A schedule-based timetable model for congested transit networks

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Abstract

This study proposes a model and method for solving the transit timetabling problem with consideration of passenger path choices within a congested and schedule-based transit network. The model has two submodels: transit timetabling and passenger-equilibrium assignment. The transit timetabling submodel generates timetables to improve passengers' journey experience based on the results of passenger-equilibrium assignment, whereas the passenger-equilibrium assignment loads passengers on transit vehicles based on a fixed vehicle capacity and timetable output from the transit timetabling. An iterative method is developed to connect these two submodels and efficiently determine an optimal solution. The iterative method includes an equilibrium assignment method to simulate passenger loading and suggests a decomposition approach for transit timetabling. The decomposition approach has a significant computational advantage when handling transit systems that involve hundreds of transit vehicles because the algorithm generates a timetable for one transit line in each step rather than for all transit lines. The decomposition approach can therefore handle relatively large problems with more transit lines. The suggested algorithms are analyzed with a hypothetical example and with the practically sized example of South China's high-speed railway in terms of efficiency, optimality, and applicability.

Key words: Transit network; Schedule-based; Timetabling; Decomposition approach

1. Introduction

Transit operators seek to improve passenger services within their operating budget to prevent customers from becoming dissatisfied and turning to alternative traffic modes. Many approaches to this have been studied (Yan et al., 2013; Szeto and Jiang, 2014b; Robenek et al., 2016; Schöbel, 2017; Tang et al., 2020; Ceder and Jiang, 2020; Zhan et al., 2020). Among these, the design of a user-friendly timetable is an important approach. When a new timetable is published, passengers may change their travel plans to maximize the utility of their trips. Passenger behavior dynamics cannot be modeled if the transit timetable does not include a passenger assignment model, which may affect the timetable quality (Borndörfer et al., 2017). We therefore propose a schedule-based model for solving transit timetabling problems (TTPs) that considers passenger path choices within a congested transit network. The transit line plan, safety headway between transit services, and time-dependent origin-destination trip demand are assumed to be given. The transit line plan considers the operating budget and determines the general framework of transit lines (e.g., operation time, stopping pattern, maximum running speed, dwell times, and number of transit services on a line). Transit services with a fixed capacity are assumed to operate precisely as detailed in the published timetable, and all passengers are assumed to have the ability to predict transit network conditions. The passengers in an urban transit network may take transit services every workday for their commute; thus, they accumulate sufficient experience to predict network information. In an intercity transit network such as the high-speed railway (HSR) network, which has great punctuality and reliability, passengers may not use HSR services as frequently as they use urban transit services, but they can rely on the HSR timetables accessible online to understand the HSR network conditions. Moreover, according to one study of the Chinese HSR (Li and Schmöcker, 2017), after their first HSR trip, most people prefer the HSR for intercity travel, with most making an average of 10 HSR trips a year. The first Chinese HSR line was opened in 2008; we assume that in the 12 years since, frequent HSR users have gained sufficient knowledge to predict HSR network conditions. With this predictive information, passengers choose paths that minimize their generalized cost, which encompasses seven components: (a) in-vehicle time (IVT), (b) walking time, (c) wait time, (d) transfer penalty, (e) access and egress time, (f) ticket fare, and (g) extra cost due to the limited capacity. With this passenger information, the model provides the operation details of each run on a line (e.g., departure/arrival times and station track allocation) to maintain the feasibility of the paths selected by passengers and minimize passenger journey time costs, which include (a) IVT, (b) wait times at the origin and transfer stations, and (c) walking time. We use a simulation-based iterative method to solve this problem.

The literature review that follows provides a brief introduction of recent studies and highlights areas that require further research attention.

1.1 Literature review

Models for public transit systems differ from those for private car systems because transit vehicles follow predetermined routes and timetables, whereas private cars have more possible options of departure times and routes (Tong and Richardson, 1984). Frequency-based and schedule-based modeling approaches have been established on the basis of these transit system attributes. Reviews of these two approaches were published by Fu et al. (2012) and by Cascetta and Coppola (2016). The frequency-based approach regards each transit line as having multiple runs with a constant headway and running speed, and it measures passenger behaviors and system performance in terms of averages. For instance, it expects that passengers wait at a station for an average period equal to half of the headway. In contrast, the schedule-based approach describes the operation of each transit service using precise departure/arrival times and allows variation in the headway between transit services and in the running speed. Accordingly, the schedule-based approach also models passenger behavior in a disaggregated manner; for example, choices to board certain transit services can be considered (Fu et al., 2012). The schedule-based approach therefore has less error in prediction than the frequency-based approach. Hence, this study adopts the schedule-based approach to solve the TTP with consideration of passenger path choices.

Some studies have used an iterative approach to consider similar problems (Sels et al., 2011; Siebert, 2011; Schöbel, 2017). The iterative method separates the model into two submodels: a timetabling model that considers passenger assignment and a passenger-assignment model based on timetables. The method iterates between these two submodels until convergence is reached, which presents two primary advantages. First, this separation reduces the computational complexity to allow the problem to be solved efficiently. Second, the iterative process captures the interaction between the passenger choices and the timetable design. Specifically, the operator publishes a timetable upon which the passengers base their choices. After the operator has gathered passenger choice information from ticket sales, the operator can improve the situation by updating the timetable. Passengers can then adjust their choices in response to the new timetable, and the operator may need to further adjust the timetable. This interaction continues until an equilibrium is reached between the passengers and operator. Due to these advantages, the iterative method is adopted here. Our iterative method presents two main differences from previous iterative methods, including (a) the use of an improved solution method for transit timetabling and (b) a different approach to assign passengers with consideration of vehicle capacity constraints.

Here, we consider only the most relevant studies that apply heuristics to solve large-scale problems (Sections 1.1.1) and those that include passenger assignment in transit timetabling models (Section 1.1.2). We refer interested readers to the following reviews for further information about TTPs: (a) Harrod (2012) summarized four model structures commonly used for timetabling; (b) Cacchiani and Toth (2012) and Lusby et al. (2018) reviewed the development of robust timetabling; and (c) Parbo et al. (2016) discussed the progress of passenger-oriented timetabling.

1.1.1 Heuristics for transit timetabling models

Many studies have noted that determining an exact globally optimal solution for the TTP is a nondeterministic polynomial-time (NP)–hard problem (Jong et al., 2013; Barrena et al., 2014; Schmidt and Schöbel, 2015; Talebian and Zou, 2015). Effective heuristic approaches are therefore needed to solve large-scale problems to yield relatively high-quality solutions in a reasonably quick manner. Extensive research has been performed in this area, and many illuminating approaches have been proposed, such as those based on modified linear programming relaxation (Wong et al., 2008), simulated annealing (Burdett and Kozan, 2010; Jamili et al., 2012; Zhou et al., 2015), genetic algorithms (Niu and Zhou, 2013; Jong et al., 2013), and decomposition approaches (Carey and Lockwood, 1995; Brännlund et al., 1998; Ingolotti et al., 2006; Caimi et al., 2009; Sinha et al., 2016; Guo et al., 2016; Zhou and Teng, 2016).

The decomposition approach decomposes a large and complex problem into smaller and simpler subproblems and then resolves the subproblems individually. The ability of the decomposition approach to handle complicated problems with efficiency and tractability of computation has led to its wide adoption in solving TTPs. One option involves the use of the decomposition approach to decompose the entire network into different zones and then generate timetables within the zones (Caimi et al., 2009; Sinha et al., 2016; Guo et al., 2016). However, this requires additional effort to coordinate operations between zones. Alternatively, decomposition can be performed at the train level (Carey and Lockwood, 1995; Brännlund et al., 1998; Ingolotti et al., 2006; Zhou and Teng, 2016), for which a timetable is obtained train by train. However, train-level decomposition for TTPs is not suitable for use with iterative methods because it may result in an unsatisfactory initial timetable at the beginning of the iterative process. For example, it is possible that several trains with the same stopping pattern will be scheduled to depart within a short period, whereas no such trains are allocated in another period. This arrangement would have a negative influence on passenger assignment, which is performed according to the initial timetable and used to guide later timetable adjustments. To avoid this, we group trains with the same stopping pattern into lines and perform decomposition at the line level for TTPs so that transit services on the same line are scheduled simultaneously according to a specific distribution constraint. This improves the starting point of the iterative method.

1.1.2 Passenger assignment in transit timetabling models

Passengers may be unable to use their preferred path due to limited vehicle capacity; hence, vehicle capacity should be considered. Some studies have included a vehicle capacity constraint in their timetabling models but have made some unrealistic assumptions. For instance, Chang et al. (2000) constrained the passenger flow per train to prevent it from violating the train's capacity; however, they assumed that all passengers would eventually board a train because the system as a whole had sufficient trains. Niu et al. (2015) followed the first-come-first-served principle to load passengers. In their model, passengers who cannot board a train will wait for the next available train, and the last train, which stops at all stations in the train corridor, can serve the remaining passengers. Yang et al. (2016) required that the total demand at a station on a train corridor fall within the total loading capacity of trains that stop at this station. However, in such a system, passengers may fail to purchase tickets for their journeys, and the railway system will then lose these customers. To relax these assumptions, Canca et al. (2014) and Robenek et al. (2018) adopted the logit model, which calculates the possibility that passengers shift to another transit mode. In addition, Robenek et al. (2018) dealt with the problem for a railway network rather than in the simpler situation of a train corridor. However, the logit model requires extra information from other available traffic modes. Robenek et al. (2016) proposed another approach to address the conflict between demand and supply in railway networks that is simpler and does not require additional information: they divided passengers into groups and assigned each group to a single path. The use of this approach means that passenger groups cannot be split, and if no path can accommodate a passenger group due to the train capacity constraint, this passenger group is assumed to travel outside the planning horizon. However, this assignment is not realistic because passengers in the same group would in reality have the ability to use different feasible paths within the planning horizon. Hence, the assignment method should be modified to increase the applicability of this simpler approach to optimize vehicle capacity. To address this problem, we allow passenger groups to be split and assign them to different paths in a transit network based on the vehicle capacity constraint. A residual path using an artificial vehicle with unlimited capacity is generated to accommodate passengers who fail to board any transit services.

Furthermore, passenger assignment must be performed under either the system optimum condition or the user equilibrium (UE) condition. Under the system optimum condition, the total passenger travel cost is minimized. In contrast, under the UE condition, passengers cannot reduce their travel cost by shifting to another path. When the vehicle capacity is assumed to be sufficient, the solutions for these two conditions are the same. Previous iterative methods (Sels et al., 2011; Siebert, 2011; Schöbel, 2017) assumed that the vehicle capacity was sufficient and assigned passengers to the shortest path; hence, the obtained solution was not only the system optimum but also the UE solution. However, vehicle capacity is generally limited in practice. Timetabling studies that have considered vehicle capacity have generally proposed models under the system optimum condition (Chang et al., 2000; Niu et al., 2015; Robenek et al., 2016; Yang et al., 2016).

In practice, the system optimum is difficult to achieve in a congested network because it requires passengers to be selfless, which means that some passengers must forgo selecting their ideal option and give up their seats for others to minimize the system cost. However, passengers are generally selfish (Bell and Lida, 1997) and maximize their own utility via different strategies. For instance, schedule-based UE models consider that passengers compete for seats with strategies such as learning from their failure-to-board experiences (Nuzzolo et al., 2012), handling supply uncertainties (Hamdouch et al., 2014), and using real-time on-board crowding predictions (Nuzzolo et al., 2016). In this case, the total passenger travel cost is higher than that under the system optimum condition, but the UE condition is more suitable to describe the real situation. For more detail, we refer interested readers to the theory presented by Gentile et al. (2016). In terms of passenger behaviors, there exist two kinds of UE model: deterministic UE (DUE) and stochastic UE (SUE). DUE models assume that passengers' perceptions are homogeneous and that they have perfect information (e.g., Xu et al. (2018)), whereas SUE models account for the heterogeneity and errors in passengers' perceptions and in operators' observations (e.g., Zhang et al. (2010), Liu and Meng (2014), and Sun and Szeto (2018)). The integration of TTP and SUE theoretically results in a better timetable than does the integration of TTP and DUE, because SUE models more realistically predict passenger flows than do DUE models. However, as suggested by Gentile et al. (2016), DUE models can identify the major passenger flows in the railway system, the outputs of which are generally easier to understand and interpret. Moreover, SUE models require much more computational effort than do DUE models. However, as far as transit timetabling is concerned, the optimized timetable may not be very sensitive to differences in the predictions of deterministic assignment and stochastic assignment models (Hartleb and Schmidt, 2019). Hence, a DUE model is adopted for passenger assignment in this study. Future studies could study the integration of TTP and SUE for a railway system with significant heterogeneity in passenger choice behavior.

1.2 Contributions

This study suggests a model and method for solving the transit timetabling problem within a congested and schedule-based transit network with consideration of passenger path choices. The model has two submodels: transit timetabling and passenger-equilibrium assignment. The transit timetabling generates timetables to improve the passengers' journey experience according to the results of the passenger-equilibrium assignment, whereas the passenger-equilibrium assignment loads passengers on transit vehicles based on the timetable output from the transit timetabling and adopts the residual-path concept and splitting strategy for passenger groups to accommodate vehicle capacity constraints.

An iterative method is developed to connect these two submodels and efficiently determine the solution. This method iterates between transit timetabling and passenger loading. To further improve the computational efficiency, the decomposition approach is adopted for TTPs. Thus, the decomposition approach decomposes a network-scale problem to the line level, which means that feasible suboptimal solutions can be obtained efficiently to address large-scale problems. Numerical tests on South China's HSR network show that our algorithm obtains satisfactory results within a reasonable time for practical applications.

The remainder of this paper is organized as follows. Section 2 describes the terminology used in the study and outlines the basic assumptions. In Section 3, we introduce the model and solution method. Section 4 tests the model and solution method using two example networks. We summarize the conclusions of the study in Section 5.

2. Assumptions and terminology

Before introducing the model's details, we first clarify our assumptions and terminology.

2.1 Network assumptions and terminology

A transit system is a system of transportation services (e.g., buses or trains) that run on fixed routes and follow published timetables for public travel. This study focuses mainly on a train-type transit system. With certain modifications, our model can also be used for bus-type transit systems. To describe a transit system in our model, we make the following definitions and assumptions.

First, a train carries passengers from its origin terminal to its destination terminal along a specific route with specific arrival and departure times and uses track at each passing station of the transit network, and a line is a group of trains that passes through the same stations, following the same order and stopping pattern. In consideration of the operating budget, a plan for a line decides the operation requirements of trains on this line, including the operating time-window, the minimum running times, and the maximum and minimum dwell times. Following the line plans, a timetable details the operation of trains on all lines. The operation details include when a train departs from or arrives at a passing station and which station track a train uses when it skips or stops at a passing station. To avoid allocation of trains too close to one another on the same line, we set a minimum departure time interval between two trains on the same line.

Second, two adjacent stations are linked by a segment consisting of two running tracks. We consider trains in both directions. Trains in one direction use one running track, and those in the other direction run on the other running track. A train station has several station tracks to serve several trains simultaneously; however, a safety headway should be maintained between two entering/leaving trains. Train overtaking can only occur at a station that has several station tracks. According to practice, not all tracks at a station can be used by a passing train. Some station tracks do not have platforms and are used only by trains that skip this station. A train timetable thus should detail the station track allocation. In addition, a station can have several yards, and each yard serves one transit corridor. For instance, Zhaoqingdong Station has two yards, one for the Guangzhounan-Guilinbei HSR corridor and the other for the Guangzhounan-Nanning HSR corridor.[1] As a result, Zhaoqingdong Station is abstracted as two substations for these two yards. Passengers can use walking links between the two substations to make a transfer.

[1] Introduction to Zhaoqingdong station: https://zh.wikipedia.org/wiki/%E8%82%87%E5%BA%86%E4%B8%9C%E7%AB%99 (Accessed August 23, 2018)

Third, sufficient resources are assumed to be available for staff and vehicles. However, the vehicle capacity is limited.

Fourth, trains are assumed to operate precisely as specified in the published timetable.

2.2 Demand assumptions and terminology

Our model considers the time-dependent demand. Thus, we group passengers according to their given origin, destination, and preferred departure time interval as different origin-destination (OD) pairs. All OD pairs are included in a set ($\boldsymbol{R}$), and an OD pair $r \in \boldsymbol{R}$ represents passengers who wish to travel from their origin zone to a destination zone at the preferred departure time ($T_r^{\text{Departure}}$), which is the end of the preferred departure time interval.

In a train system, a path is a route that connects a given origin zone and destination zone by using a series of selected trains. Passengers on a path follow the route and timetable of the selected train to travel from the boarding station to the alighting station, and the IVT of a selected train is the time difference between the alighting time and the boarding time. Passengers need to pay a ticket fare to take a train from the boarding station to the alighting station. All ticket fares are assumed to be preset and unaffected by changes in the train timetable. For a given path, we add the ticket fares of individual train journeys to determine the total ticket fare of the path. Passengers may need to wait before the selected train departs from the boarding station. There are two types of waiting: (a) waiting at stations and (b) waiting in the origin zone. Passengers generally prefer to wait in the origin zone, where they have access to more activity options than at the origin station. We assume that passengers wait only for a certain time duration (T^{Wait}) at the origin station. This value is given based on the passengers' preferences. If the passengers on path $q \in \boldsymbol{Q}_r$ ($\boldsymbol{Q}_r$ is the set of paths used by OD pair $r \in \boldsymbol{R}$) select train n to depart from the origin station s and have to wait for longer T^{Wait} in the origin, they choose to stay in the origin zone z. The wait time in the origin zone z is $T^{\text{Wait}} \times i_q^{\text{Wait}}$, where i_q^{Wait} is a nonnegative integer variable. We can easily change the method to calculate the wait time in the origin zone z, if necessary. Thus, the departure time from the origin zone z of path q is $T_r^{\text{Departure}} + T^{\text{Wait}} \times i_q^{\text{Wait}}$, and the wait time at the origin station s is $t_{ns}^{\text{Departure}} - (T_r^{\text{Departure}} + T^{\text{Wait}} \times i_q^{\text{Wait}} + T_{sz}^{\text{Zone}})$, where $t_{ns}^{\text{Departure}}$ is the time at which train n departs from station s, and T_{sz}^{Zone} is the travel time between zone z and station s. T_{sz}^{Zone} is assumed to be known in advance and is unaffected by any changes in the train timetable. In addition, if a path includes more than one selected train, the passengers on that path will alight from a selected train and board the next selected train at a transfer station. The total number of transfers is equal to the number of selected trains minus one. The passengers on the path may need to walk for some distance because these two trains may not stop on the same track. The walking time for a transfer is considered the average walking time from the alighting platform to the boarding platform. The average walking time between two platforms is also assumed to be known in advance and unaffected by changes in the train timetable, but the walking time for a transfer between two trains may change when a train timetable change involves different track allocations for the two trains. Furthermore, passengers may need to wait for the next selected train. This waiting time is the time difference between the arrival time of the passengers and the departure time of the next selected train, and the arrival time of passengers equals the arrival time of the last selected train plus the walking time for this transfer.

The generalized cost of path q is the weighted sum of (a) the IVT, (b) the wait time, (c) the walking time, (d) the transfer penalty, (e) the access and egress time, (f) the ticket fare, and (g) the extra cost due to the limited capacity. The weighted sum of the first six cost components (a−f) is the uncongested cost. Weight coefficients are used to reflect the passengers' perceptions of these costs. For example, IVT is usually considered to be less bothersome than wait time at stations; hence, the weight coefficient for IVT is smaller. Practical applications can obtain weight coefficients with the use of calibration methods such as the stated preference method.

Our model assumes that passengers have full predictive network information and are rational, which means that they always choose the path that minimizes the generalized cost. A residual path $\tilde{q}$ with a sufficiently large path cost is included in $\boldsymbol{Q}_r$. The residual path $\tilde{q}$ is a virtual path that uses an artificial transit vehicle that runs directly from the origin station to the destination station without stopping and without occupying any physical track, and the capacity of the artificial transit vehicle equals the number of passengers in OD pair r to accommodate passengers who fail to board any trains. When the transit network cannot provide a feasible path for OD pair r, all passengers are loaded on the residual path $\tilde{q}$. Alternatively, when the transit network provides feasible paths for OD pair r but some passengers cannot use any feasible paths due to vehicle capacity, these remaining passengers are assigned to the residual path $\tilde{q}$.

3. Model formulation and solution method

The model contains two submodels: a timetable optimization model (TOM) and a passenger assignment model (PAM). The PAM constructs a time-space network based on the timetable information (including the train arrival/departure times at stations and the station track allocation) output by the TOM, after which it identifies feasible paths in the network and assigns passengers to these paths. The TOM then considers the passenger assignment (i.e., the passenger flow on a path and path routing, e.g., which train is used) output by the PAM to determine a timetable, as follows:

1) To maintain the feasibility of the selected passenger paths on which the passenger flow is greater than zero, the TOM does not allow trains to depart earlier than the arrival time of the passengers who use these trains.
2) The TOM attempts to reduce the walking and wait times of the selected paths by changing the trains' arrival/departure times and station track allocations.
3) The passenger flow on a path acts as a weight when the TOM needs to make trade-offs between reducing the travel cost of one path or another.

Therefore, the TOM outputs timetable $\boldsymbol{T}$ based on passenger assignment $\boldsymbol{A}$, and the PAM, which generates passenger assignment $\boldsymbol{A}$, is limited by timetable $\boldsymbol{T}$:

$$\text{Timetable optimization: } \boldsymbol{T} = \text{TOM}(\boldsymbol{A}); \tag{1}$$

$$\text{Passenger assignment: } \boldsymbol{A} = \text{PAM}(\boldsymbol{T}). \tag{2}$$

Thus, the model can be formulated as follows:

$$\boldsymbol{T} = \text{TOM}\big(\text{PAM}(\boldsymbol{T})\big). \tag{3}$$

An iterative method is used to determine the solution. The basic idea of the iterative method is to generate a starting point using the initial timetabling model (ITM) and improve the timetable via iteration between the TOM and the PAM. In our studied problem, the line plan and OD demand are given, but the details of the passenger paths are not known in advance. The ITM aims to generate an initial feasible timetable when information on passenger assignments is unknown. There is no strict rule for the objective function. Minimization of the total train running time is widely used in transit timetabling (Zhou and Teng, 2016); therefore, we adopt this approach to generate a starting point for the iterative method. The PAM generates the passenger assignment based on the timetable of the ITM. Based on this passenger assignment, the TOM details a timetable that minimizes the passengers' total weighted travel time; that is, the TOM not only keeps the timetable feasible but also ensures the feasibility of the paths used by the passengers. The PAM then generates a new passenger assignment based on the improved timetable, and the TOM may need to be run again to improve the timetable.

We terminate the iterative method when the relative difference between the total generalized costs between two iterations falls below the termination criterion. The termination criterion is based on the operators' desired balance between computation effort and level of optimality. If the problem is complex and large, even a tiny reduction in the objective may substantially prolong the computation time. In this case, we may consider establishing a relatively larger termination criterion to reduce the computation effort. In addition, due to the limited vehicle capacity, we may encounter a paradox in which the decrease in the travel time caused by the timetable change could increase the total generalized cost. The details of this paradox are given in Section 3.4. If the paradox exists, we opt for the solution with the lowest total generalized cost. Otherwise, the final output is the solution in the last iteration that has the lowest total generalized cost. The procedure of the iterative method is shown in Table 1.

Table 1. Iterative method.

Iterative method
Input: Network information, line plan, and passenger information **Output:** Transit timetable **Step 1**. Set the iteration count $e = 0$. **Step 2**. If $e = 0$, then run the ITM to optimality and identify the initial timetable $\boldsymbol{T}_0$; or else $\boldsymbol{T}_e = \mathrm{TOM}(\boldsymbol{A}_{e-1})$. **Step 3**. $\boldsymbol{A}_e = \mathrm{PAM}(\boldsymbol{T}_e)$. **Step 4**. If the relative difference of the total generalized cost between two iterations falls below the termination criterion, the iteration ends, and the solution with the lowest total generalized cost is output as the final solution; otherwise $e = e + 1$ and return to Step 2.

ITM and TOM are both transit timetabling models. Two solution methods are used to manage transit timetabling models, the branch-and-bound algorithm and the decomposition approach, which are introduced in Sections 3.2.1.1 and 3.2.1.2, respectively. The iterative method that adopts the branch-and-bound algorithm is called the IM-BBA, whereas the iterative method that uses the decomposition approach is called the IM-DA. The IM-BBA ensures that global optima of the ITM and TOM can be found when the problem is simple; however, the computational efficiency deteriorates when the problem size increases. The IM-DA cannot guarantee that the generated solutions for the ITM and TOM are globally optimal, but it provides a strong computational advantage in large-scale problems. The decision to apply IM-BBA or IM-DA depends on practical need. The PAM formulates the UE assignment problem as a linear programming (LP) problem and solves the LP problem using a column-generation method. The following section provides details of the three models (ITM, TOM, and PAM) and their newly specified solution methods.

3.1 Notations

The notations used in the models are introduced in Table 2.

Table 2. Notations used in the models.

Component	Type	Definition
r	Element	OD pair
q	Element	Path
$\tilde{q}$	Element	Residual path
l or l'	Element	Line
n or n'	Element	Train
n_q^{Origin}	Element	Train used by passengers on path q to leave the origin
$\Lambda_{qn}^{\text{Transfer}}$	Element	Train from which passengers on path q transfer to train n
s or s'	Element	Station
s_{qn}^{Alight}	Element	Station where path q considers passengers who alight from train n
s_{qn}^{Board}	Element	Station where path q considers passengers who board train n
s_l^{Origin}	Element	Origin terminal of line l
$s_l^{\text{Destination}}$	Element	Destination terminal of line l
$\xi_{ns}^{\text{Downstream}}$	Element	Downstream station of station s for train n
p or p'	Element	Track
z	Element	Zone
z_r^{Origin}	Element	Zone that is the origin of OD pair r
$\boldsymbol{R}$	Set	Set of OD pairs
$\boldsymbol{Q}_r$	Set	Set of paths for OD pair r
$\boldsymbol{N}_l$	Set	Set of trains that belong to line l
$\boldsymbol{N}_q^{\text{Path}}$	Set	Set of trains used by path q
$\boldsymbol{S}_l$	Set	Set of stations used by line l
$\boldsymbol{L}$	Set	Set of lines
$\boldsymbol{P}_{ns}$	Set	Set of tracks that can be used by train n at station s
A_q	Real variable	Passenger inflow on path q
C^{TO}	Real variable	Objective of the timetable optimization model
c_q	Real variable	Generalized cost using path q
c_{qm}	Real variable	Cost components of path q: (a) $m = 1$, IVT; (b) $m = 2$, time waiting at stations; (c) $m = 3$, time waiting in the origin zone; (d) $m = 4$, time walking; (e) $m = 5$, total number of transfers; (f) $m = 6$, travel time between zones and stations; (g) $m = 7$, ticket fare.
$c_{nss'}^{\text{Extra}}$	Real variable	Extra cost for using train n on a segment between station s and station s' due to limited vehicle capacity
c_q^{Extra}	Real variable	Extra cost of path q
c_q^{Unc}	Real variable	Uncongested cost of path q
$\tilde{c}_r$	Real variable	Equilibrium travel cost over all paths of OD pair r
F	Real variable	Objective of the LP model
T^{Total}	Real variable	Total train running time
$t_{qnn'}^{\text{Walk}}$	Real variable	Walking time that passengers on path q spend on the transfer from train n to train n'
t_{qn}^{Wait}	Real variable	Wait time for train n at the boarding station on path q

t_{ns}^{Arrival}	Real variable	Time of arrival of train n at station s
$t_{ns}^{\text{Departure}}$	Real variable	Time of departure of train n from station s
i_q^{Wait}	Integer variable	Number of T^{wait} in the origin zone for path q
$b_{n'ns}$	Binary variable	Equals 1 if train n' reaches station s before train n, or 0 otherwise
$b'_{n'ns}$	Binary variable	Equals 1 if train n' leaves station s before train n, or 0 otherwise
$b_{nn'}^{\text{Line}}$	Binary variable	Equals 1 if train n begins to operate before train n' (trains n and n' belong to the same line), or 0 otherwise
b_{np}^{Track}	Binary variable	Equals 1 if train n uses track p, or 0 otherwise
$b_{qnss'}^{\text{Path}}$	Binary variable	Equals 1 if path q using train n on a segment between station s and station s', or 0 otherwise
c^{Residual}	Parameter	Uncongested cost of a residual path
D_r	Parameter	Number of persons in OD pair r
M	Parameter	Sufficiently large integer
Π_n^{Capacity}	Parameter	Capacity of train n
$T_{pp'}^{\text{Platform}}$	Parameter	Average time that passengers spend walking from the platform serving track p to that serving track p'
$T_r^{\text{Departure}}$	Parameter	Preferred time for departing from the origin zone of OD pair r
T_{sz}^{Zone}	Parameter	Journey time between station s and zone z
T^{Wait}	Parameter	Longest time for which passengers will wait at the origin station
T_l^{Depart}	Parameter	Earliest time of departure for trains on line l
T_l^{Arrival}	Parameter	Latest time of arrival for trains on line l
T_{sl}^{Dmin}	Parameter	Minimum dwell time of trains on line l at station s
T_{sl}^{Dmax}	Parameter	Maximum dwell time of trains on line l at station s
T^{H}	Parameter	Minimum headway between two trains
T_l^{Lmin}	Parameter	Minimum departure time interval between two trains on line l
$T_{lss'}^{\text{Run}}$	Parameter	Minimum running time of line l on a segment between station s and station s'
w_{rm}	Parameter	Weight coefficient for cost component c_{qm} ($\forall q \in \boldsymbol{Q}_r$) of OD pair r

3.2 Transit timetabling

Our timetabling model generates a daily timetable based on passenger path choices with consideration of vehicle capacity. It can be formulated as a mixed-integer model, and the constraints are based on a mixed-integer sequencing linear program, which is introduced and explained in the tutorial by Harrod (2012). In addition to the travel time and safety headway constraints mentioned in the tutorial, our model also includes constraints for station track allocation, dwell time, overtaking, distributing trains on the same line, calculating path costs, and ensuring path feasibility as follows:

Travel time constraints ($\forall l \in \boldsymbol{L};\ \forall n \in \boldsymbol{N}_l;\ \forall s, s' \in \boldsymbol{S}_l:\ s' = \xi_{ns}^{\text{Downstream}}$):

$$T_{lss'}^{\text{Run}} \le t_{ns'}^{\text{Arrival}} - t_{ns}^{\text{Departure}}; \tag{4}$$

$$\begin{cases} T_l^{\text{Depart}} \le t_{ns}^{\text{Departure}} \\ t_{ns}^{\text{Arrival}} \le T_l^{\text{Arrival}} \end{cases}; \tag{5}$$

Safety headway constraints ($\forall l, l' \in \boldsymbol{L};\ \forall n \in \boldsymbol{N}_l;\ \forall n' \in \boldsymbol{N}_{l'};\ \forall s, s' \in \boldsymbol{S}_l \cap \boldsymbol{S}_{l'}:\ s' = \xi_{ns}^{\text{Downstream}}$):

$$t_{ns}^{\text{Arrival}} + T^{\text{H}} \le t_{n's}^{\text{Arrival}},\ \text{if}\ b_{nn's} = 1; \tag{6}$$

$$t_{ns}^{\text{Departure}} + T^{\text{H}} \le t_{n's}^{\text{Departure}}, \text{ if } b'_{nn's} = 1; \tag{7}$$

$$b_{nn's} + b_{n'ns} = 1; \tag{8}$$

$$b'_{nn's} + b'_{n'ns} = 1; \tag{9}$$

Station track allocation constraints ($\forall l, l' \in \boldsymbol{L};\ \forall n \in \boldsymbol{N}_l;\ \forall n' \in \boldsymbol{N}_{l'}$):

$$t_{ns}^{\text{Departure}} + T^{\text{H}} \le t_{n's}^{\text{Arrival}}, \text{ if } b_{np}^{\text{Track}} = b_{n'p}^{\text{Track}} = b_{nn's} = 1;\ \forall p \in \boldsymbol{P}_{ns}; \forall s \in \boldsymbol{S}_l \cap \boldsymbol{S}_{l'}; \tag{10}$$

$$\textstyle\sum_{p \in \boldsymbol{P}_{ns}} b_{np}^{\text{Track}} = 1;\ \forall s \in \boldsymbol{S}_l; \tag{11}$$

Overtaking constraint ($\forall l, l' \in \boldsymbol{L};\ \forall n \in \boldsymbol{N}_l;\ \forall n' \in \boldsymbol{N}_{l'};\ \forall s, s' \in \boldsymbol{S}_l \cap \boldsymbol{S}_{l'}:\ s' = \xi_{ns}^{\text{Downstream}}$):

$$b'_{n'ns} - b_{n'ns'} = 0; \tag{12}$$

Dwell time constraint ($\forall l \in \boldsymbol{L}; \forall n \in \boldsymbol{N}_l;\ \forall s \in \boldsymbol{S}_l$):

$$T_{sl}^{\text{Dmin}} \le t_{ns}^{\text{Departure}} - t_{ns}^{\text{Arrival}} \le T_{sl}^{\text{Dmax}}; \tag{13}$$

Constraints distributing trains on the same line ($\forall l \in \boldsymbol{L};\ \forall n, n' \in \boldsymbol{N}_l:\ n \ne n';\ s = s_l^{\text{Origin}}$):

$$t_{ns}^{\text{Departure}} + T_l^{\text{Lmin}} \le t_{n's}^{\text{Departure}}, \text{ if } b_{nn'}^{\text{Line}} = 1; \tag{14}$$

$$b_{nn'}^{\text{Line}} + b_{n'n}^{\text{Line}} = 1; \tag{15}$$

Path cost calculation formulas ($\forall r \in \boldsymbol{R};\ \forall q \in \boldsymbol{Q}_r$):

$$c_{q1} = \textstyle\sum_{n \in \boldsymbol{N}_q^{\text{Path}},\, s = s_{qn}^{\text{Alight}},\, s' = s_{qn}^{\text{Board}}} (t_{ns}^{\text{Arrival}} - t_{ns'}^{\text{Departure}}); \tag{16}$$

$$c_{q2} = \textstyle\sum_{n \in \boldsymbol{N}_q^{\text{Path}}} t_{qn}^{\text{Wait}}; \tag{17}$$

$$c_{q3} = T^{\text{Wait}} \times i_q^{\text{Wait}}; \tag{18}$$

$$c_{q4} = \textstyle\sum_{n \in \left(\boldsymbol{N}_q^{\text{Path}} - n_q^{\text{Origin}}\right),\, n' = \Lambda_{qn}^{\text{Transfer}}} t_{qn'n}^{\text{Walk}}; \tag{19}$$

$$t_{qn}^{\text{Wait}} = \begin{cases} t_{ns}^{\text{Departure}} - (T_r^{\text{Departure}} + c_{q3} + T_{sz}^{\text{Zone}}), \text{ if } n = n_q^{\text{Origin}} \\ t_{ns}^{\text{Departure}} - (t_{n's'}^{\text{Arrival}} + t_{qn'n}^{\text{Walk}}), \text{ if } n \in \left(\boldsymbol{N}_q^{\text{Path}} - n_q^{\text{Origin}}\right) \end{cases};\ z = z_r^{\text{Origin}};\ s = s_{qn}^{\text{Board}};\ s' = s_{qn'}^{\text{Alight}};\ n' = \Lambda_{qn}^{\text{Transfer}}; \tag{20}$$

$$t_{qn'n}^{\text{Walk}} = \textstyle\sum_{p \in \boldsymbol{P}_{ns}} \sum_{p' \in \boldsymbol{P}_{n's'}} (T_{p'p}^{\text{Platform}} \times b_{n'p'}^{\text{Track}} \times b_{np}^{\text{Track}});\ s = s_{qn}^{\text{Board}};\ s' = s_{qn'}^{\text{Alight}};\ \forall n \in \left(\boldsymbol{N}_q^{\text{Path}} - n_q^{\text{Origin}}\right);\ n' = \Lambda_{qn}^{\text{Transfer}}; \tag{21}$$

Constraint ensuring the feasibility of paths ($\forall r \in \boldsymbol{R};\ \forall q \in \boldsymbol{Q}_r$):

$$t_{qn}^{\text{Wait}} \geq 0;\ \forall n \in \boldsymbol{N}_q^{\text{Path}}. \tag{22}$$

Constraint (4) sets the necessary running time between two stations. Constraint (5) requires that trains operate within the operating time-window. Constraints (6) and (7) impose the necessary headway between arrivals/departures between two trains at stations according to the arrival/departure order. Constraints (8) and (9) enforce a logical set of arrival/departure orders of two train at stations. Constraints (10) and (11) arrange the track allocation at the stations and ensure that sufficient time is given between two trains that use the same track at a station. Constraint (12) is set so that trains can overtake each other at stations but not between stations. Constraint (13) limits the necessary dwell time of trains at stations for passengers boarding and alighting. For a non-stopping but passing station, the necessary dwell time equals zero ($T_{sl}^{\text{Dmin}} = T_{sl}^{\text{Dmax}} = 0$). Constraints (14) and (15) prevent a situation in which all trains that belong to the same line are scheduled within a short period and no other time slot includes any trains on this line.

The fare, the number of transfers, and the access/egress times are fixed for a path, whereas the IVT (c_{q1}), the wait times at stations or in zones (c_{q2} or c_{q3}), and the walking time (c_{q4}) are limited by the timetable. Hence, the objective function of the TOM includes c_{qm} ($m \in [1, 4]$) to improve the passengers' journey experience. The calculation formulas of c_{qm} ($m \in [1, 4]$) are given by Eqs (16)–(19), whereas Eq. (20) gives the calculation formulas for the wait times at the origin station and the transfer stations, and Eq. (21) calculates the walking time for a transfer and reflects the influence of station track allocation on the walking time. Constraint (22) guarantees that the wait times at the stations on a path are not negative so that passengers will not miss their trains.

With these constraints, the ITM and TOM are detailed in Sections 3.2.1 and 3.2.2. respectively.

3.2.1 Initial timetabling

The ITM is formulated as follows.

$$\min T^{\text{Total}} = \sum_{l \in \boldsymbol{L}} \sum_{n \in \boldsymbol{N}_l,\, s = s_l^{\text{Origin}},\, s' = s_l^{\text{Destination}}} (t_{ns'}^{\text{Arrival}} - t_{ns}^{\text{Departure}}) \tag{23}$$

s.t. Constraints (4)–(15).

T^{Total} is the total running time of the trains, and s_l^{Origin} and $s_l^{\text{Destination}}$ are the origin terminal and destination terminal of train $n \in \boldsymbol{N}_l$, respectively. The input parameters of the ITM include the headway between two trains (T^{H}), the departure time interval between two trains on the same line (T_l^{Lmin}), the operating time-window (T_l^{Depart} and T_l^{Arrival}), the minimum running times ($T_{lss'}^{\text{Run}}$), and the maximum and minimum dwell times (T_{sl}^{Dmax} and T_{sl}^{Dmin}); and the decision variables of the ITM consist of the train operation order ($b_{n'ns}$, $b'_{n'ns}$, and $b_{nn'}^{\text{Line}}$), the train arrival/departure times at the stations (t_{ns}^{Arrival} and $t_{ns}^{\text{Departure}}$), and the station track allocation (b_{np}^{Track}).

Constraints (6), (7), (10), and (14) are logical constraints that can be converted into linear constraints by mathematical transformations. For instance, constraint (6) can be linearized by introducing a sufficiently large integer value of M ($\forall l, l' \in \boldsymbol{L};\ \forall n \in \boldsymbol{N}_l;\ \forall n' \in \boldsymbol{N}_{l'};\ \forall s \in \boldsymbol{S}_l \cap \boldsymbol{S}_{l'}$):

$$t_{ns}^{\text{Arrival}} + T^{\text{H}} \leq t_{n's}^{\text{Arrival}} + (1 - b_{nn's}) \times \text{M}. \tag{24}$$

The ITM can therefore be formulated as an integer linear programming (ILP) problem, and the two solution methods for this, the branch-and-bound algorithm and the decomposition approach, are presented in Sections 3.2.1.1 and 3.2.1.2, respectively. These two methods are also applied to solve the TOM, and a comparison of these two methods is presented in Section 4.2.1.

3.2.1.1 Branch-and-bound algorithm

ILP problems can be solved with a classic method: the branch-and-bound algorithm. In this algorithm, a rooted tree with possible solutions is based on a root node that is an initial possible solution. Each branch of the rooted tree is a solution subset, and rather than examining all feasible solutions on a branch, a branch is checked according to the estimated lower and upper bounds of the optimum. If the solutions in this branch cannot be better than the best solution found so far, the search for this branch stops.

The computation time of the branch-and-bound algorithm is influenced by the estimated bounds and the initial feasible solution. Although the algorithm is guaranteed to achieve the global optimum, it may have low efficiency when solving a complex ILP problem. Therefore, the algorithm is generally used to handle simple ILP problems and provides a benchmark for other ILP solution techniques.

Many mature commercial software packages such as CPLEX[2] use the branch-and-bound algorithm to solve ILP problems. CPLEX is widely used in studies of transit networks (Caimi et al., 2011; Siebert and Goerigk, 2013; Burggraeve and Vansteenwegen, 2017; Dollevoet et al., 2017; Sparing and Goverde, 2017). In this study, we use CPLEX Studio 12.7 to run the branch-and-bound algorithm to solve ILPs including the ITM, the TOM, and timetabling problems for a single line generated using the decomposition approach. We follow the default settings except for the relative gap tolerance. In CPLEX, the relative gap is defined as follows[3]:

$$\text{Relative gap of CPLEX} = \frac{|\text{The objective of the best node remaining-The best integer objective}|}{|\text{The best integer objective}|+1\text{e-}10}. \tag{25}$$

When the relative gap falls below the relative gap tolerance, we terminate the CPLEX computation, because it is often the case that much of the computation is devoted to narrowing the gap after the eventual optimal solution has been found. We set the relative gap tolerance of CPLEX as 0.05 in this study. This relative gap tolerance of CPLEX can be easily changed by the operators according to their needs.

3.2.1.2 Decomposition approach

The ITM model can be written in a matrix form as follows:

$$\min \text{Objective}$$

[2] The branch-and-bound algorithm is known as “branch and cut” in CPLEX: https://www.ibm.com/support/knowledgecenter/SSSA5P_12.7.1/ilog.odms.ide.help/refcppopl/html/branch.html (accessed June 25, 2020).

[3] The relative mixed integer programming gap tolerance in CPLEX is explained at https://www.ibm.com/support/knowledgecenter/SSSA5P_12.7.1/ilog.odms.cplex.help/CPLEX/Parameters/topics/EpGap.html (accessed June 25, 2020).

$$\text{s.t. Objective} = \sum_{l \in \boldsymbol{L}^{\mathrm{TM}}} \sum_{l' \in \boldsymbol{L}^{\mathrm{TM}}} \sum_{n \in \boldsymbol{N}_l} \sum_{n' \in \boldsymbol{N}_{l'}} \mathbf{con}_{nn'}^{1} \mathbf{var}_{nn'} \tag{26}$$

$$\mathbf{con}_{nn'}^{2} \mathbf{var}_{nn'} = \mathbf{con}_{nn'}^{3};\ \forall l, l' \in \boldsymbol{L}^{\mathrm{TM}};\ \forall n \in \boldsymbol{N}_l;\ \forall n' \in \boldsymbol{N}_{l'}; \tag{27}$$

$$\mathbf{con}_{nn'}^{4} \mathbf{var}_{nn'} + \mathbf{con}_{nn'}^{5} \geq \mathbf{0};\ \forall l, l' \in \boldsymbol{L}^{\mathrm{TM}};\ \forall n \in \boldsymbol{N}_l;\ \forall n' \in \boldsymbol{N}_{l'}; \tag{28}$$

$$\mathbf{var}_{nn'} \geq \mathbf{0};\ \forall l, l' \in \boldsymbol{L}^{\mathrm{TM}};\ \forall n \in \boldsymbol{N}_l;\ \forall n' \in \boldsymbol{N}_{l'}; \tag{29}$$

$$\boldsymbol{L}^{\mathrm{TM}} = \boldsymbol{L}. \tag{30}$$

$\mathbf{var}_{nn'}$ is a matrix for variables related to trains n and n'; $\mathbf{con}_{nn'}^{1}$, $\mathbf{con}_{nn'}^{2}$, $\mathbf{con}_{nn'}^{3}$, $\mathbf{con}_{nn'}^{4}$, and $\mathbf{con}_{nn'}^{5}$ are matrices for constants related to trains n and n'; and $\boldsymbol{L}^{\mathrm{TM}}$ is a set of lines considered by the timetabling model.

We perform decomposition at the line level to separately create a timetable for each line and then apply an iterative adjustment, as shown in Table 3. If a single line has too many trains, the computation time to obtain this line's timetable may be long. Therefore, we split this line into several sublines.

Table 3. Decomposition approach for timetable generation.

Decomposition approach for timetable generation

Input: Network information and line plan
Output: Timetable
Step 1. Generate a timetable for line l_j ($j \in [1,\ \Pi^{\mathrm{Line}}]$, where Π^{Line} is the number of lines on the network):
<u>Step 1a</u>: Run the timetabling model to determine the variables of line l_j, while the line set $\boldsymbol{L}_j^{\mathrm{Fixed}}$ (l_1 to l_{j-1} when $j > 1$; $\emptyset$ when $j = 1$) is fixed and the line set $\boldsymbol{L}_j^{\mathrm{NotIncluded}}$ (l_{j+1} to $l_{\Pi^{\mathrm{Line}}}$ when $j < \Pi^{\mathrm{Line}}$; $\emptyset$ when $j = \Pi^{\mathrm{Line}}$) is not included.
if *the solution is found* **then**
| <u>Step 1b</u>: If $j = \Pi^{\mathrm{Line}}$, go to Step 2; if not, $j = j + 1$, and return to Step 1a.
else
| <u>Step 1c</u>: Generate all possible line combinations by selecting lines from $\boldsymbol{L}_j^{\mathrm{Fixed}}$
| **for** *each line combination* $\boldsymbol{L}^{\mathrm{Combination}}$ **do**
| | <u>Step 1d</u>: Set $\boldsymbol{L}^{\mathrm{Combination}}$ and line l_j as a modified line set.
| | <u>Step 1e</u>: Run the timetabling model to determine the variables of the modified line set.
| | **if** *the solution is found* **then**
| | | <u>Step 1f</u>: If $j = \Pi^{\mathrm{Line}}$, go to Step 2; if not, $j = j + 1$, and return to Step 1a.
| | **end**
| **end**
end
Step 2. Improve the solution:
while *the gap between the latest two iterations' objectives is not smaller than the gap allowance* **do**
| **for** *each* j *from 1 to* Π^{Line} **do**
| | Run the timetabling model to determine the variables of line l_j while fixing the other lines.
| **end**
end

In Step 1a, the timetables of the lines in $\boldsymbol{L}_j^{\text{Fixed}}$ have already been generated, and the lines in $\boldsymbol{L}_j^{\text{NotIncluded}}$ are awaiting timetabling. A timetabling model is constructed based on the matrix-form ITM, but it adds a new constraint and modifies constraint (30) as follows:

min Objective

s.t. constraints (26)–(29), and

$$\mathbf{var}_{nn'} = \mathbf{con}_{nn'}^{6};\ \forall l, l' \in (\boldsymbol{L}^{\text{TM}} - l_j);\ \forall n \in \boldsymbol{N}_l;\ \forall n' \in \boldsymbol{N}_{l'}; \tag{31}$$

$$\boldsymbol{L}^{\text{TM}} = \boldsymbol{L}_j^{\text{Fixed}} \cup l_j. \tag{32}$$

$\mathbf{con}_{nn'}^{6}$ is a matrix for constants related to trains n and n'. $\mathbf{con}_{nn'}^{6}$ takes the values based on feasible timetables of lines in $(\boldsymbol{L}^{\text{TM}} - l_j)$, which were obtained in the previous calculation. Hence, the timetabling model excludes the variables associated with line $l_{j'} \in \boldsymbol{L}_j^{\text{NotIncluded}}$, sets constant values to variables only associated with lines in $\boldsymbol{L}_j^{\text{Fixed}}$, and determines the variables linked to line l_j. As an ILP, the timetabling model can then be solved by the branch-and-bound algorithm using CPLEX.

When a conflict appears, an additional approach is required (see Steps 1c–1f). For instance, if no feasible solution is obtained for line l_j, we check whether some lines of $\boldsymbol{L}_j^{\text{Fixed}}$ make it impossible to identify a solution. Hence, lines are selected from $\boldsymbol{L}_j^{\text{Fixed}}$ to generate all possible line combinations for checking. For example, if $\boldsymbol{L}_j^{\text{Fixed}} = \{l_1, l_2, l_3\}$, all possible line combinations are $\{l_1\}$, $\{l_2\}$, $\{l_3\}$, $\{l_1, l_2\}$, $\{l_1, l_3\}$, $\{l_2, l_3\}$, and $\{l_1, l_2, l_3\}$, and these line combinations are checked individually. The checking process is to build a modified line set that includes line combination $\boldsymbol{L}^{\text{Combination}}$ and line l_j. A timetabling model is constructed to identify a feasible solution to accommodate $\boldsymbol{L}^{\text{Combination}}$ and line l_j at the same time, and it is the same as the one in Step 1a except that the line set of constraint (31) is changed from $(\boldsymbol{L}^{\text{TM}} - l_j)$ to $(\boldsymbol{L}^{\text{TM}} - \boldsymbol{L}^{\text{Combination}} - l_j)$ so that no tracks and times are assigned to $\boldsymbol{L}^{\text{Combination}}$ and l_j in advance. If relaxing the current line combination's constraints is not sufficient to obtain a feasible solution, the next line combination will be examined until a feasible timetable of line l_j is found.

In the last timetabling model solved in Step 1, $\boldsymbol{L}_j^{\text{Fixed}} \cup l_j = \boldsymbol{L}$. The only difference between this model and the matrix-form ITM is that this model has constraint (31), which does not affect the fact that the solution of this model is a feasible solution of the ITM model. Hence, when competing Step 1, a feasible solution is found for the ITM.

In Step 2, an iterative process is used to improve the solution given by Step 1 with respect to the feasibility of the solution for all lines. When we create a new timetable for line l_j, we fix other lines. That is, we add constraint (31) to the matrix-form ITM and solve it via CPLEX. The solution obtained is a feasible solution of the ITM, and it is not worse than the solution before adjustment. When no further significant improvement is gained, the algorithm is terminated, and timetables are created for all lines.

The algorithm may only obtain a locally optimal solution. We address this problem by generating multiple possible timetables. We can reorder the lines and run Step 1 to easily seek a different possible timetable. Mutual influence exists between lines because the lines scheduled later cannot be scheduled at a given spatiotemporal point if those scheduled earlier already occupy that spatiotemporal point. Thus, different line orders may result in different timetables and may affect the optimality of the final solution. If possible, different line orders should be trialed to obtain better results.

In addition, Step 2 can be performed alone to improve the timetable if a possible solution is available as the starting point. The decomposition approach adjusts the lines individually until the objective difference between two consecutive iterations is less than the gap allowance, which is specified by the operators to balance the computation effort and the level of optimality.

3.2.2 Timetable optimization

The TOM is based on the result of the PAM, which gives path information such as path routing and passenger flow on a path (A_q). The IVT (c_{q1}), the wait times at stations or in zones (c_{q2} or c_{q3}), and the walking time (c_{q4}) are restricted by the timetable. The passengers' journey experience can be improved by reducing these costs; therefore, the objective function of the TOM aims to generate a feasible timetable that minimizes them. The TOM can be formulated as follows.

$$\min C^{\mathrm{TO}} = \sum_{r\in \boldsymbol{R}} \sum_{q\in \boldsymbol{Q}_r} A_q \times \left(\sum_{m\in[1,\,4]} w_{rm} c_{qm}\right) \tag{33}$$

s.t. Constraints (4)–(22).

The input parameters of the TOM contain not only those of the ITM that are related to the supply side, but also those related to the demand side, including the passengers' preferred departure time ($T_r^{\mathrm{Departure}}$), access/egress times (T_{sz}^{Zone}), the longest wait time at the origin station (T^{Wait}), the walking times between platforms ($T_{pp'}^{\mathrm{Platform}}$), and the weight coefficient for cost component (w_{rm}). The TOM consists of two types of decision variable. The first type is related to the timetable and includes the train operation order ($b_{n'ns}$, $b'_{n'ns}$, and $b_{nn'}^{\mathrm{Line}}$), the train arrival/departure times at stations ($t_{ns}^{\mathrm{Arrival}}$ and $t_{ns}^{\mathrm{Departure}}$), and the station track allocation (b_{np}^{Track}). The second type is related to the passengers and includes the IVT (c_{q1}), the wait time (c_{q2}, c_{q3}, t_{qn}^{Wait}, and i_q^{Wait}), and the walking time (c_{q4} and $t_{qnn'}^{\mathrm{Walk}}$). With the known passenger assignment and mathematical transformation of constraints (6), (7), (10), (14), and (21), the TOM can be formulated as an ILP problem; therefore, it can be resolved with a branch-and-bound algorithm. In addition, the TOM can be rewritten in matrix form that is the same as that of the ITM, so it can be solved with the decomposition approach. To apply the decomposition approach to solve the TOM, we adopt Step 2 of the decomposition approach discussed in Section 3.2.1.2, and the previous timetable is used as the starting point for adjustment of the timetable. Before adjusting the timetable, we suggest that the lines be ordered in descending order of passenger volume. If a line is used by more passengers, the line is given higher priority so it will be adjusted earlier. However, different line orders should be tested to achieve better results.

3.3 Passenger-equilibrium assignment

3.3.1 Model formulation of the PAM

The PAM consists of two parts: path enumeration and UE assignment. All available paths are generated via path enumeration based on the timetable output by the ITM or TOM, and the value of $b_{qnss'}^{\mathrm{Path}}$ ($\forall r \in$

$\boldsymbol{R}$; $\forall q \in \boldsymbol{Q}_r$; $\forall l \in \boldsymbol{L}$; $\forall n \in \boldsymbol{N}_l$; $\forall s, s' \in \boldsymbol{S}_l$: $s' = \xi_{ns}^{\text{Downstream}}$) can then be determined. $b_{qnss'}^{\text{Path}}$ is an indicator that equals 1 if path q uses train n on a segment between stations s and s', and 0 otherwise. The PAM then determines the passenger inflow on path q (A_q) considering the UE condition, capacity-related condition, and flow conservation condition.

The UE condition is based on the definition given by Poon et al. (2004)—a UE for the passenger assignment is obtained when, for all passengers in an OD pair with a given preferred departure time, the generalized costs of all selected paths are equal and are not higher than that of any unselected path. Hence, a flow pattern meets the UE condition if and only if the following constraint holds ($\forall r \in \boldsymbol{R}$; $\forall q \in \boldsymbol{Q}_r$):

$$c_q \begin{cases} = \tilde{c}_r, \text{ if } A_q > 0 \\ \geq \tilde{c}_r, \text{ if } A_q = 0 \end{cases} \tag{34}$$

where c_q is the generalized cost of path q, and $\tilde{c}_r$ is the equilibrium travel cost over all paths of OD pair r. Here, c_q is defined as follows ($\forall r \in \boldsymbol{R}$; $\forall q \in \boldsymbol{Q}_r$):

$$c_q = c_q^{\text{Unc}} + c_q^{\text{Extra}}. \tag{35}$$

The first component is the uncongested cost (c_q^{Unc}), and the second is the extra cost of this path (c_q^{Extra}). For a residual path $\tilde{q}$, $c_{\tilde{q}}^{\text{Unc}}$ is assumed to be a sufficiently large c^{Residual}, whereas for non-virtual paths, c_q^{Unc} is the weighted sum of the IVT, wait times at stations and in zones, walking time, number of transfers, access/egress times, and fare. Therefore,

$$c_q^{\text{Unc}} = \begin{cases} c^{\text{Residual}}, & \text{if } q = \tilde{q} \\ \sum_{m \in [1,\, 7]} w_{rm} c_{qm}, & \text{else} \end{cases}; \ \forall r \in \boldsymbol{R}; \ \forall q \in \boldsymbol{Q}_r. \tag{36}$$

For a known timetable, c_q^{Unc} is fixed and is regarded as an input to the PAM, but c_q^{Extra} is affected by passenger flows. c_q^{Extra} is the sum of costs for guaranteeing that passengers can use their chosen train via each segment along the path. Hence,

$$c_q^{\text{Extra}} = \sum_{l \in \boldsymbol{L}} \sum_{n \in \boldsymbol{N}_l} \sum_{s, s' \in \boldsymbol{S}_l:\, s' = \xi_{ns}^{\text{Downstream}}} \left(b_{qnss'}^{\text{Path}} \times c_{nss'}^{\text{Extra}} \right); \ \forall r \in \boldsymbol{R}; \ \forall q \in \boldsymbol{Q}_r, \tag{37}$$

where $c_{nss'}^{\text{Extra}}$ is the cost that passengers should bear to ensure that they can use train n on a segment between stations s and s'. For a transit system without seat reservations (such as a metro system), when the capacity limitation of a train is reached, no passengers can board. Passengers may need to squeeze themselves on the train and avoid being pushed out of the train when other passengers alight from the train. In contrast, for a transit system with seat reservations (such as the HSR), when the capacity limitation is reached, no seats are available. Passengers may incur an advance booking cost (Xu et al., 2018) to secure a seat. For example, prospective passengers may need to make their way to the ticket counter much earlier than the counter's opening time and stand in long queue, or they may need to download various ticket-purchasing software applications and even pay extra money to secure seats. This boarding effort or advance booking cost is modeled by $c_{nss'}^{\text{Extra}}$. We adopt the approach proposed by Szeto et al. (2013) and Xu et al. (2018) to compute $c_{nss'}^{\text{Extra}}$, as follows ($\forall l \in \boldsymbol{L}$; $\forall n \in \boldsymbol{N}_l$; $\forall s, s' \in \boldsymbol{S}_l$: $s' = \xi_{ns}^{\text{Downstream}}$):

$$c_{nss'}^{\text{Extra}} \begin{cases} = 0, \text{ if } \Pi_n^{\text{Capacity}} > \sum_{r \in \boldsymbol{R}} \sum_{q \in \boldsymbol{Q}_r} b_{qnss'}^{\text{Path}} \times A_q \\ \geq 0, \text{ if } \Pi_n^{\text{Capacity}} = \sum_{r \in \boldsymbol{R}} \sum_{q \in \boldsymbol{Q}_r} b_{qnss'}^{\text{Path}} \times A_q \end{cases}, \tag{38}$$

where Π_n^{Capacity} is the vehicle capacity of train n. $c_{nss'}^{\text{Extra}}$ is zero if Π_n^{Capacity} is greater than the passenger flow on train n serving a segment between stations s and s'. If Π_n^{Capacity} equals the passenger flow on train n serving a segment between stations s and s', it could be zero or positive. Hence, the value of $c_{nss'}^{\text{Extra}}$ is affected by whether the passenger flow on train n serving a given segment equals the train capacity, and the actual value of $c_{nss'}^{\text{Extra}}$ is estimated by solving the PAM.

The capacity-related condition limits the passenger flow on train n via a segment between station s and station s' to no more than the vehicle capacity of train n, as follows ($\forall l \in \boldsymbol{L}$; $\forall n \in \boldsymbol{N}_l$; $\forall s, s' \in \boldsymbol{S}_l$: $s' = \xi_{ns}^{\text{Downstream}}$):

$$\Pi_n^{\text{Capacity}} - \sum_{r \in \boldsymbol{R}} \sum_{q \in \boldsymbol{Q}_r} b_{qnss'}^{\text{Path}} \times A_q \geq 0. \tag{39}$$

The flow conservation condition ensures that the sum of passenger flows on the paths of OD pair r equals the total number of passengers, D_r ($\forall r \in \boldsymbol{R}$):

$$\sum_{q \in \boldsymbol{Q}_r} A_q - D_r = 0. \tag{40}$$

The UE assignment problem of the PAM can then be expressed as a problem as follows:

Finding $[A_q,\ \tilde{c}_r,\ c_{nss'}^{\text{Extra}}]$

s.t. constraints (34)–(40).

This problem can be formulated as an LP model:

$$\min F = \sum_{r \in \boldsymbol{R}} \sum_{q \in \boldsymbol{Q}_r} A_q c_q^{\text{Unc}} \tag{41}$$

s.t. constraints (39), (40), and

$$A_q \geq 0. \tag{42}$$

This LP model for passenger assignment is called the LPPA. With the input of path routing ($b_{qnss'}^{\text{Path}}$), uncongested costs (c_q^{Unc}), the total number of passengers in an OD pair (D_r), and the train capacity (Π_n^{Capacity}), the LPPA determines passenger flows on paths (A_q). Szeto et al. (2013) and Xu et al. (2018) both proved that the UE assignment problem can be formulated as such an LP model. Appendix A shows the proof that the LPPA is equivariant to our proposed UE assignment problem.

3.3.2 Solution method of the PAM

We adopt the column-generation method based on the work of Szeto et al. (2013) to solve the LPPA. As shown by Szeto et al. (2013), this method guarantees optimality and finite convergence. Moreover, the column-generation method reduces the computation complexity because it only generates paths when needed, instead of storing all path variables. The details of the column-generation method are listed in Table 4, and the analysis of convergence is given in Appendix B.

Table 4. Column-generation method for passenger-equilibrium assignment.

Column-generation method for passenger-equilibrium assignment
Input: Timetable and passenger information **Output:** Passenger assignment **Step 1**. Find the path with the lowest uncongested cost (q^*) for OD pair r, $\forall r \in \boldsymbol{R}$. If the capacity constraint is violated, go to Step 2; if not, the algorithm ends. **Step 2**. Set the iteration count $g = 0$. **Step 3**. Set the largest uncongested cost on the found non-virtual paths as the uncongested cost of q^*; the equilibrium travel cost is M, and let the path set include q^* and the residual path (i.e., $\dot{c}_r^g = c_{q^*}^{\text{Unc}}$, $\tilde{c}_r^g = \text{M}$, $\boldsymbol{Q}_r^g = \{q^*, \tilde{q}\}$, $\forall r \in \boldsymbol{R}$). **Step 4**. If one of the following two conditions is met—(a) all paths in the network are included in the corresponding path set; (b) not all paths are included in the corresponding path set, but $\dot{c}_r^g > \tilde{c}_r^g$, $\forall r \in \boldsymbol{R}$—then the algorithm ends; if not, go to Step 5. **Step 5**. Randomly select an OD pair r, $\forall r \in \boldsymbol{R}$, if it does not meet any of the condition stated in Step 4. **Step 6**. Generate $\lvert \boldsymbol{Q}_r^g \rvert$ (number of paths in $\boldsymbol{Q}_r^g$) lowest-uncongested-cost paths for OD pair r. If no new path is generated, return to Step 4. **Step 7**. For new path q', calculate its uncongested cost $c_{q'}^{\text{Unc}}$. Set $\dot{c}_r^{g+1} = \max(\dot{c}_r^g, c_{q'}^{\text{Unc}})$ and then update the path set: $\boldsymbol{Q}_r^{g+1} = \boldsymbol{Q}_r^g \cup q'$. **Step 8**. Set $g = g + 1$. **Step 9**. Use CPLEX[4] to solve the LPPA, whose path sets are $\boldsymbol{Q}_r^g$, $\forall r \in \boldsymbol{R}$, and update $\tilde{c}_r^g$. Return to Step 4.

If $\lvert \boldsymbol{Q}_r^g \rvert = 1$, that is, $\boldsymbol{Q}_r^g$ only includes the residual path, Step 6 identifies the path with the lowest uncongested cost. If $\lvert \boldsymbol{Q}_r^g \rvert > 1$, Step 6 uses the K-shortest path algorithm (e.g., van der Zijpp and Fiorenzo Catalano (2005)) to generate $\lvert \boldsymbol{Q}_r^g \rvert$ shortest paths. The algorithm determines the first lowest-uncongested-cost path in the network, after which it creates subnetworks based on the link elimination rule and determines the lowest-uncongested-cost paths in the subnetworks. The algorithm compares these paths to identify the path with the lowest uncongested cost as the second lowest-uncongested-cost path. The algorithm continues the process of dividing the subnetwork that contains the kth ($k < \lvert \boldsymbol{Q}_r^g \rvert$) lowest-uncongested-cost path to several new subnetworks and comparing the lowest-uncongested-cost paths in all subnetworks to identify the (k+1)th lowest-uncongested-cost path until $\lvert \boldsymbol{Q}_r^g \rvert$ lowest-uncongested-cost paths are identified. Tong and Richardson (1984) developed a promising schedule-based algorithm to identify the lowest-uncongested-cost path that was improved upon to make it more practicable (Tong and Wong, 1999; Khani et al., 2014; Xie et al., 2017). This algorithm first uses Dijkstra's algorithm (Dijkstra, 1959) to determine the shortest path and then sets the uncongested cost of the shortest path as the upper bound of the path search. The path search uses the upper bound to cut the search space and improve search efficiency. After the path search, the lowest-uncongested-cost path can be identified. Please refer to these respective studies for more details. In this study, we adopt the improved version presented by Xie et al. (2017).

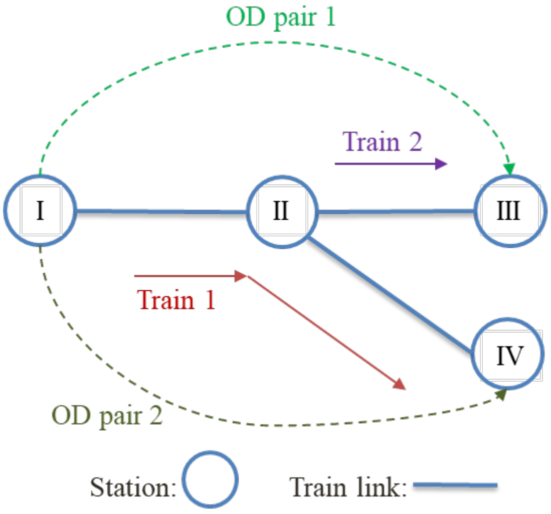
[4] To solve the LP problem, we use CPLEX Studio 12.7 with default settings, except that we terminate the computation of CPLEX when the relative gap is less than 0.05.

3.4 Analysis of solution methods

First, the convergence of the iterative method is discussed. The characteristics of the TOM and the PAM guarantee finite convergence. The objective of the TOM is to minimize the total weighted travel time (including the IVT, the walking time, and the wait time), and the objective of the LPPA equivariant to the UE assignment problem of the PAM is to minimize the total uncongested cost. The TOM attempts to decrease the total weighted passenger travel time of the path solution identified by the PAM in the last iteration, and the total uncongested cost of this identified path solution is thus not increased. In the next iteration, the PAM identifies a path solution with a non-increasing total uncongested cost. Because the total uncongested cost cannot decrease infinitely, the iterative method is convergent in finite iterations.

The iterative method cannot guarantee that the final solution is globally optimal, but it is an acceptable approach to handle complex problems and thus identify one of the practical solutions when identification of the global optimum is very difficult.

Furthermore, due to the limited capacity, the total generalized cost of the convergent solution cannot be guaranteed to be less than that of the other solutions identified in earlier iterations. When the capacity is sufficient, the generalized cost equals the uncongested cost and is non-increasing with each iteration. However, when the capacity is insufficient, a paradox may occur in which adjustment of the TOM may increase the total generalized cost. This paradox can be illustrated with a two-train example (Fig. 1). The two trains are prepared to serve two OD pairs: (a) 120 passengers in OD pair 1 wish to travel from station I to station III and arrive at station I at 7:45; (b) 80 passengers in OD pair 2 wish to travel from station I to station IV and arrive at station I at 7:50. Both trains 1 and 2 have 100 seats. In this example, except that weight coefficient for the number of transfers is 0, all weight coefficients are 1, the fares are free, and the uncongested cost of a residual path is 200 min.



Timetables	Train	Station	Arrival time	Departure time
Timetable created by the ITM	1	I	-	8:15
		II	8:45	8:47
		IV	9:57	-
	2	II	-	8:32
		III	9:02	-
Timetable created by the TOM	1	I	-	7:50
		II	8:20	8:22
		IV	9:32	-
	2	II	-	8:32
		III	9:02	-

Fig. 1. Two-train example illustrating the transit paradox.

According to the timetable created by the ITM, the PAM assigns all passengers of OD pair 1 to the residual path and those of OD pair 2 to train 1. Hence, the total generalized cost is (9:57 − 7:50) × 80 + 200 × 120 = 34,160 min. The TOM adjusts the timetable based on the assignment result. In the adjusted timetable, if the passenger flows did not change, the total generalized cost and total uncongested cost are the same: (9:32 − 7:50) × 80 + 200 × 120 = 32,160 min. However, the PAM changes the assignment because a lower total uncongested cost [200 × (80 + 20) + (9:02 − 7:45) × 100 = 27,700 min] can be obtained if 100 passengers of OD pair 1 are assigned to the path served by trains 1 and 2, and the rest passengers of OD pair 1 and passengers of OD pair 2 are assigned to their respective residual paths. The numbers of passengers onboard do not exceed the train capacity, meaning that the capacity constraint is fulfilled. According to this new assignment and constraints (34)–(40), the PAM calculates the extra costs. The extra cost of using train 1 on a segment between station I and station II is 98 min, that of using train 1 on a segment between station II and station IV is 0 min, and that of using train 2 on a segment between station II and station III is 25 min. Thus, for OD pair 1, the extra cost of the path served by trains 1 and 2 is 123 min, while for OD pair 2, the extra cost of the path served by train 1 is 98 min. Moreover, the PAM calculates the generalized path costs and equilibrium travel costs of the OD pairs, and these costs have the same value, that is, 200 min. In addition, this new assignment fulfills the UE condition, but the total generalized cost of this new assignment is 200 × (80 + 120) = 40,000 min, a larger value than that obtained before the timetable adjustment.

As suggested by Szeto and Jiang (2014a), we should consider people's selfishness and their responses to the new adjustments to avoid the transit paradox, in which operators wish to improve service quality (e.g., by reducing wait time) but deteriorate the network performance in terms of the total generalized cost. In this case, the timetable before adjustment shows better performance than the adjusted timetable, which means that the unadjusted timetable should be used as the output if another solution with a lower total generalized cost cannot be found in the later iterations.

Second, for the PAM, it is difficult to guarantee the uniqueness of the solution for the LPPA because its objective function is not strictly convex with respect to variable A_q ($\forall r \in \boldsymbol{R}$, $\forall q \in \boldsymbol{Q}_r$). Ensuring the solution uniqueness of the conventional UE is nearly impossible with the present technique and limited passenger information. The non-uniqueness gives varied passenger flow patterns, which may result in different timetables providing varying levels of improvement in passenger service. As a simple example of this non-uniqueness, consider that an OD pair has two paths (path 1 and path 2) with the same cost. Regardless of how the passengers are assigned to these two paths, the result satisfies the UE condition. The path solution of assigning more passengers to path 2 may result in a greater extent of timetable improvement (e.g., a greater reduction in the total generalized cost) than that of assigning more passengers to path 1. When possible, different path solutions of the PAM should be trialed to achieve better timetabling improvement.

4. Numerical examples

In this section, two examples (a hypothetical example and South China's HSR example) are used to demonstrate the capability of model and methods. These examples shared the following settings:

1) The shortest headway between two consecutive trains (T^{H}) was set to 4 min, based on previous studies (Kroon and Peeters, 2003; Zhan et al., 2015; Xie et al., 2020).
2) The longest wait time that passengers spend at the origin station (T^{Wait}) was set to 200 min.

3) The passenger preference parameters for OD demand were assumed to be homogeneous ($\forall r$): (a) $w_{r1} = 1.0$; (b) $w_{r2} = 1.8$; (c) $w_{r3} = 0.5$; (d) $w_{r4} = 2.0$; (e) $w_{r5} = 1.0$ min; (f) $w_{r6} = 1.0$; (g) $w_{r7} = 2.0$ min/yuan.

We used a personal computer (PC) with Windows 10 Enterprise, an Intel Core i7-4790U 3.60 GHz processor, and 16 GB RAM to perform computations for all cases except that shown in Section 4.2.1a; in this case, the PC failed to determine an optimal solution within a reasonable time, and thus a high-performance computing system, HPC2015, was used. This HPC2015 system was a 64-bit heterogeneous Linux cluster based at the University of Hong Kong, upon which we used one computer node with 96 GB memory and two 10-core Intel Xeon E5-2600 v3 (Haswell) processors.

4.1 Tests with a small hypothetical network

In this section, we describe how a small-scale hypothetical network was used to test the proposed model. Section 4.1.1 introduces the example settings. Section 4.1.2 demonstrates that the proposed model provides passengers with better service by smoothing the transfer process and decreasing the wait times. In Section 4.1.3, the proposed method for passenger loading is analyzed.

4.1.1 Example setting

The network (Fig. 2) has four zones. Each zone is served by one station; for example, station I serves zone Z1. The access and egress times (T_{sz}^{Zone}, $\forall s$, $\forall z$) are shown on the left-hand side of Fig. 2.

Access and egress times

		Station			
		I	II	III	IV
Zone	Z1	0	+∞	+∞	+∞
	Z2	+∞	0	+∞	+∞
	Z3	+∞	+∞	0	+∞
	Z4	+∞	+∞	+∞	0

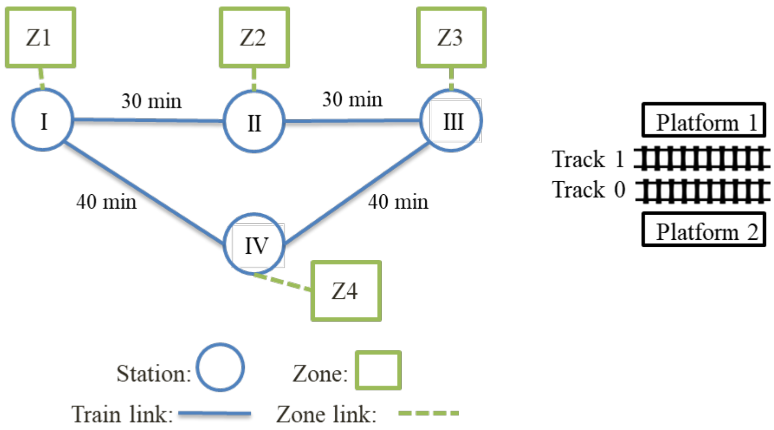


Fig. 2. Small hypothetical network.

Each station has two platforms. Each platform serves one track, as presented on the right-hand side of Fig. 2. Because passengers may carry large pieces of luggage and climb stairs to leave or arrive at a platform, we assumed that passengers walk 10 min from one platform to another (i.e., $T_{pp'}^{\text{Platform}} = 10$ min, for tracks p and p' at station s, $\forall s$). The maximum running speed for all trains in this example is identical, and the minimum running time on a segment is marked next to the respective segment in Fig. 2. For example, if trains on line 1 run on the segment between stations I and II, the minimum running time $T_{1,\text{I},\text{II}}^{\text{Run}}$ is 30 min.

The basic line settings are shown in Table 5.

Table 5. Basic line settings for small hypothetical network.

Lines	Trains on the line	Stations
L1	1, 2, 3	I→II→III→IV→I
L2	4, 5, 6	I→IV→III→II→I
D1	7, 8, 9	I→II→III
D2	10, 11, 12	I→IV→III
U1	13, 14, 15	III→II→I
U2	16, 17, 18	III→IV→I

In the line name, "L", "D", and "U" denote loop lines, lines of downstream trains, and lines of upstream trains, respectively. The operating time-window is [7:00, 12:00] (i.e., T_l^{Depart} = 420 min, T_l^{Arrival} = 720 min, $\forall l$, because the time unit was set to 1 min and 0:00 was transformed to be 0 min) and the minimum departure interval between two trains on a line is 30 min (i.e., $T_l^{\text{Lmin}} =$ 30 min, $\forall l$). According to the timetable shown on China Railway's official website[5], the dwell time generally ranges from 2 to 4 min, so the minimum and maximum dwell times at a station were set as follows ($\forall s \in \boldsymbol{S}_l, \ \forall l$):

$$T_{sl}^{\text{Dmin}} = \begin{cases} 0, \text{ if } s = \text{II}, l = \text{D1 or U1} \\ 2 \text{ min}, \quad \text{else} \end{cases}; \text{ and}$$

$$T_{sl}^{\text{Dmax}} = \begin{cases} 0, \text{ if } s = \text{II}, l = \text{D1 or U1} \\ 4 \text{ min}, \quad \text{else} \end{cases}.$$

The vehicle capacity is 100 passengers (i.e., $\Pi_n^{\text{Capacity}} =$ 100 passengers, $\forall n$). The fares (shown in Table 6) were set according to the train running time, that is, a higher fare was set for a longer train running time.

Table 6. Ticket fees for a small hypothetical network.

Line	Origin-destination stations	Fee (yuan)	Line	Origin-destination stations	Fee (yuan)
L1	I-II	5	L2	I-II	17
	I-III	10		I-III	14
	I-IV	17		I-IV	7
	II-I	17		II-I	5
	II-III	5		III-I	10
	II-IV	12		III-II	5
	III-I	14		IV-I	17
	III-IV	7		IV-II	12
	IV-I	7		IV-III	7
D1	I-III	10	U1	III-I	10
D2	I-III	14	U2	III-I	14
	I-IV	7		III-IV	7
	IV-III	7		IV-I	7

The information on OD pairs is shown in Table 7.

[5] China's official railway website: http://www.12306.cn/mormhweb/ (Accessed January 5, 2017)

Table 7. Trip demand.

OD	Preferred start time ($T_r^{\text{Departure}}$)	Origin zone	Destination zone	Number of passengers (D_r)
OD1	8:00	Z1	Z2	30
OD2	7:00	Z1	Z3	150
OD3	8:45	Z2	Z3	90
OD4	8:00	Z2	Z4	90
OD5	7:30	Z3	Z1	80
OD6	7:00	Z3	Z4	80
OD7	7:00	Z4	Z2	120

4.1.2 Solution analysis

The IM-BBA and IM-DA were applied to this example. The gap allowance to end the iteration process was set to zero. The IM-BBA took 7.69 s to obtain the final solution, whereas the IM-DA required 41.14 s. In this example, the problem size of the timetabling was relatively small, so the computational advantage of the IM-DA is not apparent. Furthermore, the IM-DA underwent more iterations (six, including the ITM and five TOMs) than the IM-BBA (three, including the ITM and two TOMs) due to the characteristics of the starting point (i.e., the initial timetable). When the starting point was near a local optimum, fewer iterations were needed to approach the convergent point. Hence, the IM-DA required more computation time. A detailed analysis of the computational efficiency of the IM-BBA and IM-DA is given in Section 4.2.

The IM-BBA obtained a local optimum, whereas the IM-DA found the global optimum, which was validated via the enumeration process. The IM-DA achieved global optimality by generating a better initial timetable (which could result in a timetable solution with a lower total generalized cost) than the IM-BBA in this case; a detailed analysis can be found below, in the description of a test using different initial timetables. The timetable output by the IM-DA is presented in Appendix C. The solutions of the IM-BBA and IM-DA are summarized and listed in Table 8. The non-in-vehicle time (NIVT) consists of the wait time at the origin and the transfer times. The IVT is limited for a passenger journey by the maximum train running speed, and fare setting is not considered in our model. Note that the travel time between a station and its served zone was set to zero for this example. Thus, the TOM focused mainly on reducing wait times and smoothing transfers with consideration of passenger perceptions. Hence, the total weighted NIVT is analyzed together with the total generalized cost in Table 8.

Table 8. Summary of solutions of the IM-BBA and IM-DA.

Solution method	Total generalized cost (min)			Total weighted NIVT (min)		
	ITM	Final TOM	Gap (%)	ITM	Final TOM	Gap (%)
IM-BBA	106,360	71,578	32.70	39,820	14,598	63.34
IM-DA	105,974	64,090	39.52	47,554	10,440	78.05

Two kinds of train timetables were chosen: those found by the ITM, which minimizes the train running time, and those given by the final TOM, which considers passenger path choices. The gap between these (as shown in Table 8) is calculated by:

$$\text{Gap} = \frac{\text{That for the ITM-That for the final TOM}}{\text{That for the ITM}} \times 100\%. \tag{43}$$

After several iterations, both solution methods reduced the total generalized cost by more than 30%, and an even larger decrease was found in the total NIVT. The IM-BBA achieved a decrease of nearly 65%, whereas that of the IM-DA was more than 75%. These reductions (shown in Table 9 and Table 10) were mainly accomplished in the following ways.

Table 9. Assignments of IM-BBA.

Path search	OD	Trains used	Waiting at origin zone (min)	Waiting at stations (min)	IVT (min)	Walking (min)	Uncongested cost (min)	Volume (passengers)
Based on initial timetabling	OD1	Train 3	0	88	30	0	198.4	30
	OD2	Train 1	0	6	62	0	92.8	100
		Train 10	0	6	82	0	120.8	50
	OD3	Train 3	0	75	30	0	175.0	90
	OD4	Train 2	0	25	72	0	141.0	90
	OD5	Train 1	0	40	82	0	182.0	80
	OD6	Train 16	0	0	40	0	54.0	80
	OD7	Trian 4	0	42	72	0	171.6	100
		Train 16 → Train 3	0	98	70	10	291.4	20
Based on the final solution	OD1	Train 3	0	30	30	0	94.0	30
	OD2	Train 1	0	0	62	0	82.0	100
		Train 10	0	6	82	0	120.8	50
	OD3	Train 3	0	17	30	0	70.6	90
	OD4	Train 2	0	2	72	0	99.6	90
	OD5	Train 18	0	0	82	0	110.0	80
	OD6	Train 16	0	0	40	0	54.0	80
	OD7	Trian 4	0	42	72	0	171.6	100
		Train 16 → Train 3	0	50	70	0	185.0	20

Table 10. Assignments of IM-DA.

Path search	OD	Trains used	Waiting at origin zone (min)	Waiting at stations (min)	IVT (min)	Walking (min)	Uncongested cost (min)	Volume (passengers)
Based on initial timetabling	OD1	Train 6	0	0	114	0	148.0	30
	OD2	Train 7	0	6	60	0	90.8	100
		Train 1	0	16	62	0	110.8	50
	OD3	Train 3	0	79	30	0	182.2	90
	OD4	Train 2	0	18	72	0	128.4	90
	OD5	Train 15	200	10	60	0	198.0	80
	OD6	Train 18	200	12	40	0	175.6	80
	OD7	Trian 4	0	42	72	0	171.6	100
		Train 5	0	72	72	0	225.6	20
Based on the final solution	OD1	Train 3	0	0	41	0	51.0	30
	OD2	Train 1	0	0	62	0	82.0	100
		Train 7	0	6	60	0	90.8	50
	OD3	Train 3	0	0	30	0	40.0	90
	OD4	Train 2	0	2	72	0	99.6	90
	OD5	Train 15	0	0	60	0	80.0	80
	OD6	Train 18	0	0	40	0	54.0	80
	OD7	Trian 4	0	42	72	0	171.6	100
		Train 11 → Train 5	0	56	88	0	213.8	20

1) Reducing wait times. For example, the passengers of the OD pair OD5 would have been required to wait for 200 min in their origin zone and for 10 min at the stations if the initial timetable generated by the IM-DA were used. After improving this train timetable, both types of wait times decreased to zero. However, the improvement in wait times may increase the IVT for some passengers, because the weight assigned to IVT is typically less than that assigned to waiting. Waiting at a station is generally considered a worse experience than staying on a train, where passengers can sit and engage in activities such as eating and reading. For example, the IM-DA reduced the generalized cost of passengers of OD pair OD7 using the foregoing approach. The headway constraint in the same line prevents the schedule for train 5 from being adjusted. The IM-DA solved this scenario by scheduling train 11 to depart from station IV at 7:48 and increasing its running time from station IV to III from 40 min to 58 min. The passengers of OD7 could then use train 11 from station IV to station III rather than using train 5, thus enjoying a lower travel cost and less wait time.
2) Smoothing transfers. For example, the passengers of the OD pair OD7 would be required to walk for 10 min to transfer from train 16 to train 3 if the initial timetable generated by the IM-BBA were used. After the use of the TOM, passengers could remain on the same platform to wait for train 3 because trains 3 and 16 were scheduled to share the same station track.
3) Finding new paths after re-timetabling. When trains are rearranged during the later rounds of the TOM, it is possible that passengers will find and use some new paths with lower costs. This occurred in the case of OD pair OD5 in the assignments of the IM-BBA and OD pair OD1 in the assignments of the IM-DA. This demonstrates the necessity of using multiple iterations.

In addition, we generated various initial timetables and optimized them to compare their final solutions. When an ILP has more than one optimum solution, CPLEX provides one of these solutions, but it may not provide the same solution if the variables and constraints are input in a different order because the order of the variables and constraints can affect the solution search process of the branch-and-bound algorithm in CPLEX. Thus, we input the lines in different orders to obtain various initial timetables. The comparison (shown in Table 11) included six initial timetables. Three were optimized by the IM-BBA, and the other three were optimized by the IM-DA. These initial timetables resulted in various final solutions. To show the differences between these final solutions, their total generalized costs are presented in Table 11 and compared with the global optimum. The difference values in Table 11 are calculated by:

$$\text{Difference}=\frac{\text{Total generalized cost-Global optimum}}{\text{Global optimum}} \times 100\%. \tag{44}$$

Table 11. Comparison of six different initial timetables.

Solution method	ID of the initial timetable	Total generalized cost in the final solution (min)	Global optimum (min)	Difference (%)
IM-BBA	1	69,178	64,090	7.94
	2	71,578		11.68
	3	72,406		12.98
IM-DA	4	64,090		0.00
	5	66,922		4.42
	6	70,006		9.23

The results show that the IM-BBA and the IM-DA identified a solution near the global optimum, and the comparison demonstrates that the initial timetable affects the quality of the final solutions. If the IM-DA has a good initial timetable, it may obtain a better final solution than the IM-BBA even though it uses a heuristic (decomposition approach) for timetabling. No technique can guarantee that the generated initial timetable will result in the globally optimal solution; hence, the final solution may converge to a fixed point that is not the global optimum. If possible, different initial timetables should be trialed to obtain better results.

In summary, the results of the IM-BBA and the IM-DA demonstrate that the final solutions of the proposed model were better than the initial solutions, which indicates that timetables that consider passenger assignment can provide better service to passengers than those that ignore passenger assignment.

4.1.3 Discussion of passenger-equilibrium assignment

The examples presented in this section are used to test the workability of the column-generation method. The passenger loading based on the initial timetable (2) (mentioned in Table 11) is given as an example. The computation time for the column-generation method is approximately 0.76 s.

To illustrate how the resulting passenger loading satisfies the capacity-related condition, we select and discuss the detailed results of two typical OD pairs, OD2 and OD7. Due to the limited vehicle capacity (100 passengers), OD pairs OD2 and OD7 must be assigned to more than one path. The details of the paths are summarized in Table 12.

Table 12. Paths used by OD pairs OD2 and OD7.

OD pair	Path		Uncongested cost (min)	Passenger volume (passengers)
	Routing	Train used		
OD2	Zone Z1→Station I→Station II→Station III→Zone Z3	Train 1	92.8	100
	Zone Z1→Station I→Station IV→Station III→Zone Z3	Train 10	120.8	50
OD7	Zone Z4→Station IV→Station III→Station II→Zone Z2	Trian 4	171.6	100
	Zone Z4→Station IV→Station I→Station II→Zone Z2	Train 16 → Train 3	291.4	20

The capacity of Train 1 was 100 passengers. Hence, not all passengers in OD pair OD2 were assigned to train 1, even though the path used by that train had the lowest uncongested cost. This was also the case with OD7. If the vehicle capacity were set to more than 150 passengers, all passengers of OD2 and OD7 would be loaded onto the same path (the one with the lowest uncongested cost). If the vehicle capacity were set to 30, more paths would be used. For example, the passengers of OD2 would be assigned to one of five paths, whereas three quarters of the passengers of OD7 would be assigned to one of three paths. The remaining passengers of OD7 would be assigned to the residual path (i.e., they could not have found a feasible path with sufficient vehicle capacity, so they may have needed to cancel their trips or change to other traffic modes to complete their trips)

4.2 Tests with South China's HSR network

Fig. 3 shows the abstracted South China HSR network.

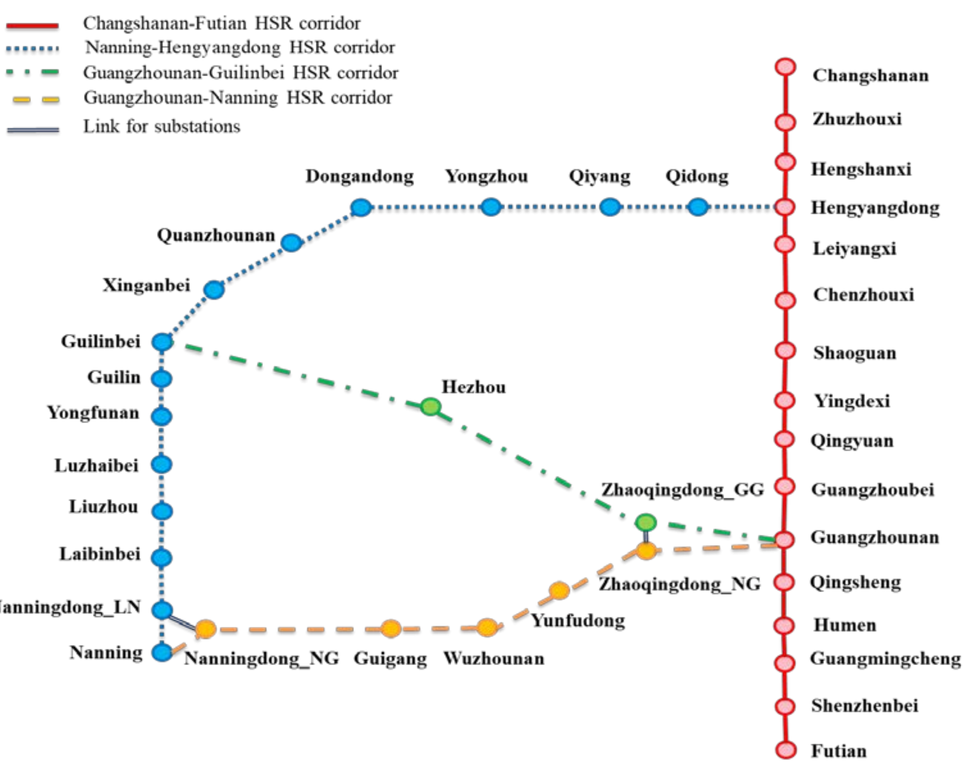


Fig. 3. Abstracted HSR network in South China.

We selected 35 main stations from the network. If a station served two train corridors simultaneously (such as Nanningdong station and Zhaoqingdong station), the station was modeled as two substations rather than one, and walking links were added to connect the substations. The network has 37 abstracted stations.

In Section 4.2.1, we discuss the results of the efficiency and optimality tests. Section 4.2.2 presents the application of the IM-DA to a large-scale problem.

4.2.1 Efficiency test

We investigated the efficiency and optimality of the IM-BBA and the IM-DA. The study period was from 6:15 to 22:30. We initially set eight lines running through the four HSR corridors shown in Table 13. In this test, the gap allowance for the generalized cost to end the iterative process was set to zero.

Table 13. Eight lines used in the computational efficiency test.

Line ID	Original terminal	Destination terminal	Number of trains on the line	Minimum departure interval between trains on the line (min)
D1	Guangzhounan	Nanningdong	1	30
D2	Guangzhounan	Guilinbei	1	30
D3	Hengyangdong	Nanning	1	30
D4	Changshanan	Futian	4	10
U1	Nanningdong	Guangzhounan	1	30
U2	Guilinbei	Guangzhounan	1	30
U3	Nanning	Hengyangdong	1	30
U4	Futian	Changshanan	4	10

a. Computation time comparison

The primary difference between the IM-BBA and the IM-DA is the method of solving the transit timetabling model. We thus compared the ability of the branch-and-bound algorithm and the decomposition approach to solve the transit timetabling models to reflect the computational efficiency of the IM-BBA and the IM-DA. The two kinds of transit timetabling models are the ITM and the TOM, where the computation time for the former is affected by the number of trains on the network. We thus tested the extent of this effect by multiplying the number of trains on a line by a scale factor to set more trains on the network and then running the branch-and-bound algorithm and decomposition approach for each case to observe the computation times.

The aforementioned PC was used to perform the test. The scale factor ranged from 1 to 10 with a step size of 1, resulting in 10 scenarios. The increase in model size with the increase in the scale factor is shown in Table 14. The branch-and-bound algorithm deals with just one ILP problem; hence, the model size is reflected by the number of variables and constraints in that problem. In contrast, the decomposition approach runs several ILP problems, and the maximum numbers of variables and constraints are used to illustrate the model size.

Table 14. Numbers of variables and constraints.

Scale factor	Number of trains	Branch-and-bound algorithm		Decomposition approach	
		Total number of variables	Total number of constraints	Maximum number of variables	Maximum number of constraints
1	14	3,841	10,574	2,097	6,693
2	28	12,626	44,490	7,457	27,641
3	42	26,355	101,644	14,041	55,335
4	56	45,028	182,096	19,401	79,095
5	70	68,645	285,816	24,761	102,855
6	84	97,206	412,774	30,121	126,615
7	98	130,711	563,030	35,481	150,375
8	112	169,160	736,524	40,841	174,135
9	126	212,553	933,316	46,201	197,895
10	140	260,890	1,153,376	51,561	221,655

The branch-and-bound algorithm obtained the global optimum, and the decomposition approach achieved similar objective values for these scenarios (all differences were within 2.5%). The trend in computation time is displayed in Fig. 4 using log 10 values for clarity.

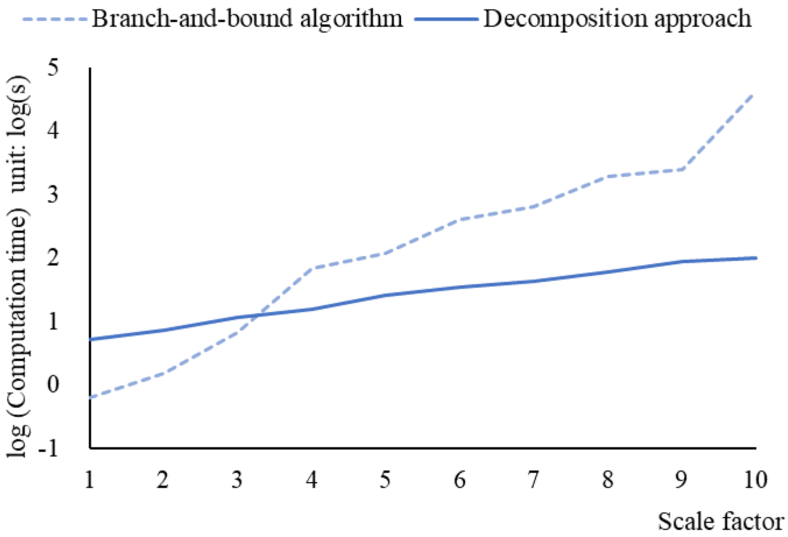


Fig. 4. Computation time for various model sizes (branch-and-bound algorithm vs. decomposition approach).

The increase in the number of trains increased the total numbers of variables and constraints (Table 14) and the computation times of the two solution methods. The decomposition approach was slower than the branch-and-bound algorithm when handling a small model. The decomposition approach handles multiple ILP problems, whereas the branch-and-bound algorithm solves a single ILP problem. However, when the model size increased, the branch-and-bound algorithm required substantially more computation time than the decomposition approach because the ILP problem for all lines was complex and large. Fig. 4 reveals that the computation time needed for the branch-and-bound algorithm increases more quickly as a function of model size than that needed for the decomposition approach. When a scenario featuring 140 trains was examined with the branch-and-bound algorithm, CPLEX reported “out of memory” after nearly 12 hours without identifying a feasible solution. Even when HPC2015 was used to run the branch-and-bound algorithm, it required nearly 3 hours to obtain the result. In contrast, the decomposition approach took no more than 2 min to obtain a result with an objective value only 0.44% larger than that of the branch-and-bound algorithm.

Compared with the ITM, which is a relatively simple problem, the TOM includes more variables and constraints for passengers and is affected not only by the number of trains on the network but also by the number of OD pairs. Passenger assignment also becomes more complicated. A further test was performed to analyze the computation time of the IM-BBA and the IM-DA with increasing numbers of OD pairs. Five OD pairs were randomly selected and added to the test one by one, and the scale factor was three (i.e., 42 trains on the network). The IM-BBA and the IM-DA achieved similar generalized costs for these scenarios (all differences were within 0.7%). The trend in computation time is displayed in Fig. 5. When the number of OD pairs is increased, the IM-DA solves the problem more quickly than the IM-BBA.

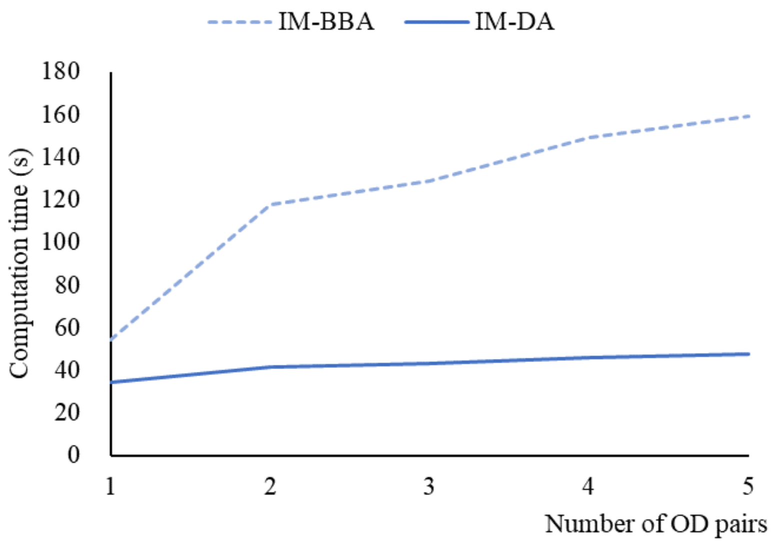


Fig. 5. Computation time for various model sizes (IM-BBA vs. IM-DA).

Hence, these two tests indicate that the IM-DA can solve a larger problem (i.e., one that includes more OD pairs and trains) in a more efficient manner than the IM-BBA.

b. Optimality comparison

To check the solution quality of the IM-BBA and the IM-DA, we conducted a further test to compare the results obtained with these two methods. The test results are presented in Table 15. The five OD pairs in the second test in Section 4.2.1a were used to analyze the optimality. The global optimal solution could be found via enumeration because only five OD pairs were involved. The gap value in Table 15 between the global optimum and the solutions obtained by the IM-BBA and the IM-DA is defined as follows.

$$\text{Gap} = \frac{\text{Solution-Global optimum}}{\text{Global optimum}} \times 100\%. \quad (45)$$

Table 15. Comparison of total generalized costs.

Scale factor	Number of trains	Number of OD pairs	Global optimum (min)	Total generalized cost of the final solution (min)		Gap (%)	
				IM-BBA	IM-DA	IM-BBA	IM-DA
3	42	1	1,530.0	1,530.0	1,530.0	0.00	0.00
3	42	2	3,339.0	3,339.0	3,354.0	0.00	0.45
3	42	3	5,206.5	5,206.5	5,239.5	0.00	0.63
3	42	4	6,615.0	6,615.0	6,648.0	0.00	0.50
3	42	5	9,357.0	9,357.0	9,390.0	0.00	0.35
4	56	5	9,357.0	9,402.0	9,372.0	0.48	0.16
5	70	5	9,357.0	9,357.0	9,375.0	0.00	0.19

The gaps are within 0.70%. Hence, the test demonstrates that the solutions obtained by the IM-BBA and the IM-DA are near the globally optimal solution. Moreover, the solutions obtained by the IM-DA are similar in quality to those given by the IM-BBA, which confirms that the IM-DA can shorten the calculation time while also generating satisfactory suboptimal timetables in practical applications.

4.2.2 Large-scale applicability

To further test the capability of the IM-DA, we applied it to a large-scale example to generate a daily timetable. We collected train data (on stopping patterns, dwell times, running times, and ticket fees) for January 5, 2017 from China Railway's official website. On that day, whose operation period lasted from 06:00 to 23:30, South China's HSR network operated 234 high-speed trains in the G-series, whose maximum running speed is 300 km/h. Considering the difference in stopping patterns, we grouped the 234 high-speed trains into 181 lines, and the shortest headway between two trains on the same line was set to 30 min. The vehicle capacity was set to 1600 based on the vehicle information for Chinese high-speed trains (*Hexie*) obtained from Wikipedia[6].

We divided the OD pairs into three categories, and their patterns are shown in Fig. 6:

1) When the shortest trip time is longer than 6 hours, the trip is long-distance.
2) When the shortest trip time is between 2 and 6 hours, the trip is medium-distance.
3) When the shortest trip time is less than 2 hours, the trip is short-distance.

The shortest trip time denotes the shortest IVT from the origin to the destination without counting the wait time, transfer time, or access and egress times. We set the peak period from 08:30 to 10:30. The preferred departure time interval was 30 min. All passengers with the same OD and preferred departure interval were regarded as a group. Due to trip time limitations (for example, passengers must leave early enough to complete their trips before midnight) and differences in zones' attractions, we generated 9076 main trips for this example. The numbers of long-distance trips, medium-distance trips, and short-distance trips were 2340, 4848, and 1888, respectively.

[6] *Hexie* (train) from Wikipedia: https://en.wikipedia.org/wiki/Hexie_(train) (Accessed November 22, 2019).

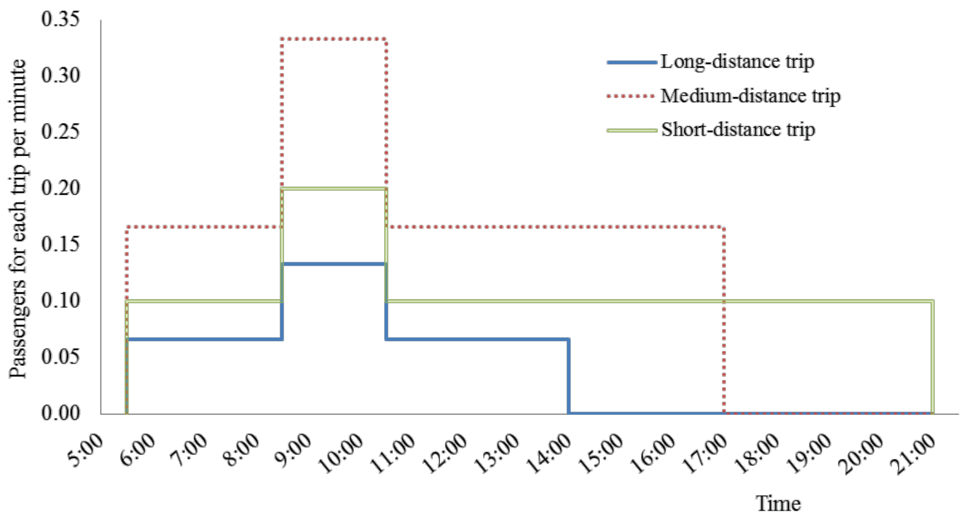


Fig. 6. Three OD patterns in the large-scale example.

The gap allowance, that is, the threshold gap value at which iteration ceases, was set to 0.01% for the IM-DA. By running the IM-DA, we were able to obtain a timetable for the whole network. At the fifth TOM, the gap between two successive iterations (0.006%) was less than 0.01%, by which time the program had run for nearly 7.6 hours. Hence, the program could have been terminated at this point because the preset gap allowance had been met. Compared with the initial timetable, which minimized the total train running time, the adjustment after five iterations decreased the total generalized cost by 360,323 min.

As stated above, the TOM focuses mainly on reducing wait times and smoothing transfers with consideration of passengers' perceptions. We therefore show the change in the total weighted NIVT (i.e., the weighted sum of the walking time and wait times in zones and at stations) for analysis. The total weighted NIVT was reduced by 7.09%. The average wait time for each passenger decreased by approximately 5.13% (8.91 min), whereas the average walking time for a passenger who made transfers during the journey was shortened by 81.33% (1.07 min). Hence, modifying the timetable according to passenger path choices can significantly improve passenger service. In particular, station track allocation improves the transfer experience by reducing the walking time.

To show the general trend of the total weighted NIVT, Fig. 7 presents the results of the ITM and the first 15 TOMs. The general trend appears to decrease. The first TOM achieves the largest part of the total weighted NIVT reduction (82.96%), the next four TOMs complete 9.81%, and the next ten TOMs provide the rest (7.22%). A non-negligible decrease was caused by TOM(6)−TOM(15). However, the gap allowance for the IM-DA had been reached in the fifth TOM; thus, the program ended, and this decrease was not included. However, this example illustrates that a timetable with better service quality can be obtained by setting a smaller gap allowance or by allowing more TOMs. In reality, the condition of ending the program can be adjusted according to operational needs.

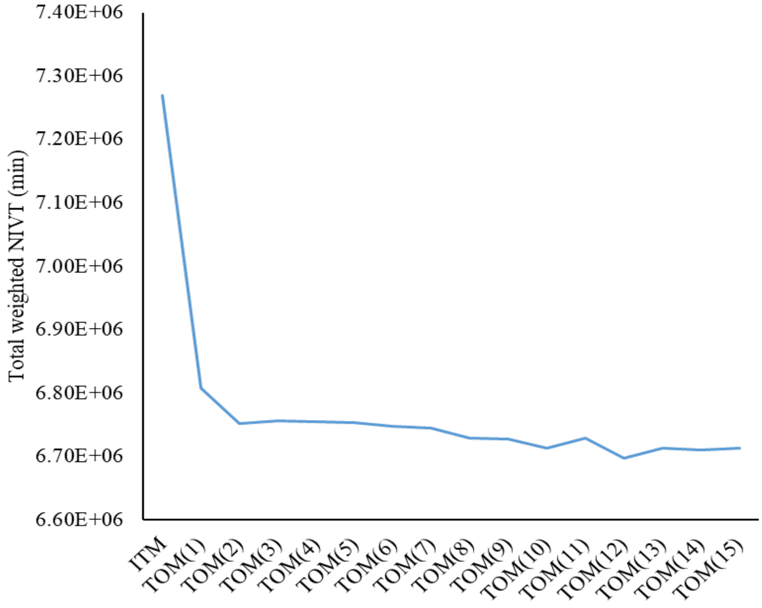


Fig. 7. Total weighted NIVT for each iteration.

We chose the Changshanan-Futian HSR corridor to demonstrate the timetable result. The timetable for this HSR corridor is shown in Fig. 8.

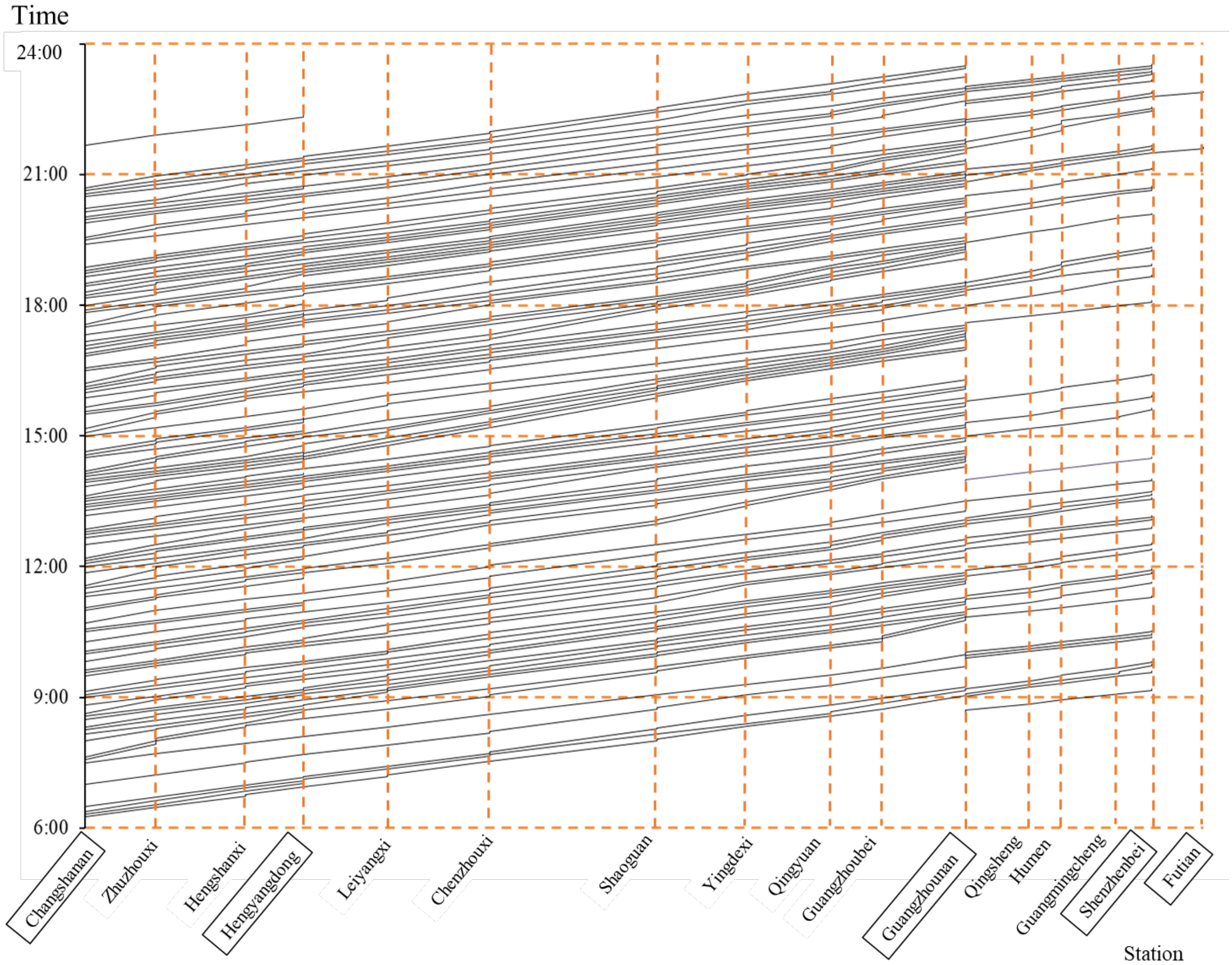


Fig. 8. Timetable for the Changshanan-Futian HSR corridor.

The black boxes in Fig. 8 frame the terminals and the main transfer stations. This corridor, which comprises the southern section of the Beijing-Guangzhou HSR corridor, is in high demand, and most departure intervals of high-speed trains are particularly short. Fig. 8 also demonstrates the movement of trans-line trains. We take one high-speed train that begins operation at 21:40 at Changshanan as an example. It leaves this HSR corridor at Hengyangdong, enters the Hengyangdong-Nanning HSR corridor, and then moves toward Yongzhou. This example indicates that the IM-DA can handle a large transit timetabling problem in a reasonable amount of time.

5. Conclusions

This study contributes to the literature by developing a schedule-based model for the transit timetabling problem that considers passenger path choices in a congested transit network. The example in Section 4.1 shows that the timetables created by our model provide improved service by shortening the travel time (such as walking time and wait time) relative to plans that minimize the total train operating time.

The study also introduces an iterative method for resolving the model. An initial timetable is generated as the starting point, and an iterative program is then designed to improve its service quality. This program has two levels. The first simulates passenger path choices based on the present timetable with consideration of the limited vehicle capacity, and the second adjusts the timetable according to the present passenger path choices. The method iterates between these two levels until convergence is reached. We suggest the use of two approaches to solve transit timetabling problems: the branch-and-bound algorithm and the decomposition approach. The branch-and-bound algorithm becomes time-consuming as the model size increases, as shown by the computation time tests described in Section 4.2.1. Hence, the branch-and-bound algorithm cannot be applied to large problems but only to simple transit networks. In contrast, the tests illustrate that the decomposition approach offers a computational advantage while generating a solution close to the globally optimal solution. The differences in the objectives calculated by the branch-and-bound algorithm and the decomposition approach are within 2.5%; thus, the decomposition approach can be used with an iterative method for large-scale problems. Although the initial timetable affects the quality of the final solution, trialing various initial timetables can help the IM-DA to identify better results. The test results show that the IM-DA can obtain good suboptimal solutions, with differences within 1.0% of the global optimum. Furthermore, our test with South China’s HSR network illustrates that the IM-DA can help obtain solutions for a practically sized problem in a reasonably short computation time. Although the obtained timetable is suboptimal, its quality is acceptable for real-world applications. Hence, the IM-DA can be applied to large transit networks, such as an intercity HSR network and mature intracity metro network.

One limitation of this model is that it assumes that passenger perceptions are perfectly captured. However, this is not always the case in real-world applications. In a future study, the authors plan to introduce uncertainties (e.g., stochastic passenger behavior) to improve the robustness of the timetabling model. Furthermore, this model does not consider the flow-dependent cost. The flow-dependent cost increases as the number of passengers onboard increases, meaning that it could reflect passengers’ attitudes toward train crowdedness. Consideration of the flow-dependent cost results in a more complex passenger assignment problem that may need another solution method (e.g., the multiplier method and the penalty function method), which is worth exploring in a future study. The supply side (e.g., the number of available vehicles) and line planning (e.g., stopping pattern design) will also be studied.

Moreover, the efficiency of the present path search algorithm, that is, the K-shortest path algorithm, decreases with increasing network scale. A more efficient path search algorithm is needed to improve the computation efficiency of our solution method and widen the applicability of our model and solution method. Hence, the authors plan to work toward improving the K-shortest path algorithm in a future study.

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References

BARRENA, E., CANCA, D., COELHO, L. C. & LAPORTE, G. 2014. Exact formulations and algorithm for the train timetabling problem with dynamic demand. *Computers and Operations Research*, 44, 66-74.

BELL, M. G. H. & LIDA, Y. 1997. Deterministic user equilibrium assignment. *Transportation Network Analysis* (pp. 83-111). Chichester, UK: John Wiley & Sons, Ltd.

BORNDÖRFER, R., HOPPMANN, H. & KARBSTEIN, M. 2017. Passenger routing for periodic timetable optimization. *Public Transport*, 9, 115-135.

BRÄNNLUND, U., LINDBERG, P. O., NÕU, A. & NILSSON, J. E. 1998. Railway timetabling using Lagrangian relaxation. *Transportation Science*, 32, 358-369.

BURDETT, R. L. & KOZAN, E. 2010. A sequencing approach for creating new train timetables. *OR Spectrum*, 32, 163-193.

BURGGRAEVE, S. & VANSTEENWEGEN, P. 2017. Robust routing and timetabling in complex railway stations. *Transportation Research Part B: Methodological*, 101, 228-244.

CACCHIANI, V. & TOTH, P. 2012. Nominal and robust train timetabling problems. *European Journal of Operational Research*, 219, 727-737.

CAIMI, G., BURKOLTER, D., HERRMANN, T., CHUDAK, F. & LAUMANNS, M. 2009. Design of a railway scheduling model for dense services. *Networks and Spatial Economics*, 9, 25-46.

CAIMI, G., CHUDAK, F., FUCHSBERGER, M., LAUMANNS, M. & ZENKLUSEN, R. 2011. A new resource-constrained multicommodity flow model for conflict-free train routing and scheduling. *Transportation Science*, 45, 212-227.

CANCA, D., ALGABA, E., BARRENA, E. & ZARZO, A. 2014. Railway rapid transit timetables with variable and elastic demand. *Procedia - Social and Behavioral Sciences*, 111, 538-548.

CAREY, M. & LOCKWOOD, D. 1995. A model, algorithms and strategy for train pathing. *Journal of the Operational Research Society*, 46, 988-1005.

CASCETTA, E. & COPPOLA, P. 2016. Assessment of schedule-based and frequency-based assignment models for strategic and operational planning of high-speed rail services. *Transportation Research Part A: Policy and Practice*, 84, 93-108.

CEDER, A. & JIANG, Y. 2020. Route guidance ranking procedures with human perception consideration for personalized public transport service. *Transportation Research Part C: Emerging Technologies*, 118, 102667.

CHANG, Y. H., YEH, C. H. & SHEN, C. C. 2000. A multiobjective model for passenger train services planning: Application to Taiwan's high-speed rail line. *Transportation Research Part B: Methodological*, 34, 91-106.

DIJKSTRA, E. W. 1959. A note on two problems in connexion with graphs. *Numerische Mathematik*, 1, 269-271.

DOLLEVOET, T., HUISMAN, D., KROON, L. G., VEELENTURF, L. P. & WAGENAAR, J. C. 2017. Application of an iterative framework for real-time railway rescheduling. *Computers & Operations Research*, 78, 203-217.

FU, Q., LIU, R. & HESS, S. 2012. A review on transit assignment modelling approaches to congested networks: A new perspective. *Procedia - Social and Behavioral Sciences*, 54, 1145-1155.

GENTILE, G., FLORIAN, M., HAMDOUCH, Y., CATS, O. & NUZZOLO, A. 2016. The theory of transit assignment: basic modelling frameworks. *In:* GENTILE, G. & NOEKEL, K. (eds.) *Modelling Public Transport Passenger Flows in the Era of Intelligent Transport Systems* (pp. 287-386). Springer.

GUO, X., WU, J., SUN, H., LIU, R. & GAO, Z. 2016. Timetable coordination of first trains in urban railway network: A case study of Beijing. *Applied Mathematical Modelling*, 40, 8048-8066.

HAMDOUCH, Y., SZETO, W. Y. & JIANG, Y. 2014. A new schedule-based transit assignment model with travel strategies and supply uncertainties. *Transportation Research Part B: Methodological*, 67, 35-67.

HARROD, S. S. 2012. A tutorial on fundamental model structures for railway timetable optimization. *Surveys in Operations Research and Management Science*, 17, 85-96.

HARTLEB, J. & SCHMIDT, M. 2019. Railway timetabling with integrated passenger distribution. *ERIM Report Series Research in Management*.

INGOLOTTI, L., LOVA, A., BARBER, F., TORMOS, P., SALIDO, M. A. & ABRIL, M. 2006. New heuristics to solve the "CSOP" railway timetabling problem. *Advances in Applied Artificial Intelligence: 19th International Conference on Industrial, Engineering and Other Applications of Applied Intelligent Systems, 2006, Annecy, France*. 400-409.

JAMILI, A., SHAFIA, M. A., SADJADI, S. J. & TAVAKKOLI-MOGHADDAM, R. 2012. Solving a periodic single-track train timetabling problem by an efficient hybrid algorithm. *Engineering Applications of Artificial Intelligence*, 25, 793-800.

JONG, J. C., CHANG, S. & LAI, Y. C. 2013. Development of two-stage hybrid method to solve high-speed rail train scheduling problem. *Transportation Research Record: Journal of the Transportation Research Board*, 2374, 44-54.

KHANI, A., HICKMAN, M. & NOH, H. 2014. Trip-based path algorithms using the transit network hierarchy. *Networks and Spatial Economics*, 15, 635-653.

KROON, L. G. & PEETERS, L. W. P. 2003. A variable trip time model for cyclic railway timetabling. *Transportation Science*, 37, 198-212.

LI, Y.-T. & SCHMÖCKER, J.-D. 2017. Adaptation patterns to high speed rail usage in Taiwan and China. *Transportation*, 44, 807-830.

LIU, Z. & MENG, Q. 2014. Bus-based park-and-ride system: A stochastic model on multimodal network with congestion pricing schemes. *International Journal of Systems Science*, 45, 994-1006.

LUSBY, R. M., LARSEN, J. & BULL, S. 2018. A survey on robustness in railway planning. *European Journal of Operational Research*, 266, 1-15.

NIU, H. & ZHOU, X. 2013. Optimizing urban rail timetable under time-dependent demand and oversaturated conditions. *Transportation Research Part C: Emerging Technologies*, 36, 212-230.

NIU, H., ZHOU, X. & GAO, R. 2015. Train scheduling for minimizing passenger waiting time with time-dependent demand and skip-stop patterns: Nonlinear integer programming models with linear constraints. *Transportation Research Part B: Methodological*, 76, 117-135.

NUZZOLO, A., CRISALLI, U., COMI, A. & ROSATI, L. 2016. A mesoscopic transit assignment model including real-time predictive information on crowding. *Journal of Intelligent Transportation Systems*, 20, 316-333.

NUZZOLO, A., CRISALLI, U. & ROSATI, L. 2012. A schedule-based assignment model with explicit capacity constraints for congested transit networks. *Transportation Research Part C: Emerging Technologies*, 20, 16-33.

PARBO, J., NIELSEN, O. A. & PRATO, C. G. 2016. Passenger perspectives in railway timetabling: A literature review. *Transport Reviews*, 36, 500-526.

POON, M. H., WONG, S. C. & TONG, C. O. 2004. A dynamic schedule-based model for congested transit networks. *Transportation Research Part B: Methodological*, 38, 343-368.

ROBENEK, T., AZADEH, S. S., MAKNOON, Y., DE LAPPARENT, M. & BIERLAIRE, M. 2018. Train timetable design under elastic passenger demand. *Transportation Research Part B: Methodological*, 111, 19-38.

ROBENEK, T., MAKNOON, Y., AZADEH, S. S., CHEN, J. & BIERLAIRE, M. 2016. Passenger centric train timetabling problem. *Transportation Research Part B: Methodological*, 89, 107-126.

SCHMIDT, M. & SCHÖBEL, A. 2015. Timetabling with passenger routing. *OR Spectrum*, 37, 75-97.

SCHÖBEL, A. 2017. An eigenmodel for iterative line planning, timetabling and vehicle scheduling in public transportation. *Transportation Research Part C: Emerging Technologies*, 74, 348-365.

SELS, P., DEWILDE, T., CATTRYSSE, D. & VANSTEENWEGEN, P. 2011. Deriving all passenger flows in a railway network from ticket sales data. *Proceedings of 4th International Seminar on Railway Operations Modelling and Analysis (IAROR): RailRome2011, Rome, Italy*.

SIEBERT, M. 2011. *Integration of routing and timetabling in public transportation.* MSc thesis, Georg-August-Universität Göttingen.

SIEBERT, M. & GOERIGK, M. 2013. An experimental comparison of periodic timetabling models. *Computers and Operations Research*, 40, 2251-2259.

SINHA, S. K., SALSINGIKAR, S. & SENGUPTA, S. 2016. An iterative bi-level hierarchical approach for train scheduling. *Journal of Rail Transport Planning and Management*, 6, 183-199.

SPARING, D. & GOVERDE, R. M. P. 2017. A cycle time optimization model for generating stable periodic railway timetables. *Transportation Research Part B: Methodological*, 98, 198-223.

SUN, S. & SZETO, W. Y. 2018. Logit-based transit assignment: Approach-based formulation and paradox revisit. *Transportation Research Part B: Methodological*, 112, 191-215.

SZETO, W. Y. & JIANG, Y. 2014a. Transit assignment: Approach-based formulation, extragradient method, and paradox. *Transportation Research Part B: Methodological*, 62, 51-76.

SZETO, W. Y. & JIANG, Y. 2014b. Transit route and frequency design: Bi-level modeling and hybrid artificial bee colony algorithm approach. *Transportation Research Part B: Methodological*, 67, 235-263.

SZETO, W. Y., JIANG, Y., WONG, K. I. & SOLAYAPPAN, M. 2013. Reliability-based stochastic transit assignment with capacity constraints: Formulation and solution method. *Transportation Research Part C: Emerging Technologies*, 35, 286-304.

TALEBIAN, A. & ZOU, B. 2015. Integrated modeling of high performance passenger and freight train planning on shared-use corridors in the U.S. *Transportation Research Part B: Methodological*, 82, 114-140.

TANG, Y., JIANG, Y., YANG, H. & NIELSEN, O. A. 2020. Modeling and optimizing a fare incentive strategy to manage queuing and crowding in mass transit systems. *Transportation Research Part B: Methodological*, 138, 247-267.

TONG, C. O. & RICHARDSON, A. J. 1984. A computer model for finding the time-dependent minimum path in a transit system with fixed schedules. *Journal of Advanced Transportation*, 18, 145-161.

TONG, C. O. & WONG, S. C. 1999. A stochastic transit assignment model using a dynamic schedule-based network. *Transportation Research Part B: Methodological*, 33, 107-121.

VAN DER ZIJPP, N. J. & FIORENZO CATALANO, S. 2005. Path enumeration by finding the constrained K-shortest paths. *Transportation Research Part B: Methodological*, 39, 545-563.

WONG, R. C. W., YUEN, T. W. Y., KWOK WAH, F. & LEUNG, J. M. Y. 2008. Optimizing timetable synchronization for rail mass transit. *Transportation Science*, 42, 57-69.

XIE, J., WONG, S. C. & LO, S. M. 2017. Three extensions of Tong and Richardson's algorithm for finding the optimal path in schedule-based railway networks. *Journal of Advanced Transportation*, 2017, 9216864.

XIE, J., WONG, S. C., ZHAN, S., LO, S. M. & CHEN, A. 2020. Train schedule optimization based on schedule-based stochastic passenger assignment. *Transportation Research Part E: Logistics and Transportation Review*, 136, 101882.

XU, G., YANG, H., LIU, W. & SHI, F. 2018. Itinerary choice and advance ticket booking for high-speed-railway network services. *Transportation Research Part C: Emerging Technologies*, 95, 82-104.

YAN, Y., LIU, Z., MENG, Q. & JIANG, Y. 2013. Robust optimization model of bus transit network design with stochastic travel time. *Journal of Transportation Engineering*, 139, 625-634.

YANG, L., QI, J., LI, S. & GAO, Y. 2016. Collaborative optimization for train scheduling and train stop planning on high-speed railways. *Omega*, 64, 57-76.

ZHAN, S., KROON, L. G., VEELENTURF, L. P. & WAGENAAR, J. C. 2015. Real-time high-speed train rescheduling in case of a complete blockage. *Transportation Research Part B: Methodological*, 78, 182-201.

ZHAN, S., WONG, S. C. & LO, S. M. 2020. Social equity-based timetabling and ticket pricing for high-speed railways. *Transportation Research Part A: Policy and Practice*, 137, 165-186.

ZHANG, Y., LAM, W., SUMALEE, A., LO, H. K. & TONG, C. O. 2010. The multi-class schedule-based transit assignment model under network uncertainties. *Public Transport*, 2, 69-86.

ZHOU, W. & TENG, H. 2016. Simultaneous passenger train routing and timetabling using an efficient train-based Lagrangian relaxation decomposition. *Transportation Research Part B: Methodological*, 94, 409-439.

ZHOU, W., TIAN, J., DENG, L. & QIN, J. 2015. Integrated optimization of service-oriented train plan and schedule on intercity rail network with varying demand. *Discrete Dynamics in Nature and Society*, 2015, 419042.

Appendix A: Proof for the UE assignment problem

This appendix proves that the UE assignment problem can be formulated as an LP model (LPPA), as shown in Section 3.3.1.

First, the UE condition (constraint (34)) can be represented as follows ($\forall r \in \boldsymbol{R}$; $\forall q \in \boldsymbol{Q}_r$):

$$\begin{cases} A_q \times (c_q - \tilde{c}_r) = 0 \\ A_q \geq 0 \\ c_q - \tilde{c}_r \geq 0 \end{cases}. \quad (46)$$

Second, the capacity-related condition (constraint (39)) can be combined with the calculation of $c_{nss'}^{\text{Extra}}$ (function (38)), and the combination is expressed as follows ($\forall l \in \boldsymbol{L};\ \forall n \in \boldsymbol{N}_l;\ \forall s, s' \in \boldsymbol{S}_l:\ s' = \xi_{ns}^{\text{Downstream}}$):

$$\begin{cases} c_{nss'}^{\text{Extra}} \times \left(\Pi_n^{\text{Capacity}} - \sum_{r\in\boldsymbol{R}} \sum_{q\in\boldsymbol{Q}_r} b_{qnss'}^{\text{Path}} \times A_q\right) = 0 \\ c_{nss'}^{\text{Extra}} \geq 0 \\ \Pi_n^{\text{Capacity}} - \sum_{r\in\boldsymbol{R}} \sum_{q\in\boldsymbol{Q}_r} b_{qnss'}^{\text{Path}} \times A_q \geq 0 \end{cases}. \quad (47)$$

Third, the flow conservation condition (Eq. (40)) can be reformulated as follows ($\forall r \in \boldsymbol{R}$):

$$\begin{cases} \tilde{c}_r \times (\sum_{q\in\boldsymbol{Q}_r} A_q - D_r) = 0 \\ \tilde{c}_r \geq 0 \\ \sum_{q\in\boldsymbol{Q}_r} A_q - D_r \geq 0 \end{cases}. \quad (48)$$

Because of constraints (46) and (48), $\tilde{c}_r > 0$ and Eq. (40) must hold.

Fourth, to show the equivalence between the UE assignment problem and the LPPA, the Lagrangian function for the LPPA is as follows:

$$F = \sum_{r\in\boldsymbol{R}} \sum_{q\in\boldsymbol{Q}_r} A_q c_q^{\text{Unc}} - \sum_{l\in\boldsymbol{L}} \sum_{n\in\boldsymbol{N}_l} \sum_{s,s'\in\boldsymbol{S}_l:\ s'=\xi_{ns}^{\text{Downstream}}} \lambda_{nss'} \left(\Pi_n^{\text{Capacity}} - \sum_{r\in\boldsymbol{R}} \sum_{q\in\boldsymbol{Q}_r} b_{qnss'}^{\text{Path}} \times A_q\right) - \sum_{r\in\boldsymbol{R}} \lambda_r (\sum_{q\in\boldsymbol{Q}_r} A_q - D_r) - \sum_{r\in\boldsymbol{R}} \sum_{q\in\boldsymbol{Q}_r} A_q \lambda_q, \quad (49)$$

where $\lambda_{nss'}$, λ_r, and λ_q are the Lagrangian multipliers of constraints (39), (40), and (42), respectively. The LPPA's Kuhn–Tucker conditions can be derived as follows:

$$c_q^{\text{Unc}} + \sum_{l\in\boldsymbol{L}} \sum_{n\in\boldsymbol{N}_l} \sum_{s,s'\in\boldsymbol{S}_l:\ s'=\xi_{ns}^{\text{Downstream}}} b_{qnss'}^{\text{Path}} \lambda_{nss'} - \lambda_r - \lambda_q = 0;\ \forall r \in \boldsymbol{R};\ \forall q \in \boldsymbol{Q}_r; \quad (50)$$

$$\begin{cases} \lambda_{nss'} \left(\Pi_n^{\text{Capacity}} - \sum_{r\in\boldsymbol{R}} \sum_{q\in\boldsymbol{Q}_r} b_{qnss'}^{\text{Path}} \times A_q\right) = 0 \\ \lambda_{nss'} \geq 0 \\ \Pi_n^{\text{Capacity}} - \sum_{r\in\boldsymbol{R}} \sum_{q\in\boldsymbol{Q}_r} b_{qnss'}^{\text{Path}} \times A_q \geq 0 \end{cases};\ \forall l \in \boldsymbol{L};\ \forall n \in \boldsymbol{N}_l;\ \forall s, s' \in \boldsymbol{S}_l:\ s' = \xi_{ns}^{\text{Downstream}}; \quad (51)$$

$$\begin{cases} \lambda_r (\sum_{q\in\boldsymbol{Q}_r} A_q - D_r) = 0 \\ \lambda_r \geq 0 \\ \sum_{q\in\boldsymbol{Q}_r} A_q - D_r \geq 0 \end{cases};\ \forall r \in \boldsymbol{R}; \quad (52)$$

$$\begin{cases} A_q \lambda_q = 0 \\ A_q \geq 0 \\ \lambda_q \geq 0 \end{cases};\ \forall r \in \boldsymbol{R};\ \forall q \in \boldsymbol{Q}_r. \quad (53)$$

Moreover, based on (50) and (53), we obtain

$$\begin{cases} A_q\left(c_q^{\text{Unc}} + \sum_{l\in\boldsymbol{L}}\sum_{n\in\boldsymbol{N}_l}\sum_{s,s'\in\boldsymbol{S}_l:\ s'=\xi_{ns}^{\text{Downstream}}} b_{qnss'}^{\text{Path}}\lambda_{nss'} - \lambda_r\right) = 0 \\ A_q \geq 0 \\ c_q^{\text{Unc}} + \sum_{l\in\boldsymbol{L}}\sum_{n\in\boldsymbol{N}_l}\sum_{s,s'\in\boldsymbol{S}_l:\ s'=\xi_{ns}^{\text{Downstream}}} b_{qnss'}^{\text{Path}}\lambda_{nss'} - \lambda_r \geq 0 \end{cases} ;\forall r\in\boldsymbol{R};\ \forall q\in\boldsymbol{Q}_r. \qquad (54)$$

If $\lambda_r = \tilde{c}_r$ ($\forall r\in\boldsymbol{R}$) and $\lambda_{nss'} = c_{nss'}^{\text{Extra}}$ ($\forall l\in\boldsymbol{L};\ \forall n\in\boldsymbol{N}_l;\ \forall s,s'\in\boldsymbol{S}_l:\ s'=\xi_{ns}^{\text{Downstream}}$), constraints (51), (52), and (54) are equivariant to the capacity-related condition (47), flow conservation condition (48), and UE condition (46), respectively. Thus, the LPPA is equivariant to the proposed UE assignment problem.

Appendix B: Analysis of the solution method for the LPPA

As the method shows, iteration (g+1) adds new paths that have higher uncongested costs than those included in iteration g. The optimal result of iteration g is then also a feasible solution of iteration (g+1). Because the objective function of the LPPA is to minimize the total uncongested cost, the objective value of a feasible solution cannot be smaller than that of the optimal result. Hence, the objective value of the found solution in each iteration is non-increasing.

Two conditions are used to terminate the iterative process. The first condition is fulfilled when all real paths have been enumerated, and the LP problem has the full set of paths in the final iteration. The second condition is fulfilled when the uncongested cost of any newly found path in an OD pair is higher than the current equilibrium cost of the said OD pair. That is, for each OD pair, the paths used have lower uncongested costs than the unused paths, so the assignment of passengers to these new paths or residual paths with higher uncongested costs cannot reduce the objective value. Hence, when the second condition is fulfilled, the current solution is the optimal solution. In the worst case, the second condition cannot be fulfilled, but the first condition must be fulfilled due to the finite number of real paths. Therefore, this method guarantees convergence within a finite number of iterations.

Appendix C: Timetable for a small hypothetical network.

Line	Train	Station	Track	Time		Line	Train	Station	Track	Time	
				Arrival	Departure					Arrival	Departure
L1	1	I	0	-	7:00	L2	4	I	1	-	7:00
		II	0	7:30	7:32			IV	1	7:40	7:42
		III	0	8:02	8:04			III	0	8:22	8:24
		IV	0	8:44	8:46			II	1	8:54	8:56
		I	0	9:26	-			I	1	9:26	-
L1	2	I	0	-	7:30	L2	5	I	1	-	7:30
		II	0	8:00	8:02			IV	1	8:10	8:12
		III	1	8:32	8:34			III	1	8:52	8:54
		IV	0	9:14	9:16			II	1	9:24	9:26
		I	0	9:56	-			I	1	9:56	-
L1	3	I	0	-	8:00	L2	6	I	1	-	8:00
		II	0	8:41	8:45			IV	1	8:40	8:42
		III	1	9:15	9:17			III	0	9:22	9:24
		IV	0	9:57	9:59			II	1	9:54	9:56
		I	0	10:39	-			I	1	10:26	-
D1	7	I	0	-	7:06	D2	10	I	1	-	7:36
		II	0	7:36	7:36			IV	1	8:16	8:18
		III	1	8:06	-			III	0	8:58	-
D1	8	I	0	-	11:00	D2	11	I	1	-	7:06
		II	0	11:30	11:30			IV	0	7:46	7:48
		III	1	12:00	-			III	1	8:46	-
D1	9	I	0	-	10:30	D2	12	I	1	-	8:06
		II	0	11:00	11:00			IV	1	8:46	8:48
		III	1	11:30	-			III	0	9:28	-
U1	13	III	1	-	11:00	U2	16	III	0	-	10:08
		II	1	11:30	11:30			IV	0	10:48	10:50
		I	1	12:00	-			I	0	11:30	-
U1	14	III	1	-	10:30	U2	17	III	0	-	10:38
		II	1	11:00	11:00			IV	0	11:18	11:20
		I	1	11:30	-			I	0	12:00	-
U1	15	III	1	-	7:30	U2	18	III	0	-	7:00
		II	1	8:00	8:00			IV	0	7:40	7:42
		I	1	8:30	-			I	0	8:22	-