a disrupted area.

Several directions for further research are possible. First, we solve our model using CPLEX. Although we obtain good solutions for a disruption with a duration of 90 min, more efficient algorithms would be helpful in solving disruptions of longer duration, and in obtaining optimal solutions. Second, we formulate the train stop deployment planning problem at a macroscopic level. A feasibility check from a microscopic point of view may be necessary. Finally, it would be worth investigating how to simultaneously solve the problem of train stop deployment planning and train rescheduling after the end of a disruption.

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